Reservoir Management Decision-Making in the Presence of Geological Uncertainty

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Reservoir management decisions are made in presence of many uncertainties. Geological uncertainty about the reservoir geometry and petrophysical properties, due to sparse sampling of the reservoir, is perhaps the most important source of uncertainty. Given the large investment required to develop a reservoir, any improvement in the development plan could represent millions of dollars.

We present an approach to incorporate geological uncertainty in the selection of the best production scenario among a set of predefined scenarios. Multiple geostatistical realizations are considered to represent uncertainty. This uncertainty is considered together with the economic profile of the company, that is, the importance of profit seeking and risk aversion. This full approach *to account for geological uncertainty leads to demonstrably better decisions. An important reservoir management problem is well placement. The concept of a quality map is introduced to locate the wells accounting for geological uncertainty. The quality map is used together with the full approach for improved reservoir management.*

An extensive case study with fifty synthetic yet realistic reservoirs and more than 450,000 flow simulations was undertaken in the course of this work. The availability of multiple realizations of multiple reservoirs permits the quantitative evaluation of the potential of the full approach *and quality map. This paper is a summary of the full work, which is documented in the first author's Ph.D. dissertation (available from University Microfilms).*

Introduction

Petroleum exploration and production are inherently risky activities. Decisions regarding those activities depend on forecasts of future hydrocarbon production revenue. Such forecasts are uncertain because of (1) uncertainty about the reservoir geometry and the spatial distribution of petrophysical properties, (2) uncertainty about the fluid properties, (3) uncertainty about the actual behavior of the rock and fluid when subjected to external stimuli, (4) model limitations, and (5) uncertainty about costs and future prices.

In this work the uncertainty scope is restricted to that of the geological model due to sparse sampling of the reservoir. Geostatistical techniques are used to model this uncertainty through multiple stochastic realizations.

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Exploration and production are sequential. First, exploration finds a promising geologic structure, making use of seismic responses and knowledge of the sedimentary basin. Then, a well is drilled to prove the existence of a hydrocarbon reservoir. If this first exploratory well succeeds in finding hydrocarbon, depending on the field size, other exploratory wells are drilled to delimit the deposit. Next, a development plan is generated to provide the necessary data for the production cash flow analysis. If the company decides to invest in that project, the development plan is implemented and hydrocarbons are produced.

There are three main types of decisions involved in the exploration and production process: (1) the decision of whether or not to drill an exploratory well, (2) the technical decision of selecting the best development plan to optimize the profitability of the reservoir production, and (3) the business decision to invest in a project or not. Decision-making in exploration typically accounts for uncertainty through decision tables which relate alternative actions to various outcomes [8].

After exploratory reservoir delimitation, a reservoir development plan is devised that determines the number, type and location of additional wells and presents the rig work schedule and the curves for injection and production of fluids. Once the development plan is defined, it is possible to transfer some aspects of data uncertainty to the production forecasts. Ballin et al. [2] considered multiple geostatistical realizations to transfer such uncertainty. The probability distribution of flow responses can be used to evaluate the expected monetary value of the project and to guide the business decision of whether or not to invest in a particular project.

The present work addresses the question of how to account for geological uncertainty in the selection of the best development plan to optimize the reservoir resources. This is by far the most important decision in reservoir management.

The "conventional" approach to define the development ignores geological uncertainty. Typically this definition is made by: (a) building a single deterministic geological model of the reservoir, (b) defining the possible production scenarios (numbers of wells, configuration for each number of wells, types of wells - vertical or horizontal, producer or injector, fluid to inject, etc.), (c) running a flow simulator for each scenario to generate the respective production/injection curves, (d) performing a cash flow analysis for each scenario and (e) selecting the scenario that provides the maximum profit.

Vasantharajan and Cullick [13] presented the concept of a quality measure of the reservoir to be used with integer programming optimization for locating wells. The measure is a combination of static characteristics of the reservoir and does not account properly for the dynamic and nonlinear interaction between the parameters.

Bittencourt and Horne [3] presented a technique based on flow simulations to optimize the scenarios using a single deterministic geological model. This "conventional" approach does not guarantee that the selected scenario is optimal for the actual field; an alternative equally-probable geological model, respecting the same available data, could lead to a different production scenario.

Experimental design and response surface methodology can be applied [4, 7, 9] to obtain the distribution of flow responses for each scenario and to retain the best scenario in presence of uncertainty; however, the consideration of a reduced number of realizations required by those methodologies yields an incomplete assessment of the uncertainty in the flow responses [12]. Moreover, no clear procedure to choose the best development scenario, after obtaining the distribution of flow responses for each one of them, is presented. Experimental design and response surface methodology can also be used to optimize well locations [1, 5, 14], but are limited to the location of only two wells at most and considering separate regions for each well.

This work aims at developing a more complete and clear methodology to incorporate geological uncertainty into the definition of the best development plan and to quantify the "worth" of this incorporation. The result of this effort is called the ***Full* approach**).

The *Full* Approach

Our goal is to chose the best development plan or production scenario among a set of possible alternative scenarios. The qualifier "full" relates to the fact that the flow responses are obtained for each scenario by running a flow simulator on every realization; no shortcuts are taken. This is in contrast to the conventional approach of defining the development plan by examining the flow responses of a single reservoir model (or realization).

The decision criterion for selecting the best scenario is economic. After each simulation run, a measure of profit is evaluated, integrating all the production and injection curves through an economic function. From the probability distribution of profit for each scenario, an estimate of profit is retained based on the minimization of a specified loss function. The best scenario is defined as the one that has the maximum retained estimate of profit.

We present a general methodology that can be applied to many types of reservoir management problems. The methodology is demonstrated with the reservoir management problem of defining the number of producer wells and their spatial configuration a large case study. This case study involved 50 different reservoirs to quantify the expected benefit of accounting for geological uncertainty. The benefit is evaluated by comparing the results of the *Full* approach with the results of the conventional approach, which uses a single deterministic model.

Methodology

The steps of the *Full* approach are illustrated in **Figure 1**:

1. Generate L geostatistical realizations of the geological model $l = 1, ..., L$. The notation for the geological model "l" is intentionally simple but actually l is a spatially distributed vector of numerical models representing top structure, lithology, thickness, porosity, permeability and fluid saturations.

2. Define the possible reservoir management scenarios: $s = 1, ..., S$. Each scenario is a complete specification of one possible solution for the problem. For example, one scenario would define the number of wells, their locations, the completion intervals, the surface facilities. The number of scenarios could be very large, but an inspection of the L realizations and prior sensitivity flow analysis based on just one realization should reduce this number substantially.

3. Establish a quantitative measure of profit P to be maximized. The measure of profit would increase with increased hydrocarbon production and would decrease as more

wells and facilities are required. The profit depends on the related costs, hydrocarbon prices and taxes. A good unit to measure the profit is the present value of the discounted cash flow.

4. Calculate the profit for each scenario and each realization: $P_{s,l},\ s = 1, ..., S; \quad l = 1, ..., L$. The fluid production and injection curves are obtained by running a flow simulator and the profit measure is calculated from the scenario specifications and curves for each case (s and l).

5. Determine the best estimate of profit $\widehat{P}_s$ for each scenario, based on minimization of a specified loss function. This summary estimate is retained as a single index to compare the scenarios. A loss function [11, 10] quantifies the impact or loss of estimating the unknown profit by a single value p^* with a given error $e = p^* - P$. The function $Loss(e)$ must be specified by the organization or person in charge of the economic decisions in the company and thus is known, but the argument e is not. Therefore, for each scenario s, an expected loss value can be determined using the distribution of P and the formula:

$$E\{Loss\}_s = \frac{1}{L} \sum_{l=1}^{L} Loss(p_s^* - P_{l,s})$$

   The best estimate of profit for the scenario s is $\widehat{P_s}$ such that the expected loss is minimum when taking $p_s^* = \widehat{P_s}$.

