A Multilevel Technique for Dynamic Data Integration in Reservoir Characterization

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Abstract

Reliable reservoir performance forecasts, with least possible uncertainty, have become imperative for reservoir management. This is particularly in view of the increasing competitiveness of petroleum energy production. Forecasting subsurface properties requires integrating all available information with varying scales, accuracy and precision of the data involved. This includes coupling of information that are static in nature with those that are dynamic. Linking these two essentially different forms of information calls for an inverse problem, where one aims to build a model based on its responses. Model parameters are mostly static except for changing geomechanical conditions, but the responses are dynamic. The space of the model parameters is essentially infinite, but limited by some forms of discrete static data (e.g., well logs, core data, seismic data, etc.) and dynamic information at certain points in time (e.g., production history, well test data, time-lapse seismic data, etc.).

This research focuses on dynamic data integration in a hierarchical manner. A coarse level optimization scheme is employed first to obtain optimal corrections of the initial model parameters. These corrections are transferred into finer grid levels through some prolongation scheme.

Introduction

Intense competitiveness in petroleum energy production and high level of uncertainty in subsurface property characterization force the reservoir management to explore various ways to optimize its operation. One option for the management is to acquire more information about the reservoir through more exploratory wells in order to reduce the uncertainty. This is not always viable as any such project requires huge capital expenditure and infrastructure. On the other hand, static data integration, to some degree, lends itself to better characterization of the reservoirs. However, dynamic nature of subsurface phenomena, multi- physics of various recovery processes, multi-scale reservoir heterogeneities, irregular geometries with internal boundaries such as lithofacies bounding surfaces and faults, etc. evoke the need for dynamic data integration in reservoir characterization.

The problem of integrating dynamic and static data in reservoir characterization is not new. Researchers have been involved in this field since the sixties, but in the "guise" of history matching. However, the approach of history matching is ideologically different from the present state of dynamic data integration in reservoir characterization. With the emergence of geostatistical tools, the prospects for better, efficient dynamic data integration have brightened. Exploring new ideas, methods to characterize difficult, complex reservoir conditions have become commonplace, although it is yet to attain a state of maturity.

Envisaging the need for better methodology for more complicated reservoir scenarios, this research was undertaken. This research will focus on early reservoir management decisions where there

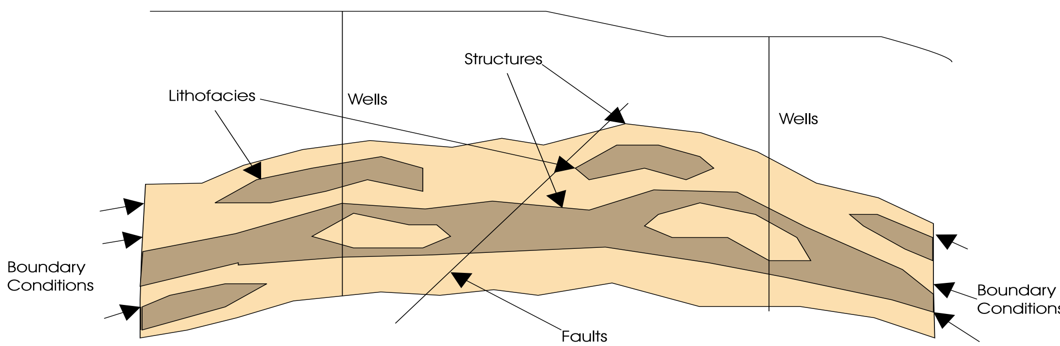

Figure 1: A schematic representation of reservoir model. Inversion for structural boundaries, boundary conditions, fault transmissibilities and lithofacies will be considered.

are sparse data and primary depletion is still the recovery process. Reservoir boundary delineation and identifying platform and drilling locations is of major concern, and a high risk is involved in the decision making process. The problem under study is an inverse problem where one is involved in the estimation of the model parameters on the basis of the dynamic responses and static information. It can be formulated as an optimization problem coupled with a forward problem. Here, a 3D 3-phase finite-difference reservoir simulator will be used for the forward problem. The reason for using finite-difference simulator is its capability of handling complicated processes which cannot be truly simulated with streamline methods [40, 43, 44, 55, 151, 153, 154, 158, 159, 160, 161, 162] or the tracer assumption [41, 45, 49, 64, 110, 112, 115, 138, 142]

