Advances in Stochastic Surface Modeling of Deepwater Depositional Systems

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Stochastic surface-based modeling is a novel geostatistical approach that allows for improved integration of geological information in deep-marine clastic turbidite reservoir models. Traditional geostatistical tools are limited to the construction of models by pixels or by stochastically placed geometric objects. Surface-based methods which model by stratigraphic layers fill available accommodation sequentially and allow for the reproduction of stacking patterns, and hierarchies of trends related to sedimentary processes. Yet, deepwater surface-based methods are in their infancy. New developments such as surface auto-picking, deterministic and stochastic surface placement, global and base levels modeling and improved hierarchical trend models result in more practical workflows and greater integration of deepwater geologic information. The result is improved numerical reservoir models of deepwater systems and, therefore, an expectation of improved reservoir performance forecasting and management.

Introduction

As a major component of deep-sea depositional systems, deepwater turbidite systems became important exploration targets since the exploration success in 1970s (Stow, 1992; Stelting et al., 2000). Research in deepwater turbidite has been active since then. The inaccessibility of deepwater environments and complicated spatial structures result in a high grade of uncertainty. Therefore, an accurate geological model is important to minimize the development risk due to the high development costs. Stochastic simulation approaches are widely applied in investigating deepwater turbidite reservoir to quantify the uncertainty by building multiple equiprobable realizations.

Conventional stochastic facies, porosity and permeability models are often cell based. These models are generally limited to reproduce one- and two-point statistics; therefore, complicated spatial structures cannot be well reproduced, which need three-point or even higher order of statistics (Pyrcz, 2005; Deutsch, 2006). However, the inference of multiple-point statistics of surfaces is difficult with limited data (Strebelle, 2002). The desire to reproduce more geologically realistic models naturally leads to the development of object- and surface-based simulation approaches. Object based techniques were introduced by Haldorsen and Lake (1984), Haldorsen and Chang (1986) and Stoyan et al. (1987). Surface based techniques were introduced recently (Xie and Deutsch, 2000; Deutsch et al. 2001; Pyrcz and Deutsch, 2003, 2004, 2005). The development of object- and surface-based simulation techniques has been important targets for many years.

Research Background and Motivation

There are many object- and surface-based turbidite modeling algorithms available in public domain. The development in this research is based on the following programs developed at the Centre for Computational Geostatistics (CCG) at the University of Alberta.

The LobeSim program, an object-based turbidite reservoir modeling program, was developed by Deutsch and Tran in 1999. A “simple” surface template was proposed and followed by later surface-based modeling techniques. This program was based on simulated annealing technique; both surface geometries and facies components can be well honored, but the algorithm is quite slow.

Xie and Deutsch (2000) proposed a surface-based modeling program, SurfSim, which is based on a rule-based scheme to enforce data conditioning. The proposed methodology stochastically builds a surface model and honors available surface picks; rules are based on volume filling, types of stratal termination and the
reproduction within unit trends. The generated surface model mimics the appearance of actual geologic bounding surfaces and it was successfully applied in an outcrop with tabular units.

Pyrcz and Deutsch (2003) proposed a hierarchical surface modeling program, TurbSim, which builds models based on user-specified surface geometry parameters. The large-scale geometry may be a third-order lobe or bounding surfaces extracted from seismic. The fine-scale geometry is a second-order lobe. Surface model may be built hierarchically, that is, both large- and small-scale lobes may be built. A rejection algorithm was proposed for well conditioning, but the convergence may be very slow with many wells. Facies are not simulated directly. As an alternative, a surface-based hierarchical petrophysical trend modeling approach was proposed to simulate reservoir petrophysical properties that followed the modeled reservoir spatial structures.

A limitation of TurbSim is that it assumes only single source location for all lobe events within given bounding surfaces. To extend the application of surface-based techniques in common geological settings without modeling third-order lobes, LE_model program was designed by Deutsch in 2006. Surfaces extracted from seismic image were applied directly as bounding surfaces and multiple source locations might be specified. Besides the flexibility, the limitations of LE_model are also apparent: (1) the entry position of a candidate surface is randomly picked without any constraints; therefore the simulated surface may not follow local bathymetry very well, and (2) the program is unconditional, that is, it cannot reproduce surface intersections at wells.

