Well Placement Algorithm Using Geo-Object Connectivity of Multiple Realizations: Methodology and Program

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Abstract

Geostatistical simulation provides a powerful tool to determine reservoir geometry and connectivity. Multiple realizations of facies and petrophysical properties provide quantitative knowledge of reservoir continuity and its uncertainty. In general, however, it is unclear how to use this knowledge for reservoir management decision making.

The connected volume (CV) of "net" reservoir within the well drainage radii may be used to determine well locations. The set of well locations that jointly maximize cumulative connected volume (CCV) is considered as a good starting point for reservoir simulation.

Several examples are presented to demonstrate the efficacy of the algorithm. An exahaustive search technique is used on a simple model to show that the proposed technique arrives at the "true" optimal well location configuration. The exhaustive technique is impossible to apply with large models; however, a series of large 2-D and 3-D examples are presented to illustrate the CPU speed of the proposed algorithm. A realistic case with multiple realizations is also presented.

KEYWORDS: connected volume, optimization, well planning, reservoir management

Introduction

Reservoir decision making depends primarily on predicted performance at possible well locations. The task of the reservoir development team is to choose well locations and production strategies to maximize production at minimum cost and meet other varied corporate objectives. Flow simulation provides a rigorous approach to predict reservoir behavior.

In order to optimize well placement, a large number of well configurations must be examined. The time needed for reservoir simulation makes it impossible to evaluate all possible combinations. Moreover, the availability of multiple geostatistical realizations, which measure the uncertainty in the geological description, make the problem even larger. Hence, there is a need for a fast technique to evaluate many possibile well configurations

and identify the best locations for further flow studies. There is also a need to select the well configurations taking into account the geologic uncertainty.

There is an increasing body of literature on ranking realizations [3, 5, 8]. These techniques could be used to reduce the number of realizations that must be considered; however, there is still a large number of possible locations that must be considered. There are papers on well placement and geostatistics [4, 9, 10]. The paper by Cullick et. al. approaches the problem from the same perspective but with a different optimization algorithm and different view to multiple realizations. Gutteridge and Gawith [5] use a quality factor to evaluate well positions. The quality factor map evaluates every possible location as a candidate; however, if there is more than one well to be planned the method does not identify the joint optimum. Other published methods suffer the same limitation. There is a need for a fast computational algorithm that will simultaneously handle multiple wells and multiple realizations.

We propose an iterative algorithm that will yield a well configuration (any arbitrary number of wells) that maximizes the expected value (over multiple realizations) of the cumulative connected reservoir volume to the well locations. The proposal uses the notion of geo-object connectivity and classical combinatorial optimization techniques.

There are some limitations. First, the method does not ensure a global optima. We can check how close the algorithm comes in some simple cases and we can repeat the algorithm with different random number seeds to see the variations in the final result. Second, we limit ourselves to vertical wells completed over the entire vertical extent of the reservoir. There is no obstacle to consider horizontal wells; we just have not done it at this point. Third, we have not incorporated some important reservoir engineering constraints such as the thickness of the oil column (concerns with coning) and geologic structure (faults ...).

The methodology will be described and then a number of examples will be presented. All of the examples have been prepared with a program written in Visual Basic with an easy to use interface. We show the result in a homogeneous reservoir to illustrate the intuitive result that the wells should be uniformly spaced. There is no guarantee that our proposed algorithm finds the "true" optimal result; however, we show on a fairly small example that our approach yields the "true" result obtained with an exhaustive search method. An example with multiple realizations is presented. Finally, a practical example with a real reservoir model is presented.

Geo-Object (Flow Unit) Calculation

A reservoir consists of connected regions, which we wish to produce with the fewest number of wells in the best locations. The different connected regions in a reservoir are called "geo-objects", that is, each geo-object is a collection of net (reservoir quality) cells that are connected together but disconnected from each other. Figure 1 illustrates three geo-objects; the volume outside of these objects is non-reservoir. By convention, in presence of n geo-objects, the geo-objects are sorted according to size, where 1 is the largest, and n is the smallest.

The paper on calculating the connectivity of 3-D objects in this volume and in [1] provides details on the calculation of geo-object connectivity. The program for geo-object calculation is integrated into the program we present for well site selection; the user input

is the facies, porosity, and permeability models required to establish the reservoir regions that are "net" reservoir quality.

Connected Volume to Well Locations

The connected volume (CV) to a well location represents the reservoir volume that can be drained by a well, that is, the reservoir volume that is reservoir quality *and* connected to the well location *and* within the drainage radius of the well. The connected volume depends on the geo-objects intersected by the well and their distribution within the drainage radius. In presence of multiple geostatistical realizations, the average or expected value of the connected volume for a well location may be determined.

Let Ω represent the set of all cells within the drainage radius of the well. The indicator variable $i(\mathbf{u}, \mathbf{u}_w)$ is an indicator variable, which represents the probability of a location $\mathbf{u}$ being connected to the well location, that is,

$$i(\mathbf{u}, \mathbf{u}_w) = \begin{cases} 1, \text{if location } \mathbf{u} \text{ is connnected to well at location} \mathbf{u}_w, \\ \quad \text{and } \mathbf{u} \text{ not connected to another well} \\ 0, \text{otherwise} \end{cases} \tag{1}$$

The connected volume for well $\mathbf{u}_w$ is the indicator-weighted sum of all cells in the drainage radius:

$$CV_w = V_c \sum_{\mathbf{u} \in \Omega} i(\mathbf{u}; \mathbf{u}_w), \text{ where } V_c \text{ is volume of a cell} \tag{2}$$

where V_c is the volume of a cell. Note that we could easily multiply this term by the porosity of each cell to get a true pore volume.

A single well can intersect several geo-objects simultaneously. The connected volume is the sum of all the cell volume that fit the criteria specified above.

Figure 1 shows a schematic example. The cells hatched are to be counted in the connected volume of the well. The cells outside the drainage radius, such as the left and right part of geo-object 2 in Figure 1, are not counted in the connected volume. Only the middle part of geo-object 2 is counted. Since the well doesn't intersect geo-object 3, although some of the cells of geo-object 3 are within the drainage radius, the geo-object is not considered.

For the multiple well configuration, we need to calculate the total CV of all the wells, the cumulative connected volume (CCV):

$$CCV = \sum_{w=1}^{n_w} CV_w = V_c \sum_{w=1}^{n_w} \sum_{u \in \Omega} i(\mathbf{u}; \mathbf{u}_w), n_w \text{ is number of wells} \tag{3}$$

The CCV can be used as a criteria for well positioning. For the same number of wells, we would prefer the well configuration with the greatest CCV.

The simplest case is when only one well location must be chosen. The CV for all candidate locations can be calculated and, then, the well is placed at the site where there is the maximum connected volume. This procedure could be repeated sequentially to place multiple wells; however, this will not in general, achieve the maximum CCV. Figure 2 illustrates this with a simple example. The multiple-well case becomes complicated because the number of combinations to consider is quite large.
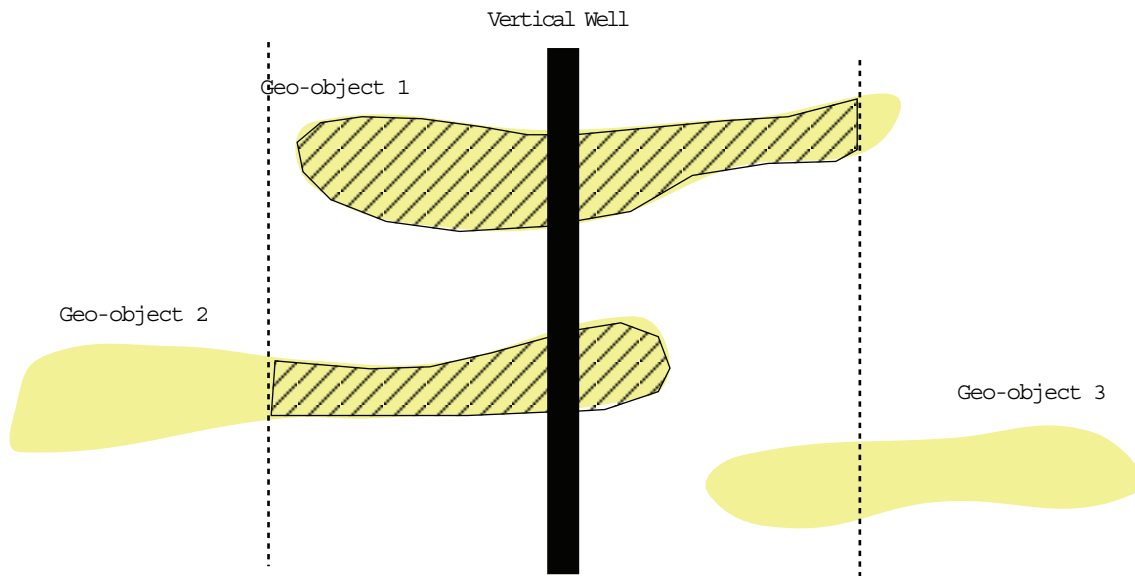
Figure 1: Schematic illustration of geo-objects and connected volume to a well location. The dashed lines represent the drainage radius.
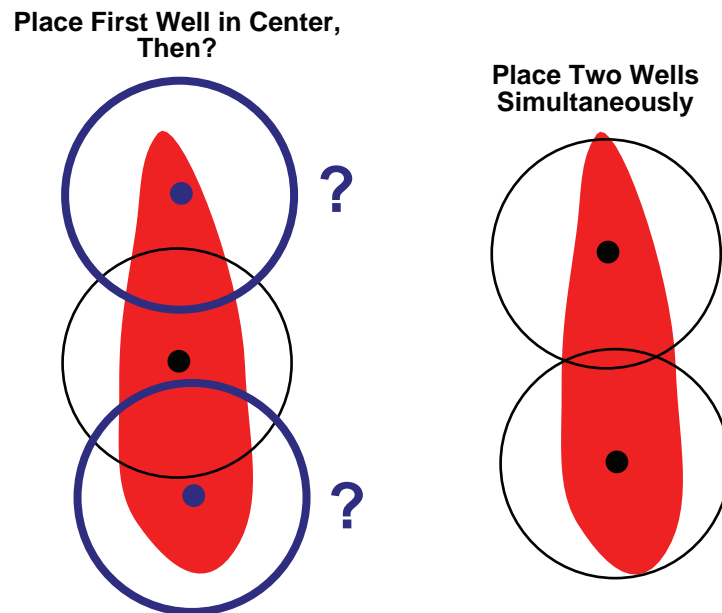

Figure 2: Schematic illustration showing that picking the wells sequentially (without regard for other wells) would be suboptimal.