Inverse problems are essentially infinite dimensional. Here, the dimensionality of the solution space (kernel) will be restricted based on a similar "master point" notion of Sequential Self-Calibration (SSC) method [159, 160, 161, 162]. The only difference is that instead of picking these points in the fine grids, entire grid locations in the coarse level will be used as the "master point"s. Further constraining of the kernel will be based on some a posteriori probability density functions or estimates of the parameter vectors at the master point locations from previous optimization or static data integration loops. The major hurdle be to overcome the ill- posedness of the inverse problem. Although master point concept reduces the level of ill-posedness, recourse of further regularization will be required for the optimization algorithm. Proper formulation of the method awaits the course of the research.

Joint inversion of permeability and porosity will be explored in this study. Although in few cases joint inversion of permeability and porosity has been performed in reservoir characterization [99, 120, 165], many of them use some simple correlations to infer one from the other [99]. Primary recovery process with multiple compressible phases will be accounted for. A permeability tensor with diagonal terms will be incorporated. Constraining the boundary conditions will be the most important issue to tackle from the objective point of view. There are some studies that have already looked into identification of geological shapes [99, 127, 128, 129]. The focus here will be on aquifer influx and support, fault transmissibilities, and structural boundaries. Incorporating seismic interpretation will be important particularly for structural boundaries and fault locations. Surface-based modeling, lithofacies proportions maps, and structure preserving algorithms will be a few options to explore. Figure 1 illustrates a schematic representation of a reservoir model. A good initialization step is important for the dynamic data integration. Otherwise, the methods can be computationally inefficient.

Some Aspects of Dynamic Data Integration

Characterization of detailed 3D reservoir models amounts to working in an almost infinite dimensional space with multitude of parameters to be estimated. The problem is transformed into a finite dimensional one, albeit non-unique. Non-uniqueness arises because of attempting to derive continuous functions or parameters from a limited number of responses. Also, inadequate modeling of the physics, flow mechanisms, or even procedures lead to non-uniqueness. For instance, one may often have to correlate some calculated or estimated parameters like acoustic impedance with some other variables such as porosity, permeability, fluid saturations affecting those measurements.

Lack of continuous dependence leads to instability and hence induces further non-uniqueness to any such parameter estimation problem. Due to its unstable nature, small errors in the data propagate into the model. Robustness which is a measure of the level of insensitivity with respect to extreme values is an important issue.

Numerous data of different types are being dealt with. There may exist some inconsistency between the data. This inconsistency may be due to different levels of accuracy within the same data type or different types of data. Also, some data may be in the time domain while some in the space domain.

Moreover, the scales or volumetric supports of various data may lead to inconsistencies. For example, well logs have a different volumetric support than well test data or core plug data. Inconsistency may also arise through application of different methods.

Literature Review

This is a compendium of the available techniques in the dynamic data integration literature. The classification of the techniques presented in subjective. A common ground of almost all the approaches is the notion of formulating a misfit or mismatch functional on which some minimization algorithm is imposed. Furthermore, in many formulations, the problem is ill-posed particularly because of the non-uniqueness of the solution space (model space) and the lack of continuous dependence. A natural consequence is that many techniques attempt to make the problem well-posed, or in mathematical parlance, regularized.

A thorough review of the subject of parameter identification in reservoir simulations is also given by Jacquard and Jain [88], Gavalas et al. [62], Watson et al. [156], Feitosa et al. [60, 61], or Oliver [119]; and by Yeh [170] and Carrera and Neuman [19] in groundwater hydrology. The classes of techniques are broadly the following:

Classical Inversion Techniques

These are early approaches to the integration of pressure transient data in geological modeling using solely inverse techniques for parameter identification or history matching [5, 12, 13, 22, 25, 26, 35, 56, 57, 66, 89, 125, 146, 148]. An important aspect of the gradient based history matching techniques is computation of the gradients or sensitivity coefficients. Schemes like perturbation methods, rigorous finite differencing of the physical flow equations, convolution integral method, optimal control theory, have been utilized.

Generalized Pulse-Spectrum Techniques and Other Regularization Based Techniques

To tackle the intrinsic problem related to stability and efficiency, a versatile technique was devised by Tsien and Chen [152]. Since its inception in 1974, the technique has been subsequently modified and improved further by others [27, 28, 29, 30, 31, 32, 71, 100, 106, 107, 108, 147]. Essentially the Generalized Pulse-Spectrum Technique (GPST) is a combination of a Newton-like iterative algorithm and Tikhonov regularization method.