   Figure 2 presents an example of the distribution of profits for two scenarios and three different types of loss function that lead to different values of the retained profit value for each scenario and to different decisions of the best scenario. The two scenarios have the same mean value, but scenario 1 has a smaller uncertainty than scenario 2. For a loss function that penalizes underestimation more than overestimation (such as the loss function in the right), the profit value that minimizes the expected loss is above the mean (for example the upper quartile of the distribution). Between the two scenarios presented in the figure, an "aggressive" company using this type of loss function would prefer the one with greater probability of high profit values (scenario 2). For a quadratic loss function where the loss due to underestimation is the same as the loss due to overestimation (loss function in the center), the profit value that minimizes the expected loss is the mean. A company using this type of loss function would prefer scenarios with high expected profit, without consideration of uncertainty (no preference between scenario 1 and scenario 2). For a loss function that penalizes overestimation more than underestimation (loss function in the left), the profit value retained would be below the mean (for example the lower quartile of the distribution). Between two scenarios with the same mean profit, a "conservative" company using this type of loss function would prefer the one with smaller uncertainty (scenario 1).

6. Define the optimal scenario s^* as the scenario that has the maximum optimal estimate of profit $\widehat{P}_s$.

Case Study

The best scenario defined with the *Full* approach takes into account the uncertainty in the geological model by using multiple models, but we must determine if it is better than the scenario that would be defined with the conventional approach of using a single deterministic model. In order to quantify the "goodness" of the *Full* approach, we must compare the "true" results (profits) of the decisions made different ways.

This case study was undertaken to demonstrate the value of considering uncertainty in reservoir decision-making. Since in practice only one development plan can be implemented, and there is no access to the "true" reservoir, we must work with synthetic reservoirs and moreover, we must consider multiple "true" (synthetic) reservoirs because, by chance, the "conventional" or the "full" method could appear better in any one particular case. The *Full* approach and two conventional approaches - using a kriged model or one single realization - were applied to each "true" reservoir and the resulting true profits of the approaches were compared.

Fifty "true" reservoirs were generated. No attempt was made to cover all the possible types of hydrocarbon reservoirs, but careful attention was given to generating reservoirs different enough to validate the conclusions of this research. Each reservoir is defined over a $90 \times 90 \times 60$ grid. There are six main stratigraphic layers, each with ten sublayers. The reservoir volumes, productivity and lithology represent medium size offshore reservoirs with sandstone/shale lithology. No faults or fractures were considered. Oil and water phases are considered and the initial saturation of the fluids is determined by the position of the oil-water contact. The position of this contact is the same for all the reservoirs but the different elevations of the top structure and the different thickness and porosity of the layers resulted in different volumes of oil and water for each reservoir. The bottom layer was generated thicker than the other layers to ensure a strong bottom aquifer for all the reservoirs.

Each "true" reservoir is sampled by five vertical wells to obtain data for the top elevation and thickness of all six layers, the lithology, porosity and permeability. A smooth image of the "true" structural top was also generated to mimic seismic data. The availability of five sampling wells and a good seismic representation of the structural top can be considered realistic for the development plan phase of an offshore reservoir.

The sample data are used to generate a kriged reservoir model and 20 simulated realizations, using geostatistical techniques different from those used to create the "true" reservoirs. Using different algorithms protects from a recursive argument.

The type of reservoir management problem chosen for the case study was the definition of the best number of producer wells to maximize the profitability of the reservoir resources. A smaller number of wells, even with smaller production, may give a higher profit if the profitability of the additional production does not pay for the cost of the additional wells. To decide the best number of wells, different spatial configurations must be considered for each number of wells, because a number of wells in good locations may produce more than a greater number of wells in poor locations.

A total of 77 different production scenarios were defined, consisting of 11 different numbers of wells and seven different configurations for each number of wells. The configurations for each given number of wells were defined using a geometric criterion to ensure a good spacing between wells and to avoid boundary effects. The scenarios are the same for all three approaches: *Full*, *Conv-1* (conventional with just one realization) and *Conv-k* (conventional with the kriged model). This case study is intended to compare the "goodness" of each approach in identifying the best scenario among a set of predefined scenarios.

A flow simulator was run for each combination of model and scenario to obtain the production curves. For this case study the flow simulator ECLIPSE was run 84,700 times, corresponding to (one true reservoir + 20 realizations + one kriged model) x 77 scenarios x 50 reservoirs. Fluid properties, well conditions, and shut-in criteria were chosen to be realistic and they were kept the same for all the runs.

The measure of profit was defined as the net present value of the oil production for 20 years of production, minus the cost of the wells. The net oil production for each period of time is the incremental oil production for that period minus the cost of processing the produced water. The economic units were expressed in volumes of oil to avoid the uncertainty in oil price, yet some results are also expressed in dollars to allow a better appreciation of the results. The price of oil used was \$100 per m^3 of oil, that is, \$16 per barrel.

With the two conventional approaches, the best scenario was defined as the one with maximum profit. For the definition of the best scenario with the *Full* approach, a quadratic loss function was considered, that is, the expected (mean) profit over all the realization results was retained for each scenario and the best scenario was defined as the one with maximum expected profit. The comparison between the approaches was done using the actual profit (calculated from the "true" reservoir) of the scenario determined as best with each approach. Access to the "true" reservoir permits this comparison otherwise impossible in practice.

Several FORTRAN programs were developed and run in combination with GSLIB and ECLIPSE programs, using UNIX script files in order to create different reservoirs, sample them, model the variograms, generate the kriged and simulated models, upscale, prepare the files for the flow simulator, run the flow simulator and evaluate the profit function automatically. UNIX script files are very useful tools, without them this research would not have been possible.

Results of Case Study

In the *Conv-1* approach, each realization could lead to a different definition of the best scenario with resulting different true profits. Instead of just presenting the result corresponding to one arbitrary realization, three results are presented for the *Conv-1* approach: the worst, the expected (mean) and the best result.

Figure 3 illustrates the comparison between the three approaches for one particular reservoir. The top four pictures of the figure show the mean profit calculated over all the realizations for each scenario (*Full*), the profit of each scenario calculated with the kriged model (*Conv-k*), with Realization 1 (given as an example of *Conv-1*), and with the "true" reservoir. These pictures presents a color-coded table where the abscissa axis gives the seven possible spatial configurations and the ordinate axis gives the 11 different total numbers of wells. A scenario is found in these tables at the intersection of the number of wells row and the configuration column, with the profit values given by the legend.

For the particular reservoir used in the example of Figure 3, the optimal scenario defined using the *Full* approach (**F**) consists of 16 wells with Configuration 1, which has a true profit

P of $6,188 Mm^3$ of oil. The optimal scenario defined using Realization 1 (**R**) consists of 13 wells with Configuration 5, which has a true profit P of $6,074 Mm^3$ of oil. The optimal scenario defined with the kriged model (**K**) consists of 12 wells with Configuration 5, which has a true profit P of $5,805 Mm^3$ of oil. None of the approaches, however, yielded the "true" best scenario (**T**), which consists of 11 wells with Configuration 2, which has profit $P = 6,291 Mm^3$ of oil. A good scale to compare those profit values is the equivalent cost of $150 Mm^3$ of oil for one offshore well as considered in the economic function.

Realization 1 was used in the center left picture of Figure 3 just as an example of the *Conv-1* approach. The realization could be any one of the other 19. The bottom left picture of Figure 3 gives the (20×77) profit results calculated over the 20 realizations (ordinate axis) for each of the 77 scenarios (abscissa axis), showing that the decision of the best scenario could be different for each realization retained for the *Conv-1* approach. The mean values over all the realizations presented at the bottom of this picture represents the *Full* approach. In order to present all the results in the same picture, the 77 scenarios shown in the abscissa axis were ordered increasing first the configuration number and then the number of wells, i.e. Scenario 1 is Configuration 1 of ten wells (the first number of wells in the range for this reservoir), Scenario 7 is Configuration 7 of the same number (ten) of wells and Scenario 77 is Configuration 7 of 20 wells (the 11^{th} number of wells in the range).