Methodology

Consider n_x to be the number of grid nodes along the X-axes of a 3-D reservoir model, n_y to be the number of grid nodes along the Y-axes, and n_w to be the number of wells. We limit ourselves to vertical wells at this time; therefore, there are $nx \cdot ny$ candidate locations for each well. The possible configurations of n_w wells can be calculated:

$$N = \text{``}n_x \cdot n_y \text{ choose } N_w\text{''} = \frac{n_x \cdot n_y!}{(n_x \cdot n_y - n_w)!} \tag{4}$$

For example, if we have a $100 \cdot 100$ reservoir model, and there are 5 well locations to be optimized, the number of possibilities may be calculated as:

$$10000 \cdot 9999 \cdot 9998 \cdot 9997 \cdot 9996 = 9.990 \times 10^{19}$$

This is a huge number. There are fewer grams of mass in the earth. The distance from the earth to the sun in microns is less than this number.

Theoretically, the "true" optimal configuration could be found by searching the the entire configuration space. Although intractable for a large number of wells, we permit the user to consider the exhaustive search technique for one, two and three wells configurations. The one and two well case were tested but the three well case would take 250,000 CPU hours on a PII/333/128M RAM. Please refer to the Example section for details. Although the exhasutive algorithm can find the true optimal configuration, CPU requirements restrict its usage. A more efficient algorithm must be developed to solve the problem.

The well location optimization problem is a combinational maximization problem. The space over which the objective function (CCV) is defined is not simply the N-dimensional space parameters. Rather, it is a discrete, but very large,configuration space, like the set of possible orders of cities in the famous traveling salesman problem. The elements in the configuration space is factorial large, so that they cannot be explored exhaustively. Furthermore, since the well location set is discrete, any useful optimization notion, such as "continuing downhill in a favorable direction.", is deprived of application in the well positioning problem. Because in this space, no such "direction" concept can be extracted.

There are a number of candidate optimization algorithms that could be considered including genetic algorithms, simulated annealing, and a limited set of derivative-based methods. We consider an iterative scheme based on simulated annealing. For many applications, however, we do not need to perform a full simulated annealing with complex decision rule [6, 7]. Some features of our problem formulation:

1. Configuration: the 3-D model has a Cartesian structure with n_x, n_y, and n_z grid nodes in each coordinate direction. To simplify the problem, we limit the well locations to fall at grid node locations.

2. Initial configuration: the initial well placement configuration of the n wells is generated randomly. For every well, we randomly select a location within the reservoir area. Wells are not placed at the same location.

3. Objective function: the objective is to maximize connected volume of the n wells. The objective function is defined as follows:

$$CCV = \max(\sum_{i}^{n} CV_i),$$

where n is the number of wells and CV_i is the connected volume of each well;

4. The initial CV is calculated.

5. Perturbation Mechanism: randomly select a well and let it move some distance within the reservoir area. The random movement is defined by the function below:

$$New\_location = old\_location + (r - 0.5) * space * 2.0$$

The movement is a random value within $[-space, space]$, where $space$ is the maximum possible radius of movement.

6. Calculate the connected volume of the new configuration:

$$CV_{try} = \sum_{i}^{n} CV_i$$

7. If CV_{try} is larger than CV then the new well configuration is saved; if not, keep the old well configuration. At this point a "true" simulated annealing decision rule could be used.

8. The perturbation process is repeated a large number of times. In practice, the CCV reaches a maximum value and does not increase any more.

After this optimization procedure, we get CV^*. If the cycles are enough and our starting poing is good enough, we will get close to the "true" optimal configuration of well locations. In practice, we repeat the procedure with different random starting points to reduce the risk of finding a local maxima.

Implementation

We use Visual Basic 5.0 32bit version under Windows95/98 as the development environment. The compiler generates a comparatively fast 32bit true Windows program. A third party chart control (OletraChart 2D from KLG Ltd.) is used to show the connected volume versus iteration times chart and the well location map. Figure 2 shows the main window of the program.

In the program, the important processes are *multi_well_CCV* and *Optimal*. The *multi_well_CCV* is used to calculate the multiple well cumulative connected volume. The input of the procedure is well locations generated by the *optimal* procedure. The *optimal* procedure is also responsible for generating the perturbations, rejecting or accepting the perturbations. There are also some other accessory procedures, such as drawing result on screen and saving the data to file or read data from 3D models.
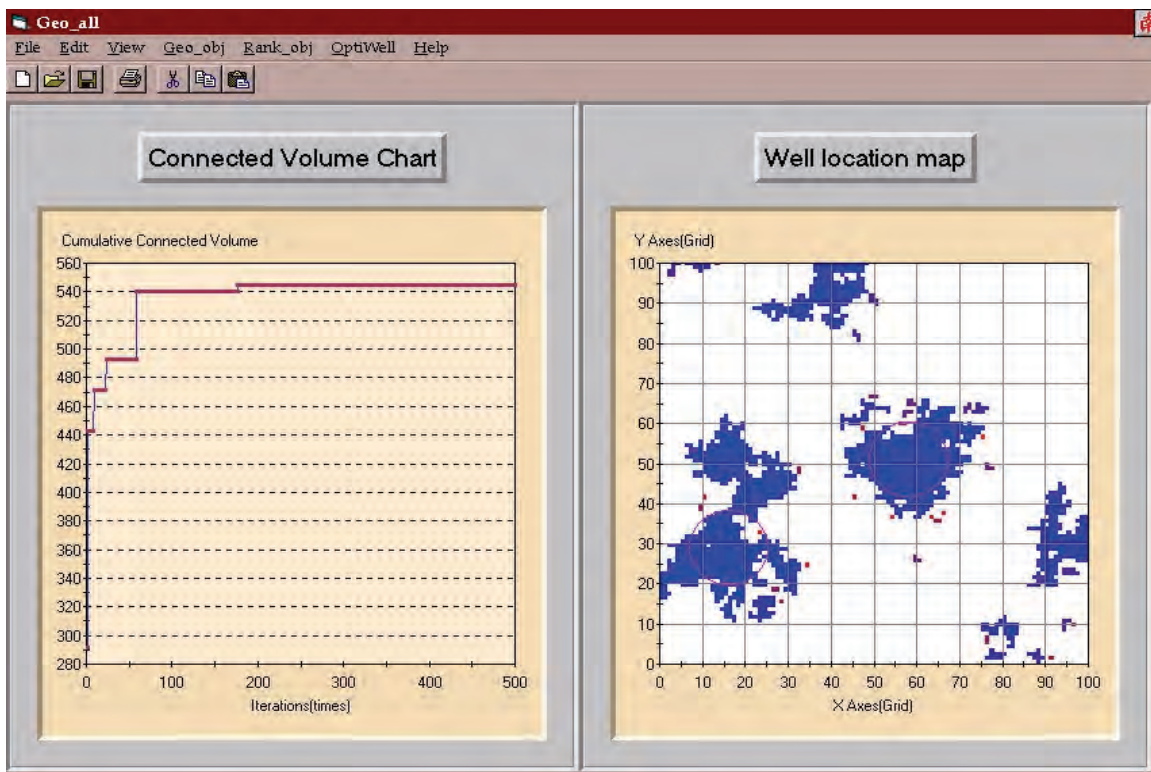
Figure 3: The main interface window to the program.

Two related programs, *geo_obj* and *geo_rank* (Deutsch, 1997) are integrated in the main menu. This facilitates all operations required for well selection. The main features of the program are:

- Two plots are shown simultaneously: (1) CCV versus iteration, and (2) well location map. The program dynamically refreshes the maps, so that the user can see how the program moves the wells to the optimal locations and how the CCV changes with each well movement.

- Typical optimization cases require several minutes with the dynamic plots.

- The program saves the results such as the well locations, CCV, average CCV and all charts. The user can easily find the intermediate data from the algorithm.

- The dynamic plots can be turned off to speed up the program without losing any data. Chart source data can be reloaded manually to be used in other operations.

- The CCV chart and well location map can be copied to the clipboard, saved to a file (Oletrachart format) or printed directly. Any of the elements of the map, such as title,footer,map color, font style, size etc., can be changed without running the program.

- The program has a procedure to transform the 3D model data into GoldenSoft Inc.'s Surfer6.0 format. Any layer of any realizations to be written to a surfer format. Hence any layers of a multiple 3D model can be viewed by Surfer in a contour or image map. The optimal well sites can be written to a Surfer source data file for posting on the contour map.

- The program can also handle multiple realizations. After the optimization, the CCV of each realization and the average CCV is given by the program.

- The program has an internal www browser connected to the "Center for Computational Geostatistics" web site at the University of Alberta. The user can browse the latest information at this site.

Examples

Several examples, including a homogeneous model, a real 3-D reservoir model, simple realistic, exhaustive example and multiple realization examples, have been tested.

Homogeneous Model

In order to check how wells get spaced within "net" reservoir, we built a $100 \cdot 100 \cdot 1$ "homogeneous" 3D model for the testing. Here 'homogeneous' means that every node in the 3D model is set to 1 (net). There is only one geo-object in this model. At constant porosity, the volume of each node is equal. We test the algorithm on this model for different numbers of wells.
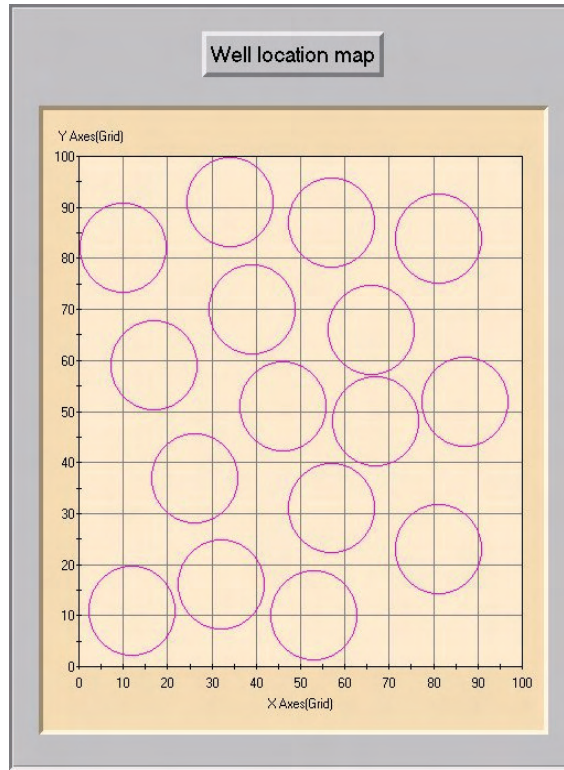
Figure 4: Well locations of 16 wells with radius 10. The wells are evenly distributed without overlapping each other.