Another numerical method based on regularization techniques proposed by Kravaris and Seinfeld [95, 96, 97] appears particularly suitable for two-dimensional single-phase simulator models [103] and for two-phase models [104]. These methods apply Tikhonov regularization method first. The well-posed problem is then solved by the partial conjugate gradient method of Nazareth [113].

Bayesian and MultiGaussian Approaches with *A Priori* Information

A Bayesian estimation framework was proposed by Gavalas et al. in 1976 [62] for reservoir modeling using dynamic production data. The underlying theory behind this technique is to reduce the statistical uncertainty by using additional prior information, for instance autocorrelation and mean values of permeability and porosity. Some later works in this area are cited in [34, 36, 38, 76, 114, 132, 141, 150].

Maximum likelihood methods [18, 19, 20, 21, 59] are also used for parameter estimation with dynamic data. This is a general non-linear technique that estimates reservoir parameters using prior estimates along with transient or steady state pressure data. Parameter estimation is performed using the maximum likelihood theory, incorporating the prior information into the likelihood function.

Reparameterization based on spectral decomposition reduces the number of the parameters to be estimated by the Gauss-Newton procedure [33, 119]. More recently, reparameterization techniques based on subspace method are presented to further improve the computational efficiency in the Gauss-Newton procedure [1, 73, 74, 122, 131].

Wu et al. [165] developed a discrete adjoint method for generating sensitivity coefficients related to two-phase flow production data. Cunha et al. [37, 121] used a hybrid Markov Chain Monte-Carlo algorithm to generate realizations of permeability conditioned to prior mean, variance and multi-well pressure data. Ates and Kelkar [6] devised a dual-loop multiphase production data inversion technique, which combines Gauss-Newton and Conjugate Gradient algorithms.

Zonation Methods

Some of the early methods [19, 34, 56, 88, 114] have already been grouped as the classical techniques for a historical perspective. While the zonation method is effective in reducing the number of unknowns, sufficient *a priori* information is not usually available to enable specification of the zones on any physical basis. Zonation methods are active research area. Amongst the newer methods are pilot point method, sequential self-calibrated method, and others.

Pilot point method [11, 46, 58, 102, 130] is a zonation method that starts by simulating a conditional transmissivity field. The generated field is then modified by adding additional or fictional transmissivity data at some selected locations, termed pilot points, to improve the calibration of the pressure data. Adjoint sensitivity analysis is used to determine the locations where additional transmissivity data should be included [101]. The iteration of adding pilot points is continued until the least-squared error criterion is met or the addition of more pilot points does not improve the calibration.

Sequential self-calibrated method [17, 63] combines geostatistical and optimization techniques. A geostatistical technique is used to generate a reservoir parameter model conditioned to local measurements of parameters. Initial model is modified to minimize the misfit function through an optimization procedure. In order to reduce the parameter dimension, the optimization is parameterized as a function of the perturbations of permeability at a few selected locations, called master points. The perturbation values at the master locations are determined from the optimization procedure by minimizing the squared difference of the simulated and observed pressures. The resulting perturbations are propagated throughout the entire reservoir domain by kriging to obtain the perturbation field that is subsequently added to the initial field. Promising results were obtained by using this approach in groundwater hydrology [157, 163, 164, 171].

Blanc et al. [14] presented a solution to the problem of constraining geostatistical models by well test pressure data similar to the pilot point method or sequential self-calibrated method. Xue and

Datta-Gupta [166] developed a two stage approach for a structure preserving inversion technique similar to pilot-point technique but incorporates the prior information in a different way. The covariance matrix is embedded in the parameterization of the permeability field.

Cokriging Based Methods

Kitanidis and his colleagues [79, 94] applied cokriging to simulate transmissivity and pressure fields using covariance or cross- covariance models based on field measurements of transmissivity and pressure. The cross-covariance between transmissivity and pressure is developed through linearization of the single phase steady state flow equation. Parameters in the covariance and cross-variance are estimated from the measured data and the linearized flow equation using a maximum likelihood method. Realizations are then constructed using Cholesky decomposition of the covariance matrix.