The research is based on the programs described above. The goal is (1) to develop an algorithm that overcomes many of the limitations that have been identified, and (2) to apply surface-based modeling technique to more environments, such as alluvial fans, which have similar spatial structures but with higher gradient.

For convenience, the development is based on the LE_model program, so the developed surface-based turbidite lobe events modeling program is still called LE_model.

Stochastic Surface Modeling

The developed stochastic surface modeling approach will be presented in this section, including (1) surface template used in this research, (2) initial bathymetry approximation approaches when bounding surfaces are highly deformed during depositional history, (3) surface positioning rules, including areal surface positioning and vertical surface positioning, (4) well conditioning approaches, including single well conditioning and multiple wells conditioning, (5) surface acceptance criterion and (6) stopping criterion.

Surface Template

As a variation of object-based modeling technique, a surface template is needed for describing the geometry of the research object, an idealized surface. During modeling, surfaces are stochastically generated based on the proposed analytical surface template. That is, the lobe length, width and height are randomly drawn based on distributions with user-specified parameters. The proper surface shape is problem-related, so user needs to tweak it when applying LE_model program. In this research, a simplified lobe geometric template is applied.

In plan view, the lobe boundary is defined by:

\[ W(x) = 4 \cdot w_{\max} \cdot \left[ 1 - \frac{x}{\text{length}} \right]^b \cdot \left[ 1 - \left( 1 - \frac{x}{\text{length}} \right)^b \right] \]  

(1)

where \( W(x) \) is the width from the centerline, \( x \) is the distance along the centerline, \( w_{\max} \) and \( \text{length} \) are the maximum lobe width and length drawn from user-specified distribution, \( b = -\ln(2)/\ln(1-a) \) where \( a \) is the relative position of maximum width (Figure 1).

The lobe geometry is quantified by a gridded surface of thickness. The thickness determination methodology adopted from the channel cross-section geometry determination methodology developed by Deutsch and Wang (1996). Thickness distribution along centerline is quantified first; cross-section thickness is determined based on the thickness at centerline location.

Thickness along center line is defined geometrically by,
where \( t(x) \) is the thickness along centerline, \( t_{max} \) is the maximum thickness drawn from user-specified distribution, \( b = -\ln(2)/\ln(1-a) \) where \( a \) is the relative position of maximum width.

Cross-section geometry is defined geometrically by,

\[
t(y) = 4 \cdot t(x) \cdot \left( \frac{y}{2 \cdot W(x)} \right)^b \cdot \left[ 1 - \left( \frac{y}{2 \cdot W(x)} \right)^b \right] 
\]

where \( t(y) \) is the thickness along a cross section, \( y \) is the distance to the lower boundary, \( y \in [0, 2 \cdot W(x)] \), \( b = -\ln(2)/\ln(a) \) where \( a \) is the relative position of maximum thickness.

During simulation, \( length \), \( w_{max} \) and \( t_{max} \) are drawn from user-specified distributions. \( a \) is deterministic, which is 0.66 in Equations (1) and (2), and 0.5 in Equation (3), which means that the position of maximum lobe width and thickness are at 2/3 position of lobe length and the geometry of lobe cross section is symmetric. A lobe built with above approach is shown in Figure 2.

**Initial Bathymetry**

As a gravity-driven mass flow, the shape and orientation of turbidite are strongly controlled by sea floor topography; therefore, the initial base surface bathymetry inference is important in surface-based modeling approaches. However, paleobathymetry is difficult to infer, especially in the presence of complicated tectonic history. Basin modeling or tectonic inversion based on seismic data and reasonable assumptions may be used. When no information is available to make reasonable assumptions, a simple surface simulation/transformation approach may be applied to rebuild the idealized initial base surface bathymetry. Three base surface transform approaches are applied in \textit{LE}_\textit{model} program.