Suppose there are 16 wells with radius 10 to be optimized. Since the overall reservoir is a $100 \cdot 100$, there is more space than the sum of the drainage areas. Thus, the optimal well configuration should be distributed without overlapping, that is, any configuration with no overlapping wells, is optimal. Figure 4 shows how the optimization algorithm distributes the 16 wells, they do not overlap.

Nine wells with radius 17 must overlap; however, the optimal configuration would minimize such overlap. In this situation,the drainage area (circles) of these 9 wells should be nearly tangent. Figure 5 shows the resulting optimized locations. The 9 well locations appear do not appear to overlap too much.

Figure 6 shows the results if increasing the radius to 20 for the example above. The wells overlap by a nearly constant amount. The resulting configuration is close to what we would expect. Figure 7 shows the CCV versus iteration through the optimization process.

We can increase the number of wells to see the "optimal" configuration. Suppose there are 16 wells with radius 20 to be distributed. The total drainage area should cover the entire reservoir field. In this case there is a total CCV of 10,000 available to be drained. Figure 8 shows the initial well location maphalf way through the optimization process, and the final well configuration. We can see how the well location configuration moves from randomly distributed to evenly distributed. The drainage area of the 16 wells totally covers the reservoir area. After 800 iterations, the algorithm has found the maximum CCV of
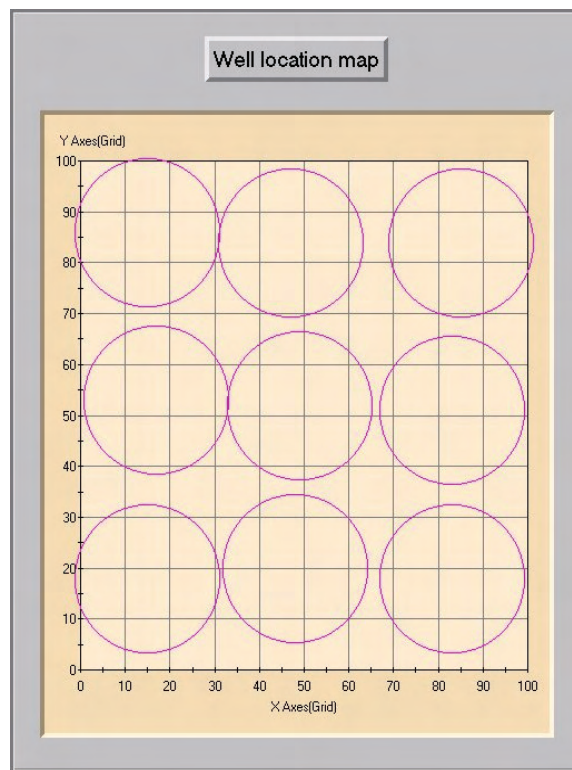
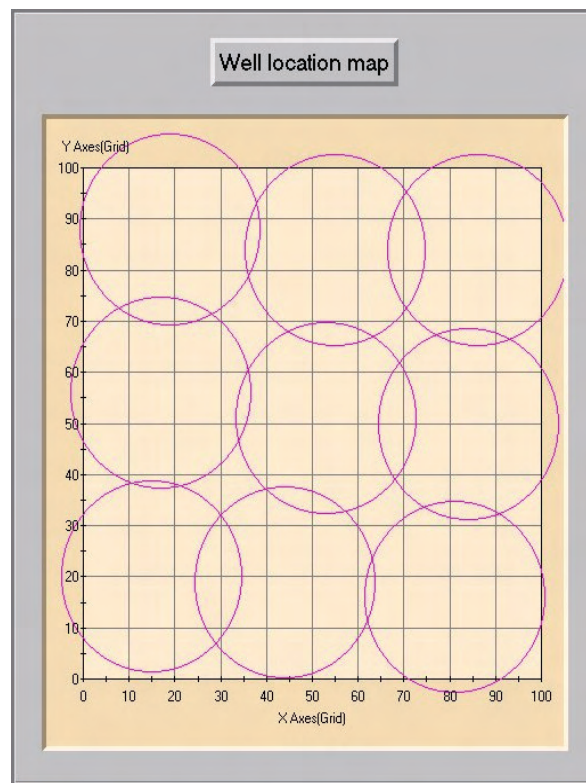Figure 5: Well location map of 9 wells with drainage radius 17. These 9 wells are nearly tangent.

Figure 6: Well location map of 9 wells with radius 20. These 9 wells distributed evenly with 3 columes by 3 lines.
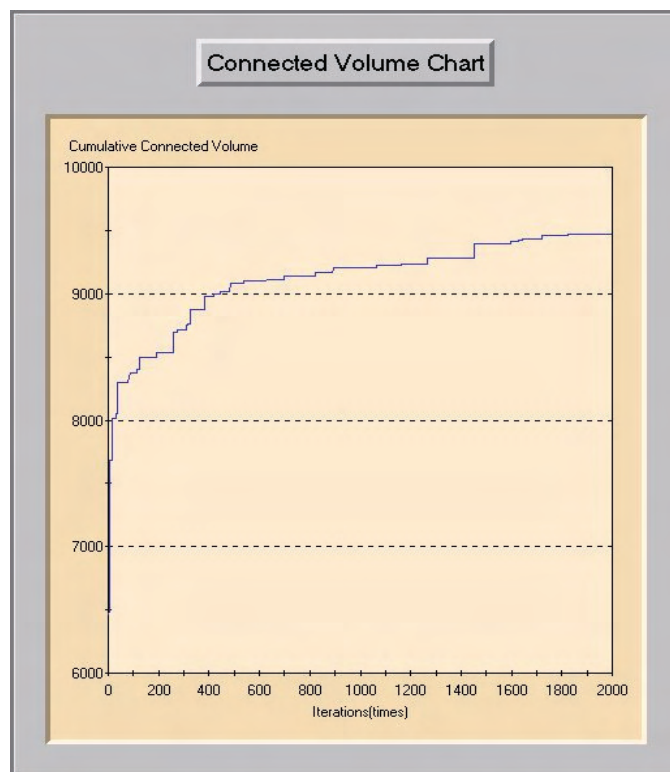
Figure 7: The CCV chart of 9 wells with radius 20.

10,000 and model does not change.

Simple Realistic Example

We use Sisim (GSLIB) to generate a $50 \cdot 50$ simple 2-D model. It is easy to judge the correctness of the optimal well locations and CCV map. Figure 9 shows the model. The blue color represents the largest geo-object and other colors represent smaller geo-objects (from 2 to 16).

Different Initial Configurations

The stochastic optimization algorithm depends on the initial configuration. Figure 10 shows three different initial configurations and the resulting optimal configurations. Although the program starts with different initial well locations, the algorithm finds nearly the same optimal configuration in all cases.

Our experience is that the optimal solution does not vary significantly with different initial realizations. Nevertheless, we advise users to consider a number of initial configurations. Also, to some extent (particularly for 2-D models) the final result can be checked visually.

Comparison with True Result

Exhaustively checking all possibilities to find the true optimal configuration is possible with small models and small numbers of wells. CPU time consuming is tested on the realistic example. For one well of radius 10, it takes 5 seconds of CPU time, see Figure 11.

Exhaustively checking all possibilities of 2 wells of radius 10 requires 10.5 hours of CPU time, while the optimization algorithm only needs 15.5 seconds. Figure 12 shows the end well location maps. The exhaustive algorithm does get 6 units of connected volume more than optimization algorithm (about 2%).

We estimate that the time needed for a 3 wells of radius 10 case would be 250,000 hours of CPU time! The time needed for optimization algorithm is only 37 seconds.

Real 3-D Reservoir Model

Figure 13 shows different views of a $100 \cdot 100 \cdot 10$ reservoir model. This is based on a real reservoir (as published by Gouveia et. al.) but shown with no scale or structural component. The different colors represent facies of different reservoir quality. There are six different geo-objects, that is, six diffent flow units. Although this model is more complex than the other 3-D models considered here, the results work the same.

Figure 14 show the optimal well locations for 1 and three wells. Any combination of wells with different drainage radii could be considered.

