A VisualBasic Program for Histogram and Variogram Scaling

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Abstract

The `VarScale` program (1) calculates dispersion variances for different support volumes, (2) performs variogram and histogram scaling, and (3) performs scaling of a linear model of coregionalization (LMC). The input variogram may be at any scale. The required dispersion variances (or average variogram values) are calculated. User input is automaticly passed between different components of program.

This paper describes the theoretical background of the scaling algorithms, the program structure of `VarScale` and presents three examples. Variogram scaling from core scale to well log scale is illustrated with data from a Danish North Sea reservoir. Histogram and LMC scaling are illustrated with data from a West Texas reservoir using porosity and seismic data.

Keywords: geostatistics, affine correction, indirect lognormal correction, average variogram, dispersion variance, upscaling.

Introduction

Scaling is important for petroleum, mining, environmental and other fields of spatial modeling. Predictions based on geostatistical models mistakenly using point scale statistics may be misleading and can drastically affect the recovery estimates of a petroleum reservoir [1, 9]. In environmental applications, the assessment of pollutant concentration varies with the volume of the sample considered. Measured data often have different volume scale than the volume of the grid cells of our model. We know that variability decreases as the volume increases. This must be accounted for in the histogram and variogram of geostatistical modeling.

`VarScale` uses conventional volume-variance scaling relationships [7, 8]. For histogram correction, there are several procedures to correct to a different volume scale or support. The procedures differ in the way that they implicitly handle the degree of symmetrization. The choice of a particular method depends largely on the degreee of symmetrization we expect. The `VarScale` program utilizes the *affine correction* and the *indirect lognormal correction* to scale distributions. The affine correction assumes no symmetrization whereas the indirect lognormal assumes a moderate amount.

There are many applications where estimates can be improved if the correlation between different variables is included. For each new variable included, one needs its variogram

(auto-variogram) and cross-variogram models between it and all other variables. The linear model of coregionalization (LMC) provides a method for modeling the auto and cross-variograms of two or more variables so that the variance of any possible linear combination of these variables is always positive. Most geostatistical modeling applications, such as cokriging, can use data at different scales; however, this flexibility requires a linear model of coregionalization at the smallest data scale. Yet, different data sources are often at different volumetric scales, for example, porosity from well logs and seismic values at a significantly larger scale. `VarScale` will perform the necessary scaling of the variogram of the primary and secondary variables and the cross-variogram to the modeling scale.

This paper presents `VarScale`, which permits histogram, variogram and linear model of coregionalization scaling. The input variogram model may be at any scale (different than the point support); then, `VarScale`, first scales down to point scale and then goes from point scale to the intended scale. It calculates dispersion variances for different support volumes. Various other features of `VarScale` will be stated in the following sections. In the Theory section, detail of the histogram correction methods and the variogram scaling technique will be discussed. Furthermore, inference of licit small-scale model of coregionalization with a combination of small-scale primary data and large-scale secondary data, will be covered in the Theory section too. Program Design part explains how all these scaling operations along with the calculation of dispersion variance are handled by `VarScale`. The application of `VarScale` will be demonstrated using data from a West Texas reservoir and a Danish chalk reservoir. Core porosity data from a West Texas reservoir will be used for histogram scaling and coupling of the core porosity with seismic data from the same reservoir will be used for getting licit small-scale coregionalization model. Core and log data, from a Danish reservoir, will be used to show how the variogram model obtained at core scale can be scaled to obtain the appropriate variogram model at log scale.

Theory

Histogram Scaling

The histogram of available sample data must be scaled to represent the volume support of the grid cells we intend to work on. This correction is made based on assumptions about how the histogram changes as volume support changes. There are various procedures for distribution correction; however, they have two common features:

- They leave the mean of the distribution unchanged, and

- They adjust the variance by a variance reduction factor, f, which comes from the variogram.

If extreme values are poorly connected, then we should choose a procedure that implicitly increases the symmetry of the distribution, if the the extreme values are well connected, then we might prefer a procedure that does not implicitly increases the symmetry of the distribution as the support increases [6].

`VarScale` considers two techniques of support effect correction. The affine correction leaves the shape unchanged which is appropriate for "good" connectivity of extremes. The indirect lognormal correction assumes an intermediate amount of symmetrization, applicable to more "disseminated" cases. The discrete Gaussian model is not included in this

version. In `VarScale`, users have the flexibility of specifying the variance reduction factor f directly or the program will calculate it from the variogram model (see later).

Affine Correction

The basic idea behind the affine correction is to move all values closer to the mean by a specified factor. The affine correction transforms a quantile (or a value) q of distribution to another quantile, q' of the variance-reduced distribution using the following linear formula:

$$q' = \sqrt{f} \cdot (q - m) + m \qquad (1)$$

The mean of both distributions is m, the variance of the original distribution is σ^2, the variance of the transformed distribution will be $f \times \sigma^2$.

By using the linear equation for transformation, the affine correction preserves the shape of the original distribution.

The main advantage of the affine correction is it simplicity, however, it produces minimum and maximum values that may not be realistic. If the variance reduction factor is not too small (greater than 0.7), then the affine correction procedure is often adequate.

