Comparing three popular optimization algorithms by considering advantages and disadvantages of each of them using a field case study

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Final goal of reservoir management is finding a high net present value during the forecast period of reservoir assessment. The first step in reservoir management is history matching. Even if we have a good history matched model, without a robust production optimization algorithm, high value of NPV cannot be found. Field-scale optimization problems consist of a highly complex reservoir model with many control variables as unknowns. So finding a high value for NPV in a reasonable time depends highly on the efficiency of optimization algorithm. There are many optimization algorithms and in this paper we study three efficient algorithms for doing optimization. These are steepest ascent (SA), sequential quadratic programming (SQP) and interior point (IP) methods. In petroleum, steepest ascent is a very popular method in maximizing NPV of reservoirs due to the small optimization cost. This method is an unconstrained optimization and due to the nature of this algorithm, it cannot find a high NPV. In contrast, sequential quadratic programming (SQP) is a robust constrained optimization algorithm and it can find a high NPV after about 20 iterations, but computation cost of each iteration is significant and it is not comparable with steepest ascent method. In this paper we introduce another optimization algorithm i.e. interior point method. Although final NPV in this method is not as high as SQP, but it is much more than steepest ascent method and computation cost of each iteration is similar to the steepest ascent method. For better realizing advantages and disadvantages of each algorithm, we consider application of these methods to the Brugge field which is a synthetic model and contains 4560 total fluid rate constraints which is pretty large.

Introduction

Closed loop optimization consists of two parts: 1-History matching 2-Production optimization. For predicting reservoir behavior, and due to the uncertainty in the reservoir parameters, history matching can be used for estimating true values of them using production data. The goal of production optimization is maximizing NPV by adjusting production from individual completions through inflow control valves in smart wells. Closed loop optimization means simultaneous history matching and production optimization. The final goal of closed loop optimization is maximizing NPV. Recently, closed loop optimization has been used widely for history matching and optimization (Brower et al., 2004; Sarma et al., 2005a,b, 2006; Saputelli et al., 2006; Van Essen et al., 2009; Chen et al, 2009; Chen and Oliver, 2010; Wang et al., 2009; Lorentzen et al., 2009).

Even if we have a good history matched model, without having an efficient optimization algorithm, high NPV cannot be obtained at the end of closed loop optimization. Although efficiency of optimization algorithm is very important, computation cost of algorithm is another challenge during optimization. An efficient method should attain to high NPV with the reasonable computational cost. Very fast method without attaining a high NPV is not desirable. Also a very robust algorithm that attain to high NPV with high computational cost is not desirable too.

Generally, optimization algorithms can be divided into two groups: 1-Unconstrained algorithms 2-Constrained algorithms. Each of these methods can be used as gradient-based methods or non-gradient-based methods. Examples of gradient-based methods are adjoint (Brower and Jensen, 2004; Sarma et al., 2005a,b, 2006; Naevdal et al., 2006; Saputelli et al., 2006; Van Essen et al. 2009; Van Doren et al., 2006; Kraaijevanger et al., 2007; Sarma et al., 2008), ensemble-based methods (Nwaozo, 2006; Chen et al., 2009; Chen and Oliver, 2009, 2010; Chaudhari et al., 2009; Su and Oliver, 2010; Wang et al., 2009; Odi et al., 2010) or simultaneous perturbation stochastic approximation (SPSA) which used by Bangerth et al., 2006; Gao et al., 2007 and Wang et al. 2009. All of these methods are based on the computing gradient of objective function respect to variables for converging to the local minima or maxima. Chen and Oliver (2009) showed that localization can be used in ensemble-based methods to reduce the effect from spurious correlations resulting from a small ensemble on the estimate of gradient. Also Chaudhri et al. (2009) showed that conjugate gradient direction can be used for faster convergence to the final solution.

Examples of non-gradient-based methods are genetic algorithm (Harding et al., 1998; Yeten et al., 2003; Tavakkolian et al., 2004), particle swarm or simulated annealing.

The focus of this work is on the production optimization using gradient-based methods. For this reason we consider application of three optimization algorithms on the Brugge field which is a synthetic model designed by TNO. Algorithms that we want to consider in this paper are 1-Steepest ascent method (SA) 2-Sequential quadratic programming method (SQP) and 3-Interior point method (IP). The first two methods have been used widely for finding optimal controls of reservoir in petroleum engineering.

Steepest ascent method is an unconstrained optimization algorithm. In terms of efficiency, this algorithm cannot find very high NPV, because at each iteration, after finding updated control values, they should be truncated based on the optimization problem constraints. In contrast, in terms of computation cost, this algorithm is very fast. Nwaozo (2006), Lien et al. (2008) and Su and Oliver (2010) used this algorithm for maximizing NPV during water flooding project. Also Chen et al. (2009), Chen Oliver (2010) and Wang et al. (2010) used steepest ascent algorithm in closed loop optimization for maximizing NPV at the end of forecasting period for models with uncertainty.

