Transfer of Grade Uncertainty into Mine Planning
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Uncertainty is always present because of sparse geological data. Conditional simulation algorithms such as Sequential Gaussian Simulation (SGS) and Sequential Indicator Simulation (SIS) are used to assess uncertainty in the spatial distribution of grades. Long-term mine planning and the management of future cash flows are vital for surface mining operations. Traditionally the long-term mine plans are generated based on an estimated input geological block model. Estimated or kriged models do not capture uncertainty and must be tuned to avoid biases. Mine plans that are generated based on one input block model fail to account for the uncertainty and its impact on the future cash flows and production targets. A method is presented to transfer grade uncertainty into mine planning. First, Sequential Gaussian Simulation is used to generate fifty realizations of an oil sands deposit. An optimum final pit limits design is carried out for each SGS realization while fixing all other technical and economic input parameters. Afterwards, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and presented. The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the krig estimate is not the expected result from all of the simulated realizations.

1. Introduction

Open pit production scheduling is the process of defining a feasible block extraction sequence that maximizes the net present value (NPV) of mining operation while meeting technical and economic constraints. There are three time ranges for production scheduling: long-term, medium-term and short-term. Long-term can be in the range of 20 – 30 years. This period is divided into several medium-term periods between 1 to 5 years. Medium-term schedules provide detailed information that allows for an accurate design of ore extraction from a special area of the mine, or information that would allow for necessary equipment substitution or the purchase of essential equipment. The medium-term schedule is also divided into 1 to 6 month periods (Osanloo et al., 2008).

In this paper, the main focus will be on long-term production planning (LTPP) in open pit mines. LTPP determines the distribution of cash flow over the mine life, the feasibility of the project, and also it is a prerequisite for medium and short-term scheduling.

Uncertainty is inevitable with sparse geological data. Geostatistical simulation algorithms are widely used to quantify and assess this uncertainty. The generated realizations are equally probable and represent plausible geological outcomes (Deutsch and Journel, 1998; Goovaerts, 1997; Journel and Huijbregts, 1981). Choosing one or some of these realizations does not realistically account for what might happen in the future. Also, the uncertainty of ore grade in block models may cause discrepancies between planning expectations and actual production. Traditional production scheduling methods that use an estimated block model as the input into the scheduling process cannot capture the risk associated with production schedules caused by the grade variability. The majority of the current production scheduling methods used in industry have two major shortcomings: (i) the production scheduling methods are either heuristic based, or derived from the mine planner’s experience; most of the current scheduling tools do not use global optimization methods, therefore the generated schedules are not necessarily optimal; and (ii) the inability to account for the grade uncertainty inherent within the production scheduling problem and as a result, there is no measure of the associated risk with the generated mine plans.

Effective open pit design and production scheduling is a critical stage of mine planning. The effects of pit design and scheduling and related predictions have major consequences on cash flows. Open pit push-back design is commonly based on the well known Nested Lerchs-Grossman algorithm. This algorithm provides a scenario of how an orebody should be mined given a set of geological, mining and economic considerations. Since 1965, several types of mathematical formulations have been considered for the LTPP problem: Linear programming (LP), mixed integer programming (MIP), pure integer programming (IP), dynamic programming (DP) and Meta-heuristic techniques.

All of these the deterministic algorithms try to solve the LTPP problem without considering grade uncertainty.
Vallee (Vallee, 2000) reported that 60% of the mines surveyed had 70% less production than designed capacity in the early years. Rossi and Parker (Rossi and Parker, 1994) reported shortfalls against predictions of mine production in later stages of production. Traditional production scheduling methods that do not consider the risk of not meeting production targets caused by grade variability, cannot produce optimal results. Therefore, the common drawback of deterministic algorithms is that they do not consider any type of uncertainty during the optimization process.

Dimitrakopoulos et. al. (Dimitrakopoulos et al., 2001) show that there are substantial conceptual and economic differences between risk based frameworks and traditional approaches. Some authors tried to use stochastic orebody models sequentially in traditional optimization methods. Dowd (Dowd, 1994) proposed a framework for risk integration in surface mine planning. Ravenscroft (Ravenscroft, 1992) discussed risk analysis in mine production scheduling. Ravenscroft used simulated orebodies to show the impact of grade uncertainty on production scheduling. He concluded that conventional mathematical programming models cannot accommodate quantified risk. Dowd (Dowd, 1994) and Ravenscroft (Ravenscroft, 1992) used stochastic orebody models sequentially in traditional optimization methods.