Indirect Lognormal Correction

While distributions may differ from the lognormal distribution, change of support may affect them in a manner similar to that described by two lognormal distributions with the same mean but different variances. The q-q curve that transforms the values of one lognormal distribution to another with the same mean but a different variance has an exponential form:

$$q' = aq^b \qquad (2)$$

The coefficient, a, and the exponent, b, are given by the following equations:

$$b = \sqrt{\frac{\ln(f \cdot CV^2 + 1)}{\ln(CV^2 + 1)}} \qquad (3)$$

$$a = \frac{m}{\sqrt{f \cdot CV^2 + 1}} \left(\frac{\sqrt{CV^2 + 1}}{m} \right)^b \qquad (4)$$

In these formulas, CV is the coefficient of variation obtained by dividing standard deviation σ by the mean m. The problem with the direct application of Equation 2 is that it does not preserve the mean if it is applied to values that are not exactly lognormally distributed. The indirect lognormal correction, for that reason, rescales all of the values from the transformation given in Equation 2 so that their mean is m:

$$q'' = \frac{m}{m'} q' \qquad (5)$$

where m' is the mean of the distribution after it has been transformed by Equation 2. This procedure requires two steps; first the values are transformed according to the Equation 2 and then, they are rescaled to the correct mean.

There are two important differences between the results of the indirect lognormal correction and the affine correction: the skewness decreases as the variance is reduced and the minimum stays at 0. It is preferable to use the indirect lognormal correction if the preservation of shape implicit in the affine correction is unrealistic.

3

Variogram Scaling and Dispersion Variance

The variogram model is associated to the volume support of the data. It is then necessary to scale the variogram to represent the volume that we are interested in.

Consider a variogram model at arbitrary scale v, where v often represents the small core scale:

$$\gamma_v(h) = C_v^0 + \sum_{i=1}^{nst} C_v^i \Gamma_v^i(h) \tag{6}$$

where $\gamma_v(h)$ is the variogram model at the v scale, C_v^0 is the nugget effect, nst is the number of nested variogram structure used to fit the variogram, C_v^i is the variance contribution of each nested structure, $i = 1, \ldots, nst$, and $\Gamma_v^i(h)$ are nested structures consisting of analytical functions. Here the "sill" of each nested structure $\Gamma_v^i(h)$ is unity, the C_v^i terms describe the variance contributions of each nested structure. The sum of the variance contribution is the variance at the v-scale and is a dispersion variance:

$$D^2(v, A) = C_v^0 + \sum_{i=1}^{nst} C_v^i \tag{7}$$

where $D^2(v, A)$ is the variance of volumes of size v in the entire area of interest A. The variance decreases as the volume increases. That is because high or low values are averaged out as the volume of investigation increases.

The following scaling relations are estabilished in geostatistics assuming that the actual shape of the variogram does not change [7]:

- The range at a large volume V, increases in volume size by:

$$a_V = a_v + (|V| - |v|) \tag{8}$$

  where $|\cdot|$ means the size of the volume in a specific direction. If V and v are the same size in one particular direction (say, horizontally), then the range does not change.

- Assuming the variogram shape does not change, we must only quantify how the variance contributions $C_v^i, i = 1, \ldots, nst$, change. The nugget effect is due to the random variation, and it decreases with an inverse relationship of the volume, i.e.,

$$C_V^0 = C_v^0 \frac{|v|}{|V|} \tag{9}$$

  where $|v|$, $|V|$ represent the volume of each scale, respectively. The nugget effect of "point" scale values is undefined. The nugget effect for any "finite" scale may be calculated by Equation 9.

- The variance contribution of each nested structure changes as following:

$$C_V^i = C_v^i \frac{1 - \bar{\Gamma}(V, V)}{1 - \bar{\Gamma}(v, v)} \tag{10}$$

  where $\bar{\Gamma}(V, V)$ and $\bar{\Gamma}(v, v)$ are the average variogram or "gamma-bar" values. Notice here the change in the variance contribution is calculated separately for each nested structure. The "gamma-bar" values are thus calculated based on the variogram of each nested structure (see below).

- When all variance contributions have been corrected for each nested structure, the sum of the coefficients $C_V^i, i = 0, 1, \ldots, nst$ give the dispersion variance implicit to the corrected variogram model.

In order to use the equations to correct the variogram models, it is necessary to have "gamma-bar" values. The "gamma-bar" value represents the mean value of $\Gamma(\mathbf{h})$ or $\gamma(\mathbf{h})$ when one extremity of the vector $\mathbf{h}$ describes the domain $v(\mathbf{u})$ and the other extremity independently describes the same domain $v(\mathbf{u})$. In mathematical notation the "gamma-bar" value is expressed as:

$$\bar{\gamma}(v(\mathbf{u}), v(\mathbf{u})) = \frac{1}{v \cdot v} \int_{v(\mathbf{u})} \int_{v(\mathbf{u})} \gamma(y - y') dy dy' \tag{11}$$

Although there exist certain analytical solutions [2, 7] to $\bar{\gamma}(v(\mathbf{u}), v(\mathbf{u}))$, the value of "gamma-bar" is usually estimated numerically by discretizing the volume $v(\mathbf{u})$ and $v(\mathbf{u})$ into a number of points and simply averaging the variogram values:

$$\bar{\gamma}(v(\mathbf{u}), v(\mathbf{u})) \approx \frac{1}{n \cdot n'} \sum_{i=1}^{n} \sum_{j=1}^{n'} \gamma(\mathbf{u_i} - \mathbf{u'_j}) \tag{12}$$

where n is the number of regular spacing point discretized for volume $v(\mathbf{u})$ and n' is the number of regular spacing point discretized for volume $v(\mathbf{u})$. Each point represents the same fractional volume of $v(\mathbf{u})$.

