A Short Note on Cross Validation of Facies Simulation Methods

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Abstract

There are many different geostatistical techniques available to construct stochastic realizations of reservoir facies. Moreover, each method requires many interdependent decisions and choices of modelling parameters. Cross validation methods may be adapted to facies or categorical variables to check the probabilistic prediction of facies techniques. This permits a refinement of the modelling parameters to improve the prediction of facies. It also permits a quantitative comparision of different facies modelling techniques. A cross validation study on a reservoir with and without seisimic data illustrates the value of seismic data and points toward the "best" approach for the facies modelling. Although the "best" method depends on the particular reservoir and data available, the methods developed to perform the check are general.

KEY WORDS: Object-Based Modelling, Sequential Indicator Simulation, Truncated Gaussian, Bayesian Updating

Introduction

Geostatistical reservoir models are being increasingly used for reservoir development planning and management. These models are typically created in a hierarchical fashion, that is, the major stratigraphic layers are first identified, then the significant facies are modelled in each stratigraphic layer, and finally porosity and permeability are assigned to each cell accounting for the stratigraphic layer and facies. Geostatistical models are preferred to conventional methods because they (1) provide a realistic representation of reservoir heterogeneity, and (2) permit the assessment of uncertainty in the reservoir response to different development strategies.

The use of geostatistical techniques, however, presents an important challenge that must be addressed: how do we decide which geostatistical technique / parameters to use? This question is addressed in the context of facies modelling with seismic and well data. Procedures are developed for the cross validation of facies modelling techniques. Indicator kriging without seisimic data, with Bayesian updating, and with block cokriging are considered. The results show the indisputable value of including seismic data. Truncated Gaussian methods and object-based modelling methods may also be considered in cross validation. Although *not* a general conclusion, the truncated Gaussian method does not perform well

in this case. Object-based methods were not applied due to a lack of clearly defined object shapes.

The goals of this study are to (1) check the probabilistic prediction of facies, (2) refine modelling parameters to improve the prediction of facies,and (3) compare different facies modelling techniques. The prediction of facies may be checked by:

1. closeness of the estimated probabilities to the true facies,
2. fairness of the local probabilities, and
3. the multivariate / spatial properties of the consequent realizations.

The first two require the "conventional" results of cross validation in a categorical variable setting, that is, the true facies types $s(\mathbf{u}_\alpha)$ and the corresponding predicted probabilities $p^*(\mathbf{u}_\alpha), \alpha = 1, \ldots, n; k = 1, \ldots, K$. The third check requires knowledge of the full underlying random function model or realizations of the algorithm. The main focus of this note is the first two checks. The procedures to check multivariate properties are more involved.

Cross Validation

The idea is to predict the probability of each facies at each data location in turn (leaving out the data at the location being estimated). There are two ways of implementing cross validation in presence of limited well data:

1. remove each sample *and* all other samples from the same well, or
2. just remove each sample (keep all other samples from the same well).

The first case is pessimistic, particularly in presence of only a few wells, because the available data are far from the locations being estimated. The second option is overly optimistic because adjacent samples are used, which would be unavailable in practice. Figure 1 illustrates the large well spacing in the case study presented later.

In practice, both approaches may be considered to provide bounds on the likely "goodness" of the prediction. The prediction in the first case with well data alone will look quite poor. Moreover, the seismic data will not appear to add much value in the second case (where very close samples are used).

Cross Validation of Indicator Facies Modelling

All variants of sequential indicator simulation provide direct estimates of the facies probabilities at the locations being (re)estimated. For example, at location $\mathbf{u}_\alpha$, the probabilities $p_k^*(\mathbf{u}_\alpha), k = 1, \ldots, K$ for all K facies are directly estimated. Order relations problems (negative probabilities and $\sum p_k^* \neq 1$) are corrected immediately following estimation.