The objective of this study is to: (i) present two methodologies to quantify the grade uncertainty transferred into production schedules; (ii) assess the impact of grade uncertainty on output parameters of mine production scheduling such as: NPV, ore tonnage, head grade, stripping ratio, amount of final product, and annual targeted production.

Kriging (Deutsch and Journel, 1998; Goovaerts, 1997) is used to estimate grades and construct the block model. Next, a final pit limit optimization study is performed using Lerchs and Grossmann (LG) (Lerchs and Grossmann, 1965) algorithm. Afterwards, a long-term life-of-mine production schedule is generated using the Whittle software (Gemcom Software International, 1998-2008); this is referred to as the krig schedule throughout the paper. Two methods to account for grade uncertainty are considered.

**Method 1-** Sequential Gaussian Simulation (SGS) (Deutsch and Journel, 1998; Goovaerts, 1997; Journel and Huijbregts, 1981) is used to generate equally probable realizations of the orebody. An optimum final pit limit design is carried out for each SGS realization with the same technical and economic input parameters as used for the kriged model. Next, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and compared against the krig schedule. This process is labelled as method number one in Figure 1.

**Method 2-** The optimal final pit limit and the krig production schedule are the basis of this approach. The same ultimate pit limit and the same krig schedule are applied to all the SGS realizations. This provides an assessment of the uncertainty in the production schedule. This process is labelled as method number two in Figure 1.

The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the krig estimate is not the expected result from all of the simulated realizations. The probability of each block being extracted in each planning period and the probability that the block would be treated as ore or waste in the respective period are calculated and can be used to assist in low range mine planning.

**2. Methodology**

The comparison of Method 1 and 2 and the respective results will be illustrated through a case study corresponding to an oil sands deposit in Fort McMurray, Alberta, Canada. In order to quantify the grade uncertainty transferred into the mine plans using Methods 1 and 2 the following steps are followed:

**3. Geostatistical Modelling**

A sufficient number of realizations must be considered for the purpose of mine planning; otherwise, there may be undue reliance on some stochastic features. The steps presented by Leuangthong et al. (Leuangthong et al., 2004) are followed for geostatistical modelling of an oil sand deposit using GSLIB (Deutsch and Journel, 1998) software to create conditional simulated realizations. The steps presented by Leuangthong et al. (2004) are:
(1) **Analyze of correlation structure**- This investigates whether a transformation of the vertical coordinate system is required, in order to determine the true continuity structure of the deposit. Determination of the correct grid is dependent on the correlation grid that yields the maximum horizontal continuity.

(2) **Decluster drillhole data distribution**- The relevant statistics must be deemed representative of the deposit prior to modelling. Declustering may be employed to determine the summary statistics that are representative of the field.

(3) **Variography**- Model the spatial continuity of the normal scores of the bitumen grade using variograms. Directional experimental variograms are calculated and fit. The Azimuths of major and minor directions are 50 and 140 degrees. Figure 3 shows the experimental and the fitted variogram models in major (Figure 3a), minor (Figure 3b) and vertical (Figure 3c) directions.

(4) **Estimation**- Perform estimation and cross validation using kriging as checks against simulation results. Ordinary kriging is used to estimate the bitumen grade (with no normal score transform) at each block location.

(5) Figure 4a and 4b illustrate the map of the bitumen grade for the kriged and the E-type models. As expected, the E-type model is smoother than the kriged model. Multiple realizations of the bitumen grade are generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid at the point scale, this method is the means of constructing uncertainty models of bitumen grades.

(6) Check simulation results against the input data and compare results against the kriged model. Histogram and variogram reproduction are checked. Figure 5 shows the histogram reproduction and Figure 6a to 6c show the variogram reproduction at major and minor horizontal, and vertical directions. Generally, Sequential Gaussian simulation would reproduce histogram and variogram of original data if it implements carefully. In this case reproduction of histogram and variograms are acceptable.

