Cross Variography Calculation of Exploratory, Infill, and Blasthole Drilling

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In the mining industry, geologic information is collected from different sources, including infill and blasthole drilling. These sources of new information are of various scales and have their own characteristic sampling errors at different degrees. At the time of using this information in building a model of the deposit, either estimated or simulated, it is difficult to integrate these sources of information in co-kriging, as this information is not collocated. The problem lies in that the cross-variography cannot be calculated. In this paper, a methodology is proposed to approach the cross-variography via the calculation of the covariance of spatial data. This methodology produces a reliable variography of diverse data that can be used in full co-kriging. An example is presented to illustrate the implementation.

Introduction

Co-kriging is a very popular geostatistical techniques. These two techniques are widely discussed by many authors, including Deutsch and Journel (1998), Goovaerts (1997), and Chilés and Delfiner (1999). Co-kriging is a more general form of kriging and is also has different types of implementation, including simple co-kriging, ordinary co-kriging. In the case of simple co-kriging, for two attributes, the estimation of the first attribute at an unsampled location is expressed as:

$$z_{SCK}^{(1)*}\left(\mathbf{u}_0\right)-m_1=\sum_{i=1}^{n_1}\lambda_i\left(z_1\left(\mathbf{u}_i\right)-m_1\right)+\sum_{j=1}^{n_2}\lambda_j\left(z_2\left(\mathbf{u}_j\right)-m_2\right), \tag{1}$$

where, $z_{SCK}^{(a)*}\left(\mathbf{u}_0\right)$ is the estimated attribute *a* at a location $\mathbf{u}_0$, m_1 and m_2 are the stationary means of attributes *a* and *b*, respectively. $z_1\left(\mathbf{u}\right)$ and $z_2\left(\mathbf{u}\right)$ are the two attributes that are sampled at different locations $\mathbf{u}$ across the domain. Similar to the case of simple kriging, the weights λ are calculated to minimize the error variance and satisfy the condition of unbiassednes. In a more general form, this expression is expanded to account for several secondary variables.

To calculate the simple co-kriging weights, the following system of equations, as presented by Goovaerts (1997), is solved:

$$\begin{cases}\sum_{\beta_1=1}^{n_1}\lambda_{\beta_1}C_{11}\left(\mathbf{u}_{\alpha_1}-\mathbf{u}_{\beta_1}\right)+\sum_{\beta_2=1}^{n_2}\lambda_{\beta_2}C_{12}\left(\mathbf{u}_{\alpha_1}-\mathbf{u}_{\beta_2}\right)=C_{11}\left(\mathbf{u}_{\alpha_1}-\mathbf{u}_0\right)\\ \alpha_1=1,\ldots,n_1\\ \sum_{\beta_1=1}^{n_1}\lambda_{\beta_1}C_{21}\left(\mathbf{u}_{\alpha_2}-\mathbf{u}_{\beta_1}\right)+\sum_{\beta_2=1}^{n_2}\lambda_{\beta_2}C_{22}\left(\mathbf{u}_{\alpha_2}-\mathbf{u}_{\beta_2}\right)=C_{21}\left(\mathbf{u}_{\alpha_2}-\mathbf{u}_0\right)\\ \alpha_2=1,\ldots,n_2\end{cases}, \tag{2}$$

where, $C_{11}\left(\mathbf{h}\right)$ and $C_{22}\left(\mathbf{h}\right)$ are the auto-covariances of $Z_1\left(\mathbf{u}\right)-m_1$ and $Z_2\left(\mathbf{u}\right)-m_2$, respectively. The $C_{12}\left(\mathbf{h}\right)$ term represents the cross-covariances between $Z_1\left(\mathbf{u}\right)-m_1$ and $Z_2\left(\mathbf{u}\right)-m_2$.