Example with Different Numbers of Wells

Sequential indicator simulation (`sisim` in GSLIB) was used to generate a 100 by 100 2-D model to test the optimization algorithm. Figure 15 shows the geo-objects resulting from
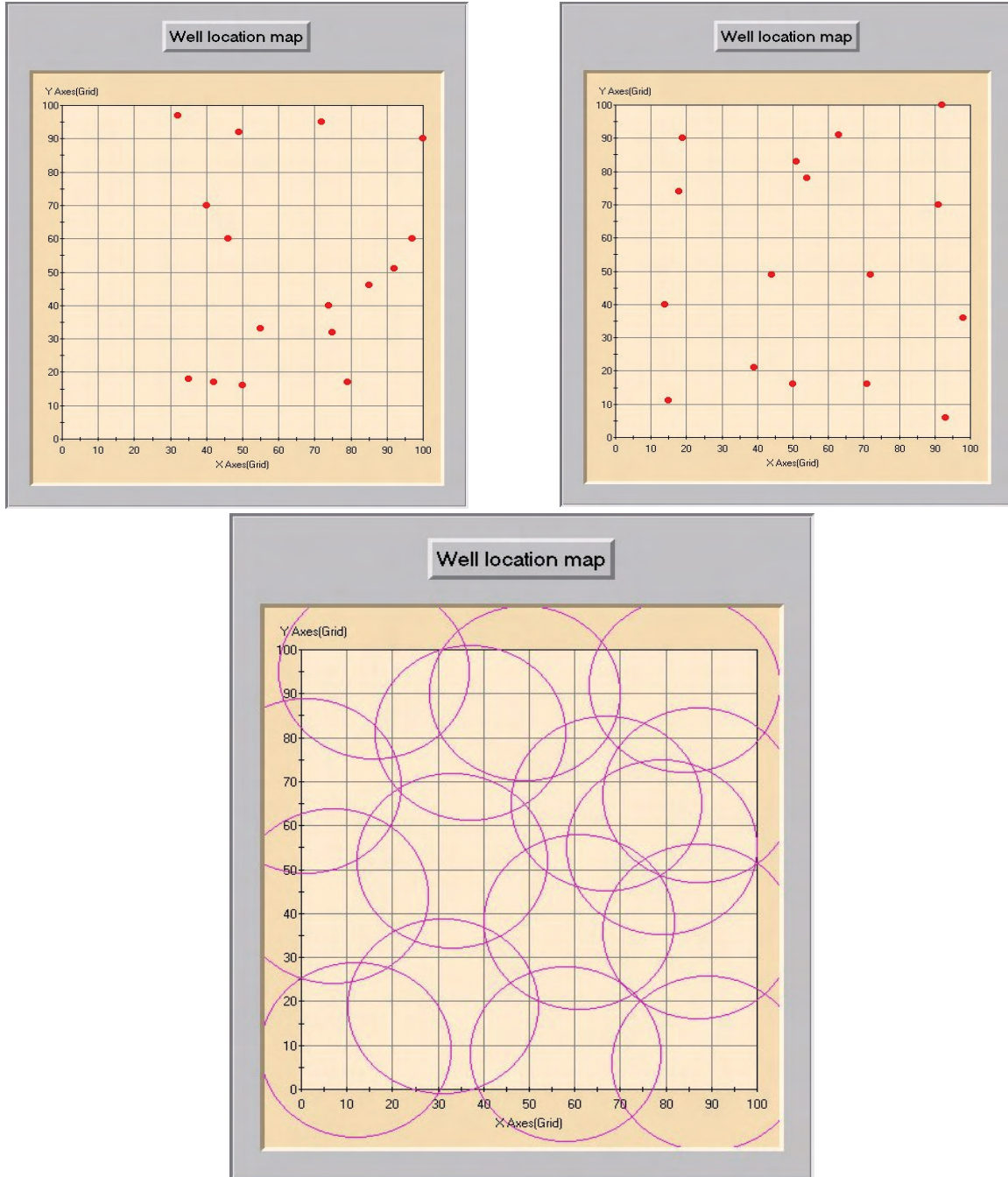
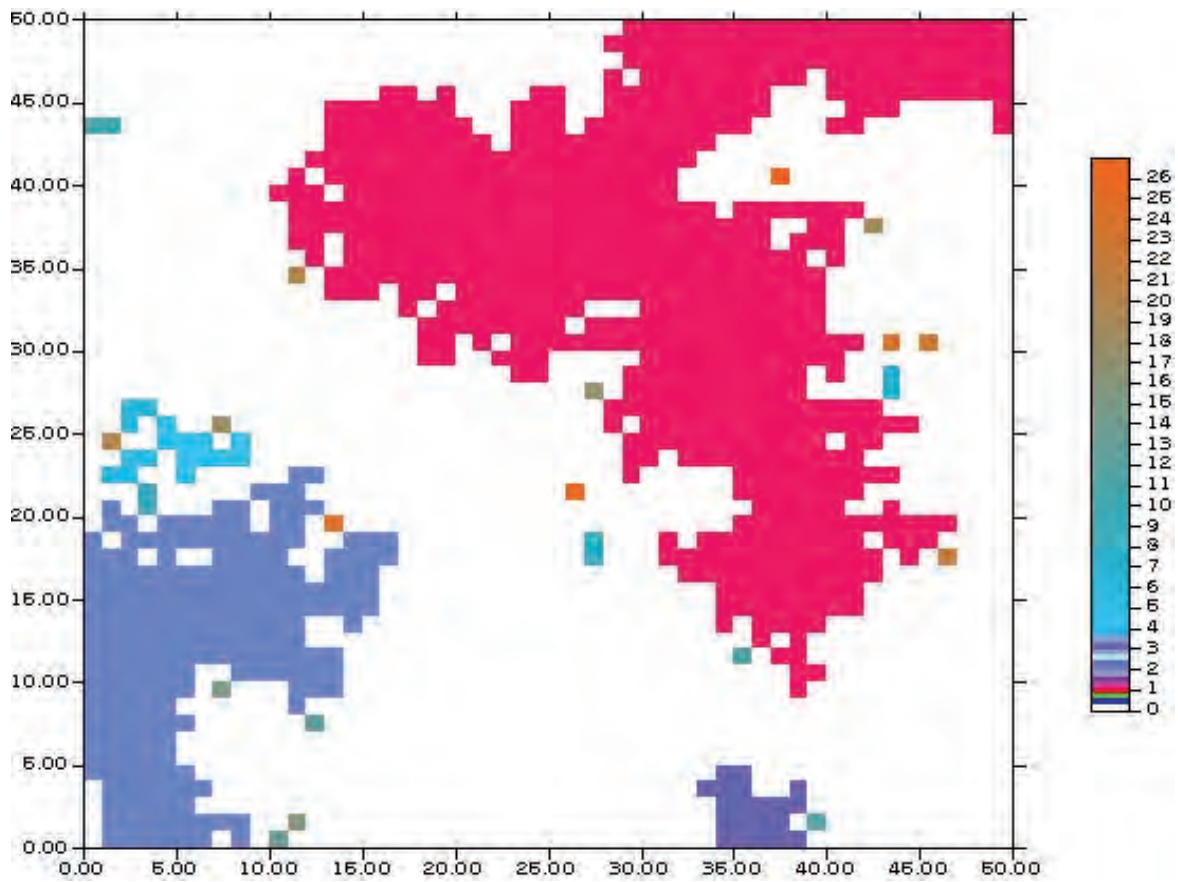Figure 8: The initial, half-way, and final well locations of 16 wells of radius 20.

Figure 9: The image map of simple model (50*50*1). The blue color represents geo-object 1. There are 26 geo-objects of this model. The colors of the color scale represents the corresponding geo-objects.
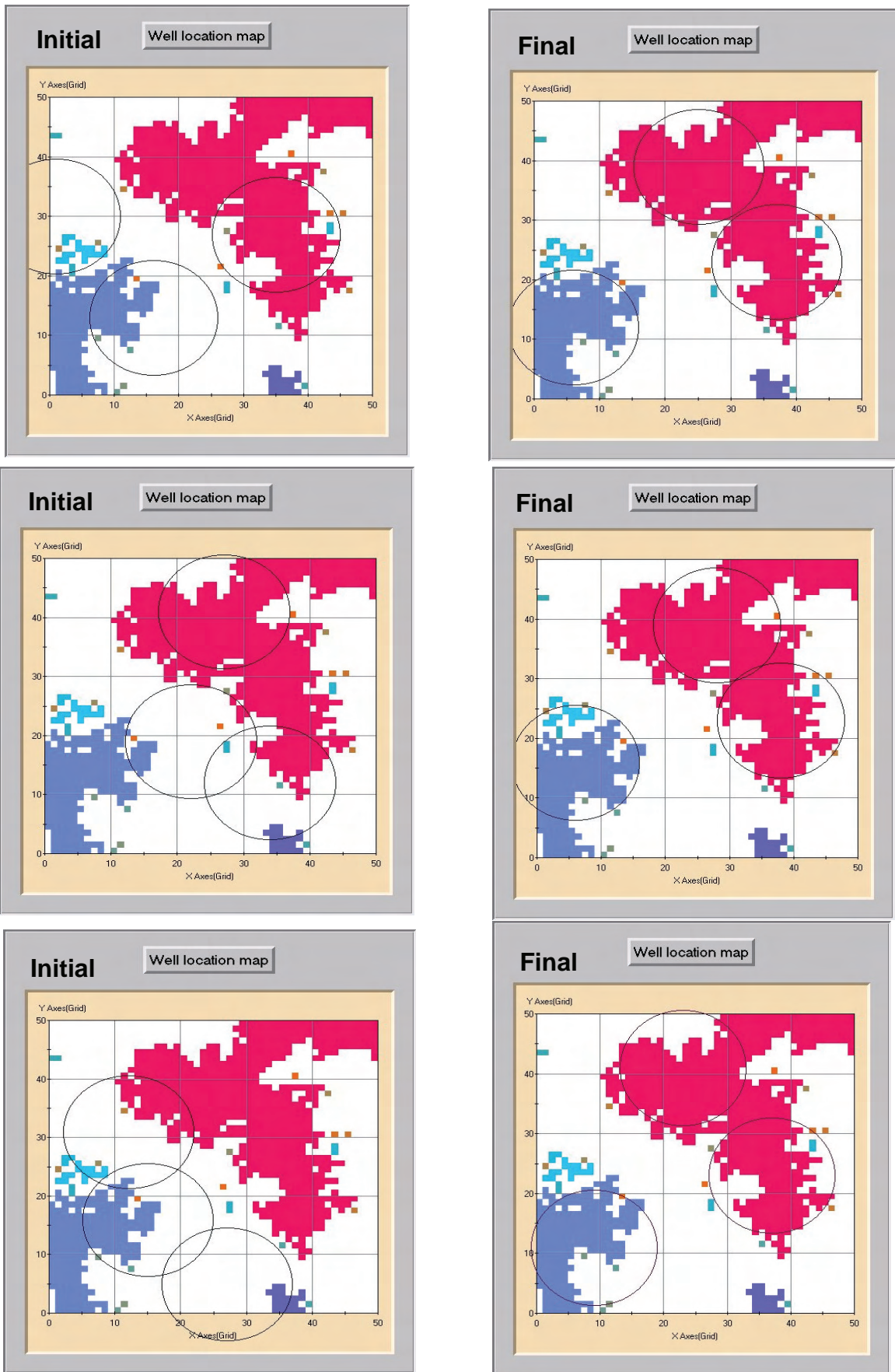
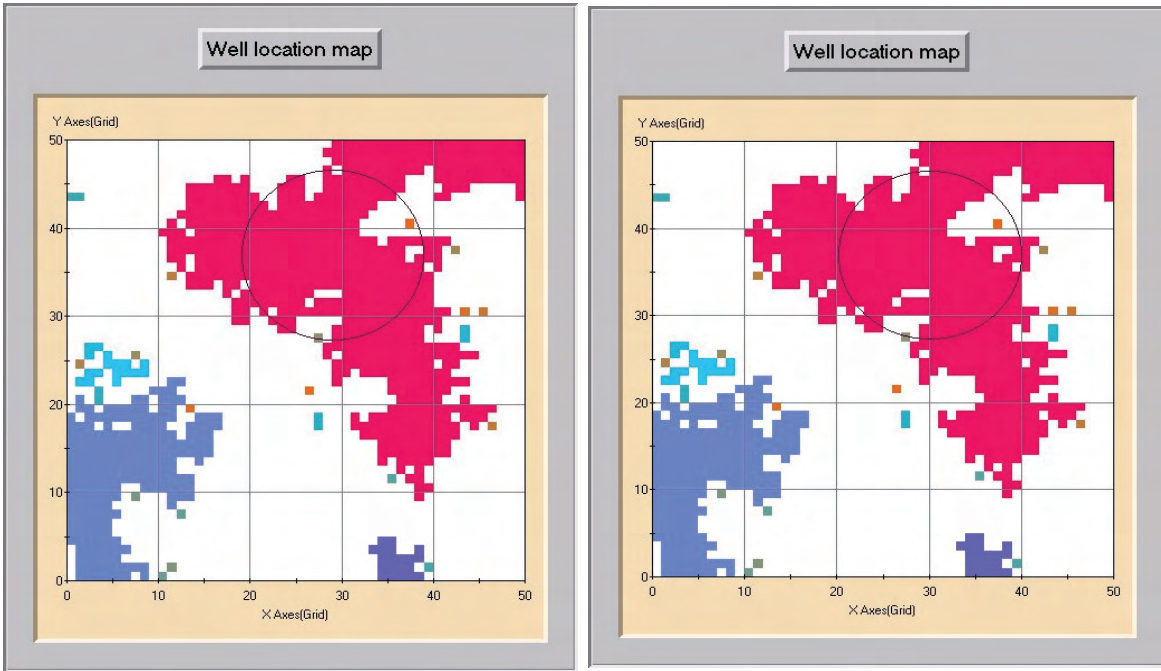Figure 10: Three different initial and final realizations. Note that the same result is obtained in all cases.