The distribution of true profits using *Conv-1* with each one of the 20 realizations is shown in the bottom right picture of Figure 3, as well the true profits (dots) of the other two approaches and the "true" best result. The worst result obtained from two of the realizations corresponds to a scenario of 16 wells with Configuration 5 yielding a true profit P of $5,224 Mm^3$ of oil, while the best result obtained from three realizations corresponds to the same optimal scenario obtained with the *Full* approach, with a true profit P of $6,188 Mm^3$ of oil. The expected true profit of the *Conv-1* approach is $5,938 Mm^3$ of oil.

These comparisons are valid only for the particular reservoir utilized for Figure 3; for a different reservoir the relative results of the three approaches considered could vary. To compare the approaches more reliably, different reservoirs should be considered: the previous exercise was repeated over 50 different reservoirs.

Since the absolute profit values are different for each reservoir, in order to better compare the relative result of each approach and present all results in a single figure, the values were scaled as follows:

$$P_{approach} = \frac{P_{approach} - P_{worst\ realization}}{P_{best\ realization} - P_{worst\ realization}}$$

Figure 4 presents the comparisons between the three approaches for the 50 reservoirs using the scaled true profits. The mean results over all the reservoirs are given in the right column of the figure. The following observations can be made:

- The *Full* and the *Conv-k* results are almost always bracketed by the worst and the best realization of the *Conv-1* approach. There were just three exceptions: *Conv-k* was worse than the worst realization for Reservoir 43, *Conv-k* was better than the best realization for Reservoir 4 and *Full* was better than the best realization for Reservoir 5.

- On average over 50 reservoirs, *Full* is better than *Conv-k* and than the expected value of *Conv-1* taken over 20 realizations, and this latter is just a little better than the

Conv-k approach.

- Models generated either by kriging or by stochastic simulations using data from only five wells do not lead to the true best decision for most of the reservoirs.

Since the best approach varies for each reservoir, an extensive attempt was made to find characteristics of the reservoirs that could be used to predict which is the best approach for any particular reservoir, but no reasonable correlation between approach suitability and model parameters or modeled variables could be found.

The probability of an approach to be better than the others is given by the number of reservoirs (in percentage) for which that approach was better than the others. Using the expected value of the *Conv-1* approach over the 20 realizations to represent the conventional approach of using a single realization taken at random, the following scores between the approaches are observed (1) the *Full* approach had better results than the expected result of the *Conv-1* approach for 64% of the reservoirs, and (2) the *Full* approach was better than the *Conv-k* approach for 70% of the reservoirs.

Each reservoir has its own distribution of *Conv-1* results corresponding to its 20 realizations. To obtain an average distribution of these results over the 50 reservoirs, the results of the 20 realizations were ranked for each reservoir and the results with the same rank order were averaged across the 50 reservoirs. **Figure 5** shows the distribution of these 20 average profit results together with the average result of the *Full* (**F**) and *Conv-k* (**K**) approaches. For reference, the average result of the best scenario obtained with the true reservoirs (**T**) is also presented.

Based on these results, the average gain of *Full* over the expected result of *Conv-1* was $63.6Mm^3$ of oil, which represents an increment of 1.22% in profit or \$6.37 million. *Conv-k* was better than 50% of the realizations and worse than 50% of the realizations. The average gain of *Full* over *Conv-k* was $64.2Mm^3$ of oil, which represents an increment of 1.23% in profit or \$6.42 million.

The influence of the specific loss function used in the *Full* approach was investigated by evaluating also the results of the *Full* approach with two other loss functions: (a) a conservative loss function for which the retained profit value is the lower quartile of the profit distribution over all the 20 realizations, and (b) an aggressive loss function for which the retained profit value is the upper quartile of that distribution. The results of the *Full* approach with different loss functions varied for some reservoirs, yet the average result over the 50 reservoirs changed very little.

There is no way to predict which approach would give the best result for any single particular reservoir because the suitability of the approach depends ultimately on the true reservoir. However, since the truth is unknown, the use of multiple realizations for decision-making decreases the risk of very bad decisions. The results of this case study show that on average over many reservoirs the *Full* approach provides higher profits than the expected profit obtained with one realization taken at random (*Conv-1* approach).

The expected gain of using multiple geostatistical realizations in decision-making through the *Full* approach (approximately \$6 million) more than justifies the additional computational costs.

The Quality Map

The parameters that govern fluid flow through heterogeneous reservoirs are numerous and most of them uncertain. Even when it is possible to visualize all the parameters together, the complex and nonlinear interaction between them makes it difficult to predict the dynamic reservoir responses to production. A flow simulator may be used to evaluate the responses for each production scenario given the geological model. The *Full* approach uses the results of flow simulations over multiple realizations to account for the geological uncertainty in the decision-making.

With the *Full* approach, the scenario defined as the best is one of the predefined scenarios. For some reservoir management problems, the number of predefined scenarios to ensure that the selected scenario is optimal would be too large. For the problem of well location, for example, the number of possible configurations for nw wells in a horizontal grid of nc cells is $\frac{nc!}{(nc-nw)!}$. For ten wells ($nw = 10$) in a 30×30 grid ($nc = 900$), for example, the number of possible configurations is 3.3×10^{29}. It would be impractical to optimize the configuration for several numbers of wells considering multiple geological models, even with the help of sophisticated optimization algorithms.

The quality map, introduced here, provides a way to optimize the configuration of a given number of wells, accounting for the geological uncertainty, with a reasonable number of flow simulations and using a simple optimization algorithm. The use of just one configuration for each number of wells significantly reduces the computational effort of the *Full* approach for the problem of defining the best number of wells and their spatial configuration.

The quality map is, by construction, a map of "how good the area is for production". The quality at a location is a measure of the expected oil production if a well was to be placed at that location with no other wells in the reservoir. The use of a flow simulator to evaluate quality ensures that the nonlinear and dynamic interactions between the parameters are taken into account. The use of multiple realizations ensures that geological uncertainty is taken into account.

Methodology

The quality map is generated by running a flow simulator multiple times with just one producer well and varying the position of the well in each run to provide a coverage of the entire horizontal grid. Each run evaluates the quality for the horizontal cell where the well is located. The units of "quality" is the cumulative oil production (N_p) after a certain time of production. The total time of production depends on the size of the reservoir but must be long enough to allow the well to approach likely economic abandonment.

In the flow simulation, the well is completed in all oil layers with automatic shut down of the layer when some water (or gas) cut limit is reached. No rate limits are imposed. Only a minimum bottom hole pressure (BHP) and a minimum oil rate must be specified in accordance with the expected limitations of the wells during actual production.

Considering the cumulative production of a vertical well placed in different positions, the three-dimensional characterization of a reservoir, involving multiple parameters, is translated into a single two-dimensional grid of values. The flow simulator accounts for all the interactions between variables and returns one single value of quality (N_p) for each position of the single well. The greater the horizontal transmissibility around the well, the higher the initial rate, the longer the production time before the minimum BHP is reached and the greater the quality value (total N_p). Also, the smaller the transmissibility between the aquifer (and/or gas cap) and the well, the smaller the water (and/or gas) production and the greater the total N_p for the same final BHP.

Figure 6 shows, as an example, some of the parameters that affect the oil production and presents the quality map (lower right corner), which integrates all these parameters. The higher the structural top, the greater the final cumulative oil production because the thicker the oil column. The larger the oil volume, the better. The larger the horizontal permeability in the upper layers where most of the oil production occurs, the better. The lower the vertical permeability between the aquifer and the production layers (k_z - Layer 3 in the figure), the better. Several other parameters also affect the flow of fluids inside the reservoir and only a flow simulator is capable of accounting for all the interactions between these parameters.