Estimation of Variance Reduction Factor

The variance reduction factor is the ratio of the block variance to the point variance:

$$f = \frac{D^2(V, A)}{\sigma^2} \tag{13}$$

The block variance is given by:

$$D^2(V, A) = \sigma^2 - D^2(\cdot, V) \tag{14}$$

Inserting the above equation into Equation 13:

$$f = \frac{\sigma^2 - D^2(\cdot, V)}{\sigma^2} \tag{15}$$

Then by simplification, we get the variance reduction factor as:

$$f = 1 - \frac{D^2(\cdot, V)}{\sigma^2} \tag{16}$$

The important task is to calculate $D^2(\cdot, V)$. The average variogram value, "gamma-bar", introduced in the previous section may be used. The dispersion variance of point values within any volume V is equal to the variogram model averaged over all possible vectors contained within that volume:

$$D^2(\cdot, V) = \bar{\gamma}(V, V) - \bar{\gamma}(\cdot, \cdot) = \bar{\gamma}(V, V) \tag{17}$$

where $\bar{\gamma}(\cdot, \cdot) = 0$, since the mean variogram value at point scale is 0. This is demonstrated in *Mining Geostatistics* [7].

Inference of Small-scale LMC Model

The linear model of coregionalization (LMC) [5, 7] provides a method for modeling the auto and cross-variograms of two or more variables so that the variance of any possible linear combination of these variables is always positive. Each variable is characterized by its own autovariogram and each pair of variables by their own cross-variogram. For a positive definite LMC, the auto and cross-variogram models of Z and Y variables must be constructed using the same basic variogram models as follows:

$$\gamma_Z(h) = C_Z^0 + C_Z^1 \Gamma^1(h) + C_Z^2 \Gamma^2(h) + C_Z^3 \Gamma^3(h) \dots$$

$$\gamma_Y(h)) = C_Y^0 + C_Y^1 \Gamma^1(h) + C_Y^2 \Gamma^2(h) + C_Y^3 \Gamma^3(h) \dots \tag{18}$$

$$\gamma_{ZY}(h) = C_{ZY}^0 + C_{ZY}^1 \Gamma^1(h) + C_{ZY}^2 \Gamma^2(h) + C_{ZY}^3 \Gamma^3(h) \dots$$

where the $\Gamma(h)$ terms are common variogram models of specified type (spherical, exponential, Gaussian, ...), range and anisotropy. Only the sill C parameters are allowed to vary between the three variogram models.

The C values must also satisfy the following constraints:

$$C_Z^i > 0 \quad \forall \ i$$

$$C_Y^i > 0 \quad \forall \ i \tag{19}$$

$$C_Z^i \cdot C_Y^i > C_{ZY}^i \cdot C_{ZY}^i \quad \forall \ i$$

In the context of variogram scaling, we have the basic structures of small-scale variogram model, $\gamma_Z(h)$ (i.e. variogram model for primary variable), then, the next step is to "downscale" the large V scale secondary (Y) variogram, $\gamma_Y(h)$ (i.e. variogram model for secondary variable) and the large V scale cross-variogram, $\gamma_{ZY}(h)$ to small, v, scale that ensures the positive definiteness constraint of linear model of coregionalization given by Equation 19. Equations 8, 9 and 10 are used for this purpose, by rearranging them in terms of small v scale. Once a small-scale LMC model has been established, the spatial averages of the variogram are used in the cokriging equations.

In the petroleum context of primary data with excellent vertical resolution and secondary data with little vertical resolution, there are some basic assumptions;

1. The shape of the cross-variogram is the same as the shape of the auto variogram at v scale. This is reasonable, since the shape of auto variogram is certainly related to the cross variogram.

2. The same type of basic covariance structure that is used to model the large-scale data is also appropriate at the small-scale.

3. Given 2-D horizontal seismic data a vertical range can obviously not be determined. In this case, a vertical range value for the seismic data must be chosen to complete 3-D small-scale LMC model. Commonly, in practice, this vertical range is taken from the small-scale variogram model.

Program Design

A flowchart of `VarScale` is presented in Figure 1. `VarScale` operates on `Windows 95/98/NT`. From the main window, which is given in Figure 2a, user can access the options of (1) Histogram scaling, (2) LMC Scaling (3) Variogram Scaling, (4) $\bar{\gamma}$ calculation, and (5) D^2 calculation. According to the chosen option(s), `VarScale` expects user to input the necessary data and parameters. In Figure 2b, 2c and 2d, those windows are presented to let the user input necessary data, parameters and commands that directs `VarScale`. From the main window, user can input/browse the output file for histogram and variogram scaling. The result of the "average variogram" and "dispersion variance" values will be printed to window and it is advisable that user see the results of them before going to histogram and variogram scaling.

