A Short Note on Geostatistical Scaling Laws Applied to Core and Log Data

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Abstract

Reconciling data from different scales is a longstanding problem in reservoir characterization. Data from core plugs, well logs of different types, and seismic data must all be accounted for in the construction of a geostatistical reservoir model. These data are at vastly different scales and it is inappropriate to ignore the scale difference when constructing a geostatistical model.

Geostatistical scaling laws were devised in the 1960s and 1970s primarily in the mining industry where the concern was mineral grades in selective mining unit (SMU) blocks of different sizes. These principles can be extended to address problems of core, log and seismic data. The adoption of these classic volume-variance or scaling relationships presents some challenges. Some specific concerns are (1) the ill-defined volume of measurement, (2) uncertainty in the small-scale variogram structure, and (3) non-linear averaging of many responses including acoustic properties and permeability.

We demonstrate the application of volume-variance relations for upscaling and downscaling techniques to integrate data of different scales. Practical concerns are addressed with data from a chalk reservoir in the Danish North Sea.

KEYWORDS: volume-variance relations, dispersion variance, upscaling, downscaling, periodicity

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Problem Statement

The integration of data at different scales with conventional geostatistical scaling laws has not seen wide application in petroleum geostatistics. This is due mainly to unfamiliarity with the techniques and scaling laws. Recalling and demonstrating such techniques will address this unfamiliarity. There remain, however, some significant assumptions with conventional techniques that must be addressed in future work.

A significant assumption is that the petrophysical property must average linearly. This is appropriate for facies indicator variables and porosity; however, acoustic properties and permeability simply do not average linearly. A power-law formalism could be used whereby the original variable is transformed to a variable that, in general, averages linearly. The widely used sequential simulation approaches require a transformation either at the beginning to the convenient Gaussian distribution or afterwards to correct for the non-reproduction of the histogram. Such transforms are not compatible with the assumptions behind the classical volume-variance relations of geostatistics.

Another significant assumption in the use of conventional volume-variance relations is that the spatial variability is completely characterized by a stationary random function using 2-point

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variogram / covariance measures of correlation. No higher order non-linear spatial connectivity is accounted for, which may pose a serious problem in real petroleum reservoirs.

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Objectives

The main objective of this note is to recall conventional volume-variance scaling relationships that were formalized in the early days of geostatistics. The scaling laws will then be applied to data from a Danish chalk reservoir. Core and well log data will be used. These data measure significantly different volumes. The volume of the core measurement is well understood; however, the volume of the interpreted well log derived porosity is less well understood. The statistics of each data type together with analytical volume-variance relationships can be used to quantify the volume of investigation of the well log data.

The ultimate goal of this work is to illustrate how data of different scales may be used simultaneously in the construction of high-resolution geostatistical models. When the different types of data are all "hard," in the sense that they do not contain significant errors or uncertainties relative to the property being modelled, it is possible to use block kriging. Certain data types such as seismic contain uncertainties related to the great distance of measurement and calibration of the measured acoustic properties to the petrophysical properties of interest. In this case, it will be necessary to use block cokriging.

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Recall of Volume-Variance Scaling Relationships

Mining Geostatistics (Journel and Huijbregts, 1978) is the classic reference for volume-variance scaling relationships. The essential results are recalled below. Details and proofs may be found in the original reference.

Consider the fitted variogram model at arbitrary scale *v*, where *v* often represents the small core scale:

$$\gamma_v(\mathbf{h}) = C_v^0 + \sum_{i=1}^{nst} C_v^i \cdot \Gamma^i(\mathbf{h}) \qquad (1)$$

where $\gamma_v(\mathbf{h})$ is the variogram model at the v scale, C_v^0 is the nugget effect, *nst* is the number of nested variogram structures used to fit the variogram, C_v^i is the variance contribution of each nested structure, *i=1,…,nst*, and $\Gamma^i(\mathbf{h})$ is the nested structure consisting of an analytical function (spherical, exponential, Gaussian, hole-effect, etc.) and six anisotropy parameters (three angles and three distance ranges). Note that the "sill" of the analytical nested structure Γ is unity; the *C* coefficients describe the variance contribution of each structure. The sum of the variance contributions is the variance at the *v*-scale also known as the dispersion variance