A full quality map could be built for a particular reservoir model with the well in each cell of the horizontal grid, as was the case of the quality map in Figure 6. However, when dealing with multiple models, it would be too CPU demanding to evaluate quality for each cell of each model. The alternative is to obtain only some points for every realization and then to interpolate the maps by kriging. The number of necessary points depends on the reservoir heterogeneity and on the grid size of the model, however between five and ten percent of the total number of cells should be sufficient, provided the points are evenly distributed over the entire grid. The sampling positions must vary for every realization in order to sample each cell at least once.

The expected value of quality over all the realizations or an L-optimal quality map can be generated if a loss function is specified. The quality value that minimizes the expected loss is retained for each cell, generating the L-optimal quality map.

Figure 7 presents the quality maps of two realizations (with the positions of the data points that were used in the kriging), the mean quality, the quality uncertainty and the lower quartile quality maps. The lower quartile map is the L-optimal map for a "conservative" linear loss function where the loss due to underestimation is three times smaller than the loss due to overestimation. This loss function is also presented at the bottom left corner of the figure.

Uses of the Quality Map

The uses of the quality maps include: (1) locating wells; (2) speeding the *Full* approach to determine the best number of wells; (3) identifying a representative realization; (4) ranking of realizations; and (5) comparing reservoir models.

An optimization program was developed to find the best configuration for a given number of wells. The objective function to be maximized is the total quality associated with the wells, as defined hereafter. Although the quality map is built using one single well, the interference between any two or more wells is taken into account by the total quality function. The evaluation of this function is very quick because it is based on the quality map and does not require any further flow simulation.

For each evaluation of the total quality, the program first visits each cell c and allocates the cell to the closest well. Then the program evaluates the quality of each well (Q_w) by adding all the quality values of the cells (Q_c) allocated to that well, weighting the quality of each cell by an inverse distance weight (w_c). The total quality (Q_t) is the sum of all the well qualities.

$$w_c = \frac{1}{(d_c + 1)^b}$$

$$Q_w = \sum_{c=1}^{nc_w} Q_c \cdot w_c$$

$$Q_t = \sum_{w=1}^{nw} Q_w$$

where: d_c is the distance from the cell c to the nearest well, nc_w is the number of cells allocated to the well w, and nw is the total number of wells.

The optimization of the configurations is made taking two wells at a time and trying all possible combinations for the positions of these two wells within an area defined by one cell on each side of the previous well location (total of nine possible locations for each well and total of 81 possible combinations). A configuration is final when no further improvement in Q_t is possible after trying all the combinations of two wells at a time.

A sensitivity analysis, with different weights w_c, was performed to establish the best values. $b = 3$ was found to be suitable after analysis of the 50 reservoirs.

Case Study

One quality map was built for each of the 20 realizations and for the kriged model. No sensitivity simulation runs were necessary to define the weighting formula for the evaluations of total quality, because the reservoirs and models are the same as above, therefore, hundreds of profit evaluations associated with different scenarios were already available.

A high correlation between total quality and profit gave us confidence to the optimization procedure based on the maximization of total quality to find the best configuration for a given number of wells.

The loss function considered in this case study was linear with the loss due to underestimation three times smaller than the loss due to overestimation. In the *Full* approach the profit retained for each scenario was the lower quartile of the distribution of profits over all realizations. The L-optimal quality map was obtained by retaining the lower quartile quality value from the distribution of qualities over all realizations for each cell.

The goodness of the well locations using the quality map was checked by comparing the results with locating the wells using a map of oil volume. The map of oil volume is obtained by summing the oil volume of all the layers for each cell of the horizontal grid. For this check no uncertainty was considered. Only the first realization (Realization 1) of each reservoir, taken as a deterministic model, was used for both methods (quality and oil volume). The methods were compared with respect to the profit obtained from production of the wells located with each map.

Figure 8 shows the locations of three different numbers of wells obtained using the quality map and with the oil volume map of Realization 1 for a particular reservoir. The profits evaluated for this realization and with each one of the scenarios are also given, showing the superiority of the quality map over the oil volume map for well locations. The average gain of using the quality map instead of the oil volume map was $653Mm^3$ of oil. The quality map provided better well locations than the oil volume map for 88% of the reservoirs. Over the 50 reservoirs, the average gain per reservoir when using the quality map instead of the oil volume map was $309Mm^3$ of oil. That gain is more than two times the cost considered for one offshore well and represents an increment of 4% in the reserves.

Let's now compare the results and the computational effort of the *Full* approach with seven predefined configurations for each number of wells with the results and computational effort of the *Full* approach where just one optimized configuration is used for each number of wells.

The decision-making here relates to the definition of the best number of wells. The same range of 11 different numbers of wells was used here but only the best configuration for each number of wells was retained. The same three approaches (*Full*, *Conv-1* and *Conv-k*) are compared, but each approach is evaluated not only by its ability to determine the best number of wells but also by its ability to find the best configuration for each number of wells.

The *Full* approach accounts for uncertainty. The lower quartile quality map was used to find the best configuration for each number of wells. A flow simulator was run over all the realizations for each number of wells and the best number of wells was defined as that with maximum expected profit over all the realizations. *Conv-1* and *Conv-k* are the two conventional approaches for deciding the best number of wells without accounting for uncertainty, i.e. using only one deterministic model.

For the *Conv-1* approach, only Realization 1 was used. The best configuration was found for each number of wells, using the quality map of that realization and the best number of wells was defined as that with maximum profit, using the flow simulation results for each number of wells.

For the *Conv-k* approach, a quality map was built using the kriged model and the best configuration was found with this map for each number of wells. The same procedure as in *Conv-1* was followed to define the best number of wells.

Each scenario, defined by the number of wells and its best configuration, was applied to the true reservoir generating the true profits. The goodness of each quality map for well location is evaluated by the average value of the true profits obtained from the 11 numbers of wells. The goodness of each decision-making approach is evaluated by the true profit of the best number of wells defined with each approach.

The comparison between the true profits obtained for well locations using the three types of quality map for all the reservoirs is shown in **Figure 9**. The comparison between the true profits obtained with the three approaches for all the reservoirs is shown in **Figure 10**. In both figures, all results were divided by the result obtained with the kriged model to provide an easier comparison.

The *Full* approach had better results than *Conv-1* for 70% of the reservoirs. The expected gain per reservoir of *Full* over *Conv-1* was $158.4Mm^3$ of oil, which represents an increment of 3.0% in profit or 15.84 millions of dollars. Comparing *Full* and *Conv-k*, the *Full* approach had better results than *Conv-k* for 64% of the reservoirs and equal results for 2% of the reservoirs. The expected gain per reservoir of *Full* over *Conv-k* was $187.9Mm^3$ of oil, which represents an increment of 3.6% in profit or 18.79 millions of dollars.

A comparison was made between the results of the *Full* approach where just one optimized configuration is used for each number of wells and the results of the *Full* approach where seven configurations were defined using a geometric criterion for each number of wells. Three measures were used in this comparison: (1) *Measure 1*: the mean profit over all configurations, numbers of wells and realizations, using the profits from the realizations (not the true profits). This measure is useful to evaluate the goodness of the well locations with the lower quartile quality map. (2) *Measure 2*: the mean profit over all configurations and numbers of wells, using the profits from the true reservoir. This measure is useful to evaluate the influence of uncertainty on the goodness of the well locations. (3) *Measure 3*: the true profits from the defined best scenario. This measure is useful to evaluate the goodness of the decisions.

The average value of *Measure 1* over all the reservoirs was $442.1Mm^3$ of oil greater when using the optimized configuration for each number of wells than when using seven configurations for each number of wells. *Measure 1* was greater with the optimized configuration for all the reservoirs, giving confidence in using the quality map to locate wells.

