Quantifying Uncertainty in Mineral Resources with Classification Schemes and Conditional Simulations

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Abstract

Mineral inventory consists in estimating the amount of mineral resources and classifying them into categories with increasing level of geologic confidence. Such classification is a crucial issue for mining companies, investors, financial institutions and authorities, but it remains subject to some confusion because of the lack of standards. The first part of this paper analyzes the impact of the criterion used to classify the resources and the parameters that define this criterion, through a sensitivity study on a Chilean porphyry copper deposit. Five criteria are compared and criticized, namely: search neighborhoods, absolute and relative kriging variances, absolute and relative conditional variances. It is shown that some criteria favor or penalize the high-grade areas if the grade distribution presents a proportional effect. In the second part, conditional simulations are used to quantify the uncertainty on the overall mineral resources. This approach is promising for risk analysis and decision-making, since it allows including a cutoff in the calculation of the resources. However, it requires paying attention to several points, such as the choice of the geostatistical model that describes the grade distribution, the division of the deposit into homogeneous domains and the accurate delimitation of the ore zone.

Introduction

The investments and development of mining projects depend on the quantity (tonnage) and quality (grades) of the mineral resources in the deposit (Vallée, 1999, 2000). Mineral inventories are based on assessing the grades and tonnages through an estimation procedure such as ordinary kriging. This allows the construction of a model where block grades are estimated, usually from exploration data. Logically, this leads to block estimates with different precisions, depending on the number and configuration of the neighboring samples used to estimate each block, as well as on the spatial continuity of the grades and their local values (Diehl and David, 1982). This observation has led the mining industry to define several categories of mineral resources, to reflect the degree of uncertainty on the estimated grades. Classification of mineral resources and mining reserves is important because reliable information is required by financial institutions, investors and authorities for fixing royalties and taxations, and for strategic decisions and investment planning.

Several international classification systems have been developed in the past decades (Rendu and Miskelly, 2001), the main ones being the American USGS Circular 531 (USGS, 1980) and SME Guide (SME, 1999), the Australian JORC Code (JORC, 1999), the Canadian CIM Guidelines (CIM, 1996) and National Instrument 43-101 (CSA, 2001), the South-African SAMREC Code
A mineral resource is accepted as a concentration of material of intrinsic economic interest such that there are reasonable prospects for eventual economic extraction. The characteristics of a mineral resource (location, tonnage, grade, continuity, etc.) are estimated from geologic evidence and knowledge, such as exploration samples from drill holes. Mineral resources are subdivided into measured, indicated and inferred categories, in order of decreasing geologic confidence.

An ore reserve is defined as the economically mineable part of a measured or an indicated mineral resource. Its assessment includes mining, metallurgical, economic, legal, environmental, social and governmental considerations, such that the extraction can be reasonably justified. Ore reserves are subdivided into proved and probable, in order of decreasing financial confidence.

The categories defined above combine the uncertainty that stems from several sources, the first one being the inherent geological uncertainty of the phenomenon. Indeed, the modeling of the deposit is nothing but an interpretation based on scarce information; the expert appreciation of the available data (samples, analyses, maps, etc.) allows constructing a model of the main geological units of the deposit, defining their extent, location and characteristics. Hence, the geologic evidence and knowledge are subject to error. A second source of uncertainty is the grade variability, which can be quantified through structural tools such as the variogram. Geostatistical theory can be used to generate estimates of the block grades using this information. Assumptions of homogeneity of the grades over each geological unit (stationarity decision) are often required. The parameters of the model are then uncertain, providing a second source for errors. Finally, there are the “modifying factors”: these include technological factors (mining, metallurgical, environmental) and external factors (economic, legal, social and governmental).

Codes for mineral inventory classification and reporting aim at accounting for all these uncertainties through the definition of confidence classes (Wober and Morgan, 1993). For instance, measured resources are defined as those for which the data locations are spaced closely enough to confirm geologic and grade continuity, while for indicated resources, there must be enough information to reasonably assume geologic and grade continuity, although this continuity is not confirmed. The definition of the categories remains quite vague and is left to the judgment of a specialist called “competent” or “qualified” person.