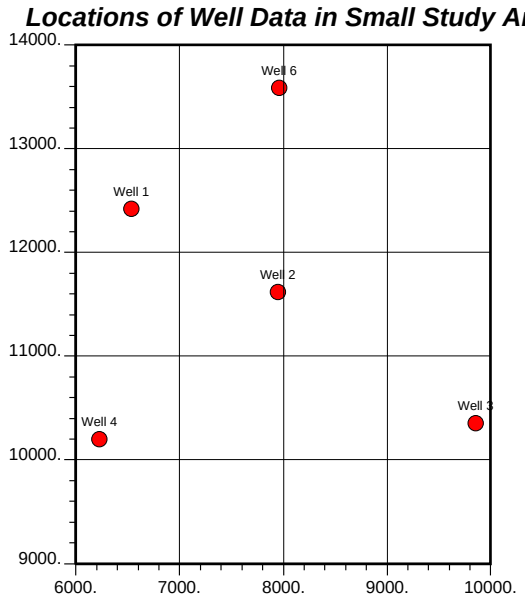

Figure 1: Location map of 5 well data used in the small case study. Note the large separation distances between the well data.

Cross Validation of Object-Based Facies Modelling

Object-based methods were not considered in the following case study due to a lack of a clearly defined object shape. Nevertheless, in many reservoir situations object-based methods (or even surface-based modelling methods) are applicable. In such cases, the cross validation exercise is quite tedious. For every well to be (re)estimated the facies modelling must be repeated many (say, 100) times to build up the probabilities $p_k^*(\mathbf{u}_\alpha), k = 1, \ldots, K$ by simulation. For $N = 5$ wells, we would require 5 sets of realizations (a total of 500 if we require 100 in each case).

Although tedious, this procedure permits a quantitative comparison with other methods and an ability to fine tune the object shape and size parameters.

Cross Validation of Truncated Gaussian Facies Modelling

The central idea of truncated Gaussian simulation is to convert the facies to a continuous variable with a normal distribution and build on the well understood and remarkably congenial properties of the Gaussian distribution. A Gaussian variable is simulated in 3-D space and then truncated at a series of $K - 1$ thresholds to arrive at the final categorical-variable realization. These thresholds can be based on the global proportions or they can vary locally to account for geological trends or seismic-derived proportion information. In our case, we use the seismic-derived probabilities to determine the thresholds.

The first requirement in cross validation of truncated Gaussian is to transform the data to normal scores values, construct the normal scores variogram, and perform a kriging to establish the predicted normal mean and variance at each location. Ideally, these steps would be performed with exactly the same parameters as used in the truncated Gaussian simulation program used for modelling.

In a cross validation setting we have the true facies $s(\mathbf{u}_\alpha), \alpha = 1, \ldots, n$, the seismic proportions $p^s(\mathbf{u}_\alpha), \alpha = 1, \ldots, n; k = 1, \ldots, K$, and the predicted Gaussian mean and variance $y^*(\mathbf{u}_\alpha), \sigma^{2*}_{SK}(\mathbf{u}_\alpha), \alpha = 1, \ldots, n$. The program `tgprob` was written to take this information and calculate the corresponding probability of each facies. The sequence of steps for a particular location $\mathbf{u}_\alpha$ (note that $G(y)$ is the standard normal cumulative distribution function, which has no analytical expression but excellent numerical approximations):

1. Convert the seismic proportions to cumulative proportions [1]:

$$P^s_k = \sum_{k'=1}^{k} p^s_{k'}, \quad k = 1, \ldots, K-1$$

Calculate the $K-1$ thresholds corresponding to these cumulative probabilities:

$$y^s_k = G^{-1}\left(P^s_k\right), \quad k = 1, \ldots, K-1$$

Correct the thresholds to account for the fact that the local distribution is not standard normal:

$$\hat{y}^s_k = \frac{y^s_k - y^*}{\sigma^*_{SK}}$$

Calculate the cumulative probabilities corresponding to the corrected thresholds:

$$P^*_k = G\left(\hat{y}^s_k\right)$$

Calculate the corresponding probability values:

$$p^*_1 = P^*_k$$

$$p^*_K = 1.0 - P^*_{K-1}$$

$$p^*_k = P^*_k - p^*_{k-1}, \quad k = 2, \ldots, K-1$$

The resulting $p^*_k, k = 1, \ldots, K$ values are exactly what are implicitly predicted by truncated Gaussian simulation with the 3-D seismic proportion values. These values may be checked in the same way as the direct estimates from indicator-kriging methods.