Most calculations need the definition of block size and discretization parameters except for user specified *variance reduction factor* case; there is an option in Block size and Discretization window (Figure 2b) that allows the user to define f directly. `VarScale` asks for input data file and histogram correction method(s) (see Figure 2d) to accomplish the histogram scaling. The output will be in GeoEAS format preserving the original data columns and adding corrected column(s) accordingly.

It is rare that we know *variance reduction factor, f* directly for histogram scaling. Therefore, estimation of f from the variogram model is conveninent and `VarScale` does the necessary calculations. This calculation requires estimation of $\bar{\gamma}$. As given in Equation 12, $\bar{\gamma}$ may be estimated by discretizeting the volume and averaging the variogram values. Since discretization may introduce numerical errors (numerical integration errors), `VarScale` will automatically choose a discretization level. From the block size and discretization window, user can selecet either automatic or manual discretization option. If the manual option is selected, then user must specify the discretization. With the automatic option, the number of discretization points will be increased continuously until a stable $\bar{\gamma}$ is obtained. Discretization will be performed in direction(s) that data and target scales differ.

Calculation of $\bar{\gamma}$ requires point scale variogram model; however, `VarScale` gives user the flexibility of specifying the input variogram at any scale. If the point scale check box in Block size and Discretization window is not checked, then `VarScale` assumes that input variogram model is not a point scale variogram model; the input variogram model is first downscaled to point scale and then upscaled to target scale. `VarScale` directly scales the nugget effect because the nugget effect at the point scale cannot be defined. If the input variogram model is a point scale variogram, then scaling to target scale will be performed directly.

The input file for histogram correction should be in GeoEAS format. For the histogram scaling, `VarScale` allows two methods; affine correction and indirect lognormal correction. Both of them can be chosen, two columns will be added in the output file, and the results can be compared.

The variogram model window (Figure 2b) has different options. The user can either input the variogram model parameters directly, which are in GSLIB [3] conventions, directly or load them from a presaved file. The *Variogram Types* button is designed to access for the *Variogram Types window* which includes the corresponding variogram model types supported by *VarScale*.

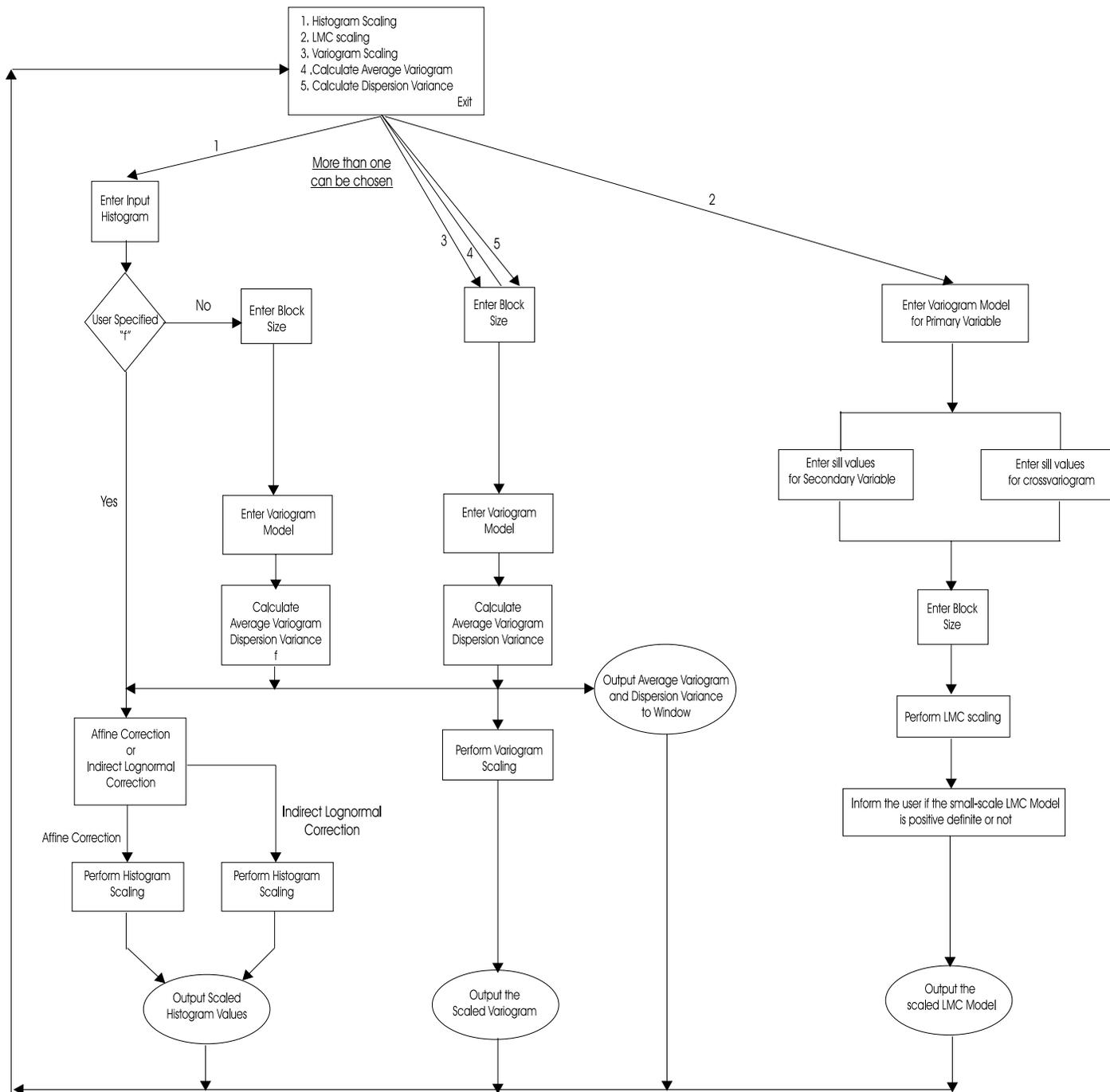The linear model of coregionalization requires that the nested structures (types, ranges),