In this work, we focus on the evaluation and classification of mineral resources and do not examine the definition of ore reserves. This study compares five criteria that are currently used in Chilean copper mining companies, based on geometrical or geostatistical considerations, and stresses the pros and cons of each criterion. In the last section, an alternative approach based on stochastic simulations is proposed for quantifying the uncertainty on mineral resources.

Presentation of the Deposit and the Data

The concepts and results will be illustrated through a case study corresponding to a porphyry copper deposit in Chile exploited by open pit. The available data are the copper grades measured on a set of 2,376 composites from diamond drill holes in an area of 400m × 600m × 130m. Their histogram has a lognormal shape with a mean grade of approximately 1.00% Cu and a coefficient of variation of 0.63. These data belong to a single geological unit. The variogram analysis shows
a greater spatial continuity along the vertical direction than in the horizontal plane, for both the copper grades and their normal scores transforms (Figure 1). Selective mining units (blocks) are 15m × 15m × 12m in size.

Two block models are constructed: one by ordinary kriging of the grades, the second by averaging a set of conditional simulations obtained by applying the turning bands method to the normal score data (Matheron, 1973, p. 461). In each case, additionally to the grade estimates, one also has a measure of uncertainty: kriging variance and conditional variance of the simulations (Figure 2). Notice that the maps of block grade estimates are slightly different, particularly at the edges of the estimated domain. These differences are due to the use of ordinary kriging in the first case, where a local mean is used to locally adjust the estimated values, whereas the second model corresponds to an average of simulated fields, where a stronger assumption of stationarity is required and a single global mean is used to adjust the estimates over the domain. Much more different are the maps of block grade variances. From the one obtained using the kriging variances, it can be seen the dependency between this measure of uncertainty and the local configurations of the data, while for the map of conditional variances (local variances of the simulated block grades), a strong dependency between the estimated grades (local mean) and the measure of uncertainty is evident. The consequences of this difference in classification are further discussed in the next section.
Mineral Resource Classification

In this section, we apply several criteria for classifying the estimated resources and comment their pros and cons.

Criteria for classifying

1) Neighborhood restrictions (applied to the kriged resources)

This first criterion consists in classifying the blocks according to geometrical constraints, referred to the number and configuration of the data located in a given neighborhood. The block grades estimated by ordinary kriging (Figure 2) are classified using two auxiliary estimation runs, corresponding to two kriging neighborhoods with different search constraints. The blocks estimated by using the most restrictive neighborhood are classified as measured resources, the blocks estimated with the second neighborhood but not the first one are classified as indicated resources, whereas the blocks that are not estimated in these two runs are classified as inferred.
resources (Sinclair and Blackwell, 2002, p. 329). Beware that the grades estimates are obtained from the original kriging run. Indeed, the amount of measured resources is likely to be inaccurate and conditionally biased if it is calculated after the most restrictive neighborhood, since it usually has a small search radius and may contain few samples.

In the present case, three tests are performed. The parameters that define the neighborhoods are given in Table 1. The search radius in the horizontal plane is smaller or equal to the radius in the vertical direction, so as to account for the anisotropy of the grade distribution (the continuity is greater along the vertical). The three tests only differ on the maximum distance without data.

<table>
<thead>
<tr>
<th>First test</th>
<th>Second test</th>
<th>Third test</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius along east direction</td>
<td>50m</td>
<td>90m</td>
</tr>
<tr>
<td>radius along north direction</td>
<td>50m</td>
<td>90m</td>
</tr>
<tr>
<td>radius along vertical direction</td>
<td>90m</td>
<td>90m</td>
</tr>
<tr>
<td>minimum number of data</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>maximum number of adjacent empty octants</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>maximum distance without data</td>
<td>15m</td>
<td>20m</td>
</tr>
</tbody>
</table>

Table 1. Parameters of the kriging neighborhoods

2) Kriging variance (applied to the kriged resources)

Here, each block is classified according to its kriging variance, which accounts not only for the quantity and configuration of the neighboring data, but also for the spatial continuity of the grades measured by their variogram. In this respect, the kriging variance combines both geometric and geological knowledge (through geostatistical parameters). In practice, the criterion requires defining a threshold variance between the categories (Royle, 1977; Sabourin, 1984; Froidevaux et al., 1986). To do so, one can compute the kriging variance of a block located at the center of a given sampling mesh, which is taken as the upper bound of the category (the calculation of the variance can be made once the grade variogram is modeled, since the kriging variance does not depend on the data values). Again three tests are performed with the parameters given in Table 2. Different mesh sizes define the threshold variances for classification.
Table 2. Definition of the sampling meshes and threshold variances