$$D^2(v,A) = C_v^0 + \sum_{i=1}^{nst} C_v^i \qquad (2)$$

where $D^2(v,A)$ is the variance of volumes of size *v* in the entire area of interest *A*. The mean value at a larger (or smaller) scale does not change, assuming arithmetic average. The variance, however, decreases as the volume increases; high and low values are averaged out as the volume of investigation or measurement increases. As the variance decreases the variogram structure also changes. The variance contributions C_v^i , *i=0,…,nst*, decrease. The shape of each nested structure $\Gamma^i(\mathbf{h})$ may also change.

Experience has shown that the variogram shape change is small. The three correlation ranges increase as the averaging volume increases; however, the actual "shape" of the variogram changes very little. The range at a larger volume V increases as the increase in volume size $(|V|-|v|)$ in each particular direction:

$$a_V = a_v + (|V| - |v|) \tag{3}$$

Depending on the shape of the larger volume V, the range may increase in some directions and stay the same in other directions. Assuming the variogram shape does not change, we have to quantify how the variance contributions C_v^i, $i=0,\ldots,nst$, change.

It can be shown that the purely random component, represented by the nugget effect, decreases with an inverse relationship of the volume, i.e.,

$$C_V^0 = C_v^0 \cdot \frac{|v|}{|V|} \tag{4}$$

In this case $|v|$ and $|V|$ represent the volume of each scale, respectively.

Average variogram or "gamma-bar" values are used to determine how the variance contribution of each nested structure decreases:

$$C_V^i = C_v^i \cdot \frac{1-\bar{\Gamma}(V,V,\mathbf{a}^i)}{1-\bar{\Gamma}(v,v,\mathbf{a}^i)} \tag{5}$$

The "gamma-bar" notation represents the average variogram for vectors where each end of the vector independently describe the volume V or v. In 3D the gammabar values may be expressed as the infamous sextuple integrals of early geostatistics. The modern approach, however, is to calculate all gamma-bar values numerically using software (such as the `gammabar` program).

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Scaling Relations with Real Data

Data from an interval in the MFB-7 well in the Dan Field will be considered. The Dan Field is an Upper Maastrichtian to Lower Paleogene chalk limestone reservoir, and is characterised by high porosities (30-40 %) and generally low permeabilities (1 mD) (Kristensen *et al*. 1995). 18 m from the vertical wellbore has been extracted, see Figure 1 on next page.

The section shows cyclic porosity variations caused by climatic variations during deposition of the pelagic chalk material (Scholle *et al*. 1998). The core measurements represent a volume of about 5x2x2 cm (vertical resolution 2 cm). The log measurements represent an average over approximately 60 cm of the well-bore with an uncertain investigation depth probably around 25 cm. Core-based air permeability measurements are also available for many of the core plugs in this section, see lower squares on Figure 1.

As expected, the core porosity values show greater variability than the log porosity values. The histograms shown on Figure 2 illustrate the difference.

The average well log-derived porosity is consistently less than the core porosity. This has been corrected by a simple shift of the log-derived porosities by 1.8 p.u. in order to match the mean value of the core porosity. Notwithstanding this bias in the log-derived porosity, the core and well log porosity values are both considered excellent measurements with litle measurement or interpretation error. Figure 3 shows a cross plot of the core versus log porosity values. The scatter on this plot is attributable to the different measurement volumes. The log-/core-porosity crossplot shows a fair correlation between the two variables. The comparison shows that the well log data does not represent some high porosity layers recorded by core analysis.