Cross Validation Results

A challenge with all cross validation exercises is to summarize the results in a way that permits comparison of different results, that is, given some true facies $s(\mathbf{u}_\alpha)$ and corresponding predicted probabilities $p^*_t(\mathbf{u}_\alpha), \alpha = 1, \ldots, N; k = 1, \ldots, K$ from different techniques or sets of input parameters $t = 1, \ldots, T$, what measures do we use to summarize the "goodness" of the predicted probabilities?
Four measures are developed here

1. *Closeness to the True Facies* measures how large the predicted probabilities of the true facies are. Ideally, the probability of the true facies $p^*_t(\mathbf{u}_\alpha; s(\mathbf{u}_\alpha))$ should be close to one.

[1] Note that the integer-coding of the facies must be chosen ahead of time to reflect any ordering / nesting.

2. *Fuzzy Closeness to the True Facies* accounts for the fact that misclassifications are different, that is, assigning the wrong type of sand is less consequential that assigning a sand facies as non-net shale.

3. *Fairness of the Local Probabilities* measures the reliability of the predicted probabilities, that is, 80% of the locations assigned an 80% chance to be facies s'_k *should be* facies s'_k.

4. *Multivariate Spatial Properties* accounts for the fact that local prediction of facies is only part of the picture; the multivariate or spatial distribution of facies has an overwhelming effect on the flow behavior of a model.

These four measures will be elaborated on below.

Closeness to the True Facies

A quantitative measure of closeness to the true probabilities may be summarized by:

$$C_k = E\left\{p(\mathbf{u}_\alpha; k) |\text{ true } = k\right\}, \quad k = 1, \ldots, K, \tag{1}$$

and

$$C = E\left\{E\left\{p(\mathbf{u}_\alpha; k) |\text{ true } = k\right\}\right\} \tag{2}$$

which may be interpreted as the average predicted probability of the true facies. This probability would be 1.0 in an ideal case of complete information and $1/K$ in the case where no information is available. In no case should this probability be less than $1/K$; this would imply the data are making the estimate worse than just guessing on the basis of randomness.

The closeness measures $C_k, k = 1, \ldots, K$ are most easily interpreted relative to the global proportions $p_k, k = 1, \ldots, K$. With no well or seismic data the closeness measures will equal the global proportions. This leads us to consider the percent improvement on the no-data case, i.e.,

$$C_k^{rel} = \frac{C_k - p_k}{p_k}, \quad k = 1, \ldots, K \tag{3}$$

Another measure of "information" or closeness to the trueth is entropy (the classical definition by Shannon in information thoery):

$$S_k = E\left\{-\sum_{k'}^{K} p^*_{k'} ln\left(p^*_{k'}\right) |\text{ true } = k\right\} k = 1, \ldots, K \tag{4}$$

and

$$S = E\left\{E\left\{-\sum_{k'}^{K} p^*_{k'} ln\left(p^*_{k'}\right)\right\}\right\}, \tag{5}$$

which may be interpreted as the average entropy or uncertainty of the predicted probabilities. The entropy would be 0.0 in the ideal coase of complete information (one of the p_k's is 1.0 and all of the others are 0.0) and $-ln(1/K)$ in the case of no information.

Fuzzy Closeness to the True Facies

The closeness measure C_k described above quantifies how "right" the probabilities are for the true facies; however, it gives no credit when a similar facies is predicted nor does it penalize the case when a drastically different facies is predicted. For example, if the true facies at a particular location is clean channel sand, it is more acceptable to assign that location as marginal sand than as shale or mudstone. Almost always there are more than two facies types and misclassifications have different consequences.

A fuzzy closeness matrix is defined to account for these consequences. The closeness value $c(i,j)$ specifies how close facies i is to facies j. By definition the closeness $c(i,j)$ is one for $i=j$, the same facies, and $c(i,j)$ is zero for completely different facies, say i =clean channel sand and j =shale. The exact values of $c(i,j)$ must be determined qualitatively considering the impact of misclassification. The need for a qualitative assignment is a shortcoming; however, some means must be used to account for the fuzzy closeness of facies.