(7) The block dimensions for mine planning are selected according to the exploration drilling pattern, ore body geology, mine equipment and anticipated operating conditions. The sizes of the blocks used in mine planning are a function of the selective mining unit (SMU). The high resolution grid is up-scaled to get the right block scale values. Arithmetic averaging of point scale grades provides the up-scaled SMU grades.

### 4. Optimal pit outline design

The final pit limit design is carried out based on the industry standard Lerchs and Grossmann algorithm (Lerchs and Grossmann, 1965) using the Whittle strategic mine planning software (Gemcom Software International, 1998-2008). The kriged, E-type, and fifty SGS realizations models are imported into the Whittle software.

The ultimate pit limit design is carried out based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). Price of oil was considered US$45 with an exchange rate of 1.25:1 equal to CAN $56.25/bbl SSB for the same time period. We assume that every two tonnes of oil sands with an average grade of 10% mass will produce one barrel of sweet blend, which is approximately 200 kg. We also assume a density of 2.16 tonne/m³ for oil sands, and a density of 2.1 tonne/m³ for waste material, including clay and sand.

Table 1 summarizes the costs used in the pit limit design. The mining cost of $12.18 is per tonne of oil sands ore, we assumed a stripping ratio of 1.8:1, and this would lead to a cost of $4.6/tonne of extracted material (ore and waste).

Table 2 shows the pit design and production scheduling input parameters. Thirty three pitshells are generated using 49 fixed revenue factors ranging between 0.1 to 2.5, based on the kriged block model. The number of pitshells is reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 3 summarizes the information related to the final pit limit based on kriged block model at 6% bitumen cut-off grade. Figure 7 illustrates the histogram of total tonnage of material within the optimal final pit limits. Ultimate pits are generated for each SGS realization. E-type and krig results are marked by a solid circle and a hollow circle respectively.

### 5. Production Scheduling

The kriged model is the basis for production scheduling. The aim is to maintain a uniform processing feed throughout the mine life. Four push backs are defined with a fixed lead of three benches between pushbacks. Five years of pre-stripping is considered to provide enough operating space and ore availability. No stockpile is defined
and the target production is set to 20 million tonnes of ore per year with a mining capacity of 35 million tonnes per year. Figure 8 illustrates the kriged block model schedule over 21 years of mine-life. This schedule is the basis of Method 2. The schedule is applied to all the 50 SGS realizations within the krig fixed optimal pit limit. In Method 1, the final pit limits is designed for E-type model and all the fifty realizations with the exact same input variables.

6. Discussion of results

Conditional simulation enables us to provide a set of production scenarios, which capture and assess the uncertainty in the final pit outline, net present value, production targets, and the head grades. The realizations provide equally probable scenarios to calculate different outcomes in terms of NPV and production. The probability of each block being extracted in each planning period and the probability that the block would be treated as ore or waste in the respective period is calculated. The histograms and box plots for the following production schedule output variables are presented and discussed for Methods 1 and 2:

(1) Overall stripping ratio.
(2) Total tonnage of ore.
(3) Average grade of ore.
(4) Net present value.

Figures 10 through 14 illustrates the uncertainty within the final pit limits. The krig final pit (the hollow circle) has 653 million tonnes which is less than the average amount of 663 million tonnes, recorded for the simulation realizations. The inter quartile limits are 647 and 678 million tonnes. The stripping ratio of the krig pit is 1.33, which is near the 1.35 stripping ratio of the lower quartile of the simulation realizations. As expected, Method 1 has a larger standard deviation compared against Method 2.

The histogram and box plot of ore at final pit is shown. The amount of ore in the kriged and Etype models is close when different schedules are generated for each of them. The tonnage of ore in the kriged model is almost the same as Etype where the same schedule is followed.

The histogram of average input grade to the mill for kiging (solid circle), Etype (hollow circle) and SGS realizations are presented. The average grade of the kriged model is more than the Etype model and also it is higher than the third quartile of the realizations of both methods. The grade uncertainty is clearly illustrated in this figure. The average grade of the Etype is less than kriged model and even less than lower quartile of average grade of realizations.