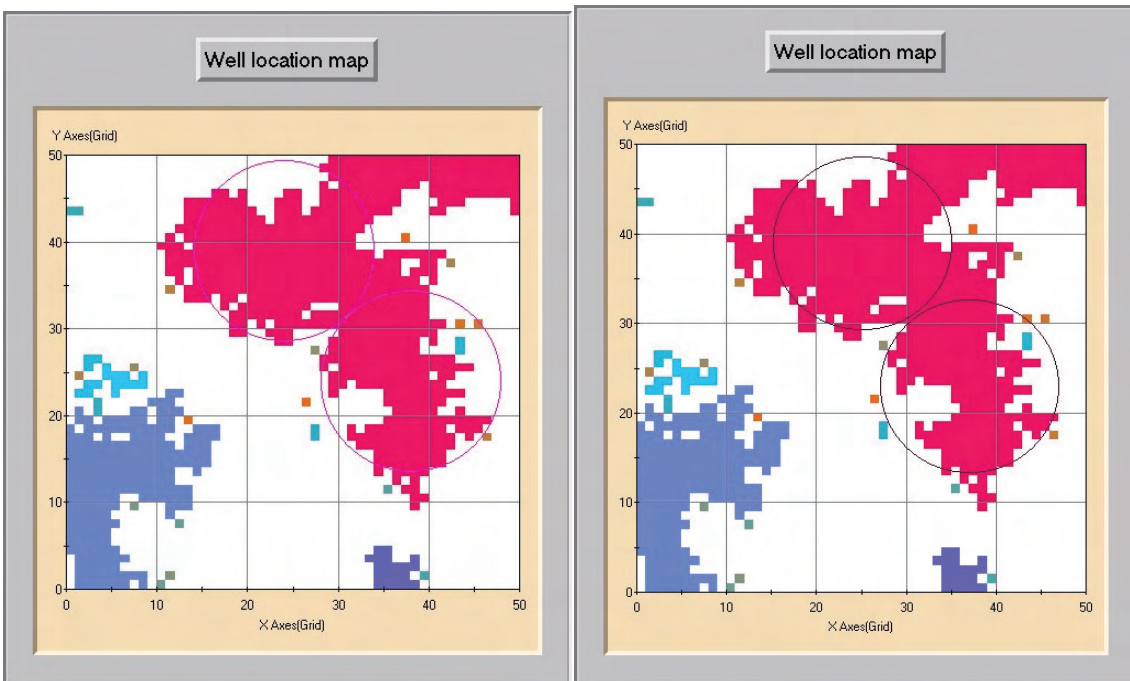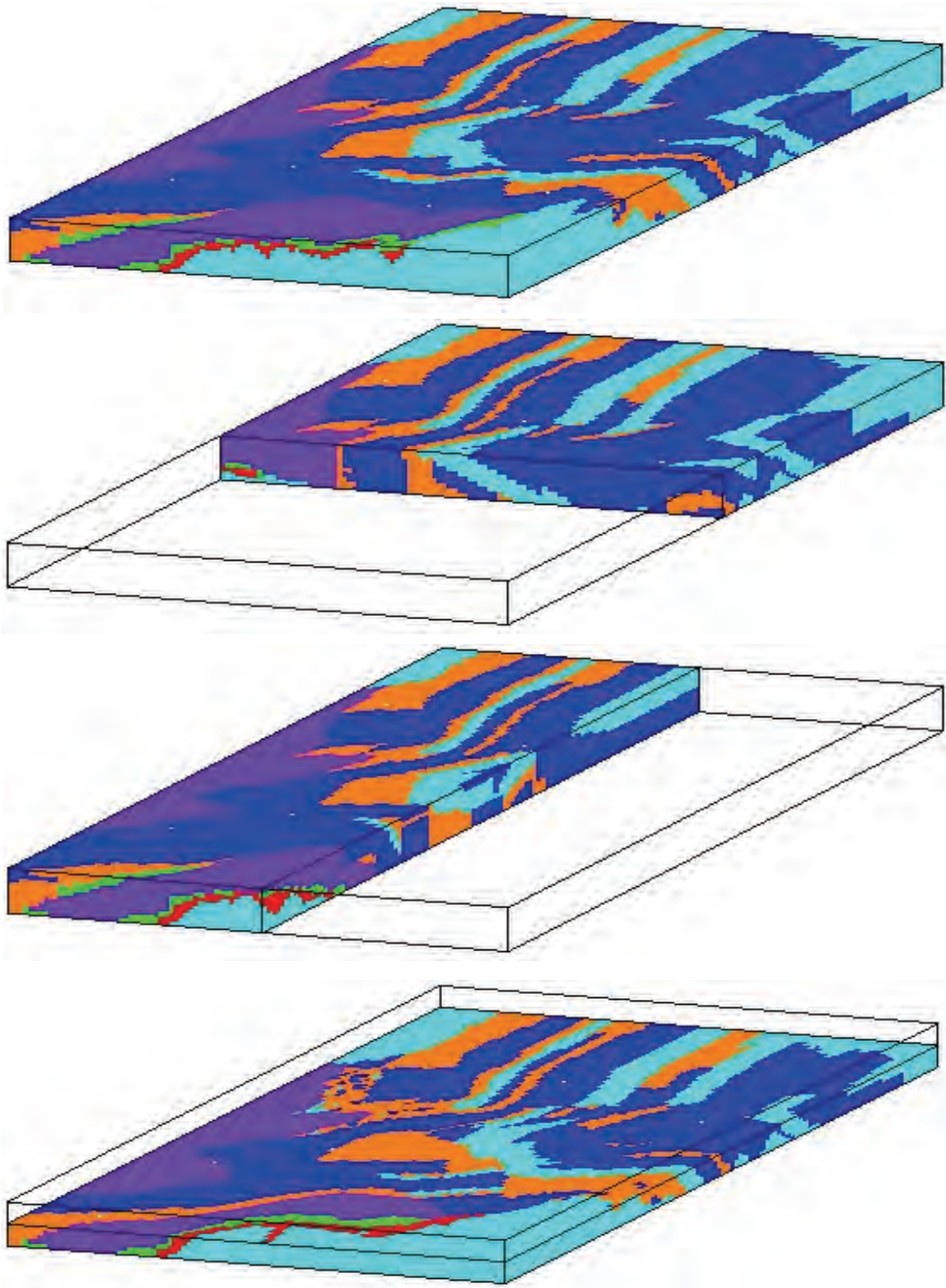Figure 11: True optimal and result of program for one well.

Figure 12: True optimal and result of program for two wells.