Sequential quadratic programming is another efficient method which has been used by several authors. Davidson and Beckner (2003) used this method for maximizing oil rate and minimizing water rate simultaneously. They called this method Optimized Rate Allocation (ORA) and tested this method on both black oil and compositional models. Diez et al. (2005) tested this method for maximizing field oil production by adjusting gas lift injection rates and chock opening on small number of wells. Alhuthali et al. (2007) used this method for maximizing sweep efficiency and delaying water breakthrough. Also Lorentzen et al. (2009) used this method for maximizing NPV of Brugge field with uncertain model parameters in a closed loop optimization. Due to the significant cost of optimization, they reduced the number of control variables by using reactive control strategy in which a zone is shut-in at the optimized maximum water cut. Dehdari and Oliver (2011) used this algorithm for maximizing NPV of Brugge field on a single realization during water flooding project. In order to make it possible to use this method for solving large optimization problems, they added different options to the optimization algorithm. QR update, MPI for matrix multiplications, eliminating non-negative constraints and localization for improving gradient approximation was the options they used for decreasing cost of optimization. They showed that SQP can attain significantly higher NPV than steepest ascent method. Even by adding these options, still computation cost of this algorithm was significantly higher than steepest ascent method.

The interior point method is another optimization method. Phale and Oliver (2011) used this method in constrained EnKF for reservoir history matching. In this paper for the first time we introduce this method for solving a petroleum optimization problem during forecast period. We show that although final NPV of this method is not as high as SQP, but it is significantly higher than steepest ascent method. Also, computation cost of this method is negligible which is desirable for us.

Methodology

In this part, we consider formulation of all of these methods. After that we talk about method we use for estimating gradients. In this paper we consider a case study which only has inequality constraints. For this reason, we only consider derivations for inequality constrained optimization problem.

Steepest ascent method

This method is an efficient unconstrained optimization method and after finding the search direction and step length α at each iteration, values of control variables can be updated. Then, these values should be truncated based on constraints on wells or completions (Chen et al., 2009). Updating formula for steepest ascent method is

$$x_{l+1} = x_l + \frac{1}{\alpha_l} C_{x,x} C_{x,s}.$$

We can approximate covariance matrices from ensembles. We use $C_{x,x}$ as a filtering (smoothing) matrix for making changes of control variables smoother. Also $C_{x,s}$ is covariance between control variables and NPV. In above formula, covariance functions can be found from the following formula:

$$C_{x,x} = \frac{1}{N_{xe} - 1} \sum_{i=1}^{N_{xe}} \left(x_{l,j} - \langle x_l \rangle \right) \left(x_{l,j} - \langle x_l \rangle \right),$$

$$C_{x,S(x)} = \frac{1}{N_{xe}-1}\sum_{i=1}^{N_{xe}} \left(x_{l,j} - \langle x_l \rangle\right)\left(S(x_{l,j}) - \langle S(x_l)\rangle\right).$$

After updating initial solution, objective function can be found by running reservoir simulator. This procedure should be terminated whenever difference between objective function in two iterations is less than the stopping criteria.

Sequential quadratic method

Suppose we want to minimize some objective or cost function, $f(x)$, subject to constraints $c_i(x) \geq 0$ for $i = 1,2, \dots, q$.

$$\text{Minimize } f(x)$$
$$\text{Subject to: } c_i(x) \geq 0 \text{ for } i = 1,2, \dots, q$$

$f(x)$ can be a linear or nonlinear objective function. $c_i(x)$ are constraints which are functions of x and can be nonlinear. $f(x)$ and $c_i(x) \geq 0$ are assumed to be continuous and have continuous second partial derivatives, and the feasible region of this problem is assumed to be nonempty. Solution of this problem can be found by writing Karush-Kuhn-Tucker conditions and solving this system of equations (Nocedal, 2006; Antoniou, 2007):

$$\nabla_x \mathcal{L}(x,\mu) = 0 \quad \text{for } j = 1,2, \dots, q$$
$$c_j(x) \geq 0$$
$$\mu \geq 0$$
$$\mu_j c_j(x) = 0$$

In above equations $\mathcal{L}(x,\mu)$ is Lagrangian and can be defined as below:

$$\mathcal{L}(x,\mu) = f(x) - \sum_{j=1}^{q} \mu_j c_j(x)$$

Where μ is vector of Lagrange multipliers. For solving this system of equation, we should write Taylor series expansion for each of these conditions and after linearizing them a new system of equation can be found which is KKT conditions of the following optimization problem:

$$\text{Minimize } \frac{1}{2}\delta^T Y_k \delta + \delta^T g_k$$
$$\text{Subject to: } A_k \delta \geq -C_k \text{ for } i = 1,2, \dots, q$$

In above optimization problem, Y_k is Hessian of Lagrangian, g_k is gradient of objective function respect to variables, A_k is Jacobian of constraints and C_k is matrix of constraints at x_k where k is iteration index. Using this method, nonlinear optimization problem converted to a quadratic optimization problem. As a result, in each iteration instead of solving a nonlinear optimization problem, only a quadratic optimization problem should be solved. This is the reason that this method has been called sequential quadratic programming. By solving this problem, δ which is search direction of original optimization problem can be found. This inequality quadratic optimization problem can be solved by converting it to the equality optimization problem by considering only active constraints. After that, this equality constraint optimization problem can be solved by converting it to the unconstrained optimization problem using variable elimination method (Antoniou, 2007). Optimized values of unconstrained optimization problem can be found easily by finding its derivative and setting it equal to zero. Initial solution can be updated using the following formula:

$$x_{k+1} = x_k + \alpha_k \delta_k$$

In order to update solution we should know how far we can move along the search direction to stay in the feasible region. For finding value of α the following one-variable multi-dimensional optimization problem should be solved:

$$\psi_p(\alpha) = f(x_k + \alpha\delta_k) - \sum_{j=1}^{q} (\mu_{k+1})_j c_j(x_k + \alpha\delta_k)$$