In linearized semi-analytical cokriging method [136, 137], a linearized form of the single phase steady-state flow equation is used to develop analytical expressions of cross-covariances of permeability and pressure assuming uniform flow and infinite domain. Transient pressure is accounted for with the linearity assumption between change of pressure and time.

Harvey and Gorelick [70] presented a cokriging method, combining numerical simulation of flow and tracer transport with a linear estimation, to construct permeability field that sequentially accounts for permeability, pressure and tracer arrival times. Yeh et al. [169] applied a similar but iterative technique to account for the nonlinear relationship between permeability and pressure in the estimation through successive linear approximation.

In another cokriging based method, FFT method is applied to the linearized steady-state flow equation [67, 68]. Transmissivity realizations conditioned to the pressure data are constructed by adding the difference between the unconditional simulation and kriged values of the unconditional simulation to the kriged values using the field data [47, 93]. Other kriging based approaches in the literature are [82, 86, 87, 143].

Simulated Annealing Based Techniques

Ouenes and his colleagues [123, 124, 144] employed simulating annealing for automatic history matching. Petrophysical and reservoir engineering parameters are estimated through an automatic and multiwell history matching using simulated annealing method. A least- square error objective function defined by the oil, gas, and water productions at each well is minimized by the simulated annealing method. At each iteration in the simulated annealing method, a limited number of reservoir parameters are adjusted. The impact of these new parameters on the objective function is evaluated by forward reservoir simulation, which is too costly for routine application for large number of parameters and iteration steps used in this approach.

In another simulated annealing approach proposed by Tauzin [149], the objective function is evaluated analytically which improves the computational time. An analytical influence function is defined to approximate the perturbation on the pressure transient due to a local heterogeneity. This influence function is derived from the analytical solution of transient pressure in an infinite homogeneous reservoir containing a single circular discontinuity from Rosa and Horne [135]. This approximation is usually sufficiently accurate to predict the direction and the order of magnitude of the pressure perturbation caused by the permeability perturbation. Other notable and improved simulated annealing based techniques are developed by Datta-Gupta et al. [40, 42, 45], Vasco et al. [154] and Maroongroge et al. [110].

Inversion Techniques for Statistical Parameters or Constraints

Most modern indirect inversion techniques fall into this category. Production data can be used to estimate statistical parameters, such as the mean, covariance, or the fractal dimension that describe the spatial distribution of reservoir properties. These parameters are subsequently used to characterize the reservoir.

These indirect techniques seek to construct geological models that honor critical features interpreted from the production data. Some relationship is first established between the production data and some reservoir parameters or their spatial variation. This relationship then serves as a constraint in the construction of the geological model so that the production data are indirectly integrated into the reservoir model.

The first step is to analyze transient production data and infer spatial heterogeneity features of the underlying reservoir model. These heterogeneity features may be in the forms of general information on the degree of heterogeneity, anisotropy and zonation of the reservoir properties; the presence of internal or external reservoir boundaries such as faults, lithofacies changes, water-oil contacts, stratigraphic pinchouts; the presence of high flow channels or low permeable zones in an area and the distance to these zones; in multiple well systems, water breakthrough time and recovery efficiencies inform connectivity between wells; effective transmissivity and facies proportions in the wellbore vicinity, etc. [7, 8, 9, 16, 23, 24, 64, 65, 72, 75, 80, 105, 109, 126, 155, 167, 168].

Once the statistical parameters are estimated, they are used in geostatistical techniques to construct reservoir models. The contribution of production data lies in the improvement in the estimation of statistical parameters describing the reservoir heterogeneity. In some cases, such as when the reservoir parameters are Gaussian, and the relationship between the reservoir parameter and pressure data are linear, the constructed geostatistical reservoir model may also directly honor the pressure data. Another approach is to infer parameters of the heterogeneous reservoir model from the production data and then constrain the reservoir models to those inferred parameters.

Effective permeability within the drainage area of the well obtained from well test data [81, 139] does not resolve local details of the spatial distribution of permeability. However, well- derived effective permeability can be regarded as the average value of the heterogeneous permeability values in the vicinity of the test well [2, 116]. Deutsch [50, 51, 52, 53, 54] presented an approach, based on simulated annealing, that integrates well test-derived effective permeabilities in stochastic reservoir models. The volume and type of averaging formed by the well test are first calibrated by forward simulating the well test on a number of stochastic reservoir models that are consistent with the geological interpretation, core, well log, and seismic data. The effective permeability from the well-test is assumed to be the power average of the heterogeneous permeability within the influence volume of the well test [2]. The optimal volume and power parameter for the averaging process are obtained from the calibration as suggested by Alabert [2]. Stochastic reservoir models are then constructed with simulated annealing to honor the well-derived average permeabilities. Results showed the improvement in characterizing permeability heterogeneity and waterflooding predictions when the effective permeabilities are constrained in the model.