Figure 13: Different views of a real 3-D reservoir model.
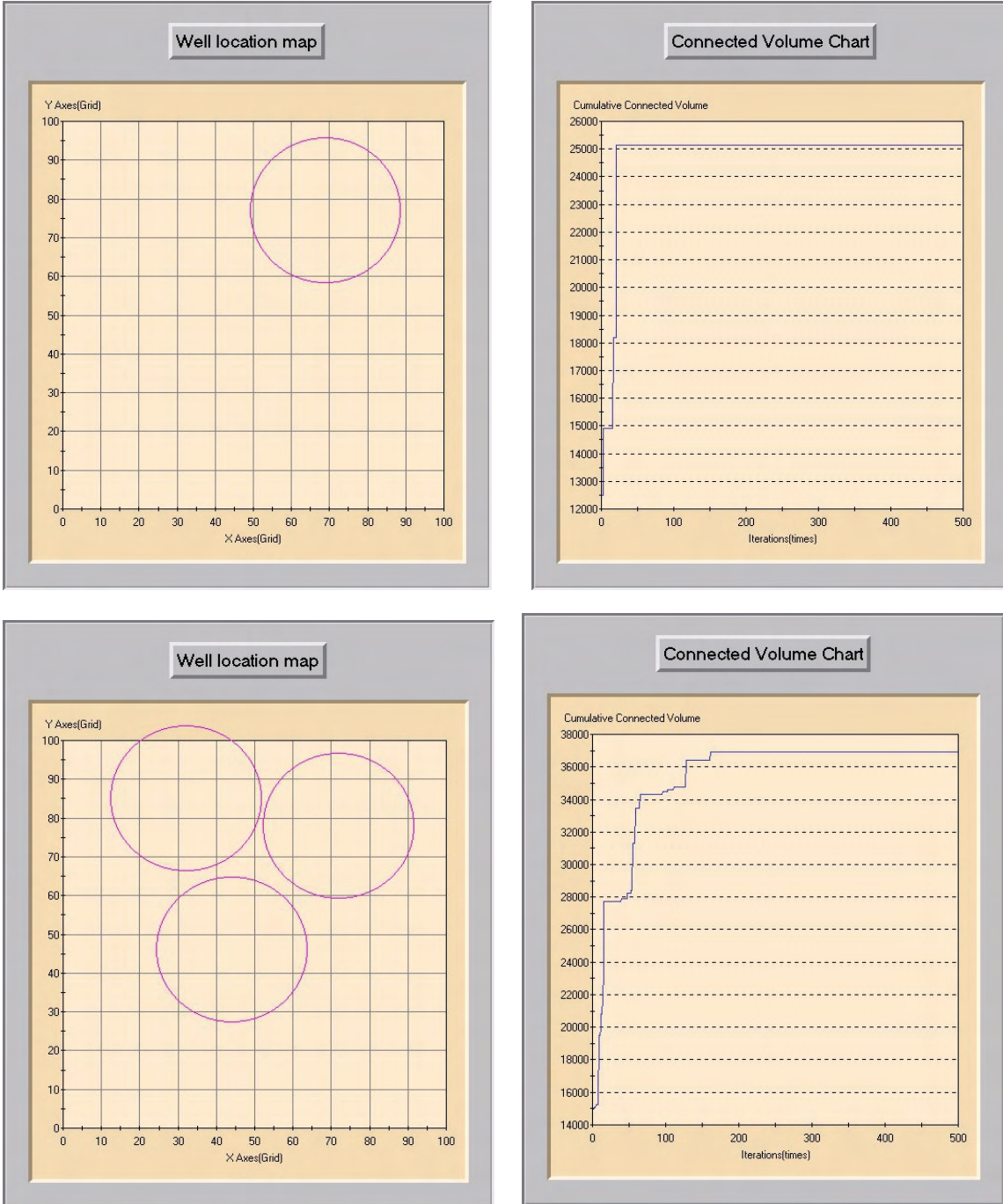
Figure 14: results of 1 and 3 wells for 3-D reservoir model.
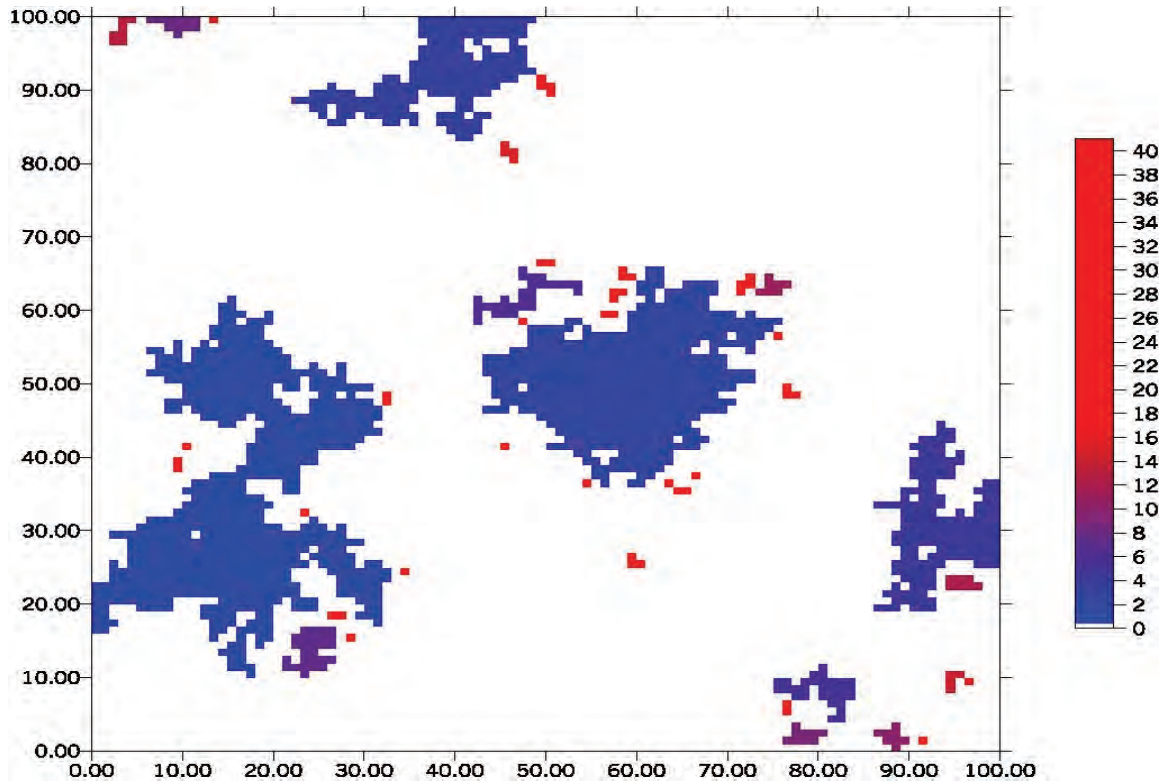
Figure 15: Geo-objects in exhastive example.

an unconditional `sisim` model. Results for using an increasing number of wells is shown on Figures 16, 17, 18, and 19.

At times, it is necessary to decide on the optimal number of wells to produce a field. The production rate and details of well productivity can be established by classical petroleum engineering calculations. The analysis we present here, however, could be used to ascertain the additional reservoir volume that could be drained by using more wells. A plot of the optimal CCV versus the number of wells could be created. Then, the incremental recovery with additional wells can be considered to determine the point where the cost of a well is not justified by the incremental recovery. Figure 20 shows the results in this case, that is, the cumulative recovery versus the number of wells.

Multiple Realization Example

In practical decision making situations we have multiple realizations. Each realization is equally likely and, by itself, would have a different optimal configuration of well locations. In practice, however, we need a single optimal configuration of wells that is "optimal" over all realizations simultaneously. As presented in the methodology section, this can be used in the objective function.

Four 2-D realizations with dimension $50 \cdot 50$ were generated by `sisim`, see Figure 22. The geo-objects are calculated for each image and the optimal well configurations (for 1, 2, and 3 wells) are then shown on Figure 23. These globally optimal configuraions are not
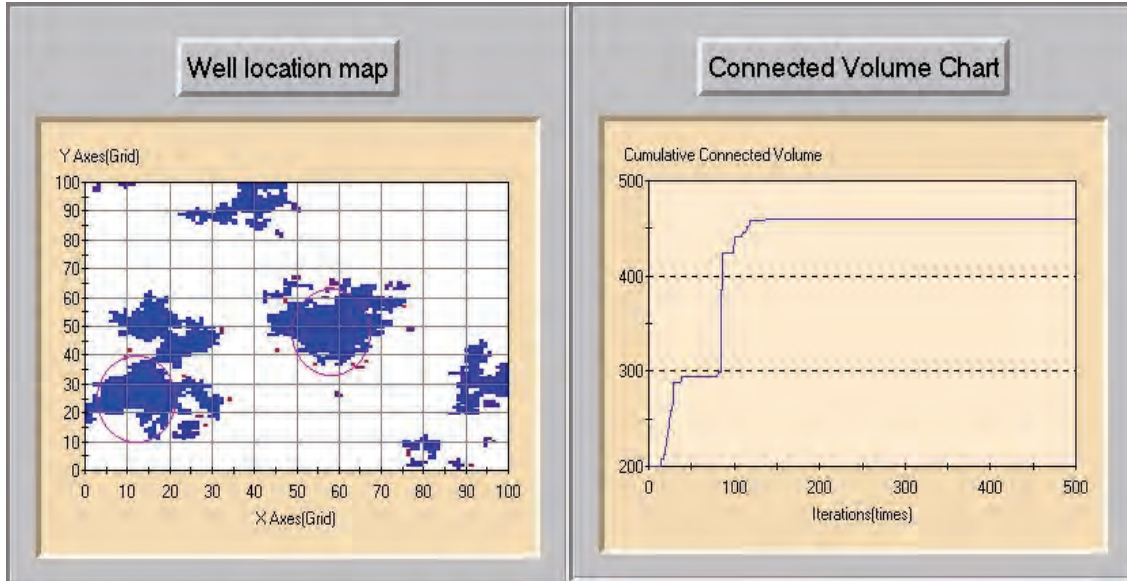
20

Figure 16: Results of 2-wells of radiues 10.

Realization Number	CCV
1	1932
2	2935
3	2794
4	2881

Table 1: CCV of each realization.

optimal for each realization, but *in expected value* over all realizations. This is precisely the goal of reservoir management, that is, to make decisions that are robust with respect to the unavoidable uncertainty. Table 1 shows the CCV for the four different realizations. These results quantify the uncertainty in the reservoir response for this particular decision.

Discussion

The well location perturbation is very important in realistic examples. If there are no optimal well locations around the search radius, then the optimal algorithm has difficulty moving from this local minimum, even with many iterations. One may have to consider the use of simulated annealing to help jump out of local maxima. Also, some rules can be used to select the initial locations to avoid areas where there is little or no connected reservoir volume.

At this time we only consider vertical wells. The approach could be modified to consider sophisticated horizontal and deviated well trajectories; however, this refinement has not been implemented at this time.