The input average head grade for each period is shown. Kriged model (bold solid line), Etype model (bold dashed line) and the simulation realizations (dashed lines). As expected, the grade fluctuations of the first method are greater than second method.

Histogram and box plot of produced ore is shown. 27.52 million tonnes of bitumen is produced by kriged block model. 26.86 and 26.97 million tonnes of bitumen was produced by Etype model for method 1 and 2 respectively. The kriged block model produced more bitumen than third quartile of realizations at both methods.

The tonnage of feed to the plant is shown. The deviation from target production is also presented. There are under productions at first years for each of two methods. Also, at first method, we have hard constraint for upper limit of plant feed, there is no over production for each of realizations. When the same schedule is followed for each of realizations, at some realizations, there is overproduction between years 8 to 16. One should take into account that the shortfall in production has happened although we have used five years of pre-stripping. The effect of the grade uncertainty on the production targets would be more severe if the pre-stripping strategy was not adopted.

A box plot illustrates the plant feed, the percentage deviation from the target production, and the probability that we would not meet the target production. There is a relatively high probability to not meeting the target production at first and last years of mine life. If the krig schedule would be followed, the probability of not meeting the target production for some middle age of mine life will be increase. There is a 1.2 and 2.1 percent probability of not meeting the target production for years 13 and 14 respectively. X

A histogram of cumulated discounted cash flow is shown. E-type and krig results are marked by solid circle and hollow circle respectively. NPV of kriged block model is 847 million dollars, where the NPV of Etype method is
724 and 780 million dollars for method 1 and 2 respectively. The NPV of kriged model is also more that third quartile of realizations.

The discounted cash flow over the years is illustrated. Cumulative discounted cash flow for 50 realizations and the kriged and Etype models are showed by solid line and dashed line respectively. The optimum final pit limit is calculated for each realization; therefore, there are some realizations that the mine life is 22 years, where for most realizations, kriged and the Etype model the mine life is 21 years. In method 2, the same schedule as kriged model is followed. Therefore the mine life is 21 years for all realizations and Etype model.

A box plot of discounted cash flow for realizations is shown. The kriged model is showed by solid line and dashed line is used for Etype model. The probability of meeting the NPV form kriged model is also presented. As it is clearly illustrated, based on 50 realizations, there is very low chance to reach the NPV of kriged model for each period. No realization exceeds the NPV of the kriged model. One realization (2%) exceeds the final NPV of kriged model for the second method. This graph shows that the schedule based on the kriged model may produce a realistic and unachievable NPV in present of grade uncertainty. It is important to tune the kriged model to get the recoverable reserves that will be achieved in practice (Isaaks, 2005).

Realization number 4 is the worse scenario because the discounted NPV is the minimum among all other realizations. The schedule of Etype mean and realization number 4 is shown. Gray columns are tonnage of waste rock that is mined during mine life. The mining capacity is 35 million tonnes per year. The yellow columns show the tonnage input to the processing. The processing capacity is 20 million tonnes which is shown as horizontal line. Shortfalls in the first years cause lower NPV.

8. Conclusions
We used two methods to show the impact of grade uncertainty at mine planning. First, using the kriged model for an oil sand deposit, an optimal final pit limit is generated. Sequential Gaussian Simulation is used to generate fifty realizations. An optimum final pit limits design is carried out for each SGS realization based on same parameters and technique that are fixed with kriged block model. Afterwards, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and presented.

At the second method, for each SGS realization, schedule that generated from kriged model was followed. The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the krig estimate is not the expected result from all of the simulated realizations.