In practice, the covariance terms are calculated from the variography of the existing dataset. The variography consists of a set of direct variograms for the individual attributes, and cross-variogram for the combination between attributes. The variography has to be positive definite to be used in co-kriging. The conventional workflow for calculating the variography is to calculate the experimental variography from the existing dataset and then fit the experimental values with a licit model. There is extensive documentation about this matter, including Journel and Huijbregts (1978), Deutsch and Journel (1998), and Chilés and Delfiner (1999). For two attributes, the respective variograms, $\gamma_1(\mathbf{h})$ and $\gamma_2(\mathbf{h})$, and cross-variogram $\gamma_{12}(\mathbf{h})$ are expressed as:

$$\begin{aligned}\gamma_1(\mathbf{h}) &= \frac{1}{2}E\left\{\left(Z_1(\mathbf{u}_i)-Z_1(\mathbf{u}_i+\mathbf{h})\right)^2\right\}\\ \gamma_2(\mathbf{h}) &= \frac{1}{2}E\left\{\left(Z_2(\mathbf{u}_i)-Z_2(\mathbf{u}_i+\mathbf{h})\right)^2\right\}\\ \gamma_{12}(\mathbf{h}) &= \frac{1}{2}E\left\{\left(Z_1(\mathbf{u})-Z_1(\mathbf{u}+\mathbf{h})\right)\left(Z_2(\mathbf{u})-Z_2(\mathbf{u}+\mathbf{h})\right)\right\}\end{aligned} \quad . \tag{3}$$

The expression of the cross-variogram in (3) corresponds to the traditional cross-variogram. An alternative representation of the cross-variogram is discussed by Deutsch and Journel (1998). Clark, Basinger, and Harper (1989) and Myers (1991) referred to this alternative cross-variogram as pseudo cross-variogram.

The difference in scale and quality of the information of the different sources of information, e.g., exploratory, infill, and blasthole drilling, make that the one attribute from these sources has to be treated as different attributes at the time of building the model of the deposit. For example, the samples of copper values of a deposit are treated differently depending on how they were collected, diamond, air-reverse or blasthole drilling. Although there is no problem in calculating the direct variograms, the cross-variogram in (3) requires that both attributes have to be homotopic. In mining, this is not the case, as exploratory drillholes are collected at different locations than infill drillholes.

Calculation of Cross-Variogram

In the proposed approach, the cross-variogram is calculated via the cross-covariance $C_{12}(\mathbf{h})$. It is expressed as:

$$C_{12}(\mathbf{h}) = E\left\{Z_1(\mathbf{u})Z_2(\mathbf{u}+\mathbf{h})\right\} - E\left\{Z_1(\mathbf{u})\right\}E\left\{Z_2(\mathbf{u}+\mathbf{h})\right\} \tag{4}$$

In equation (4), it is not required that the attributes are homotopically sampled. The conversion of the cross-covariance in terms of the cross-variogram is necessary as it is more difficult to fit covariances than variograms. The majority of the commercial software available in mining is designed to work with variograms than covariances. The relationship between the traditional cross-variogram and the cross-covariance, as presented by Deutsch and Journel (1998), is:

$$\gamma_{12}(\mathbf{h}) = C_{12}(0) - \frac{1}{2}\left(C_{12}(\mathbf{h}) + C_{21}(\mathbf{h})\right). \tag{5}$$

It is important to consider the potential impact of the 'lag effect', that is $C_{12}(\mathbf{h}) \neq C_{21}(\mathbf{h})$, which occurs due to geologic conditions in mineral deposits (Isaaks & Srivastaba, 1989). This condition has been discussed by several authors, including Journel and Huijbregts (1978) and Deutsch and Journel (1998). In this approach it is assumed that $C_{12}(\mathbf{h}) = C_{21}(\mathbf{h})$. Thus the relationship between the traditional cross-variogram and the cross-covariance used is:

$$\gamma_{12}(\mathbf{h}) = C_{12}(0) - C_{12}(\mathbf{h}) \tag{6}$$

As $C_{12}(0)$ is not accessible, the sill of the cross-variogram is inferred. The positive definite requirements of the linear model of coregionalization is used to approach $C_{12}(0)$. In Figure 1, a sketch of the cross-covariance and cross-variogram is presented. The $C_{12}(0)$ term is approached from the nugget effect of the cross-variogram (see Figure 1-right). Although the nugget effect is not accessible, the limits of the nugget effect can be approached from the positive semi-definite condition, that is:

$$b_{11}b_{22} \geq \left(b_{21}\right)^2 \tag{7}$$

where, for the case of the nugget effect, b_{11}, b_{22} are the contributions of the nugget effect in the direct variograms, and b_{21} is the contribution of the nugget effect in the cross-variogram. Any value of the nugget effect taken within this range affects the co-kriging estimation only when the location $\mathbf{u}_0$ is at the location where the secondary variable is sampled. In mining, models of the deposit a built at a much larger scale than the conditioning data. Thus the above mentioned situation would have little effect in the estimated value.