**Flatten base surface**

The simplest solution is to flatten the base surface. The transformed stratigraphic coordinate can be expressed as (Deutsch, 2002):

\[
z_{rel} = \frac{z - z_{cb}}{z_{ct} - z_{cb}} \cdot T 
\]

where \( z_{rel} \) is the relative stratigraphic coordinate, \( z_{cb} \) is the elevation of base surface, \( z_{ct} \) is the elevation of top surface, and \( T \) is the average thickness of the strata. The coordinate \( z_{rel} \) is 0.0 at the stratigraphic base and thickness at the stratigraphic top.

This transform may be reversed by:

\[
z = z_{cb} + \frac{z_{rel}}{T} \cdot (z_{ct} - z_{cb}) 
\]

While, this transform is not recommended when there is additional information on the reservoir thickness and geometry.

**Transform base surface to a paraboloid**

The base surface may be transformed to an idealized shape based on global base surface bathymetry, which represents for the idealized initial bathymetry. The transformed base surface works as a trend surface that removes high frequency and possible unreliable noise in the bathymetry.

The proposed method is to approximate the base surface by an analytical surface of second order:

\[
z = ax^2 + bxy + cy^2 + dx + ey + f
\]
where \(a, b, c, d, e\) and \(f\) are the parameters of the paraboloid, \(x, y\) and \(z\) are the coordinates of the original base surface.

The coefficients of the paraboloid are obtained by least-squares solution of an over-determined system of linear equations (Krsek et al. 1998).

The stratigraphic coordinates are transformed based on the difference between the base surface and the fitted paraboloid. That is,

\[
z_{\text{rel}} = z - (z_{\text{ch}} - z_{\text{parab}})
\]

where \(z_{\text{parab}}\) is the elevation of the fitted paraboloid.

This transform may be reversed by:

\[
z = z_{\text{rel}} + z_{\text{ch}} - z_{\text{parab}}
\]

Transform base surface to an incline

When the paleogeometry is approximately linear, an incline may be more suitable for representing the idealized base surface. The analytic surface of first order may be expressed as:

\[
z = ax + by + c
\]

The incline transform and back transform procedures are similar to the paraboloid transform.

The paraboloid and incline base surface transform are recommended because, (1) the global geometry is kept after transformation, (2) resulting surface models are realistic, (3) honoring stacking patterns becomes possible because the global gradient information is kept that may be applied to construct the probability distribution of surface happening, and (4) the variation of base surface will be brought into final surface model, therefore, the influence of tectonic movements may be reproduced (Figure 3). Modeler may plot the base surface and visualize it in 3D view to find out the proper transformation approach (Figure 4).

Empirical studies with the research code have shown that paraboloid and incline transforms are stable in common geological settings, even in high gradient environments. Therefore, the LE_model program may be applied to model other fan bodies with similar spatial structures, such as alluvial fans.

Surface Positioning

Surface placement includes the determination of attitude and location. For convenience, we will discuss the two aspects in two sections. Surface positioning is rule-based in this research. A reasonable surface positioning rule should (1) respect geologic information with respect to the interrelationship of the architectural elements described by the surfaces, (2) avoid boundary artifact and vertical stacking artifact, (3) result in a reasonable probability of the candidate surface being accepted and (4) be computationally efficient (Pyrcz, 2004).

**Vertical Surface Positioning Rule**

According to sequence stratigraphic principles, a candidate surface should ideally be positioned on a base level. Base level is generally regarded as a global reference surface to which long-term continental denudation and marine aggradation tend to proceed (Catuneanu, 2006). Base level is not real physical surface; it is dynamic, moving upward and downward through time.

In this research, the local base level concept is applied for fine-scale surface positioning, but the scale is much smaller than the local base levels used in conventional sequence stratigraphy research. These base levels are formed by the bathymetric healing process of turbidity current, so they are not related to any large-scale controlling parameters. Figure 5 illustrates the simplified sketch of a large turbidity current, which may be divided into head, body and tail regions. Finer component will move backward from the head to the body and then to the tail. Along flow path, turbidity current releases loads from both head and tail, and absorbs new load into head by eroding former deposition as compensation. If the internal balance between the head and the body is maintained, the turbidity current may keep moving. The balance is dynamic and results in the evolution of local base level (Figure 6). In practice, a flooding event may only
smooth topographic relief to some extent. A local base level is the final trend of this process. In TurbSim program, moving window smooth is applied to smooth the local topography, which represents for the seafloor bathymetry after bathymetric healing, and it is used as the reference surface for vertical surface positioning. But in this research, the final status of the smoothing, local base levels, are applied as the reference surfaces for vertical surface positioning. This may eliminate the artifact caused by high gradient.