Additional restrictions on the objective function such as connectivity to injection wells,
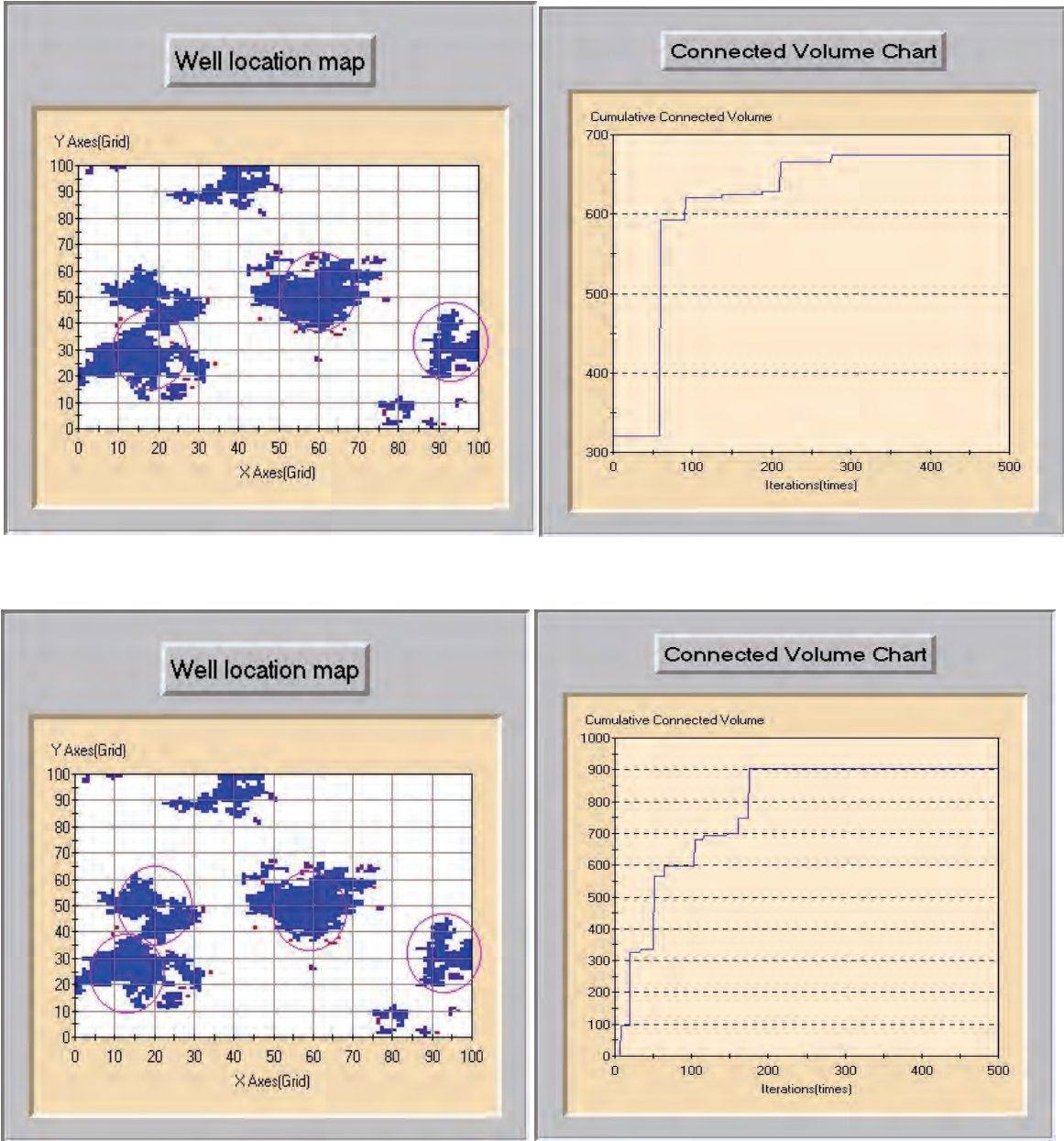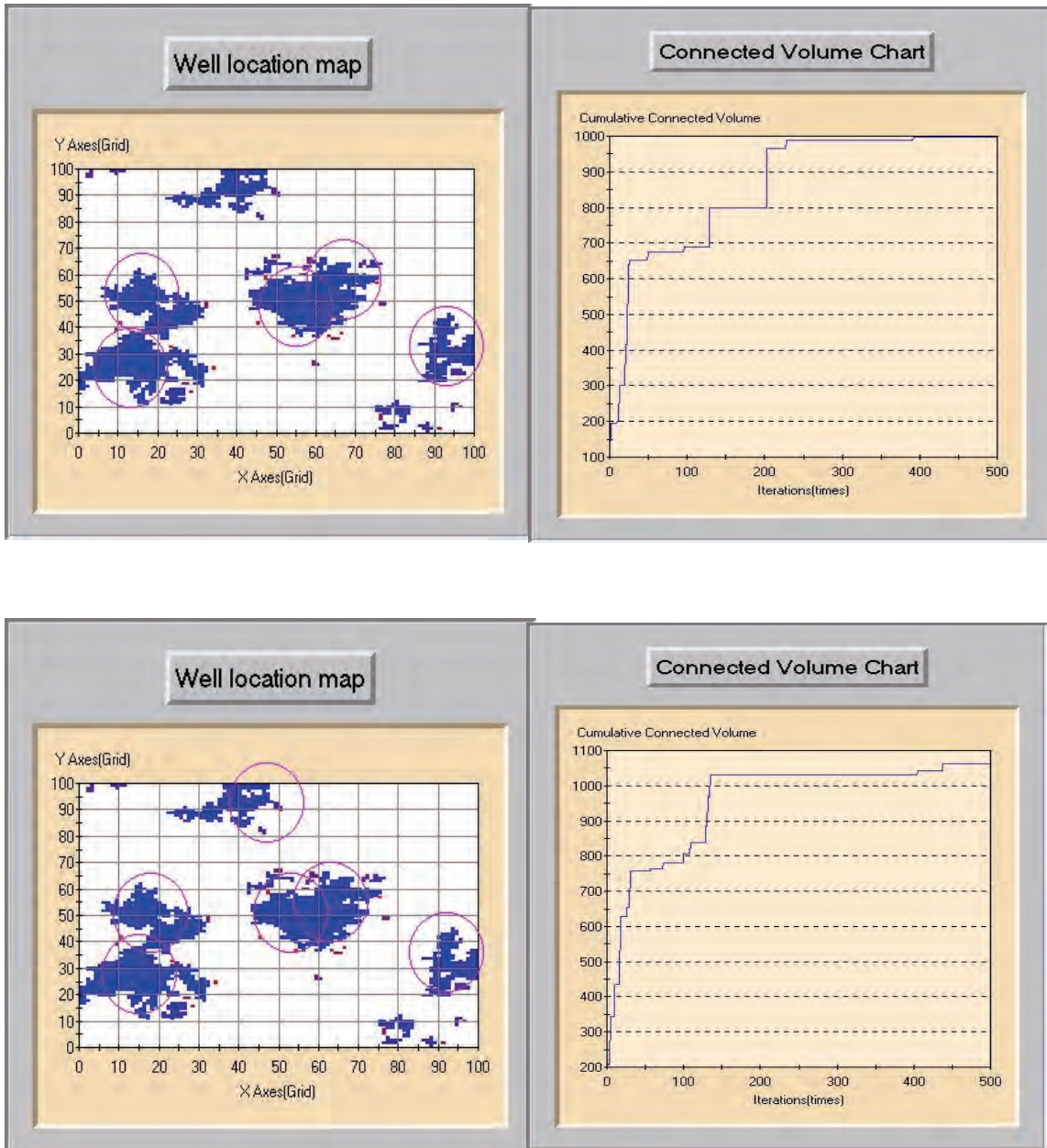
Figure 17: Results of 3 and 4 wells of radiues 10.
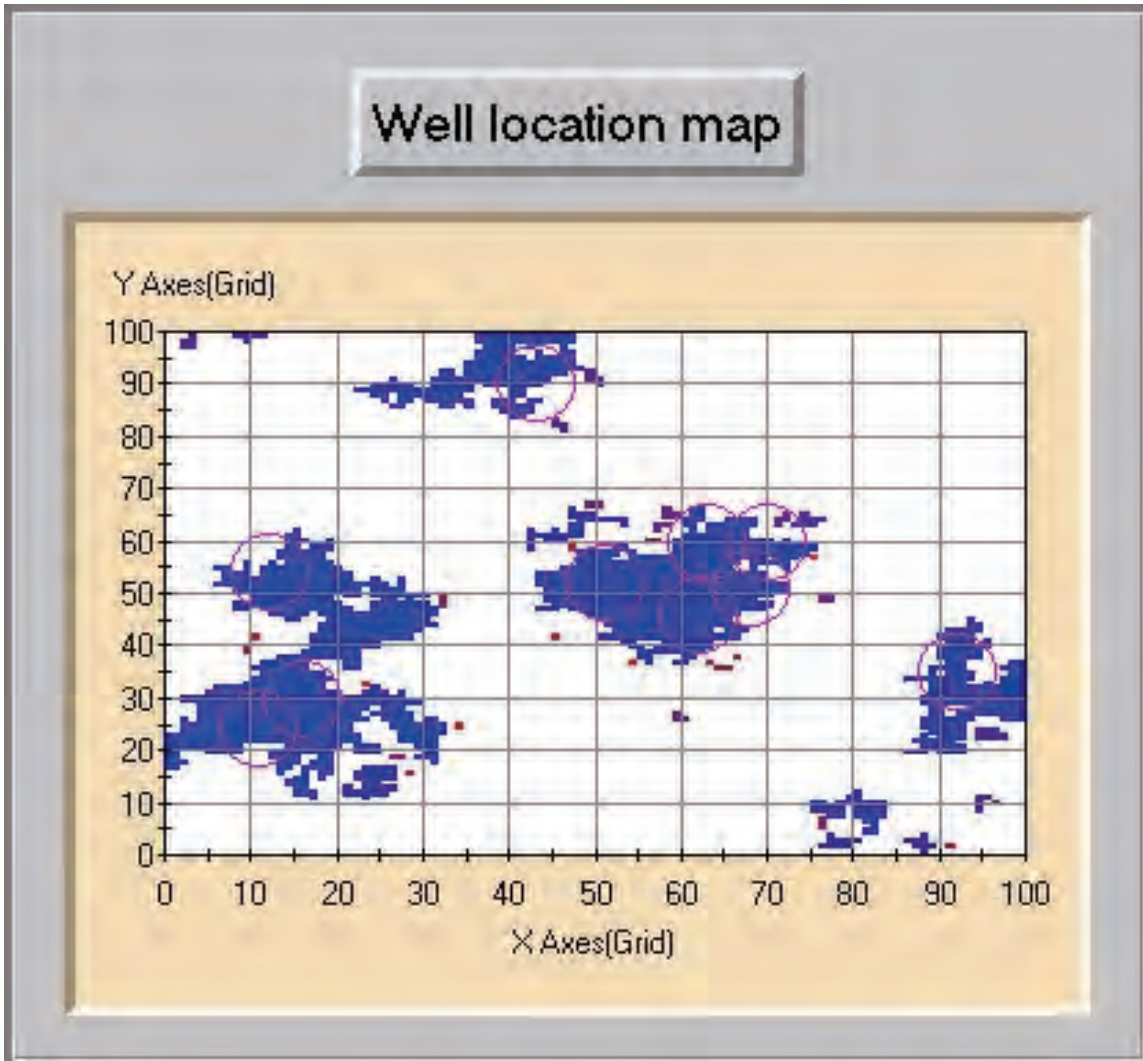
Figure 18: Results of 5 and 6 wells of radiues 10.

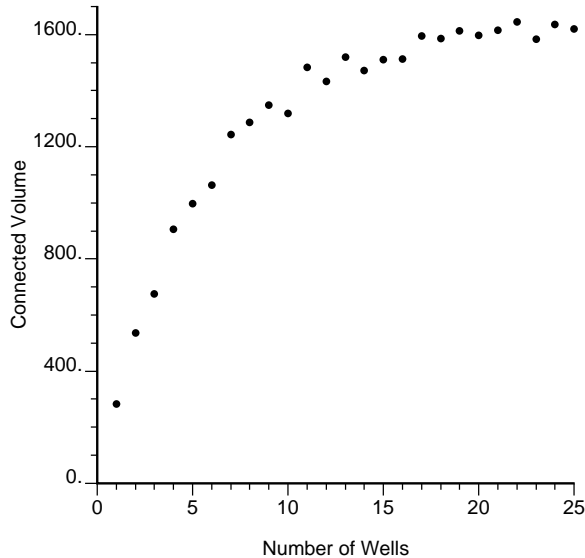Figure 19: Results of 10 wells of radiues 5.

Figure 20: Cumulative recovery for different numbers of wells. Not that the non-monotonic behavior is due to some randomness introduced by different initial realizations. The solid curve is a polynomial fit to the data.

structural complications such as faults, and other factors could be added to the objective function. These factors will be essential for the full fledged practical application of this methodology.

As described in our examples, this algorithm is directly applied to find the optimal well locations, such as production well sites. Yet, the tool can also be used to help provide more directed input to reservoir simulation. The suitable configurations can be decided before costly reservoir simulation. The CCV statistic could also be used to rank multiple geostatistical realizations for uncertainty assessment with full reservoir simulation.

There is a need to rigorously account for the predictions of flow simulation in reservoir decision making. The work presented here uses "static" connectivity measures as a proxy for the full "dynamic" response of the reservoir. The optimization algorithm presented here is intended to complement conventional flow simulation and geological input to well location selection.