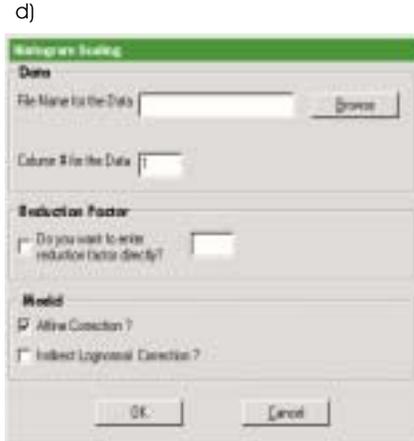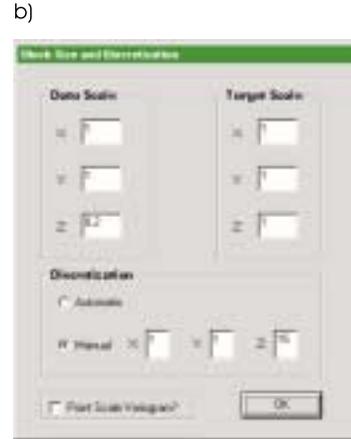Figure 1: Flowchart for VarScale.

Figure 2: Program interface: (a) main window, (b) block size and discretization window, (c) variogram model window, (d) histogram scaling window, and (e) LMC Scaling window.

fitted to the small-scale primary data, be kept for the small-scale v variogram of the secondary and for the small-scale v cross-variogram, only nugget and the sill components of these variograms have to be determined by the analytical model. Therefore, `VarScale` asks user to input complete small-scale variogram model for primary data (nugget effect, sill values, types and ranges), whereas, only nugget effect and sill values are needed for the large-scale variogram model of secondary variable and large-scale cross-variogram model between primary and secondary variable. LMC scaling window of `VarScale` is given in Figure 2e. Just like the option at the variogram model window, user can either input the variogram model parameters for primary variable, which are in GSLIB [3] conventions, directly or load them from a presaved file.

The types and ranges of small-scale, primary data, variogram model will be used to calculate the average variograms at small-scale and large-scale. The large-scale, V and small-scale, v are specified in the Block size and Discretization window. It is important to remind here that, since we are downscaling to small scale v, in the Block size and Discretization window, the small-scale v should be input to the "Target scale" section and the large-scale V should be input to the "Data scale" section (i.e. just the reverse of upscaling case). Once, average variogram values for small-scale and large-scale have been obtained, then, sill values for the small-scale v variogram of the secondary variable Y and the small-scale v cross-variogram between variables Z and Y will be calculated using Equation 10. The range and nugget effect values are downscaled using the Equations 8 and 9 respectively. The small-scale linear model of coregionalization will be checked for positive definiteness by applying the Equation 19. The result of this check will be printed to window to inform the user. After small-scale linear model of coregionalization has been obtained, then the results will be printed to a file that user selects from the LMC scaling window. The output format is suitable to be used by the other GSLIB programs; such as `cokb3d`.

Example

Histogram Scaling

Data from West Texas field was used. The dataset consists of 3303 porosity data with a mean of 8.33 and standard deviation of 3.37. Kupfersberger et al. [8] worked on this field and according to their result; 3-D normal score porosity variogram model has zero nugget effect and two nested structures: (1) a exponential structure with sill equal to 0.4 and range of values in x, y and z directions are 1000.0 m, 3000.0 m and 12.0 m respectively and (2) a spherical model with sill equal to 0.6 and range of values in x, y and z directions are 25000.0 m, 5000.0 m and 50.0 m respectively. Our aim is to adjust the original porosity data distribution to reflect upscaling, when the original vertical scale of 1 m (core values averaged to 1 m blocks) is replaced with 10 m and 30 m. `VarScale` will calculate the *variance reduction factors* for each upscaling from the above variogram model assuming without point variogram model. For simplicity, only the *affine correction* will be applied to get the new scaled histograms. The following steps are performed:

1. Go to Variogram Model window by pressing Variogram Model button. The response of `VarScale` and the complete input variogram model are shown in Figure 3.