For convenience, paraboloid and incline are applied to represent local base levels. The selection is based on (1) paraboloid or incline mimic the idealized local surface bathymetry well, (2) a candidate surface will not change its inclination locally after positioning on a paraboloid or incline, therefore, no surface stacking artifacts arise, and (3) there are many algorithms available in public domain for paraboloid or incline fitting, such as Yu (2001), and the fitting is computationally efficient.

The local base levels are dynamically fitted, which are functions of former simulated surfaces. The surface shape which meets proposed surface acceptance criterion better is selected and no user interactions are needed. The surface acceptance criterion will be discussed later.

The workflow for vertical surface positioning is (1) a candidate surface is drawn, (2) the local base level for this candidate surface is fitted based on local topography, (3) the candidate surface is added on the local base level based on its thickness, and (4) the volume below former surfaces is truncated under the assumption that no erosion happens.

**Areal Surface Positioning Rule**

The areal surface positioning rule amounts to the selection of an entry location. The entry position may or may not be a function of previous surfaces. In LE model, entry positions are assumed independent of previous surfaces. A reasonable surface model should honor compensational stacking pattern, which is the tendency of flow event deposit to fill topographic lows and to smooth topographic relief. Some additional constraints are added for a reasonable surface model.

The areal position is selected by following procedures: (1) an areal surface occurrence probability field is constructed first. The probability is a function of areal relative positions, simulated thickness and remaining thickness. A random number within (0.0, 1.0) is drawn; and (3) the ratio of cumulative probability over total probability is calculated with specific searching path and the position where the ratio is equal to the random number is selected as the entry location.

**Well Conditioning**

A dual-spline error surface interpolation algorithm is proposed for fast well conditioning, which is based on the rejection algorithm developed by Pyrcz (2004). The surface acceptance criteria and optimal surface selection methodology are modified as follows.

**Surface acceptance criteria**

a) If the candidate surface does not pass through any wells, it is accepted only when it meets other constraints. For example, the surface \( A \) in Figure 7.

b) If the candidate surface passes through only one well, it is acceptable only when available surface pick is located within the surface. For example, the surface \( C \) in Figure 7; surface \( B \) is rejected.

c) If the candidate surface passes through more than one well, it is acceptable when all available well picks are located within the surface. For example, the surface \( D \) in Figure 7.

**Surface conditioning methodology**

a) If the candidate surface passes through only one well, (1) residual of available well pick to candidate surface is calculated, and (2) the surface is shifted downward based on the residual (Figure 8).

b) If the candidate surface passes through multiple wells, (1) the residual of each surface pick to the candidate surface is calculated, (2) the minimum residual \( r_{\text{min}} \) and maximum residual \( r_{\text{max}} \) are calculated, (3) the candidate surface is shifted downward by distance \( (r_{\text{max}} + r_{\text{min}}) / 2 \), which is the optimal position for the sum of new residuals should be close to zero, and (4) a dual-spline error surface is interpolated based on new residuals and it is added back to new candidate surface for exactitude (Figure 9).
The optimal surface position is selected by shifting a candidate surface instead of trial-and-error; therefore, it is more efficient than the rejection algorithm.