The average value of *Measure 2* over all the reservoirs was $330.0Mm^3$ of oil greater for the optimized configuration. *Measure 2* was greater for 78% of the reservoirs, showing that most often the goodness of locating the wells with the quality map was transfered to the true reservoirs. In some cases, though, due to unrepresentative models, it was better to consider a set of geometric configurations for each number of wells than to use a single optimized configuration.

The average value of *Measure 3* over all the reservoirs was $110.7Mm^3$ of oil greater for the optimized configuration. *Measure 3* was greater for 62% of the reservoirs, showing that most often the goodness of the location of wells was transfered to the true results of the decision of the best number of wells, but not always.

Even though 900 flow simulations are necessary to build the quality maps for all the realizations and the quality map, the total number of flow simulations for optimized well location is only 73% of the number of flow simulations in the first case study. Moreover, the flow simulations to build the quality maps, where only one well is used, are simpler and faster than the flow simulations required to compare the scenarios, where all the wells are used.

Ranking realizations

Ideally a ranking methodology should lead to the same rank as obtained with the flow response of interest. Typically, there is good correlation between different types of flow responses and the profit is a good summary of all of them. A ranking of the 20 realizations was done using the total quality associated with the wells (Q_t) for each of the 11 scenarios for all the 50 reservoirs. The same weighting formula used for well locations was applied here. Another ranking of the 20 realizations was obtained using the profits, and the correlation coefficient between the two ranks was evaluated for each case. Just for comparison and to provide a feeling of the goodness of ranking with total quality, the same exercise was repeated using the oil volume maps and ranking the realizations by the total oil volume associated with the wells.

Figure 11 shows the distribution of the correlation coefficients between the rank using total quality and the rank using profits. **Figure 12** shows the correlation coefficients between the rank using total oil volume and the rank using profits. The distribution of correlation coefficients between the rank using quality shows a mean of 0.578, a median value of 0.627 and the most frequent value is between 0.70 and 0.75. None of the cases displayed a negative correlation. These numbers indicate that, for most of the cases, the ranking of realizations using total quality is good enough to choose low-side, expected and high-side for the realizations, see Deutsch and Srinivasan [6].

Using the total oil volume associated with the wells to rank the realizations, the mean correlation coefficient with the correct ranking (using profit) was just 0.292, with negative correlation in several cases, indicating that static parameters work poorly to approximate ranking from flow responses.

Figure 14 presents the comparison between reserves and OOIP to show that the previous quality measure is better correlated with reserves than OOIP. The correlation coefficient between reserves and OOIP is only 0.592. The goodness of the average value of the map of quality uncertainty to characterize the uncertainty in flow responses was checked by its correlation with the uncertainty in reserves. The uncertainty in reserves was defined as the standard deviation over the 20 realization reserves. **Figure 15** presents the comparison between these two evaluations of uncertainty, showing that the correlation coefficient is high at 0.719.

Figure 16 shows that the standard deviation of OOIP is a much poorer estimation of the uncertainty in reserves; the correlation coefficient is only 0.418. The correlation between the uncertainty estimated with the quality maps and the uncertainty in flow responses "in general"' was also calculated showing a correlation coefficient of 0.711, very similar to the result with reserves. The uncertainty in flow responses "in general" was evaluated by calculating the standard deviation of the distribution of realization profits for each scenario and then taking the expected value of the standard deviations over all the scenarios.

Remarks

The quality map integrates all the variables involved in the flow of fluids through a heterogeneous reservoir into a two-dimensional visualization of "how good each specific area is for production." The quality map along with a simple optimization algorithm can be used to determine good locations for vertical producer wells.

The L-optimal quality map, obtained by building a quality map for each realization and integrating all of them with a loss function, can be used for well location accounting for the geological uncertainty and for the profit desire and risk aversion profile of the company. Comparing different types of quality map for well location based on the average results over 50 reservoirs, it was found that: (1) the L-optimal (lower quartile, in this case study) quality map is better than the quality map of a realization taken at random and than the quality map of the kriged model, and (2) The quality map of a realization taken at random is better than the quality map of the kriged model.

Comparing the three approaches to define the best number of wells, finding the best configuration with the associated quality map and using only the optimized configuration for each number of wells, it was found that taking the results obtained using either Realization 1 or the single representative realization as representatives of *Conv-1*, the following comparisons can be made: (a) between *Full* and *Conv-1*, *Full* is better; (b) between *Conv-k* and *Conv-1*, *Conv-k* has a higher probability of better decisions but the expected result of *Conv-k* is smaller than *Conv-1* because *Conv-k* has a higher risk of very poor decisions. *Full* is clearly better than *Conv-k*.

Using the lower quartile quality map and finding the best configuration for each number of wells provides better results and requires less computational effort than using several configurations for each number of wells, for the definition of the best number of wells and their spatial configuration.

The realizations can be ranked using the total quality (Q_t) associated with the wells. This ranking permits low-side, expected and high-side realizations to be identified for each production scenario. The average value of the mean quality map has good correlation with the production potential of the reservoir and the average value of the map of quality uncertainty has good correlation with the uncertainty in flow responses. These two average values may be used to help comparing reservoirs.

Future work

Each quality value is obtained by running a flow simulator with just one well. Thus, by construction, there is no consideration of the interferences between more than one well producing at the same time in the quality map. These interferences are considered to some extent in the optimization algorithm. Allocating the cells to the closest well, weighting the quality values with an inverse distance to the well, and seeking the maximum total quality associated with the wells ensures that interference between the well locations is taken into account. The results of the case study showed that this methodology provides good results for the joint location of several wells.

A more explicit way to account for the interference between wells could be tried following the alternate methodology (1) record the pressure drop (ΔP) after certain time in all the cells due to the production of the single well during the generation of one quality value, (2) average, somehow, the pressure drops of all the layers to obtain just one value of pressure drop for each position i, j of the horizontal grid due to the production of the well in the cell i_w, j_w ($\Delta P_{i,j}^{i_w,j_w}$), (3) use the following formula to evaluate the total quality, weighting the quality value of each cell by the ratio of the total pressure drop in that cell due to the production of the total number of wells (nw) and the pressure drop in that cell due to the production of the well in that cell, without necessity for allocating the cells to the closest well:

$$Q_t = \sum_{i=1}^{nx} \sum_{j=1}^{ny} Q_{i,j} \frac{\sum_{w=1}^{nw} \Delta P_{i,j}^{i_w,j_w}}{\Delta P_{i,j}^{i,j}}$$

This methodology seems attractive since it uses the superposition concept to add up the effects of the production of several wells in the pressure drop of a particular cell. Nevertheless, there are some points that still need investigation, such as the time for recording the pressures of all the cells and the pressure averaging formula over all layers.

Different quality units can be considered for different problems. As one example, for the problem of horizontal well location, two or three quality maps can be built, fixing the layer for the single well completion when building each map.

The problem of locating injector wells after the definition of the producer well configuration can be solved with the help of a quality map built with all the producer wells and one single injector well and by varying the position of the injector well.

The quality unit can be a measure of profit, instead of cumulative oil production, to incorporate different costs of wells in different areas of the reservoir or to compare reservoirs with different well costs.