Solving this problem could be possible using different methods such as Strong Wolfe conditions (Oliver et al., 2008). For solving this problem, we need to know the values of Lagrange multipliers. Lagrange multipliers can be found by solving the following equation which is based on linearizing the first KKT condition.

$$\mu_{k+1} = \left(A_{ak} {A_{ak}}^T\right)^{-1} A_{ak}(Y_k \delta_k + g_k).$$

After finding search direction and step length, initial solution can be updated. This procedure should be terminated whenever the difference between the objective function in two iterations is less than the stopping criteria.

Interior point method

Again, the optimization problem that we want to solve is similar to the SQP problem. For converting inequality constraints to the equality constraints, slack variables should be added (Antoniou, 2007):

$$\text{Minimize } f(x)$$
$$\text{Subject to: } c_i(x) - y_i = 0 \text{ for } i = 1,2,\dots,q$$
$$y_i \geq 0 \text{ for } i = 1,2,\dots,q$$

In above equations, inequality constraints are converted to equality constraints by adding slack variables y_i, but still there are some nonnegative constraints related to slack variables. For eliminating these constraints, a logarithmic barrier function (Antoniou, 2007) can be added to the objective function. Using this method, the new optimization problem has the following form:

$$\text{Minimize } f(x) - \tau \sum_{i=1}^{q} \ln y_i$$
$$\text{Subject to: } c_i(x) - y_i = 0 \text{ for } i = 1,2,\dots,q$$

In this formulation $\tau > 0$ is the barrier parameter. For solving this optimization problem the KKT conditions should be written and by solving that system of equations, the optimization problem can be solved. Based on the objective function and constraints, the Lagrangian formulism can be defined as the following formula:

$$\mathcal{L}(x, y, \lambda, \tau) = f(x) - \tau \sum_{i=1}^{q} \ln y_i - \lambda^T [c(x) - y]$$

In the above equation, λ is the Lagrange multiplier. Using Lagrangian formula, KKT conditions can be written as:

$$\nabla_x \mathcal{L} = \nabla f(x) - A^T(x)\lambda = 0$$
$$\nabla_y \mathcal{L} = -\tau \mathrm{Y}^{-1}\mathrm{e} + \lambda = 0$$
$$\nabla_\lambda \mathcal{L} = c(x) - y = 0$$

In above equations, $A(x)$ is the jacobian of constraints:

$$A(x) = [\nabla c_1(x) \quad \cdots \quad \nabla c_q(x)]^T$$
$$\mathrm{Y} = \mathrm{diag}\{y_1, y_2, \dots, y_q\}$$
$$\mathrm{e} = [1 \quad 1 \quad \dots \quad 1]^T$$

At the *k*th iteration, the set of vectors $\{x_k, y_k, \lambda_k\}$ is updated to $\{x_{k+1}, y_{k+1}, \lambda_{k+1}\}$ as

$$x_{k+1} = x_k + \alpha_k \Delta x_k$$
$$y_{k+1} = y_k + \alpha_k \Delta y_k$$
$$\lambda_{k+1} = \lambda_k + \alpha_k \Delta \lambda_k$$

Δx_k, Δy_k and $\Delta \lambda_k$ can be found after writing the Taylor series expansion for the nonlinear terms in the KKT conditions, linearizing them and solving the new system of equation.

Also α_k in each iteration can be found by minimizing a merit function:

$$\psi_{\beta,\tau}(x, y) = f(x) - \tau \sum_{i=1}^{q} \ln y_i + \frac{\beta}{2} \|c(x) - y\|^2$$

In this equation, β is a sufficiently large number greater than or equal to zero.

After finding the search direction and step length, the initial solutions can be updated. This procedure should be terminated whenever difference between objective function in two iterations is less than the stopping criteria.

Method for estimating the gradient

Computing the gradient matrix is the most time consuming part of field optimization as it generally requires running the reservoir simulator many times. In this study, we approximate the gradient matrix from the ensembles. For this reason, we need to generate different realizations from Gaussian distribution and by applying these realizations to the reservoir model and by running the simulator we can calculate NPV of each realization.

Using this method, the gradient matrix can be approximated from the following formula (Nwaozo, 2006; Chen, 2008, Dehdari and Oliver, 2011):

$$g = C_{x,x}^{-1} C_{x,S(x)}.$$

where

$$\langle x_l \rangle = \frac{1}{N_{xe}} \sum_{i=1}^{N_{xe}} x_{l,j},$$

and

$$\langle S(x_l) \rangle = \frac{1}{N_{xe}} \sum_{i=1}^{N_{xe}} S(x_{l,j}),$$

Definition of covariance matrices is similar to the definitions in the steepest ascent method. $C_{x,x}^{-1}$ can be found using singular value decomposition method. Computing the Hessian matrix from an approximate gradient and using modified BFGS algorithm is easy. In this problem, we used 50 realizations of control variables to estimate the gradient of the objective function with respect to control variables. The number of realizations and random numbers were the same for all of three methods.