Similar philosophy in a broader perspective has been implemented in the literature [3, 4, 10, 15, 39, 48, 49, 60, 61, 69, 77, 78, 80, 83, 84, 85, 90, 91, 92, 99, 111, 117, 118, 120, 127, 128, 129, 133, 134, 145, 140, 155]. Indirect methods provide flexibility to account for production data in the construction of reservoir models with less computational effort than full inversion. However, the success of these techniques in constraining reservoir models essentially relies on the quality of the interpretation of production data in retaining reservoir heterogeneous features.

Proposed Methodology

The philosophy of the research focus is simple and realistic. The objective is to devise a method that can integrate the dynamic and static data under consideration in an efficient manner which will supplement to our knowledge of the reservoir system and the pertaining heterogeneity. Ideally one should be able to integrate the entirety of the information about the system. This is not pragmatic. The course of the research is segmented in modules to render the process more tractable. However, efforts are made to ensure the compatibility of these modules.

The main stages of the research are:

- Static data integration.
- Algorithm for assignment of master-point location.
- Options for forward simulator.
- Initialization for the optimization problem.
- Implementation of the optimization algorithm.
- Interpolation of the optimal parameters to the finest grid.
- Additional cycles for iterative improvement.
- Inversion for fault transmissibilities.
- Inversion for lithofacies bounding surfaces and structural boundaries.
- Inversion for aquifer influx and support.
- Simultaneous inversion for many parameters.

Static Data Integration

The parameters we are primarily interested in are the permeability fields ($k_{x_{i,j,k}}$, $k_{y_{i,j,k}}$, $k_{z_{i,j,k}}$) and porosity ($\phi_{i,j,k}$). There is very little static information available for the saturation which precludes its inclusion at this inversion stage. The information considered will be well log and seismic data mainly. Some indicator formalism for lithofacies will be implemented at this stage.

The static integration technique is laid out below:

- Indicator simulation of lithofacies.
- Cosimulating of porosity and seismic data with full block cokriging approach.
- Cosimulating of permeability fields and porosity with full cokriging.

To account for the different volumetric supports of various data, histogram correction and variance correction will be performed, particularly at the stage of static data integration. The algorithms and resources to implement the above is, to a large extent, available in the literature [54, 98].

Algorithm for Master-Point Location

Similar "Master point" concept from the SSC approach is adapted here [159, 160, 161, 162]. Essentially, this can be considered as a 'contraction' of the domain. The only difference is that instead of picking these points in the fine grids, entire grid points in the coarse grid will be taken as master points. This reduces the dimensionality of the parameter space for the inverse problem. Optimization, discussed later, will be performed on the master point perturbations. The optimal perturbations at the master points will be projected into the entire space. This projection is analogous to 'prolongation' of the restricted space into the original domain.

In this research, the masters points will be considered as the points in the coarsest grid. This covers all the finer grids in the system. From this standpoint, this formulation of the master points will also be a surjective (onto) mapping of the entire domain. The dimensionality of the master points will be governed by the number of levels to be considered in the simulator, and to some extent by the overall fine grid dimensionality.

Options for Forward Simulator

Integration of dynamic data in reservoir characterization is essentially a parameter estimation problem or more precisely an inverse problem. A core issue in resolving an inverse problem is the development of a corresponding forward problem. Sensitivity of the underlying physical process or its numerical model to the desired parameters to be estimated is directly associated to the gradients in the forward numerical model. Thus in order to estimate the optimal or true parameters, a tedious amount of effort is directed to the development of the numerical model.