<table>
<thead>
<tr>
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<th>First test</th>
<th>Second test</th>
<th>Third test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>indicated</td>
<td>measured</td>
</tr>
<tr>
<td>sampling mesh (east direction)</td>
<td>30m</td>
<td>60m</td>
<td>60m</td>
</tr>
<tr>
<td>sampling mesh (north direction)</td>
<td>30m</td>
<td>60m</td>
<td>60m</td>
</tr>
<tr>
<td>sampling mesh (vertical direction)</td>
<td>12m</td>
<td>12m</td>
<td>12m</td>
</tr>
<tr>
<td>maximum kriging variance</td>
<td>0.081</td>
<td>0.233</td>
<td>0.233</td>
</tr>
</tbody>
</table>

3) Conditional variance (applied to the simulated resources)

This criterion is similar to the previous one, except that the kriging variance is replaced by the conditional variance calculated from the realizations conditioned to the available data. Since the values of the kriging and conditional variances lie in a similar range (between 0.0 and 0.5 (%Cu)^2), the same cutoffs are applied to define the three categories. However, results will differ since, as previously mentioned, the conditional variance strongly depends on the local mean while the kriging variance only depends on the spatial configurations and is independent of the data values, thus independent also of the local mean.

4) Relative kriging variance (applied to the kriged resources)

This criterion uses the kriging variance of each block divided by the square of its estimated grade, providing a standardized and dimensionless measure of uncertainty that can be used to define whether a block is classified as a measured, indicated or inferred resource (David, 1988, p. 203). Blackwell (1998) suggested threshold relative variance values for porphyry copper deposits; however, here we choose more restrictive values which are closer to the ones commonly used in Chilean copper deposits (Table 3).

Table 3. Definition of thresholds for the relative kriging variances

<table>
<thead>
<tr>
<th></th>
<th>First test</th>
<th>Second test</th>
<th>Third test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>Indicated</td>
<td>measured</td>
</tr>
<tr>
<td>relative standard deviation</td>
<td>15%</td>
<td>25%</td>
<td>20%</td>
</tr>
<tr>
<td>relative kriging variance</td>
<td>0.0225</td>
<td>0.0625</td>
<td>0.04</td>
</tr>
</tbody>
</table>
5) Relative conditional variance (applied to the simulated resources)

The classification is performed according to the values of the relative conditional variance (variance of the realizations divided by the square of their average); the same cutoffs as in the previous criterion are applied to define the categories. This standardization can be understood as removing the effect of the local mean grade in the measure of uncertainty, making it similar to the kriging variance.

Results and comments

For each criterion and each test, the tonnages and metal quantities of the three categories are calculated. The results, reported in Figures 3 and 4, call for several comments. First of all, great differences are observed when changing the criterion that defines the categories, or when the values of the parameters that rule a given criterion are modified. It is important to mention that all these methods can be considered valid methods in the current framework defined by international codes. Therefore, we can conclude that results from mineral resource evaluation are still very dependent on the way the uncertainty is modeled and how it is taken into account (absolute versus relative variances). Although the sensibility study is not presented, the authors have also noted that the classification strongly depends on the size of the selective mining units (blocks): indeed, when this size increases, the kriging variance and conditional variance decrease due to of the support effect, so the quantity of measured resources is likely to increase, as pointed out by David (1988, p. 194). The notion of “uncertainty” on which the classification relies depends not only on the amount of neighboring information but also on the volume of the estimated units. All these observations prove that the classification is always subjective and should be clearly documented so as to be reproducible by an external auditor.