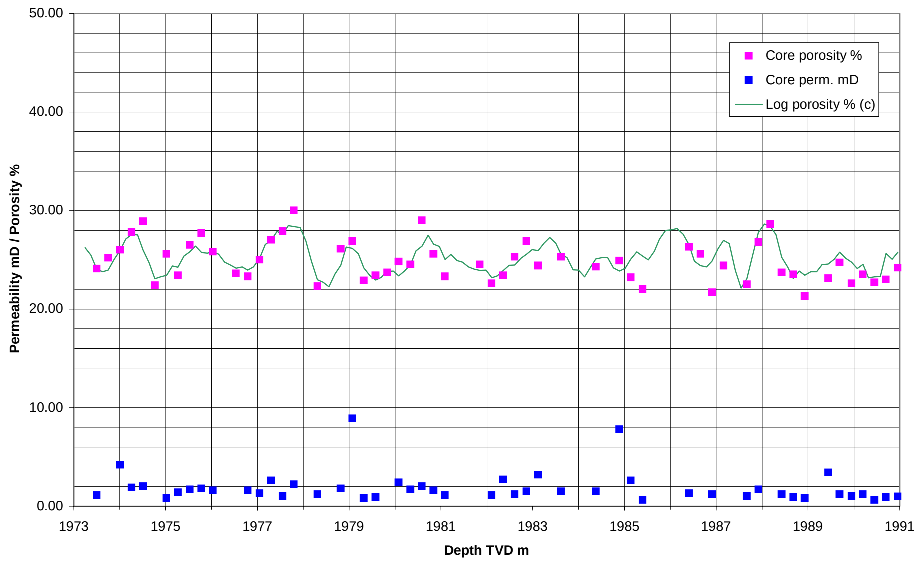
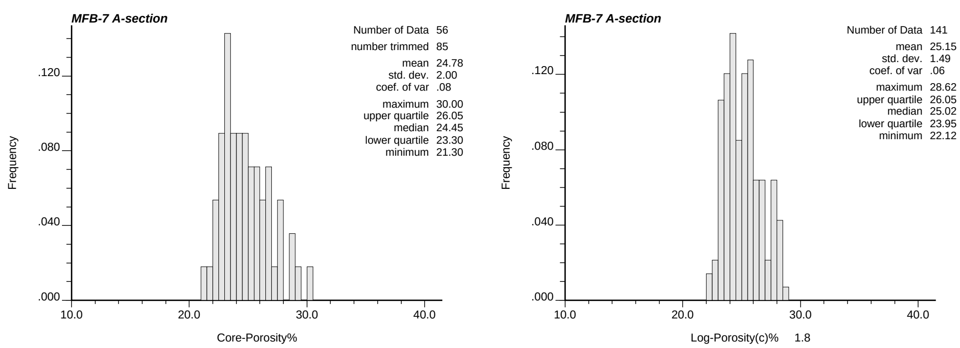


Figure 1: profile of core and log data from MFB-7. Note the general agreement of the core and log porosity data, the greater variability in the core data, and the generally low permeability values.



	Mean	Variance	Coeff. var.
Core	24.78	4.02	0.08
Log	25.15	2.23	0.06

Figure 2: histogram of core and log data from the MFB-7 interval. Note that the log data has lesser variance.

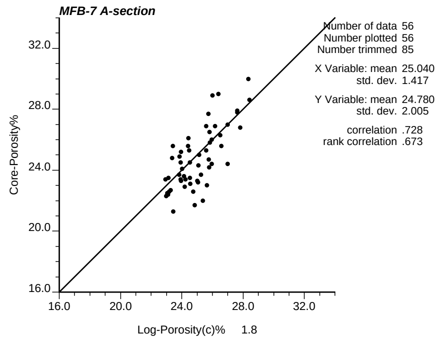


Figure 3: cross plot of log and core data from the MFB-7 interval. Note the good correlation and the difference in variance.

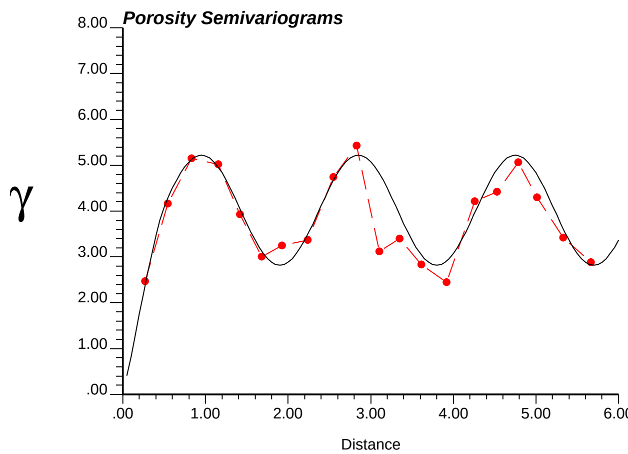


Figure 4: experimental variogram of core data from the MFB-7 interval. The experimental variogram has been fitted with nested spherical and hole effect variogram models.