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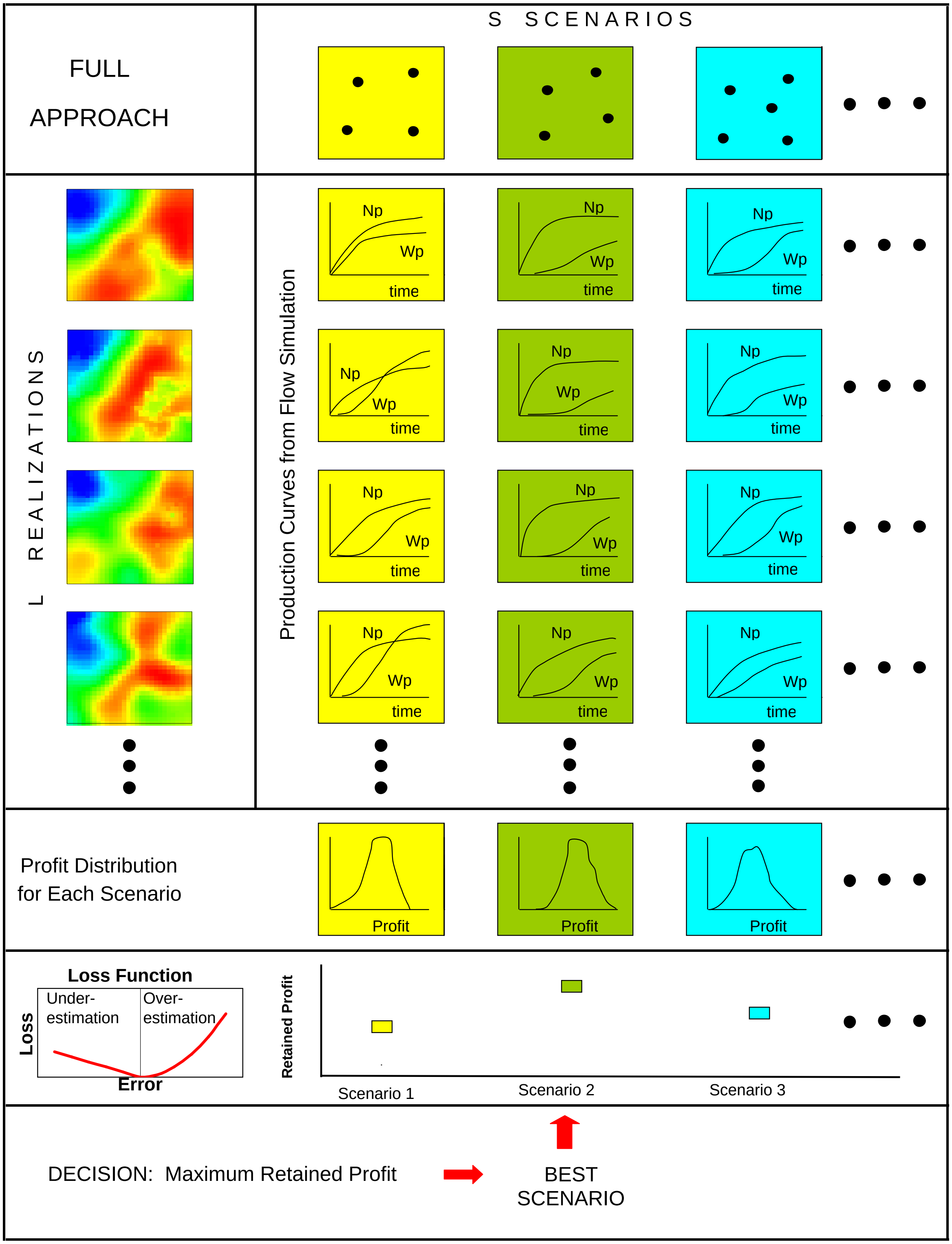

Figure 1: *Full* approach methodology.

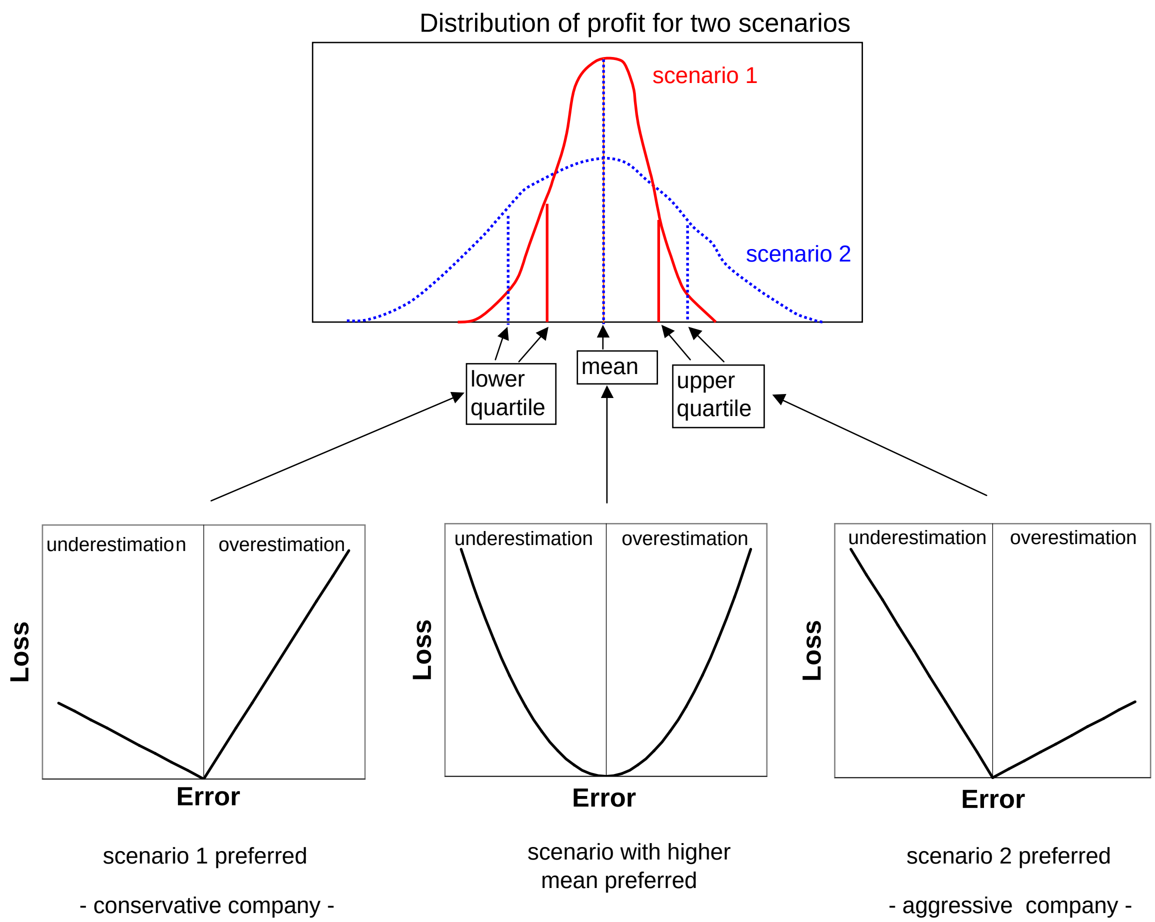

Figure 2: Example of probability distribution of profit for two scenarios and three types of loss function that yield different values of the retained profit value for each scenario and different decisions about the best scenario.