Furthermore, by examining the mean grade per category (Table 4), one notes that the relative kriging variance criterion favors the high grades and that the conditional variance criterion favors the low grades: for these criteria, the measured resources have a mean grade equal to 2.01%Cu and 0.85%Cu respectively, whereas the overall mean grade is 1.00%Cu. The explanation of this observation stems from a property of the grade spatial distribution known as proportional effect, according to which the high-grade areas have a greater variability than the low-grade areas. For lognormal distributions (which is almost the case for the deposit under study), theory even states that the local dispersion is proportional to the square of the local mean, i.e. the local standard deviation is proportional to the local mean, hence the name of “proportional” effect (Journel and Huijbregts, 1978, p. 187; Chilès and Delfiner, 1999, p. 56). This effect appears in deposits of precious metals for which the grades have a positively skewed distribution (copper, gold, silver, etc.); the higher variability affects the low grades instead of the high grades for negatively skewed grade distributions (e.g., for iron deposits).

The kriging variance does not depend on the data values and therefore does not account for the proportional effect (Journel and Huijbregts, 1978, p. 308), hence it underestimates the “true” grade uncertainty in the high-grade areas. However, this underestimation compensates the request of the miner to allow a greater estimation error for the high grades than for the low grades (the reliability in the mineral resources is rather defined in terms of relative errors instead of absolute errors), hence the criterion is “neutral” (not favorable nor detrimental to high or low grades). In contrast, the relative kriging variance divides the kriging variance by the squared grades and turns to be more favorable to the high grade areas: the richer portions of the deposit tend to be declared as measured, which explains why the proportion of measured metal quantity is greater.
than the proportion of measured tonnage with this criterion. This situation would be acceptable if the sampling were denser in the high grade areas; however, this is not really the case for the present study (the declustered mean grade is 1.00%, whereas the arithmetic data mean is 1.05%) and the differences between the proportions of measured tonnage and measured metal proves that the relative kriging variance favors the high grade in the classification.

Figure 3. Percentage of the total tonnage per category (each test is represented by a vertical bar)

Figure 4. Percentage of the metal quantity per category (each test is represented by a vertical bar)
Contrarily to the kriging variance, the **variance of the realizations** is conditioned to the data values and therefore accounts for the proportional effect. A classification based on this parameter penalizes the high grade areas, which tend to be declared as “indicated” or “inferred” resources: in this case, the proportion of measured metal quantity is much smaller than the proportion of measured tonnage, which is quite unsatisfactory. Finally, the **relative conditional variance** measures a relative error and appears as a neutral criterion, i.e. it does not penalize nor favor the high or low-grade areas. It performs similar to the kriging variance criterion in terms of being neutral.

In conclusion, the criterion chosen to establish the resource classification should be suited to the requirements of the miners, authorities and investors, so as not to favor nor penalize the high or low-grade areas. The measures of uncertainty can incorporate different levels of information:

- geometrical considerations (kriging neighborhood);
- global properties of the grade spatial distribution (kriging variance);
- local properties such as a proportional effect (relative conditional variance).

The results let us foresee that standardization of the criterion for classification and reporting will be difficult, and most likely, national codes will remain as subjective as they are now in this matter: expert judgment will always be required for classifying and reporting mineral resources and reserves.

### Quantifying Uncertainty by Simulations

In this section, we propose an alternative approach that quantifies the overall uncertainty on the mineral resources by means of conditional simulations of the grades and that can also be used to classify each block as measured, indicated or inferred resource. More precisely, a set of 49 realizations are drawn on the 15,644 selective mining units that constitute the block model. Then, for each realization, the tonnage-cutoff and metal-cutoff curves are computed, assuming an average rock density of 2.7 t/m³ (Figure 5).
Ideally, each realization constitutes a plausible scenario for the real deposit, so that the true tonnage and metal curves should lie within the set of simulated curves. We now have an image of the uncertainty on the total amount of mineral resources: a risk analysis can be performed with the simulated curves in order to assess the worse and the best scenarios, with no need for classifying. Furthermore, one can assess the uncertainty on the mineral resources after applying a cutoff without generating biased conclusions, whereas the previous approach is limited to the total resources (or equivalently, a zero cutoff) because of the smoothing effect of the estimators based on kriging or averaging realizations.

The simulation approach is quite promising for quantifying the uncertainty on resources, but it requires taking care of three points:

1) The data at hand should be representative of the whole deposit. The user must look over the sampling strategy, check the sample quality and beware of preferential samplings that may lead to an overestimation of the resources. This is not peculiar to the simulation approach; it is valid for any resource estimation and classification (Annels and Dominy, 2003). Declustering techniques for inference of the histogram and variogram for Gaussian simulation have been proposed by the authors (Emery and Ortiz, 2004).