At the onset of this dynamic data integration research two lines of thoughts have been considered. One option was to develop an indigenous reservoir simulator and compute the required gradient matrix using simulation runs. The other was to explore reliable commercially available simulators to obtain the gradients. There are advantages and disadvantages for both the avenues. A notable advantage of the first option is that everything within the simulator is known and can be manipulated as needed. Disadvantages are that it requires a considerable amount of time and effort, and in spite of that it is not possible to have sophisticated features available in the commercial simulators. Validation of the developed model can be another disturbing factor. On the contrary, an enormous amount of man-hours have been exerted to wield reliable and proven performances from simulators like Eclipse, CMG or VIP. Credibility of the results of these simulators is ubiquitous. Only disadvantage is one probably has to use these simulators as engine or 'black box' to derive the desired outputs. Both the options were explored at the beginning. The viability of this option has been reconsidered, and the option discontinued at the moment. However in future if it is essential to build the simulator, it will be pursued from the documented state of being.

Nonetheless, the algorithm for the Jacobian is presented below. The finite-difference formulation of the flow equations for 3D 3-phase reservoir simulation can be written as:

$$[A][P]^{n+1} = [B][P]^n + [C] = [f] \tag{1}$$

where $[P]^{n+1}$ is the solution vector with $[P]^{n+1} = [P_o \;\; S_w \;\; S_g]^T$, and $[A]$, $[B]$ and $[C]$ are coefficient matrices which are also implicit functions of $[P]^{n+1}$, which makes the equation strongly nonlinear and coupled.

The sensitivity matrix or the Jacobian for the optimization problem with respect to any parameter matrix, $[\Theta]$, can be formulated as

$$\frac{\partial [P]^{n+1}}{\partial [\Theta]}$$

which can be computed indirectly from Equation 1. For the objective this thesis, $[\Theta]$ consists of the parameter space for which the inversion is being performed, that is, $[\Theta] = [[k_x] \;\; [k_y] \;\; [k_z] \;\; [\phi]]$. The dimension of the parameter space will then be $n_{k_x} + n_{k_y} + n_{k_z} + n_\phi$. For the multigrid scheme $n_{k_x} = n_{k_y} = n_{k_z} = n_\phi = n_{G^0}$. Here n_{G^0} is the dimension of the coarsest grid G^0.

The sensitivity matrix will be defined in the the coarsest grid G^0 by the following equation.

$$\frac{\partial [P]^{n+1}_{G^0}}{\partial [\Theta]} = [A]^{-1}_{G^0} \left[\frac{\partial [f]_{G^0}}{\partial [\Theta]} - \frac{\partial [A]_{G^0}}{\partial [\Theta]} [P]^{n+1}_{G^0} \right] \tag{2}$$

where

$$\frac{\partial [f]_{G^0}}{\partial [\Theta]} = \frac{\partial [B]_{G^0}}{\partial [\Theta]} [P]^n_{G^0} + [B]_{G^0} \frac{\partial [P]^n_{G^0}}{\partial [\Theta]} + \frac{\partial [C]_{G^0}}{\partial [\Theta]} \tag{3}$$

$[A]^{-1}_{G^0}$ is already computed in the forward simulation stage. Matrices $\frac{\partial [A]_{G^0}}{\partial [\Theta]}$, $\frac{\partial [B]_{G^0}}{\partial [\Theta]}$, and $\frac{\partial [C]_{G^0}}{\partial [\Theta]}$ can be computed directly from the expressions in matrices $[A]_{G^0}$, $[B]_{G^0}$, and $[C]_{G^0}$, respectively with $\frac{\partial [P]^0_{G^0}}{\partial [\Theta]} = 0$.

Initialization for the Optimization Problem

Almost all the optimization algorithms are based on measuring the effects on the kernel for small changes, that is, perturbation at certain locations. Optimal perturbations are determined on a 'restricted' space based on some norm sense, in most cases L_∞ or L_2 norms. These optimal perturbations are propagated or 'prolonged' to the entire discretized space. The validity of these methods rely on the low magnitude of the perturbations. This notion emphasizes the need for a good initialization to make the method computationally more efficient. Although the algorithm should converge with any initial model, a good starting point will improve the process.

The initialization step will account for the dynamic data in a crude manner along with all the static data. As production/injection takes place, the observed flowrate and pressure values inform about the instantaneous influence areas of the observation points. The initialization will focus on well tests and not on the production history. This is because well tests are the most common reservoir engineering diagnostic tools for the reservoir system. The time derivative of the pressure is a good indicator of the reservoir heterogeneity in the vicinity of the wells. For a homogeneous system, the pressure derivative will be almost constant provided all other conditions remain the same. Any change in the permeability or porosity (to lesser extent) distribution will be reflected on the pressure derivative. A brief algorithm is given below to incorporate this dynamic data in the initial characterization stage.