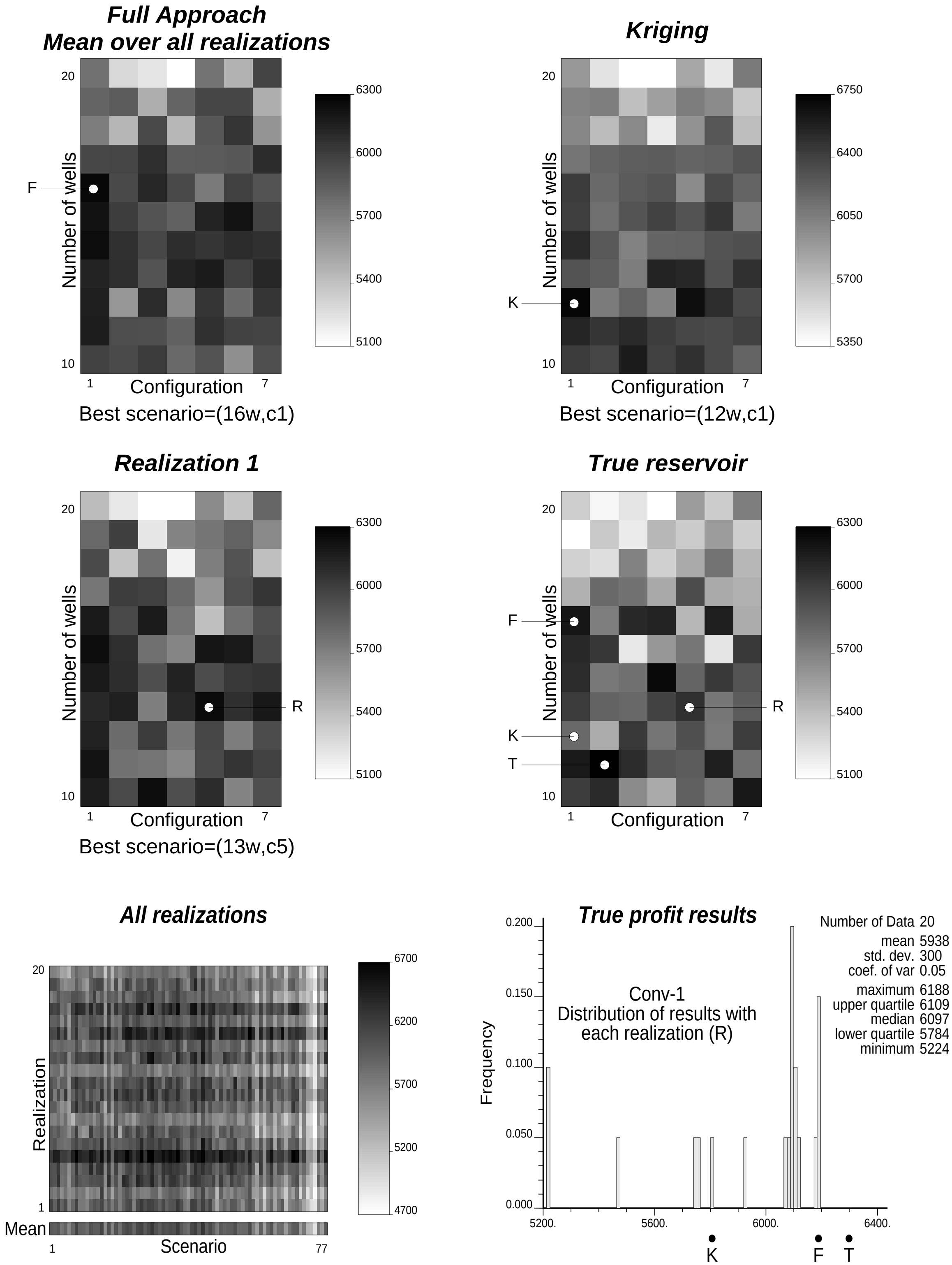

Figure 3: Scenario profits (Mm^3 oil) obtained from each approach and from the true reservoir. Best scenario and corresponding true profit from: F=*Full* approach, K=Kriging, R=Realization 1, T=True reservoir.

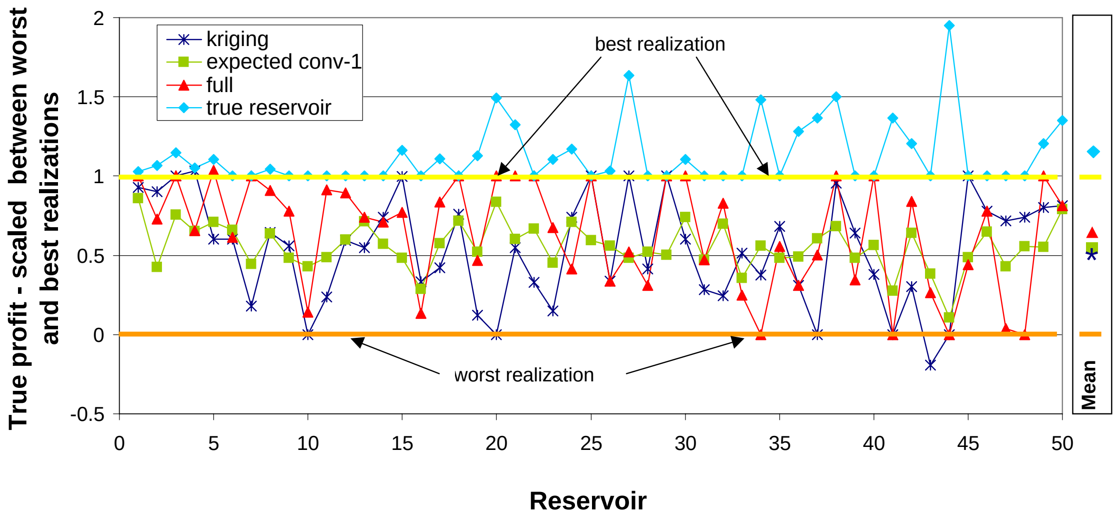

Figure 4: Comparison between the approaches for 50 reservoirs.

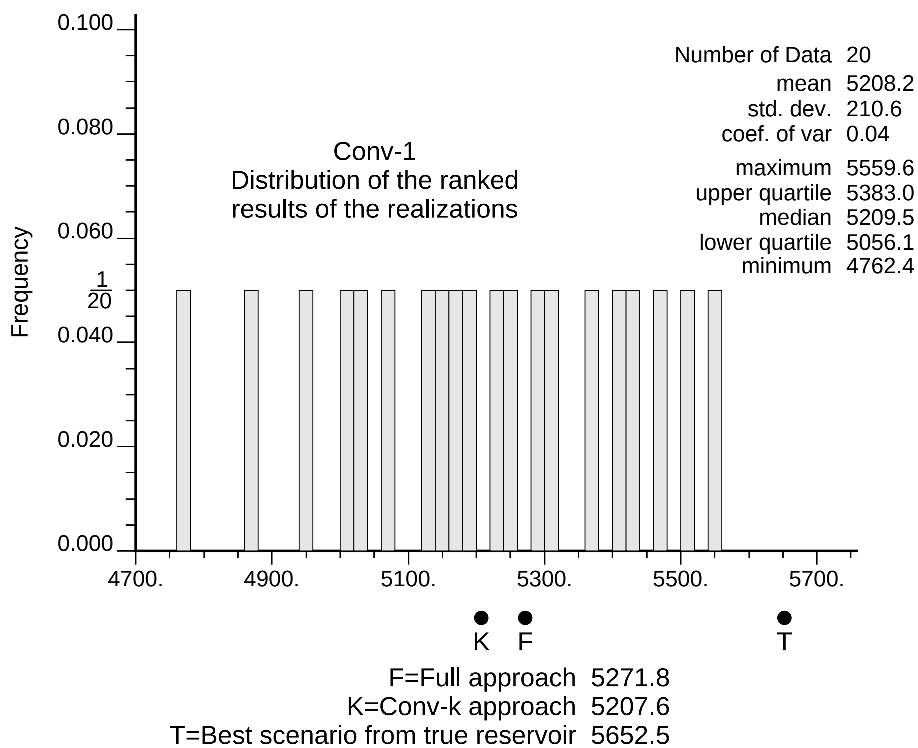

Figure 5: True profits (Mm^3 oil) averaged over 50 reservoirs.