Example

The example consists of two separate datasets with sampled data from two unconditional realizations, which correlation coefficient is 0.8. It is considered that this information is unknown during the exercise. Each dataset consists of 72 point data (see Figure 2). The two datasets were split randomly from an original configuration over a regular grid of 40 x 40 m. No samples of each dataset occur at the same location.

The two direct variograms are presented in Figure 3. The nugget effect of each variogram is inferred at 0.2. From equation (7), the valid range of the nugget effect in the cross variogram is from 0 to 0.2. In Figure 4-left, the cross-covariance is presented. The projected covariance, based on the experimental values, at $\mathbf{h}$ = 0 is 0.6. Considering the nugget effect is the maximum value of the calculated interval, that is 0.2, the $C_{12}(0)$ is inferred as 0.8, which corresponds to the correlation coefficient of the initial referential unconditional realizations. However, any value from between 0 and 0.2 could have been selected. The approached cross-variogram is presented in Figure 4-right.

References

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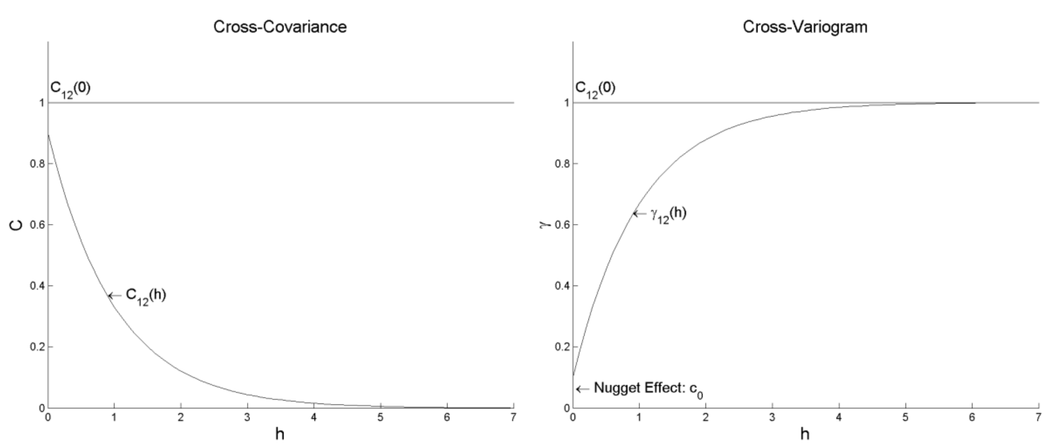