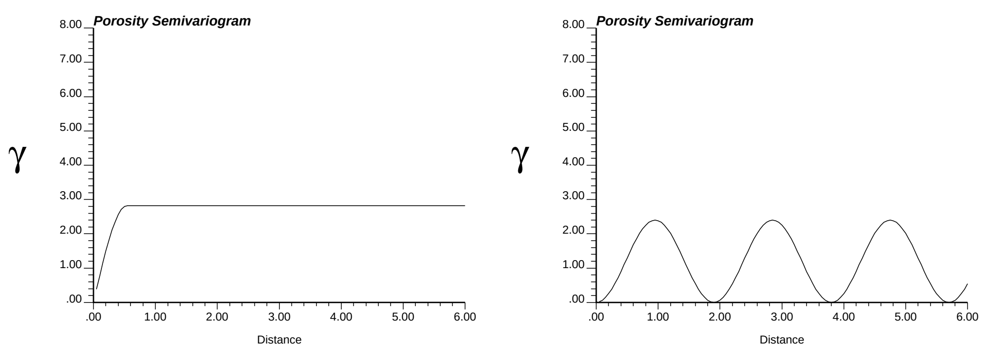


Figure 5: individual variogram structures for downscaling.

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The core-porosity semivariogram shown below on Figure 4 shows clear cyclic variations with a period at about 1.90 m. The variogram model, shown by the solid line, has no nugget effect and two nested structures: (1) a spherical structure with sill equal to 2.82 and range of 0.54 m, and (2) a hole effect model with amplitude 1.2 and peak at 0.95 m. The fit is quite good. Figure 5, below, shows the two elementary nested structures to be considered in the scaling relationships.

Derivation of Point-Scale Variogram

The scaling laws developed above may be applied on each nested structure in the variogram. The scaling laws are concerned with the changes to the nugget effect, the variogram range and the sill. For the downscaling to point-scale from our data, we need to consider all three elements.

The core-scale data shows that no nugget effect is needed to describe the correlation structure. From more detailed investigations at mm^3-scale it also seems that no nugget is present. Therefore the point-scale variogram is assumed to have no nugget.

For the spherical structure the correction to point variogram range a_p based on the core-scale range a_v is: $a_p = a_v - (|v| - |p|)$, see equation (3) above. Where p = point scale, which is zero and v = plug scale, which is 0.02 m. Thus, the corrected range for the shperical nested structure is = 0.54 - (0.02 - 0) = 0.52 m

For the hole effect structure, the wavelength for the periodic structure is not changed, and the peak distance used for the modelling is kept constant at 0.95 for the point scale variogram.

The sill of each basic structure is corrected according to equation (5) presented above. The `gammabar` program is used to calculate all needed $\bar{\Gamma}(v,v)$ values. The mean variogram value at the point scale is, of course, 0.0. For all calculations of mean variogram values we assume one dimensional averaging of the data. This entails that our well data are only averaged in the vertical direction, which is a fair assumption given that we have a layered formation with large horizontal continuity. Therefore, the investigation depth is not to be considered in this case.

The value for $\bar{\Gamma}(v,v)$ for the spherical structure can be calculated with the `gammabar` program, using the point-scale variogram description as unit variogram between 0 and 1 with range 0.52 m as derived above. For calculation of $\bar{\Gamma}(v,v)$, the volume v is defined as the core-plug volume of 5x2x2 cm with a vertical length scale measure of 2 cm, giving a $\bar{\Gamma}(v,v)sph$ value of 0.0192, and a resulting point-scale sill C_p of 2.88 for the spherical structure, as compared to the core-scale sill value of the spherical structure of 2.82.

The same procedure is used for the hole effect variogram for the amplitude scaling, giving $\bar{\Gamma}(v,v)hole$ = 0.00036, and therefore a virtually unchanged variance contribution of 1.2 for the point scale variogram.