- Identify the starting time t_{min} and end time t_{max} of the well test data using standard correlations.
- For all time intervals $\in (t_{min}, t_{max})$ obtain initial k_{eff} values. It should be noted that the k_{eff} values at any interval will have annular area of influence.
- Compute weighting parameters based on the kernel functions (Oliver [117, 118, 119]).
- Assign all the grids (fine grids in the composite grid scheme) the weighted permeability values. It must be noted that the global fine grids are assigned with the permeability and porosity values in the static data integration stage. At this stage, the fine grid values will be reassigned or improved. The dimensionality of the local fine grids (Ω_F) can also be defined based on the influence area of the well tests.

Directionality in the permeability distribution at the fine grid cannot be identified with this method.

Implementation of the Optimization Algorithm

The main objective of this research is to generate reservoir models that honor static and dynamic data. This problem is formulated as an optimization problem where a mismatch function needs to be minimized. The mismatch function is calculated based on the errors, that is, the differences between the calculated and the observed pressures and fractional flow data weighted with the covariance matrices of the observation errors. Engineering heuristics and data inaccuracy can be accounted for with the weights. In that case, weights will not be based solely on the covariance matrices of the errors. Thus the mismatch or objective function can be generalized by:

$$O = \sum_i \sum_t W_p(i,t) \left[p_i^{obs}(t) - p_i^{cal}(t)\right]^2 + \sum_k \sum_t W_f(k,t) \left[f_k^{obs}(t) - f_k^{cal}(t)\right]^2$$

$$+ \sum_{l=1}^{2} \sum_c \sum_t W_{S_l}(c,t) \left[S_{l,c}^{obs}(t) - S_{l,c}^{cal}(t)\right]^2 \quad (4)$$

where $p_i^{obs}(t)$ and $p_i^{cal}(t)$ are the observed and simulated pressure at well i at time t. $f_k^{obs}(t)$ and $f_k^{cal}(t)$ are the observed and simulated fractional flow rate at well k at time t. The third term in

Equation 4 is required for the inversion of saturation data. Although inversion of saturation data is not a focus at the initial stage of the research, it is shown here to give a more general definition of the objective function. $S_{l,j}^{obs}(t)$ and $S_{l,j}^{cal}(t)$ are the observed and simulated saturation of phase l at cell c at time t. $W_p(i,t)$, $W_{S_l}(c,t)$ and $W_f(k,t)$ are weights assigned to pressure, saturation of phase l, and fractional flow rate data at respective locations at different times. In the case of 3-phase reservoir simulation the fractional flow should be modified to account for any 2 phases, as the fractional flow of the third phase will be redundant. Inversion of saturation data on every grid cell of the domain or its subdomain is possible. However, it will not be implemented at the initial stage of the research.

The objective function is a strongly nonlinear function of the kernel of the inverse problem. A first order linearization can be done on the objective function using Taylor's approximation of the pressure data, that is,

$$[P]_{G^0}^{n+1,1} \approx [P]_{G^0}^{n+1,0} + \frac{\partial [P]_{G^0}^{n+1}}{\partial [\Theta]} [\triangle\Theta] \tag{5}$$

where $[P]_{G^0}^{n+1,0}$ and $[P]_{G^0}^{n+1,1}$ are simulator response variables (i.e., pressure, water and gas saturation) at the coarsest grid G^0 at time interval $n+1$ before and after introducing a perturbation matrix $[\triangle\Theta]$. With this linear approximation, a simplified form of the objective function (Equation 4) can be obtained. This linearization will be done to accelerate the optimization process.

The above formulation is a standard quadratic optimization problem. It will be solved using a modified gradient projection method with additional local and global constraints which are yet to be formulated. At each iteration of the optimization process, the search direction is obtained by projecting the gradient of the objective function on the null space of the gradients of the binding constraints [63].

Interpolation of the Optimal Parameters to the Finest Grid

As stated earlier, optimization will be done at the coarsest grid level. Now the issue is to interpolate the coarse grid optimal perturbations into the finest grid. This can be done in a similar manner as in the multi-grid technique in the forward simulation problem. The 'prolongation' or 'interpolation' matrices I^i at different grid level i can be used. The global covariance should not be altered with this method, because the coarse grid values will take into account the global distribution functions of the parameters. However, the local distribution might be changed. To avoid this, a weighted averaging interpolation will be formulated.