Problem statement

For comparing efficiency of different methods, the Brugge field is considered as a test case. Brugge field is a synthetic water flooded model designed by TNO. This model has been used in SPE applied technology workshop on closed loop optimization in June 2008 at Brugge, Belgium. Different groups worked on this model to discuss their history match and production strategies from a common basis. Objective of this contest was maximizing NPV at the end of forecast period, but accuracy of history matching had effect on results of optimization. Peters et al. (2010) compared results of different participants by comparing final NPV they obtained to show the efficiency of methods they used for optimization. Between all of the methods, the best optimization results were related to the participants that used gradient-based algorithms for production optimization. All of these groups also used EnKF for history matching. This field has 10 injectors and 20 producers with a total of 84 separate completion intervals, each of which can be controlled separately. All producers and injectors are smart wells with vertical flow control. All injectors have 3 completion intervals and each producer has at most 3 completion intervals. The optimization starts at the end of year 10 when the reservoir has been under peripheral water flood for about 8 years. Forecast period assumed to be 20 years. During forecast period, the total number of control steps is 40, as we only allow the controls to be adjusted at 6 month intervals. The total number of control variables in optimization is equal to $84 \times 40 = 3360$. There are different constraints on each well. During the forecast period, the bottom hole pressure constraint on the wells is $725 \leq BHP \leq 2611\ psi$ and fluid rate constraint on each well is $3000\ bbl/day$ for each producer and $4000\ bbl/day$ for each injector. The reservoir simulation model contains 60,048 gridblocks, 44,550 of which are active. The dimension of simulation model is $139 \times 48 \times 9$. 104 realizations provided by TNO for using in history matching.

Figure 1 shows a side view of reservoir. From this view, the main fault location in the north of reservoir can be seen. Colors show the depth of top of the reservoir. Based on the location of the main fault, it is obvious that water injection from the well on the right of the fault cannot sweep oil in the locations which are left of the fault. Figure 2 can gives us a better understanding of fault location and also depth of layers at different locations (there is exaggeration in the z direction).

The goal of this paper is comparing results of different optimization algorithms. For this reason we assumed that geology of reservoir is known a priori and just one of these realizations has been selected randomly. For this reason, results of this paper are not comparable with results in Peters et al. (2010) and Chen and Oliver (2010) which are based on history matched model. The objective of this work is maximizing NPV. Because SQP and interior point optimization algorithms are minimization algorithms, objective function changed to minimizing the negative of the NPV. As a result, objective function is

$$S(x) = \sum_{i=1}^{N_t} \frac{v_w Q_{w_i}(x) - v_o Q_{o_i}(x)}{(1 + r_\tau)^{t_i/\tau}}$$

where i is the time step index, N_t is the total number of control steps, r_τ is the discount rate in terms of time span τ, t_i is the cumulative time since the start of the production, υ_o and υ_w are the price of oil and the cost of water, Q_{o_i} and Q_{w_i} are the total oil and water production over the time step Δt_i, and x is vector of control variables. In this problem we assumed Oil price is $80\ bbl$ and water injection or production price is $5\ bbl$. Also discount rate is 10% per year with reference year of year 10.

Results and discussion

Computation cost of optimization can be divided into two parts: 1-Gradient estimation 2-search direct and step length computations. As we mentioned before, adjoint method can be used for gradient estimation with just two simulation runs. If there is only one realization this method is better than other methods, but for closed loop optimization, which is doing history matching and optimization simultaneously, gradient information in history matching can be used for optimization. For this reason ensemble-based method is more efficient than adjoint method. As an advantage, using ensemble-based methods, reservoir simulator can be used as a black box, and there is no need to access simulator source code, and any type of simulator can be used for estimating gradient. In this paper we try to find an optimization method which is fast and have good results which can be used in closed loop optimization. In closed loop optimization, optimization code should be called several times and finding a fast optimization code is very important. For this reason we use ensemble-based method for estimating gradient. 50 realizations used for this purpose. BHP constraints set on the simulator. For each well in each time step there is one constraint. Total number of well constraints for 30 wells during 40 time steps is 1200. Also there is one non-negative constraint for each completion, because completions cannot have negative rate. Total number of non-negative constraints is $84 \times 40 = 3360$. As a result total number of constraints is 4560 which is pretty large. Steepest ascent method is an unconstrained optimization method which does not depends on the number of constraints. At each iteration, after finding ascent direction and updating controls, their values should be truncated based on the constraints. For this reason this method is very fast and doesn't depend on the number of constraints. SQP is very efficient optimization method, but it depends highly on the number of constraints. 4560 constraints is pretty large number and computational cost of this method is very high. Effect of number of constraints on the interior method is not significant. Increasing the number of constraints can increase size of matrices that should be worked with them numerically. The most important cost in each iteration of this method is related to the matrix multiplications. Dehdari and Oliver (2011) showed that using MPI and Lapack library, this cost can be decreased significantly, for this reason cost of matrix multiplications in this method is not significant. As an advantage of this method, large number of constraints can be used without any significant effect on the computation costs. For comparing efficiency of these methods, we run different methods with the same seed number. Ensemble-based methods depend on the seed number, because gradient can be estimated using stochastic method. By changing the seed number, final NPV would be different. Figure 3 shows results of different methods for two different seed numbers. There are two stopping criteria in these three methods: 1-maximum number of iterations which is 20 in this case. 2-amount of increasing NPV at each iteration which if $(NPV_{new} - NPV_{old})/NPV_{old} \leq 5 \times 10^{-6}$ algorithm will stop.