2) The geostatistical model used to simulate the grades should describe accurately their spatial distribution. In particular, the model must include structural features like anisotropies, trends, proportional effect, destructuring of the extreme grades or, on the contrary, connectivity of the high grade values (Goovaerts, 1997, p. 401; Chilès and Delfiner, 1999, p. 460; Emery, 2002; Lantuéjoul, 2002). If need be, the deposit should be divided in several domains with a homogeneous grade distribution according to geological features such that rock type, mineral type or alteration.

3) The boundaries of the ore zone to simulate should be clearly defined by geologic, economic and mining considerations: indeed, if the simulations are performed outside the mineralized area (i.e. in waste areas), the quantity of resources will increase unrealistically (the principle of stochastic simulation is to draw grade values according to a specified model, so it “creates” non-existing resources if the simulated domain increases too much). To the authors’ opinion, the correct delimitation of the deposit is the most crucial issue for the simulation approach to be successful. Actually, this delimitation is inherent to the problem at hand: the definition of the resources implicitly refers to a specific spatial domain, and a modification of this domain.

Figure 5. Simulated tonnage-cutoff and metal-cutoff curves
alters the values of the mineral resources (tonnages, metal quantities, mean grades) (Sinclair and Blackwell, 2000). Expert judgment is once again necessary.

Conditional simulations can also help to classify the resources. We suggest three possible approaches:

a) classify each block on the basis of a measure of local uncertainty, for instance the relative conditional variance or the selectivity index (Gini coefficient), which is the ratio between the dispersion indicator and the mean and belongs to the interval \([0,1]\) (Matheron, 1984, p. 431; Chilès and Delfiner, 1999, p. 424);

b) classify each block according to a confidence level. For instance, a block corresponds to a measured resource if the relative difference between its simulated grade and its estimated grade (average of all the realizations) is less than 10% nineteen times out of twenty. It is an indicated resource if this relative difference is less than 20% nineteen times out of twenty. Otherwise, it is an inferred resource;

c) classify each block according to a confidence level considering a production volume. The idea is to consider the error on the grade over quarterly and annual production. A block can be classified as a measured resource if the grade over a quarterly production volume lies within the interval defined by the estimated value \(\pm 10\%\) nineteen out of twenty times. Indicated resources can be defined similarly, but considering an annual production volume. Otherwise, the block is classified as an inferred resource. Some of the issues of this approach are how to consider the production volume. An option is to scale down the block variance to account for the change of support (from a single selective mining unit to a much larger volume). Another option is to consider the uncertainty for a particular configuration of blocks that will not necessarily be related to the actual quarterly production volumes: these will often be irregular disconnected shapes (several work faces).

One advantage of such approaches is to provide a classification in which the expected amount of resources depends on the cutoff, a characteristic that cannot be obtained from the traditional approach based on a kriged block model because of the smoothing effect of kriging. A second advantage is that these approaches are easily interpretable and provide an intuitive measure of uncertainty. As a final comment, we should state that subjectivity has not and cannot be removed from the classification procedure.

Conclusions
Classification of mineral resources into measured, indicated and inferred is always subjective and strongly depends on the criterion chosen by the competent person, hence it should be clearly documented and justified. It is also important to be aware that some criteria penalize and others favor the high-grade areas, e.g. classification based on the relative kriging variance or the conditional variance. Other criteria are more neutral, such as the relative conditional variance or the kriging variance. The former is more complete than the latter as it can account for the local features of the grade distribution like a proportional effect; it could also constitute a standard criterion applicable to any deposit since it just requires defining percentage bounds for the relative error.

Conditional simulations enable to provide a set of simulated selectivity curves and constitute an alternative (or a complement) to the traditional classification framework. This approach offers greater flexibility such as the possibility to quantify the uncertainty on the amount of resources at
a given cutoff, a feature that the classification on a kriged block model cannot provide due to the smoothing effect of kriging. Several points should be taken with care to make the simulation approach successful, in particular the choice of the geostatistical model and its parameters, the division of the deposit into homogeneous geological units, and the correct delimitation of the ore zone. All these points may be considered as additional sources of uncertainty and be included in the simulation paradigm.

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References