2. Averaging will be done in vertical direction. Data scale in vertical direction is 1 m and the target vertical scales are 10 m and 30 m. Using the Block size and Discretization
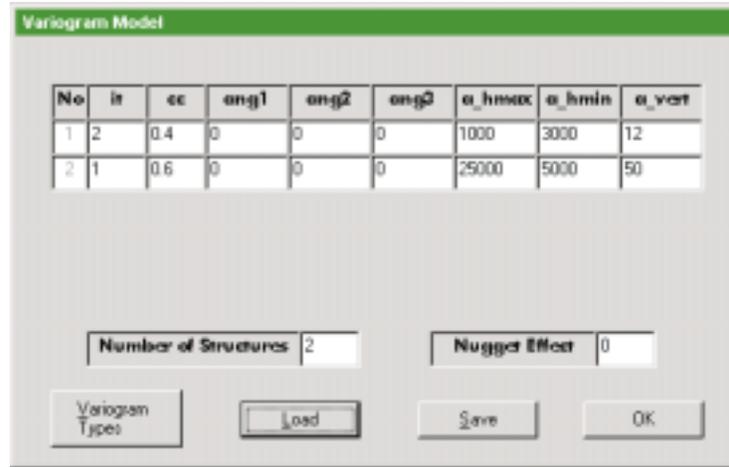
Figure 3: The Variogram Model Window showing the input variogram for Histogram scaling example

window (by pressing the Block size and Discretization button from the Main window), data scale and target scale configurations are defined (see Figure 4). In Figure 4, it is seen that the same values are assigned to x and y data and target scale, whereas, 1 m and 10 m are assigned to z direction of data and target scale accordingly. This directes `VarScale` to do only vertical averaging. Since the input variogram model is not a point scale variogram, the point scale variogram option was not selected. This will direct the `VarScale` to down scale the input variogram from data scale to point scale and then upscale from point scale to target scale.

3. In Figure 5, the histogram scaling window is presented. A data file was defined with the data column number for the considered data. Since the *variance reduction factor* will be calculated from the available variogram model, user defined *variance reduction factor* option was not selected. For the histogram correction method, *affine correction* option was selected.

4. Final step to start histogram correction is to select the histogram correction option from the Main window (see Figure 6). The result of the scaled distribution will be printed to a file called Wtexas.out, in GEOEAS fromat. The output file is specified in the Main window.

Variance reduction factors of 0.72 and 0.50 were obtained for the 10 m and 30 m scaling, respectively. The results of the new corrected distributions are given in Figure 7 along with the original distribution. It is seen that the shape of corrected distributions are similar to the original one which is the typical result of affine correction. On the other hand, it can be checked from the Figure 7 that, variances of the two corrected distributions are reduced by 0.72 and 0.50 respectively comparing to the variance of the original distribution. For example, for the 10 m scale, dispersion variance is equal to the; $D^2 = \sigma^2 \times 0.72 = 8.18$. It is important to point out here that these two scaled histograms are "target histograms" for simulation purposes.
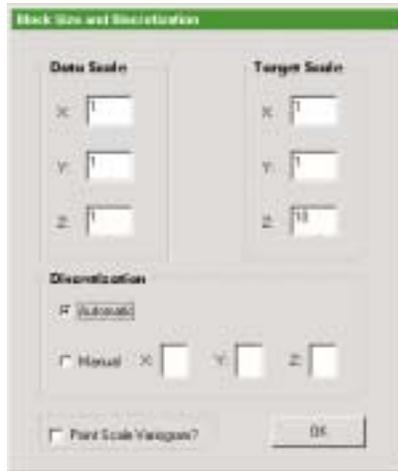
Figure 4: The Block Size and Discretization Window showing the data and target scale along with the discretization option(s) for Histogram scaling example
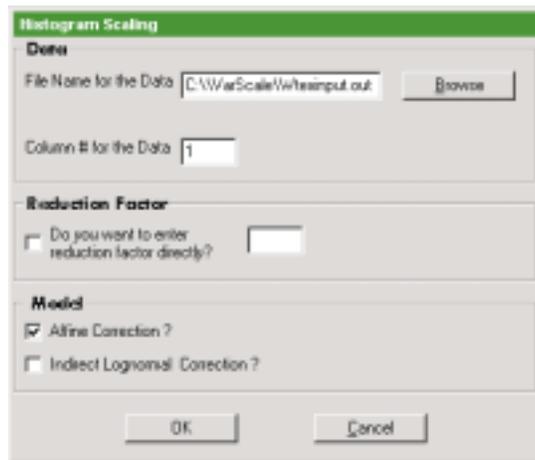

Figure 6: Main window showing the histogram scaling option and the output file specification
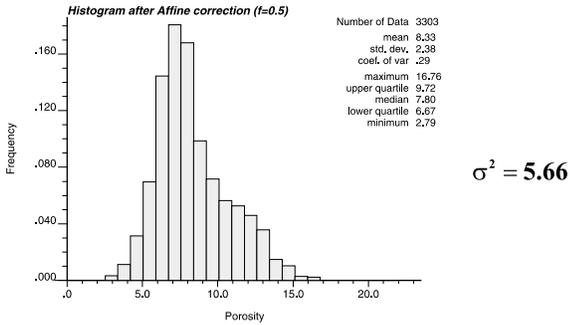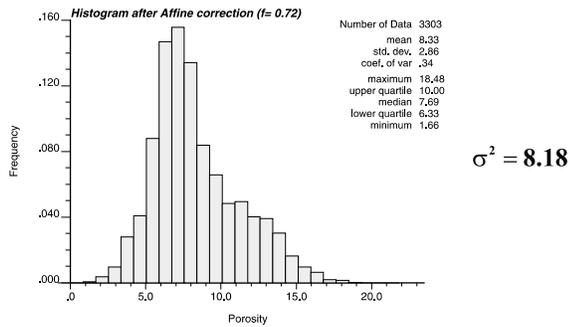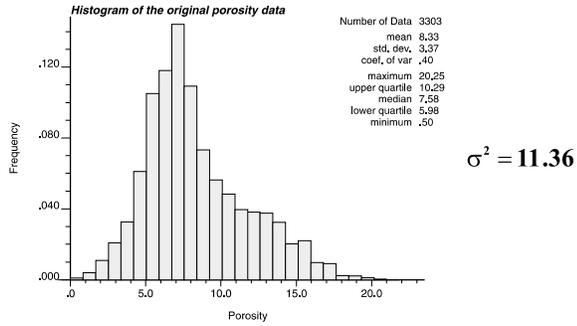
Figure 7: Original distribution along with the two adjusted distributions for Histogram scaling example