As you can see in Figure 3a, the best result is related to SQP method and the worst result is related to the steepest ascent method. Steepest ascent method stopped at iteration 8. This stopping may have one of these two reasons: 1-insufficient increase in NPV compare to the previous iteration 2-not having increase in NPV at iteration 9 due to the truncating controls after updating them. This truncation can cause decrease in NPV compare to the previous iteration. Stopping steepest ascent algorithm in Figure 3a is because of the second reason. Based on my experience, stopping SQP and interior point methods due to the insufficient increase in NPV is very rare. Especially in the SQP method, always there is increase in NPV which is above the stopping criteria, although it is not significant. Due to the computational cost, we stopped running SQP algorithm at the end of iteration 20. Even computational cost of 20 iterations is significant. Later you can see that cost of each iteration in steepest ascent and interior point methods is not significant compare to the SQP method. Figure 3b shows result of these methods for 20 iterations. In this case result of SQP and interior point are significantly better than steepest ascent method. Result of interior point is not as good as SQP, but still it's about 0.1 billion dollar better than steepest ascent method. Although final NPV in interior point is lower than SQP, but later you can see that computational cost of interior method is so much less than SQP method. Rate of NPV increase at early iterations in SQP method is higher than two other methods, but rate of increase in other two methods are comparable with each other.

Although we compared results of these methods for two different seed numbers, but using these two set of runs, we cannot draw any conclusion about efficiency of these two methods. For this reason we run each method for 40 different seed numbers. Figure 4 show results of these runs. In Figure 4, the bounds of each box are 25% and 75% quantiles, the whiskers are the extremes and the line in the box is the median. The first box plot (from left) in this figure is related to the results of the steepest ascent method. The second and third plots are related to the results of the SQP and the interior point methods. This figure shows results of steepest ascent method depends highly on the seed number. As you can see its results show wide range of NPV change with changing the seed number. Dependency of SQP and interior point methods to the seed number is less than steepest ascent method. Based on the results of Figure 4, as we expected, SQP can reach to the higher NPV than interior point method. In average, results of interior point is 0.1 billion dollar better than steepest ascent, and result of SQP is 0.05 billion dollar better than interior point method.

Usually, the most efficient method is not the method that finds the highest NPV. Most of the time there is deadline for doing a project and by delaying start of a field development project we may loss lots of money in that field. Especially in closed loop optimization, optimization and history matching code should be called several times for considering uncertainty during field development. For this reason we considered running time of each method separately. In Table 1, you can see the average running time of each method for one iteration using one or five processors. These computation costs are excluding gradient computation cost. As you can see, the fastest method is steepest ascent method. In this method after finding gradient, there is not any other time consuming step in updating value of controls. Although SQP can achieve to the highest NPV compare to the other methods, computation time of this method in only one iteration is significant. Using MPI with 5 processors computation cost can be decreased a little bit, but still it is significant. Consider a case that we need to call optimization code several times for considering reservoir uncertainty, for each run SQP needs around 15 iterations for finding high NPV values. Total computational time would be significant. In closed loop optimization, for increasing speed it is desirable to compute gradient using MPI and after that using each processor, run the optimization code separately. In this case, it is important for us to use a method that work fast with only one processor. Interior point method is a fast method and only by improving mathematical operations, using Lapack library (Anderson et al., 1999), we can decrease its computation cost significantly. In this method, even using only one processor, we can update controls very fast. Brugge is a synthetic 2 phase back oil model and simulation running time of this method is very fast. We used 50 realizations for computing gradients and the total running time was about 20 minutes (using 5 processors).

Table 1: Running time of each method for one iteration

Method	**CPU time (minute)**	
	1 Processor	**5 Processor**
Steepest ascent	2	2
SQP	221	147
Interior point	15	11

In all above results, we used only one realization to test different optimization algorithms. Most of the time efficiency of algorithm does not depend on the model we use. But for making sure about efficiency of different methods, we tried another realization for confirming above results. Generally, this realization has higher porosity and permeability in different layers. Figure 5 shows permeability maps of these two realizations for active cells of two different layers.

Figure 6 shows results of different methods for two different seed numbers. Similar to the first realization, in this case the best results belong to the SQP and the worst result is related to the steepest ascent method. In steepest ascent method, due to truncating controls at each iteration, stopping algorithm is a common problem due to the insufficient increase in NPV. For better understanding about the results of different methods, in Figure 7 you can see results of running different methods for 20 different seed numbers. Again the results are similar to the first realization. Range of changing NPV by changing seed number is similar to the first realization too. It shows that efficiency of these methods does not depend on the realization we use for optimization.