Figure 1: Sketch of cross-covariance and cross-variogram

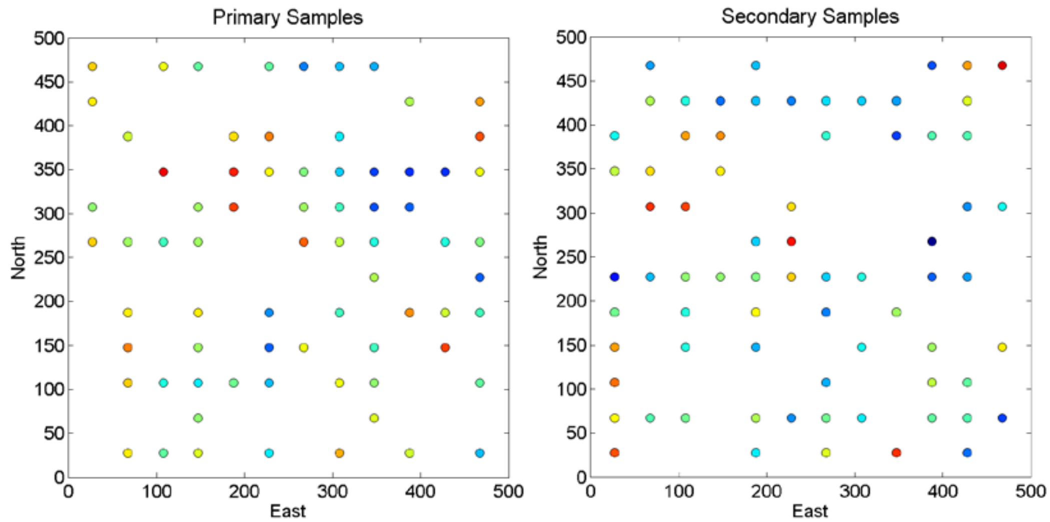

Figure 2: Spatial configuration of primary and secondary datasets; the two datasets are sampled at different locations

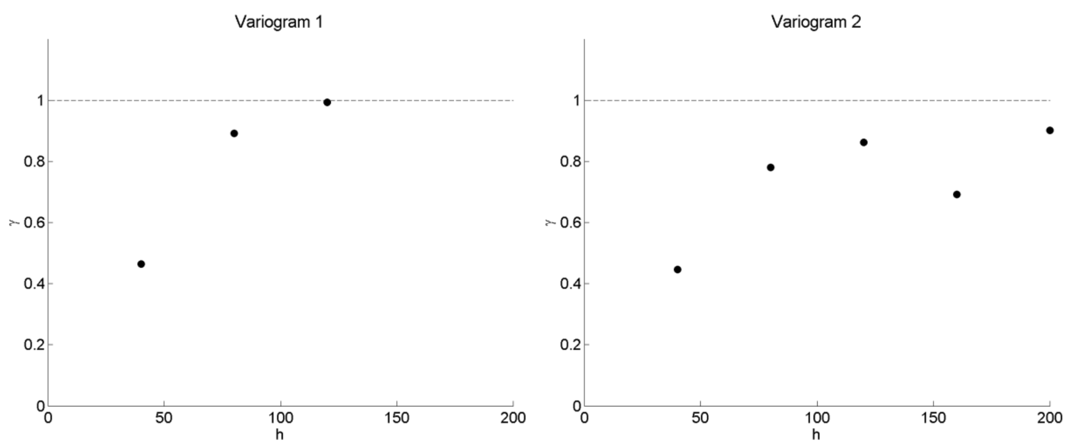

Figure 3: Direct variograms of attributes 1 (left) and 2 (right)

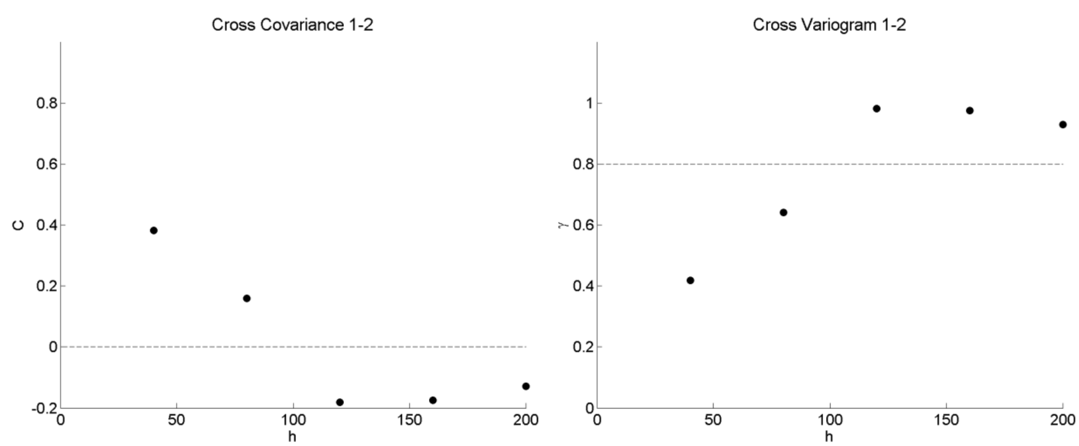

Figure 4: Cross-covariance (left) and approached cross-variogram (right) of attributes 1 and 2