Following is a straightforward example with four facies: (1) clean channel sand, (2) lesser-quality levee sand, (3) calcareous cemented sands, and (4) overbank / floodplain shale:

	$i=1$	$i=2$	$i=3$	$i=4$
$j=1$	1.0	0.5	0.2	0.0
$j=2$	0.5	1.0	0.2	0.2
$j=3$	0.2	0.2	1.0	0.4
$j=4$	0.0	0.1	0.4	1.0

The somewhat arbitrary assignment of $c(i,j)$ values accounts for the fact that the two sand types are close, the cemented sands are somehow related to the clean sands, and the there is little consequence if the two last facies are misclassified.

With this measure of fuzzy closeness we can calculate a revised measure of global closeness to the true probabilities:

$$FC = \frac{1}{N \cdot K} \sum_{\alpha=1}^{N} \sum_{k'=1}^{K} p(\mathbf{u}_\alpha; k') \cdot c(k', s(\mathbf{u}_\alpha)) \tag{6}$$

where $\alpha = 1, \ldots, N$ are the locations being cross validated, $k' = 1, \ldots, K$ are the different facies types, and $s(\mathbf{u}_\alpha)$ is the true facies at location $\mathbf{u}_\alpha$.

Fairness of the Local Probabilities

The local probabilities are "fair" if the accurately reflect the true fraction of times the predicted facies occurs. For example, consider all locations where we predict a 651 ($p_1^* = 0.65$); there should be 65in facies 1.

p_k^*	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$	$\hat{p}_4$	$\hat{p}_k$
					(ideal)
0.0 - 0.1	-	-	-	-	0.05
0.1 - 0.2	-	-	-	-	0.15
0.9 - 1.0	-	-	-	-	0.95

We construct this table from the predicted probabilities $p(\mathbf{u}_\alpha), \alpha = 1, \ldots, n; k = 1, \ldots, K$ and the true values $s(\mathbf{u}_\alpha)$.

Some probability intervals for some facies will have too few (or no) data. In presence of sparse data the predicted probability of a particular facies may alwats be the global probability p_k, that is, only one class in the table above will have data.

The check is qualitative. We are actually looking for cases where the actual fraction departs significatly from the predicted probability. Two useful interpretation guidelines:

1. when $\hat{p}_k < p_k^*$ we are too optimistic, that is, we are injecting too much spatial correlation *or* false correlation with seismic.

Multivariate Spatial Properties

The checks discussed up until now only consider the point-by-point goodness of the predicted facies probabilities. We must also check the spatial properties that have an impact (probably the most important impact) on the resulting flow behavior.

The "correct" way to check the flow results is to run a flow simulation code on a number of realizations generated by each candidate technique. Then, the average flow behavior can be compared against any actual flow data. In this way the *full* multivariate character can be assessed. This is not often practical because (1) there is rarely sufficient flow data to allow unambiguous results, and (2) the time / effort required to perform this check is significant.

Instead of checking the full multivariate properties, we settle for lower order statistics that have not been used by any of the candidate techniques. The simplest multivariate property that is not directly used by any conventional method are two-point transition probabilities:

$$p_{k,k'}(\mathbf{h}) = Prob\{\text{transition from } k \text{ to } k' \text{ for lag } \mathbf{h}\} \tag{7}$$

We compute these probabilities from the available well data $\hat{p}_{k,k'}(\mathbf{h})$ and from a candidate realization $p^*_{k,k'}(\mathbf{h})$. A cross plot of the results shows the difference. A summary mean squared difference can also be calculated:

$$msd = \sum_k \sum_{k'} \sum_{\mathbf{h}} \left(\hat{p}_{k,k'}(\mathbf{h}) - p^*_{k,k'}(\mathbf{h}) \right) \tag{8}$$

An Example

This example relates to a petroleum reservoir where most results are being kept proprietary. Tables 1, 2, and 3 show the preliminary results of the study. The Bayesian updating approach looks quite good! We have an indication that the block cokriging approach is more sensitive to the larger number of input parameters. Another striking observation is that the seismic data shows dramatic improvement in the case where the entire well is removed (the most realistic case). The seismic data also makes a significant contribution in the case where the well is left, which is more surprising.