Summary and conclusions

In the past years, different optimization methods have been used for finding a suitable schedule during forecast period of petroleum reservoirs. Steepest ascent method is the first one which is an unconstrained optimization method and value of controls should be truncated at the end of each iteration based on the constraints. Advantage of this method is its computational cost which is very low compare to the other methods. Unfortunately due to the truncating controls, high value of NPV cannot be obtained using this method. For improving the results, different persons tried to use constrained optimization methods. Between all of them SQP method is very popular and using this method, high NPV can be obtained. The only problem related to this method is its computational cost which is significant for each iteration. Dehdari and Oliver (2011) showed that by adding different options to this method, computational cost can be decreased significantly. Even by adding these options, computational cost of each iteration is significant and it's not comparable to the steepest ascent method. For this reason, finding a method that give high NPV value with reasonable computation cost was one the biggest challenges in petroleum engineering. In this paper, we introduced interior point method in optimization during the forecast period for the first time. This method can attain to the high NPV with reasonable computational cost for using in closed loop optimization. In closed loop optimization, optimization algorithm should be called several times and using SQP is very difficult and time consuming. As an advantage, this method does not depend on the number of constraints highly, and by increasing the number of constraints, computational cost will not change significantly, but increasing number of constraints can increase computational cost of SQP significantly. As a disadvantage, interior point method cannot reach to the high NPV as quick as SQP method.

In this problem we used ensemble base method for computing gradients. This method is very suitable for using in closed loop optimization and allows us to use reservoir simulator as a black box. In this paper we considered Brugge field as a case study for comparing results of different methods. Brugge is a synthetic 2 phase model. For this reason, running simulator and computing gradient is not very time consuming. Probably for large 3 phase models which gradient computation dominate other optimization costs, SQP is a better method, but if model is not 3 phase or when we have 2 phase model with large number of constraints, interior point method is more efficient than SQP method. In all cases, steepest ascent method which is based on unconstraint optimization is not a very efficient method.

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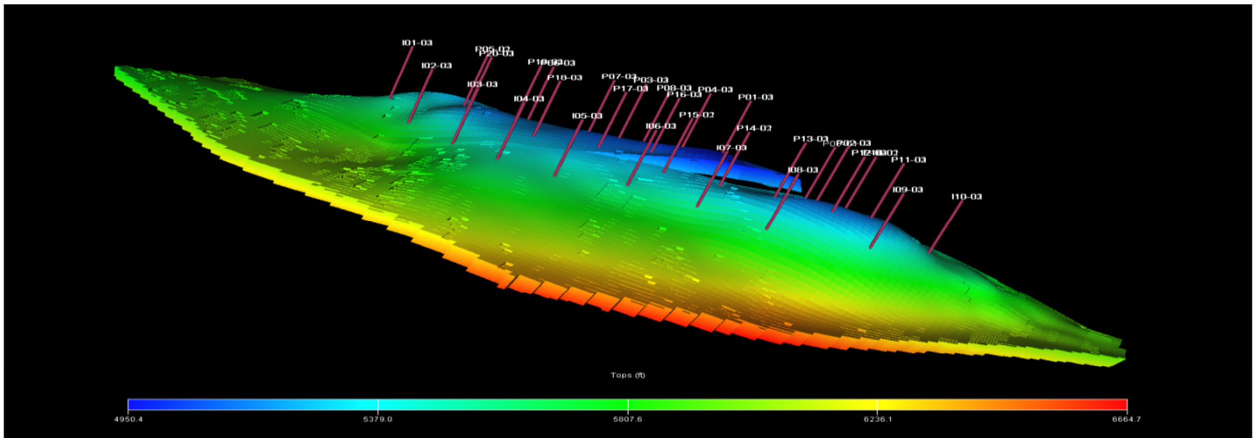


Figure 1: Depth of top of the reservoir

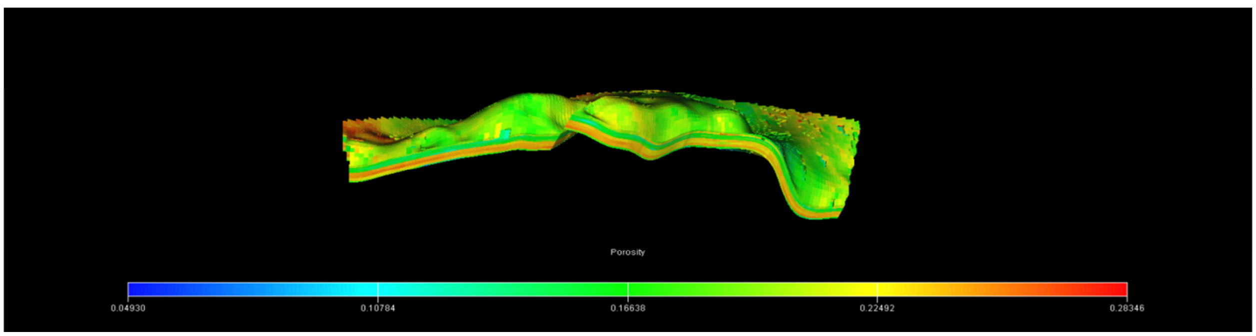
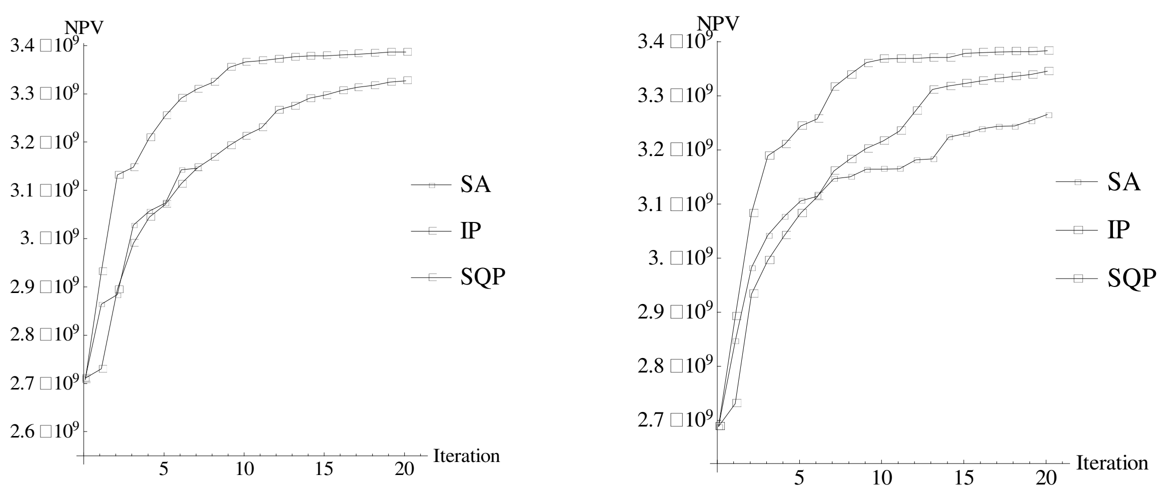