Variogram Scaling

The integration of different data sources simultaneously in the construction of high-resolution geostatistical models prompts researchers to work on data of different scales. Core and well log measurements are commonly used in constructing geostatistical models of an oil field. The volume of core measurement is well understood; however, the volume of well log derived porosity is less well understood.

An example application of `VarScale` will be given for the core and well log derived porosity from a Danish chalk reservoir. Details may be found in Frykman and Deutsch [4]. Not only the existence of correlation between core porosity and well log porosity, but also the reliance of core volume measurement make it convenient to predict log-scale variogram from core-scale variogram. The practical importance of this prediction is that once we have chance to calibrate the log scale variogram then we can use it efficiently in modeling since log data are more prevalent than core porosity measurements. The objective of this example application is to show how to do this prediction via `VarScale`.

In the afforementioned study, the authors came up with a variogram model derived from core porosity measurements. The variogram model has no nugget effect and two nested structures: (1) a spherical structure with sill equal to 2.82 and a range of 0.54 m and (2) a hole effect model with amplitude 1.2 and peak at 0.95 m. Core scale, which is also the data scale, is 0.05m by 0.02m by 0.02m and log scale is 0.6 m in vertical direction.

The complete input variogram model is shown in Figure 8. Let's proceed step by step and see how to estimate log scale variogram from core scale variogram using `VarScale`:

1. Set the core and log scale in the Block size and Discretization window. User can access this window by pressing the Block size and Discretization button from the Main window. This window is given in Figure 9. Well data are only averaged in vertical (i.e. z) direction. From Figure 9, this was accomplished by assigning the same values to x and y directions and different values to z direction of the data and target scale boxes. Then, the automatic discretization option will do the discretization only in z direction. Since the input variogram model is not a point scale variogram, the point scale variogram option was not selected.

2. After providing the needed data and parameters we can direct `VarScale` to perform variogram scaling along with the calculations of average variogram and dispersion variance by checking the appropriate boxes and pressing the calculate button. The result of the calculations for average variogram and dispersion variance will be printed to window, see Figure 11. Besides, the result of the variogram scaling, which is log scale variogram model for our example, will be printed to the file called Danish.out (user can specify the output file for variogram scaling from the Main window.)

The results of the disperson variance and the average variogram for the log scale are 2.295 and 1.780 respectively (see Figure 11). The log scale variogram model has the range values of 1.12 for the spherical and 0.95 for the hole effect structure and corresponding sill value for spherical structure is 1.439 and 0.855 for the hole effect structure (Figure 12).

LMC Scaling

The application of linear model of coregionalization scaling will be demonstrated with West Texas data. Seismic data will be used for secondary data along with the primary data

Figure 8: The Variogram Model Window showing the input variogram for Variogram scaling example.
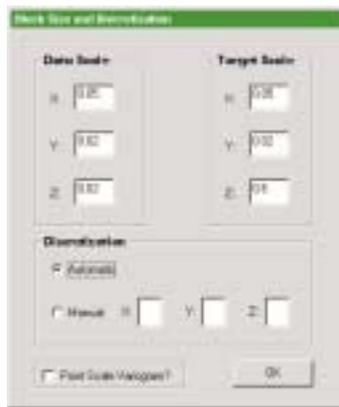

Figure 9: The Block Size and Discretization Window showing the data and target scale along with the discretization option(s) for Variogram scaling example.

Figure 10: Main window showing the desired boxes checked to accomplish variogram scaling and estimate average variogram and dispersion variance

Figure 11: Output window with the Dispersion variance and Average variogram for Variogram scaling example.

```
2  0                  \nst, nugget effect
1  1.439  0  0  0     \it,cc,ang1,ang2,ang3
       0.54 0.54 1.12  \a_hmax, a_hmin, a_vert
5  0.855  0  0  0     \it,cc,ang1,ang2,ang3
       0.95 0.95 0.95  \a_hmax, a_hmin, a_vert
```

Figure 12: Output result for Variogram Scaling (Danish.out).

(porosity). Kupfersberger et al. [8] obtained the 3-D porosity normal scores variogram model, the variogram model for normal score seismic data and the cross variogram between vertically averaged porosity and seismic attribute (normal score). In the Histogram scaling section, previously, the 3-D porosity normal scores variogram model was presented. The fitted sill values for each variogram component were given 0.29 and 0.71 for the large-scale seismic normal scores variogram and 0.11 and 0.52 for the large-scale cross-variogram.