The point-scale variogram structure has a zero nugget and two nested structures: (1) a spherical structure with range 0.52 m and sill of 2.88, and (2) a hole effect structure with peak at 0.95 and a variance contribution of 1.2.

Application of Scaling Laws

The point variance within an arbitrary volume v is equal to the mean value $\bar{\gamma}$ of $\gamma(h)$ for all h and all directions within that volume, where $\gamma(h)$ is the point-scale variogram consisting of all nested structures, that is,

$$\sigma^2(\bullet, v) = \bar{\gamma}(v,v) \tag{6}$$

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Furthermore the additivity of variance entails that,

$$\sigma^2(\bullet, R) = \sigma^2(\bullet, v) + \sigma^2(v, R) \qquad (7)$$

for any volume v. In words, the variance of points • in a region is equal to the variance of points within a larger volume v plus the variance of that larger volume v within the region R.

Let's consider two different volumes v and V (e.g. core and log scale volumes). $\sigma^2(\bullet, R)$ is the global stationary point scale variance. Applying relation (7) to volumes v and V, the experimental average variogram $\bar{\gamma}(V,V)$ for the log-scale volume may be expressed as:

$$\bar{\gamma}(V,V) = \bar{\gamma}(v,v) + \sigma^2(v,R) - \sigma^2(V,R) \qquad (8)$$

In our case:

$$\bar{\gamma}(V,V) = \bar{\gamma}(v,v) + 4.02 - 2.23$$

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$\bar{\gamma}(v,v)$ can be calculated with the `gammabar` program given the real point scale variograms as defined earlier, and as the sum of the contribution from the two structures in the nested model, we obtain:

$$\begin{aligned}\bar{\gamma}(V,V) &= \bar{\gamma}(v,v) + 4.02 - 2.23 \\ &= \bar{\gamma}(v,v)sph + \bar{\gamma}(v,v)hole + 4.02 - 2.23 \\ &= 0.055 + 0.00044 + 4.02 - 2.23 \\ &= 1.85\end{aligned}$$

This gives us an independent assessment of the variance within log-scale volumes. Such volumve variance scaling laws may also be applied to check the volume V.

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Assessment of Tool Investigation Volume

Given the small-scale core variogram and the experimental log-scale variogram we can determine the well-log volume of measurement. This volume is determined in an iterative fashion until the gammabar prediction of variance matches the actual well log-derived variance. The volume was found to be 62cm, which is almost exactly what is predicted from the physics of the logging tool. The tool resolution is reported to be 30 cm, the measurements are reported at 15 cm, and the effective resolution in the logged data is 60 cm due to tool movement.

Provided we have a reliable estimate of the point-scale variogram it is possible to calculate the theoretical $\bar{\gamma}(V,V)$ for a range of different volume scales. Then, the volume scale can be determined where the theoretical $\bar{\gamma}(V,V)$ matches the experimental $\bar{\gamma}(V,V)$.

Applying this procedure with the point-scale variogram model derived above leads to the results on Figure 6. The cross plot shows the match for the scale of 0.62 m.

This can be double checked by calculating the $\bar{\gamma}(V,V)$ from the point scale variogram model

$$\bar{\gamma}(V,V) = \bar{\gamma}(V,V)sph + \bar{\gamma}(V,V)hole = 1.47 + 0.37 = 1.84$$

which matches exactly the value derived from the experimental calculation. From the relation between the two data types at different scales, we can predict a logging data resolution of 62 cm, which is very close to the assumed resolution of approximately 60 cm.

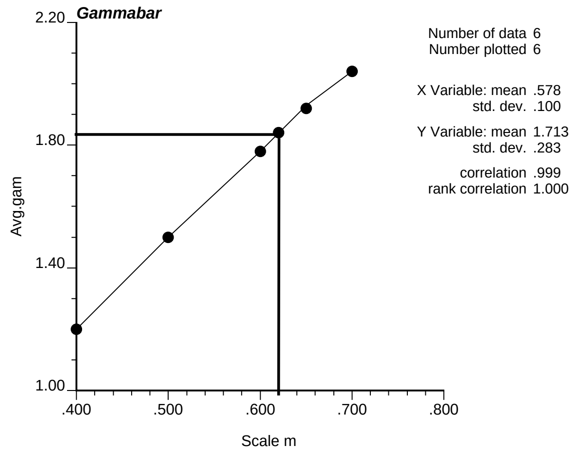

Figure 6: cross plot of the average variogram $\overline{\gamma}(V,V)$ versus the volume V for length scales close to the tool investigation volume. The average variogram value of 1.84 is known experimentally permitting an estimate of the scale of investigation, that is, 0.62m

Prediction of Log-Scale Variogram from Core-Scale Variogram

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The theoretically derived variogram for log-scale may be calculated and compared to the experimental variogram from log-scale data. The closeness of the match is a measure of the efficacy of the scaling relations described above.