With Well	Facies	IK	BU	BC
	1	0.663	0.660	0.690
	2	0.437	0.385	0.411
	3	0.425	0.459	0.425
	4	0.241	0.347	0.288
	all	0.511	0.522	0.526
Remove Well	1	0.470	0.502	0.491
	2	0.217	0.205	0.206
	3	0.181	0.219	0.224
	4	0.157	0.272	0.240
	all	0.320	0.358	0.349

Table 1: Closeness measures $C_k, k = 1, \ldots, K$ as described in equation 1.

	IK	BU	BC
With Well	0.591	0.597	0.605
Remove Well	0.437	0.463	0.458

Table 2: Fuzzy Closeness measures FC as described in equation 6.

With Well	Facies	IK	BU	BC
	1	47.3	46.7	53.3
	2	108.1	83.3	95.7
	3	136.1	155.0	136.1
	4	60.7	131.3	92.0
	all	77.6	86.1	82.4
Remove Well	1	4.4	11.6	9.1
	2	3.3	-2.4	-1.9
	3	0.6	21.7	24.4
	4	4.7	81.3	60.0
	all	3.5	20.8	17.1

Table 3: Percentage improvement over the global probabilities for indicator-based methods.

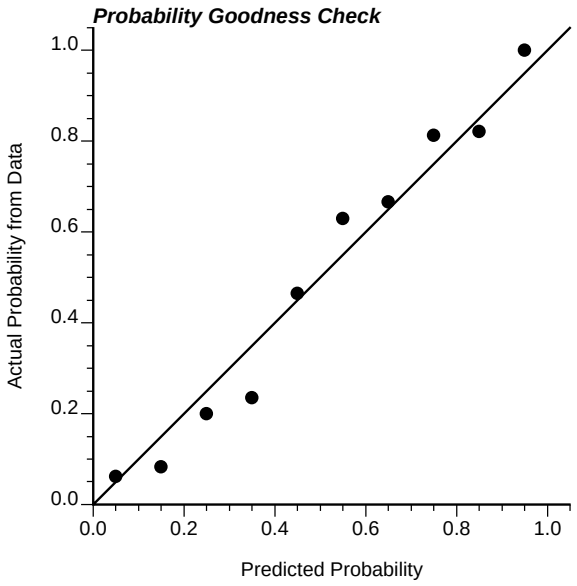

Figure 2: Example cross plot of actual probabilities versus the predicted probabilities. The closeness of the results to the 45 degree line attests to the goodness of the probabilies. These results are for block cokriging in all facies. We could also look at the results on a by-facies basis.

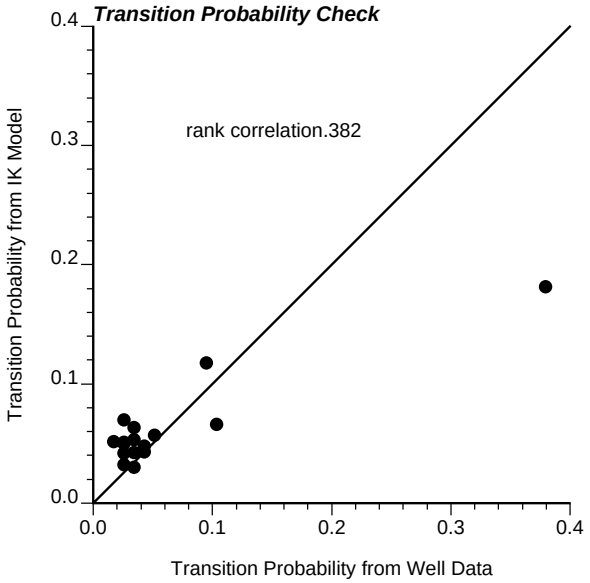

Figure 3: Example cross plot of transition probabilities from 3-D model (indicator simulation without seismic data) and probabilities from well data. The difference is significant; however, it can only be judged relative to other techniques.

APPENDIX: Programs for Cross Validation

The program `ik3d_br` was custom written during the visit to implement Bayesian Updating, Block Cokriging, and to perform cross validation with the option of removing the entire well.

The program `chkprob` was written to calculate the summary checks described above. It calculates the measures of closeness and the the tabular information required to check the fairness of the probabilities.