Figure 2: Main fault of reservoir



(a) Seed A

(b) Seed B

Figure 3: Results of different methods for 2 different seed numbers

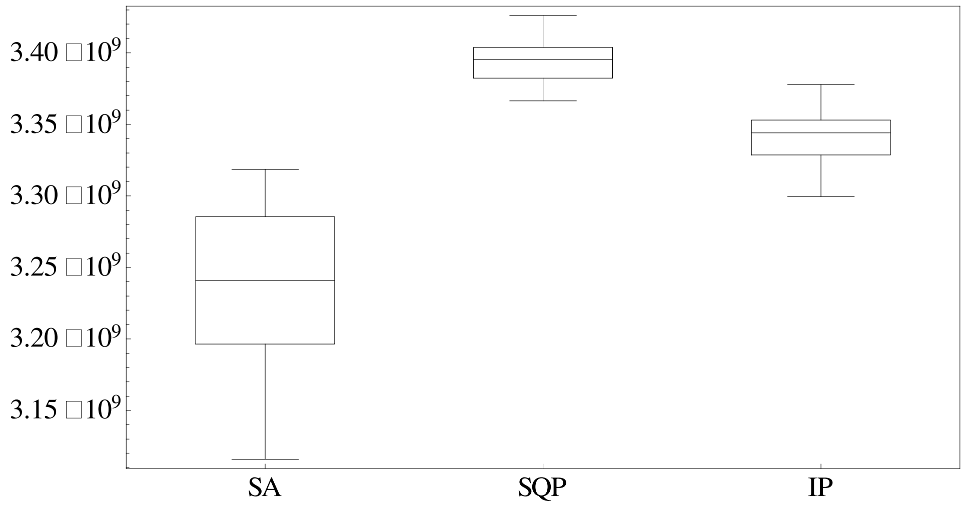
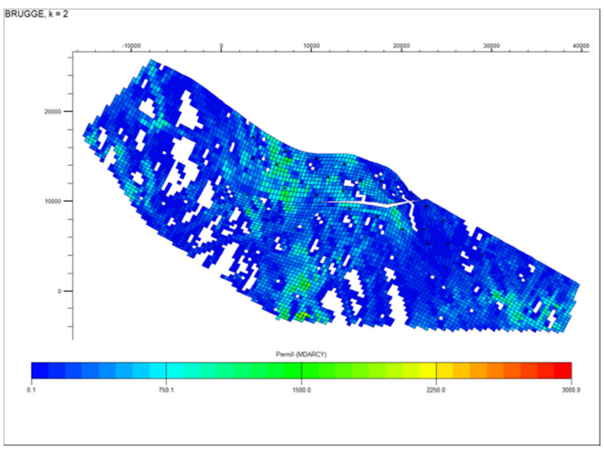
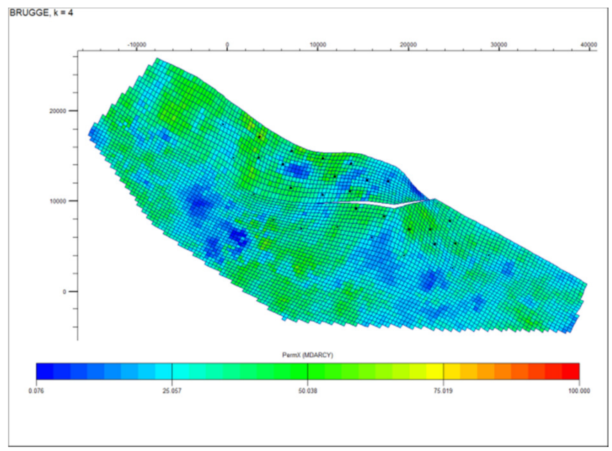