Additional Cycles for Iterative Improvement

In the above formulation of the reservoir characterization problem, there is a possibility that the final models generated at the finest grid might not exactly replicate the situation of minimized objective function. In order to attain better match with the fine scale model, it is required to implement a practical 2- or 3-cycle additional recursions of the entire algorithm. It is believed that the additional cycles will improve the overall fine scale model resolution with reduced observational errors.

Inversion for Fault Transmissibilities

Simply faulted reservoirs will be considered. Fault transmissibilities can be principally defined by the directional permeabilities at the faulted grid blocks. Refined global grid will be used to resolve the transmissibilities better. Figure 2 illustrates an example of a faulted grid to be used in the multi-level grid scheme. The faulted grids could have higher transmissibilities than the neighboring grid blocks or lower if shales or clay have been "dragged" into the fault zone. Also, the transmissibility along the strike (plane) of the faults may be higher than perpendicular to it. These notions will used in the inversion algorithm for fault transmissibilities.

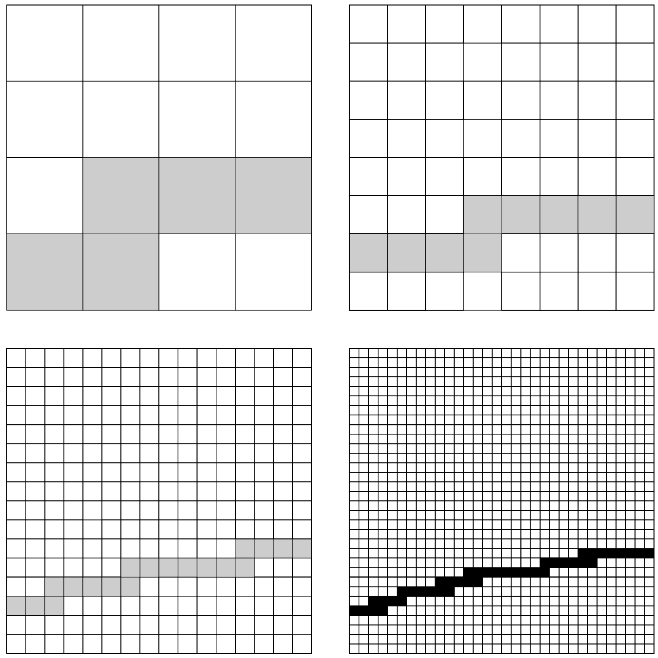

Figure 2: A schematic representation of the faulted multigrids to be used for the inversion of fault transmissibilities. Light shades in the coarser grids indicate the fault grids. Dark grids in the finest grid scale denote the fault grids.

The locations of the faults can be approximately determined from seismic data. Refined grid will improve the resolution of the fault locations. In the optimization stage, the perturbation parameter space will be modified to include the order transmissibility ratios between the faulted and regular grid. A sequential optimization loop can be implemented to account for the fault grid locations at the finest scale and then the transmissibilities.

Inversion for Lithofacies Bounding Surfaces and Structural Boundaries

Inversion for structural boundaries and lithofacies bounding surfaces can be implemented in a similar manner to simply faulted reservoirs. Grid refinement will be used, as for the inversion of faults, to resolve the boundary grids. An initial delineation of the structural boundaries and also the lithofacies bounding surfaces will based on the seismic data on a coarse scale. Optimization will be performed for higher resolution boundary grid locations. The directional permeabilities and porosity will be sufficient with respect to the perturbation parameter space to be used to tackle the optimization algorithm.

Inversion for Aquifer Influx and Support

Inversion for aquifer influx and support will be relatively difficult. Influx can possibly be identified with an optimization on the directional permeabilities and the porosities. However, the aquifer support will be extremely difficult to determine. A simple approach will be to analyze the the changes in the influx over time. Notions from Hurst-van Everdingen and Fetkovich aquifer models may be applied to invert for the aquifer issues. Another option will be to treat the aquifer grids as wells; thereby inversion can be performed for permeability and porosity of these grids.

Simultaneous Inversion for Many Parameters

An integrated approach to allow for simultaneous inversion of complex heterogeneous system with the above three characterization issues will be implemented. However, the time frame of this research and manageable computational algorithm will dictate the implementation.

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