In the West Texas field, the large-scale V comprises 55 m in the vertical direction (average well length), and the small-scale v represents 1 m in the vertical direction. Since LMC requires that types and ranges, fitted to the small-scale primary data, be kept for the small-scale v variogram of the seismic data and for the small-scale v cross-variogram; thus, only the nugget effect and the sill components of the seismic variogram and the cross-variogram have to be determined. In order to get the small-scale LMC via `VarScale`:

1. In the LMC scaling window, complete the input for porosity (primary data) variogram model, sill values for each nested structures of seismic (secondary data) variogram model and cross-variogram model (between porosity and seismic), see Figure 13. The output file is defined in this window as LMC.out

2. The data-scale, target-scale and the discretization configurations are given in Figure 14. Automatic discretization option was selected. Our aim is to downscale from large-scale 55 m in the vertical direction to small-scale 1 m in the vertical direction therefore, 55 m and 1 m are defined in the z direction of data-scale and target-scale sections accordingly.

3. From the main window, selecting the LMC scaling option and pressing to the calculate button, will direct `VarScale` to complete small-scale LMC. Before printing to the output file, `VarScale` will inform user about the result of the positive definitness of this small- scale LMC. For our case, small-scale LMC is positive definite. The output file is given in Figure 15. The first small-scale variogram model is for porosity, the second one is for seismic and the final is for cross-variogram between porosity and
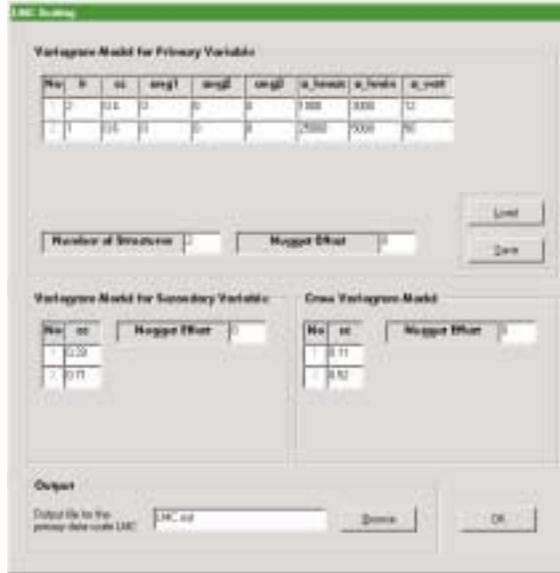
Figure 13: The LMC Scaling Window showing the input variograms for LMC scaling example.
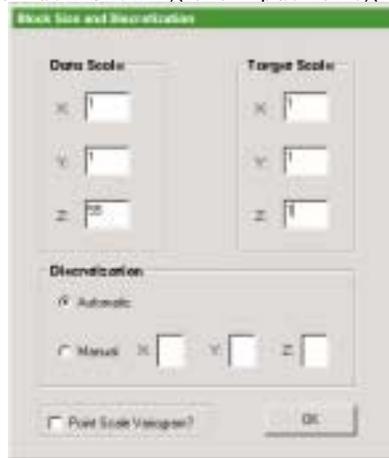

Figure 14: The Block Size and Discretization Window for LMC scaling example.

seismic.

Conclusion

Histogram and variogram scaling are important steps in geostatistical calculations. This is the first public easy-to-use software that performs such scaling. The use of Visual Basic and a common "Microsoft-like" interface makes it directly applicable on the ubiquitous PC. Availability of the source code will help software vendors incorporate functionality in commercial codes.

User input is automatically passed between the different components of VarScale. The option of "non-point variogram" in VarScale allows input variograms at any scale. Automatic discretization option does the required numerical integration to arrive at stable average variogram values. In addition to scaling the variogram for a single variable, the LMC scaling option of VarScale permits the inference of a small-scale linear model of coregionalization. Then, results can be used for cokriging. Calculations can be repeated for

```
1    1                \semivariogram for 'i' and 'j'
2   0                 \nst, nugget effect
2  0.4  0   0   0     \it,cc,ang1,ang2,ang3
      1000  3000   12  \a_hmax, a_hmin, a_vert
1  0.6  0   0   0     \it,cc,ang1,ang2,ang3
      25000  5000  50  \a_hmax, a_hmin, a_vert
2    2                \semivariogram for 'i' and 'j'
2   0                 \nst, nugget effect
2  0.584  0   0   0   \it,cc,ang1,ang2,ang3
      1000  3000   12  \a_hmax, a_hmin, a_vert
1  0.416  0   0   0   \it,cc,ang1,ang2,ang3
      25000  5000  50  \a_hmax, a_hmin, a_vert
1    2                \semivariogram for 'i' and 'j'
2   0                 \nst, nugget effect
2  0.265  0   0   0   \it,cc,ang1,ang2,ang3
      1000  3000   12   \a_hmax, a_hmin, a_vert
1  0.365  0   0   0   \it,cc,ang1,ang2,ang3
      25000  5000  50  \a_hmax, a_hmin, a_vert
```

Figure 15: Result of the small-scale LMC.

different volumes easily for all options in `VarScale`.

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