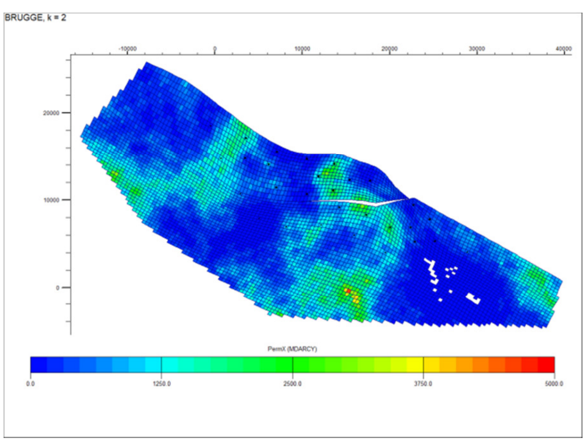
Figure 4: Results of different method for 40 different seed numbers



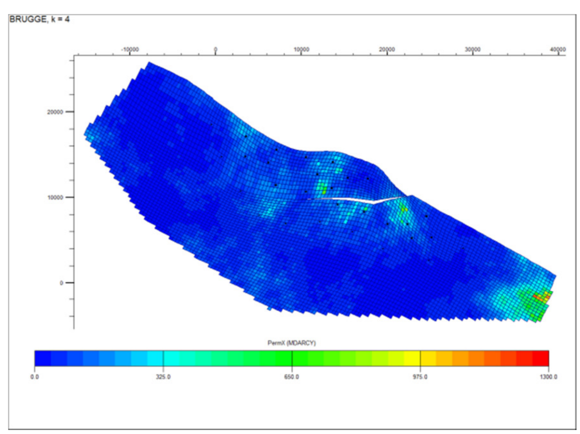
(a) Layer 2 realization 1



(b) Layer 4 realization 1

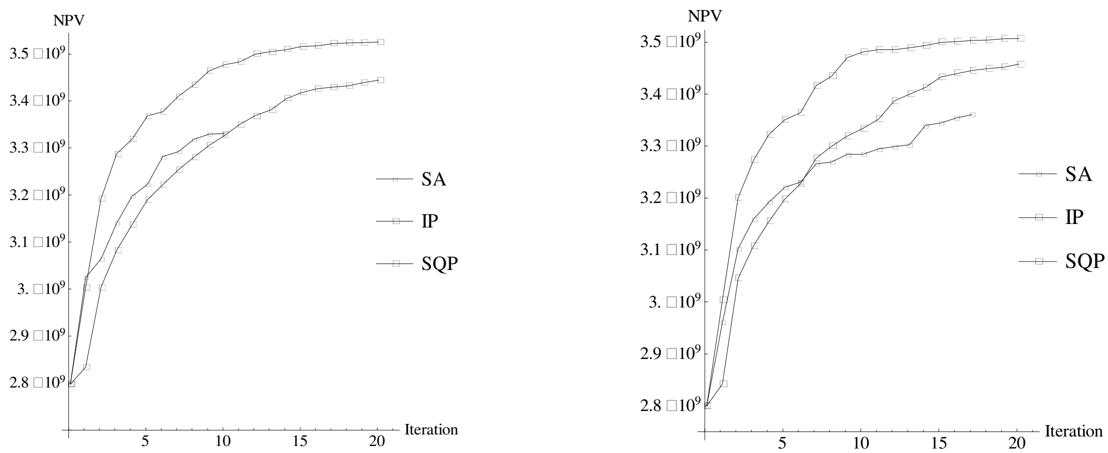


(c) Layer 2 realization 2



(d) Layer 4 realization 2

Figure 5: Permeability maps for 2 different layers for each realization



(a) Seed A

(b) Seed B

Figure 6: Results of different methods for 2 different seed numbers (realization 2)

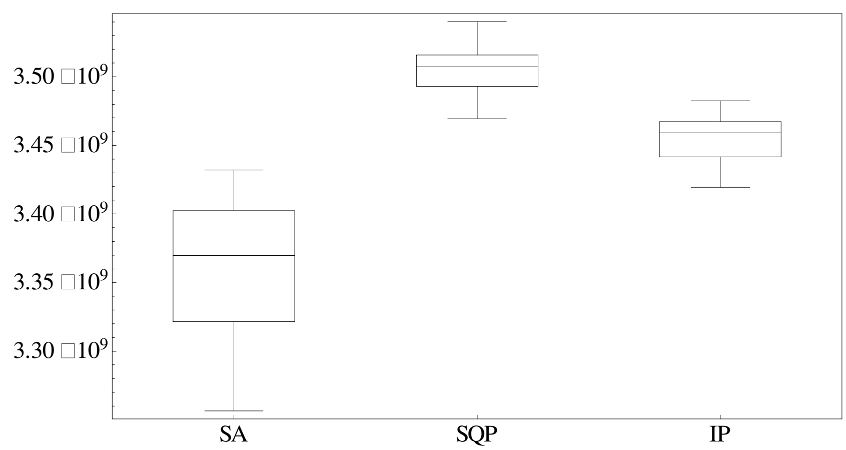


Figure 7: Results of different method for 20 different seed numbers (realization 2)