`chk2p` was written to calculate the two-point transition probabilities from the input well data and a 3-D realization for a specified lag vector and tolerance.

```
                  Indicator Kriging with Seismic Data
                  ***********************************

START OF PARAMETERS:
3                               -kriging: 1=IK, 2=bu, 3=bc
20.0                            -thickness of seismic averaging
4                               -number thresholds/categories
1     2     3      4            -   thresholds / categories
0.45  0.21  0.18   0.15         -   global cdf / pdf
clayprop.dat                    -file with data
5 2  3  4  10     6  7  8  9    -   cols for DH,X,Y,Z, true, prob
-1.0e21   1.0e21                -trimming limits
1                               -debugging level: 0,1,2,3
ik3d.dbg                        -file for debugging output
ik3d.out                        -file for kriging output
40 6017.5   100.0               -nx,xmn,xsiz
50 9085.5   100.0               -ny,ymn,ysiz
27    0.0    2.5                -nz,zmn,zsiz
1    8                          -min, max data for kriging
4000.0 4000.0 12.0              -maximum search radii
 0.0   0.0   0.0                -angles for search ellipsoid
0                               -max per octant (0-> not used)
0   2.5                         -0=full IK, 1=Median IK(category)
2 0.22 0.15 0.32                -One   nst, nugget (i,i-s,s)
1 0.30 0.21 0.26 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
          728.  360. 10.26      -      a_hmax, a_hmin, a_vert
1 0.48 0.34 0.42 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
         4557. 2278. 10.26      -      a_hmax, a_hmin, a_vert
2 0.26 0.09 0.39                -Two   nst, nugget (i,i-s,s)
1 0.31 0.11 0.25 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
          646.  318.  7.72      -      a_hmax, a_hmin, a_vert
1 0.43 0.16 0.36 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
         5017. 2564.  7.72      -      a_hmax, a_hmin, a_vert
2 0.18 0.11 0.52                -Three nst, nugget (i,i-s,s)
1 0.31 0.19 0.18 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
          424.  262.  6.45      -      a_hmax, a_hmin, a_vert
1 0.51 0.32 0.30 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
         4502. 2251.  6.45      -      a_hmax, a_hmin, a_vert
2 0.63 0.42 0.39                -Four  nst, nugget (i,i-s,s)
1 0.22 0.15 0.37 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
          572.  286.  6.51      -      a_hmax, a_hmin, a_vert
1 0.15 0.10 0.24 0. 0. 0.       -      it,c,ci-s,cs,ang1,2,3
         4502. 2251.  6.51      -      a_hmax, a_hmin, a_vert
```

Figure 4: Parameter file for the revised indicator kriging program that implements Bayesian updating and block cokriging.

```
                  Parameters for CHKPROB
                  **********************

START OF PARAMETERS:
4                               -number of categories
1  2  3  4                      -integer codes for categories
ik3d.out                        -file with input true / probabilities
5  1  2  3  4                   -column for true, probabilities
chkprob.out                     -output file
0.1                             -probability increment
1.0 0.5 0.0 0.0                 -
0.5 1.0 0.0 0.0                 - matrix of "fuzzy closeness"
0.0 0.0 1.0 0.5                 -
0.0 0.0 0.5 1.0                 -
```

Figure 5: Parameter file for `chkprob` - the program written to calculate summary statistics to check predicted probabilities.

```
                  Parameters for CHK2P
                  ********************

START OF PARAMETERS:
4                                  -number categories
1     2     3      4               -   categories
clayprop.dat                       -file with well data
2     3     4     10               -   cols for X,Y,Z, category
sis_br.out                         -file with 3-D facies model
40 6017.5   100.0                  -   nx,xmn,xsiz
50 9085.5   100.0                  -   ny,ymn,ysiz
27    0.0    2.5                   -   nz,zmn,zsiz
0.0  0.0  2.5                      -lag distance  :X,Y,Z
1.0  1.0  1.0                      -lag tolerance
chk2p.out                          -file for output
```

Figure 6: Parameter file for `chk2p` - the program written to calculate two-point transition probabilities from the input well data and a simulated realization.