As stated above on equation (3), the range of the log-scale variogram range a_V may be calculated based on the core-scale range a_v, that is, $a_V = a_v + (|V| - |v|)$. Where v is the core-scale of dimension 2 cm and V is the coarse-scale at 60 cm resolution (corresponding to the vertical logging-tool resolution). This results in a range correction for the spherical structure from the core-scale to log-scale as follows:

$$a_V\, sph \;= 0.54 + (0.60 – 0.02) = 1.12 \text{ m}$$

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For the hole effect structure the peak distance is not changed from the original 0.95 m.

The sill of each basic structure in the variogram model is modified as in equation (5). The required $\overline{\Gamma}(v,v)$ and $\overline{\Gamma}(V,V)$ values can be calculated with the `gammabar` program, using as input the point-scale variogram which has been derived earlier. For the spherical structure, $\overline{\Gamma}(v,v)sph =$ 0.0192 and $\overline{\Gamma}(V,V)sph =$ 0.499. The variance for the core data for the spherical structure $C_v sph =$ 2.82, and we therefore derive the log-scale sill for the spherical structure $C_V sph =$ 2.82 *(1-0.499/1-0.0192) = 1.44.

Likewise for the hole effect structure, $\overline{\Gamma}(v,v)hole =$ 0.00036 and $\overline{\Gamma}(V,V)hole =$ 0.29. The variance for the core data hole effect $C_v hole =$ 1.2, and we therefore get the log-scale sill for the hole effect structure $C_V hole =$ 1.2 *(1-0.29/1-0.00036) = 0.85.

The sill value for the log scale data is predicted to be the sum of these two contributions of 1.44 and 0.85 = 2.29, which compares closely to the actual sill of 2.23 found in the log porosity data set. The theoretical variogram model derived is compared to the experimental data from the log-scale data, see Figure 7.

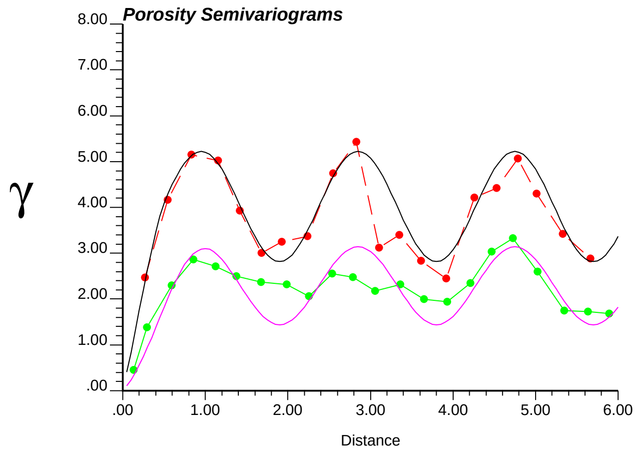

Figure 7: Core scale experimental variogram (upper bullets connected by dashed line), core scale variogram model (upper solid line), log-scale experimental variogram (lower bullets connected by line), and theoretically derived log-scale variogram (lower solid line). Note the close agreement between the theoretical prediction and the experimental points.

Discussion

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The theoretical upscaling seems to match very well the actual data despite some uncertainties regarding the size of the log-scale averaging volume. In fact, the back-calculation of the averaging volume from the spatial statistics could prove a useful supplement to analytical calculations regarding well log tool response.

This short note on geostatistical scaling laws opens a number of research avenues that will be explored in the future. In particular, a closer look at the underlying assumptions such as linear averaging, no shape-change in the variograms and the adequacy of two-points statistics.

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