**Surface Acceptance Criterion**

To build geologically realistic surface models, (1) all available surface picks should be honored, (2) compensational stacking pattern should be honored, and (3) input surface geometry statistics should be honored. A minimum volume ratio (MVR) criterion is proposed for a realistic surface model. MVR is the ratio of lobe volume, $V_{lobe}$, over the gross strata volume, $V_{gross}$. Because $V_{gross}$ is problem-related, with the same $V_{lobe}$, MVR may be different for different projects. To establish a reasonable MVR, the modeler needs to set up all surface modeling parameters and run LE_model one time. During surface modeling procedure, (1) $V_{gross}$ is calculated, (2) an ideal lobe with geometry of all means parameters is built, (3) ideal lobe volume, $V_{ideal}$, is calculated, and (4) the ratio of $V_{ideal}/V_{gross}$ is a good reference to set up the MVR parameter. Ideally, $(0.5-0.8) \times V_{ideal}/V_{gross}$ is a good starting point. Under MVR, a larger surface is likely to be drawn, which accelerates the well conditioning process significantly.

**Stopping Criteria**

Surface modeling will stop when one of the following criteria is met:

1) User-specified volume filling proportion is reached.
2) No available surface pick exists.

**Case Study**

The bounding surfaces were synthetically constructed. The slope gradient is about 2 degrees, which is common in distal continental slope environment. The stratum is roughly 60m (198ft) thick and pinches out towards west and north. The initial bathymetry is a northwest-southeast trending submarine depression, and it is assumed that 36 vertical wells are available (Figure 10). The 36 wells are regularly distributed (1000 m). Figure 11 shows a long section and a cross section with different base surface transforms. Local topology has great influence on surface positioning without base surface transform. The surface model follows structure variation very well when transform base surface to a paraboloid or incline. Two conditional surface models were built, which are quite reasonable (Figure 12). Only about 1 minute was needed for one realization.

**Conclusions**

Deepwater surface-based methods are under development. The paper documents new developments, such as deterministic and stochastic surface placement, global and local base levels modeling and improved well conditioning approach, which result in more practical workflows and greater integration of deepwater geologic information. The result is improved numerical reservoir models that may be applied as constraints in facies and petrophysical modeling.

**References**


Pyrcz, M. J., and C. V. Deutsch, 2003, Stochastic surface modeling in mud rich, fine-grained turbidite lobes (abs.), AAPG Annual Meeting, Salt Lake City, Utah.


Xie, Y., and C. Deutsch, 2000, Surface geometry and trend modeling for integration of stratigraphic data in reservoir models, Sixth International Geostatistics Congress, Cape Town.


Figure 1: Parameters needed to describe the 3-D lobe geometry.
Figure 2: a lobe built with proposed analytic surface template showing the idealized lobe geometry.

Figure 3: a diagram illustrating that the paraboloid and incline base surface transform may reproduce the influence of tectonic movements.
Figure 4: An example base surface and transformed base surfaces. (a) original base surface, (b) transformed paraboloid base surface, and (c) transformed incline base surface.
Figure 5: Simplified sketch of a large turbidity current, divided into head, body and tail regions. Setting from the wake behind the head produces a lateral size grading in the flow (after Pickering et al. 1989).

Figure 6: A schematic diagram showing the local base profile forming procedure. The vertical scale has been exaggerated to illustrate the local erosion and new deposition process.

Figure 7: A synthetic diagram illustrating the acceptance criteria for well conditioning.

Figure 8: Schematic diagram illustrating the well conditioning methodology with one well case. The residual of the available well pick with the candidate surface is calculated first (left plot); then the candidate surface is “lower” down to pass through the available well pick based on the residual (right plot).
Figure 9: Schematic diagram illustrating the well conditioning methodology with multiple wells case. The minimum and maximum residuals are calculated first, i.e., \( r_{\text{min}} \) and \( r_{\text{max}} \) on the left plot; then the candidate surface is “lower” down in-between the minimum and maximum residual; after that, the residual of each available well pick to the new surface, \( r_1 \), \( r_2 \), and \( r_3 \), are calculated (the right plot).

Figure 10: Well location map with thickness distribution map as the background. Regular well pattern is assumed and 36 synthetic wells are designed.

Figure 11: A long section and a cross section showing the unconditional simulation results with different base surface transform: (a) without base surface transform; (b) flatten base surface; (c) transform base surface to a paraboloid; and (d) transform base surface to an incline. 45, 82, 45 and 68 surfaces were generated, respectively.
Figure 12: The thickness map of the strata and the location map of 36 synthetic wells. Two conditional realizations were built to illustrate possible spatial structures.