One of the main aspects of this study is to show the impact of grade uncertainty on mine planning. The study is not aimed to compare the simulation with kriging, because it is well-known that kriging is conditionally biased (Isaaks, 2005) and on the other hand “there is no conditional bias of simulation when the simulation results are used correctly” (McLennan and Deutsch, 2004). Conditional biasness of kriging can be reduced by tuning estimation parameter but it cannot be eliminated (Isaaks, 2005). The Grade-Tonnage curve is a good tool to check the impact of kriging biases. Figure 20 shows the grade tonnage curve of simulation realization (dashed lines), krig (bold solid line) and Etype (bold dashed line). The systematic biasness of kriging was tried to be minimized but still the there are differences between kriging and simulation results. Also Etype is slightly different than kriging; Theoretically Etype model is identical with simple kriging result at Gaussian space (Journel and Huijbregts, 1981).
References
### Tables

**Table 1:** Summary of costs used in pit limit design.

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<th>Description</th>
<th>Value</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>Mining Costs (CAN $/ bbl SSB)</td>
<td>24.35</td>
<td>Mining Costs (CAN $/tonne)</td>
<td>12.18</td>
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<td>Upgrading Costs (CAN $/ bbl SSB)</td>
<td>10.05</td>
<td>Upgrading Costs (CAN $/tonne)</td>
<td>5.025</td>
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<tr>
<td>Others (CAN $/bbl SSB)</td>
<td>1.5</td>
<td>Others (CAN $/tonne)</td>
<td>0.75</td>
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<tr>
<td>Total Costs (CAN $/ bbl SSB)</td>
<td>35.9</td>
<td>Total Costs (CAN $/tonne)</td>
<td>17.28</td>
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**Table 2:** Final pit limit and mine planning input parameters.

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<th>Description</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>Cutoff grade (%mass bitumen)</td>
<td>6</td>
<td>Processing limit (M tonne/year)</td>
<td>20</td>
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<tr>
<td>Mining recovery fraction</td>
<td>0.88</td>
<td>Mining limit (M tonne/year)</td>
<td>35</td>
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<tr>
<td>Processing recovery factor</td>
<td>0.95</td>
<td>Overall slope (degrees)</td>
<td>20</td>
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<td>Minimum mining width (m)</td>
<td>150</td>
<td>Pre-stripping (years)</td>
<td>5</td>
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**Table 3:** Material in the final pit using the kriged block model.

<table>
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<tr>
<td>Total tonnage of material (M tonne)</td>
<td>653.61</td>
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<tr>
<td>Tonnage of ore (M tonne)</td>
<td>280.5</td>
</tr>
<tr>
<td>Tonnage of material below cutoff (M tonne)</td>
<td>37.4</td>
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<tr>
<td>Tonnage of waste (M tonne)</td>
<td>335.71</td>
</tr>
<tr>
<td>Bitumen recovered (M tonne)</td>
<td>27.52</td>
</tr>
<tr>
<td>Stripping ratio (waste:ore)</td>
<td>1.33</td>
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Figures

**Figure 1:** Flow diagram of the study.

**Figure 2:** Geostatistical modelling stages.
Figure 3: Experimental directional variograms (dots) and the fitted variogram models (solid lines), distance units in meters.
Figure 4: Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.
Figure 5: Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line)

Figure 6: Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).
Figure 7: Histogram and boxplot of total tonnage of material within the final pit limits at million tonnes. E-type and krig results are marked by a solid circle and a hollow circle respectively.

Figure 8: Production schedule based on the kriged block model.
Figure 9: Histograms and box plots of overall stripping ratio. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations. E-type and krig results are marked by solid circle and hollow circle respectively.
Figure 10: Histograms and box plots of total tonnage of ore. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations. E-type and krig results are marked by solid circle and hollow circle respectively.
Figure 11: Histograms and box plots of average head grade in bitumen %mass. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations. E-type and krig results are marked by solid circle and hollow circle respectively.
Figure 12: Head grade of simulation realizations, kriged, and E-type models. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations. The krig result is marked by solid line.
Figure 13: Histograms and box plots of tonnage of bitumen produced. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations. E-type and krig results are marked by solid circle and hollow circle respectively.
Figure 14: Plant feed (realizations dash lines), kriged model (solid line) and E-type (blue dash line). (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations.
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Figure 18: Box plot of the Cumulative discounted cash flow for 50 realizations (dashed lines) and kriged model (solid line). Probabilities of reaching the NPV of kriged block model over realizations are presented in percentage. (a) schedules are generated for each block model separately. (b) krig schedule applied to all realizations.
Figure 19: Production schedule based on a. the Etype block model and b. realization number 4 as the worse case.
Figure 20: Grade tonnage curve of simulation realizations, kriged, and Etype block models