The efficacy and speed of the optimization algorithm provides us a tool to optimize drilling plans and development schemes, which permits improved economics and reduced risk.

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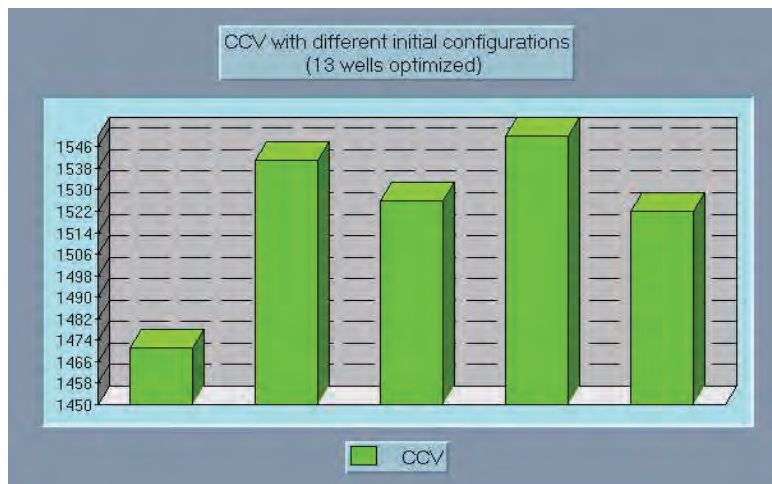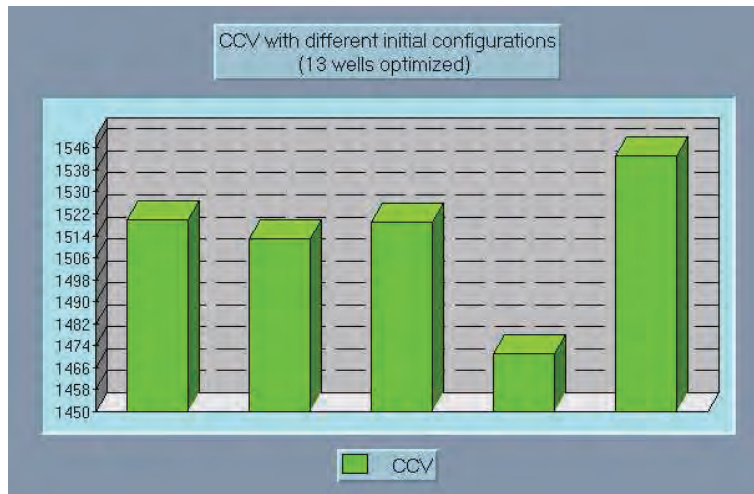
Figure 21: Multiple results with different initial configurations for 13 wells and 14 wells. Note the greater maximum value for the 14 well case.
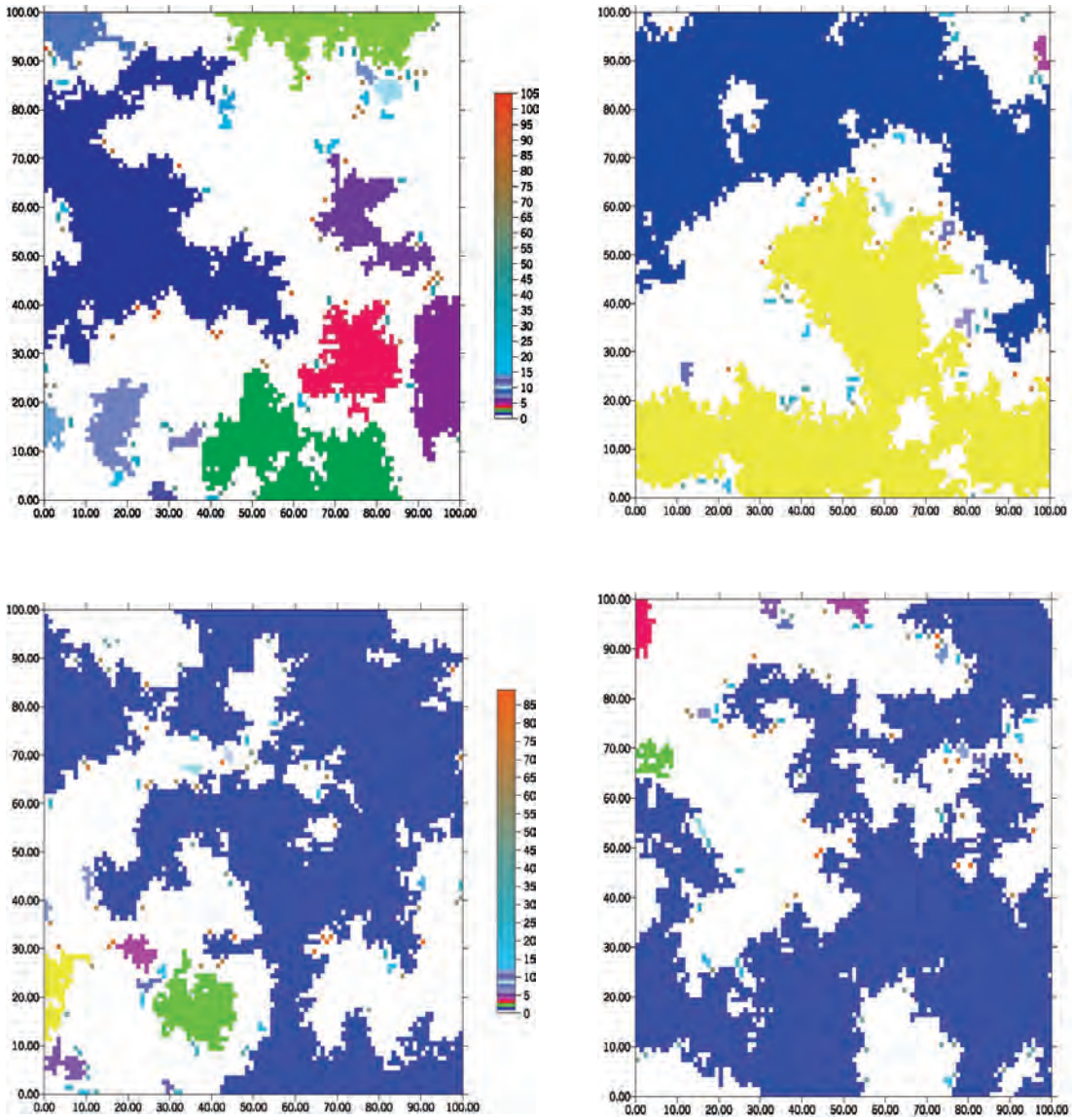
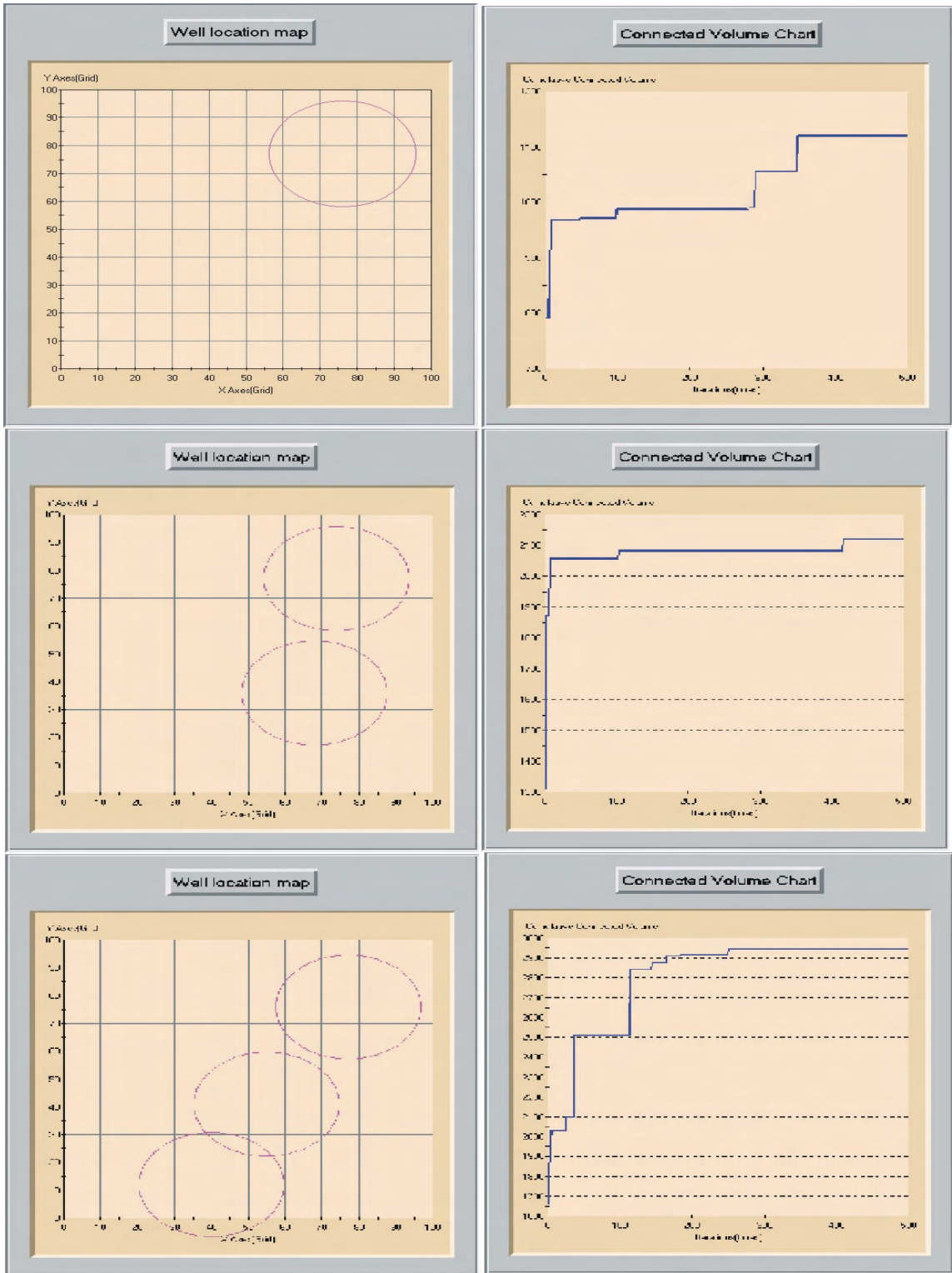Figure 22: Four realizations for multiple realization example.

Figure 23: Optimal result for 1, 2, and 3 wells, respectively.

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