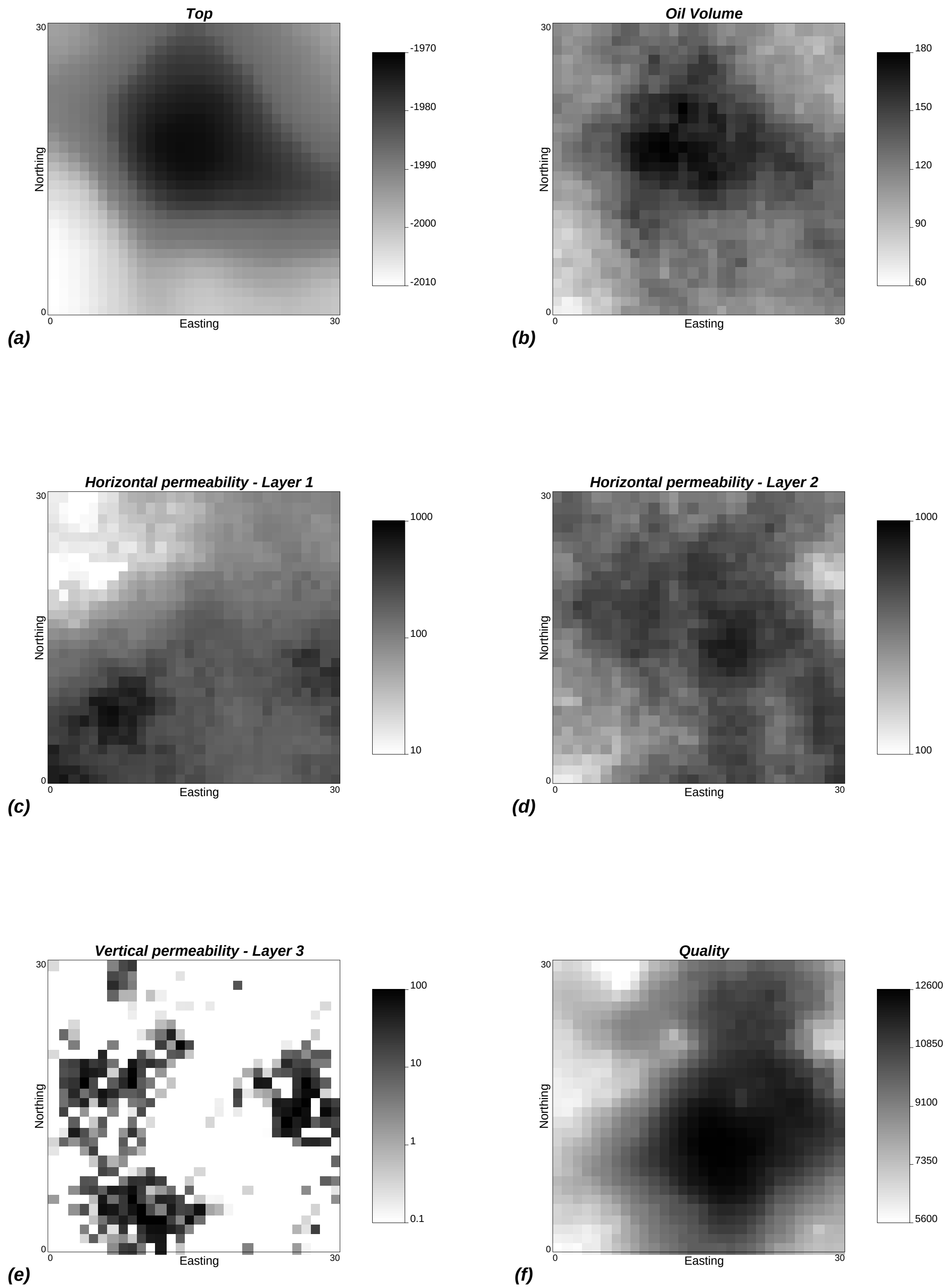

Figure 6: Presentation of the quality map. Several variables, including: (a) top (m), (b) oil volume (Mm^3 oil), (c)(d) horizontal and (e) vertical permeabilities (md), are integrated into (f) the quality map (Mm^3 oil).

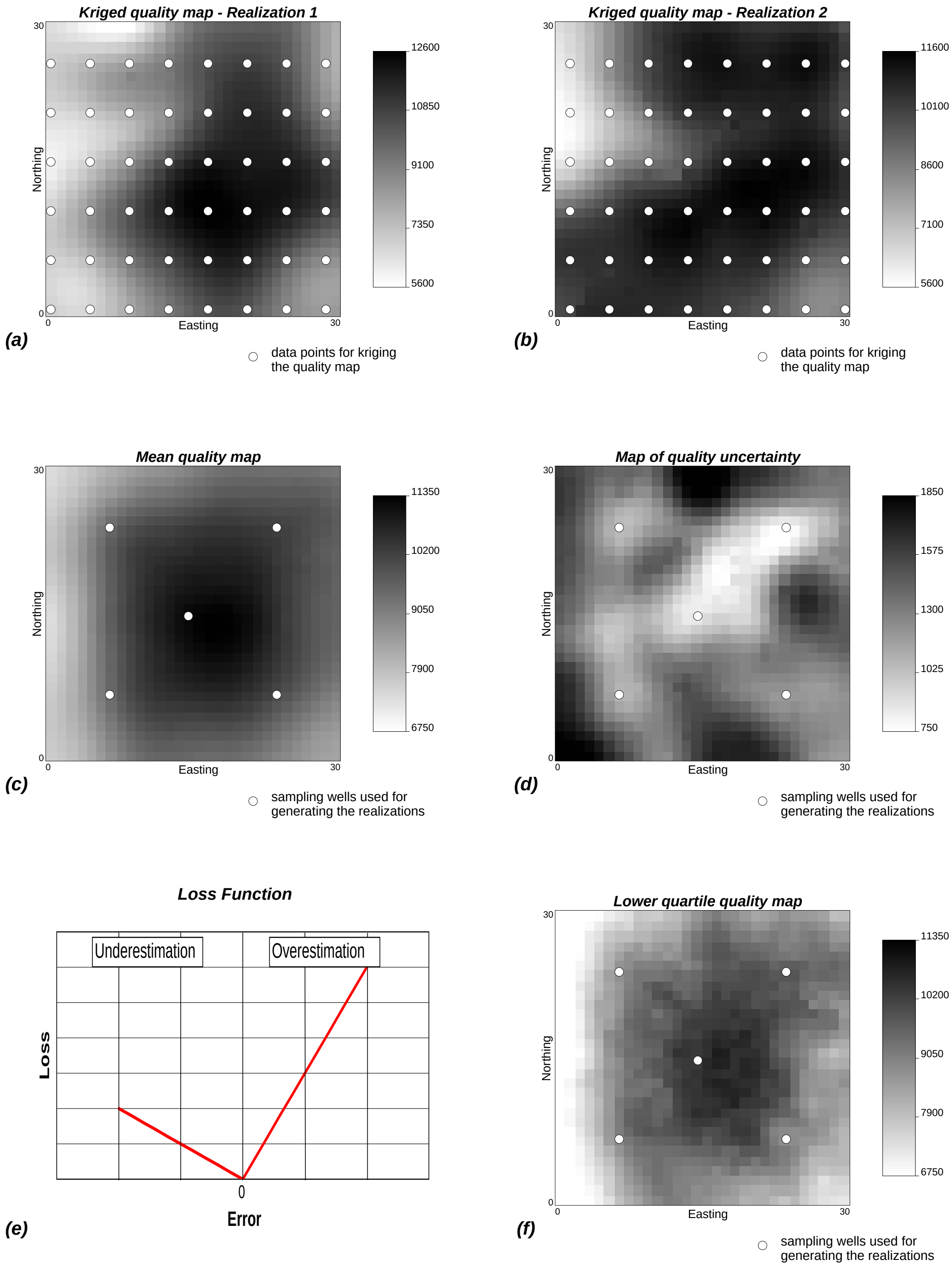

Figure 7: Types of quality map: kriged quality map of the first two realizations (a)(b), mean quality map (c), map of quality uncertainty (d) and lower quartile quality map (f). The loss function (e) was used to define the L-optimal quality map as the lower quartile quality map. The unit of quality is Mm^3 of oil.

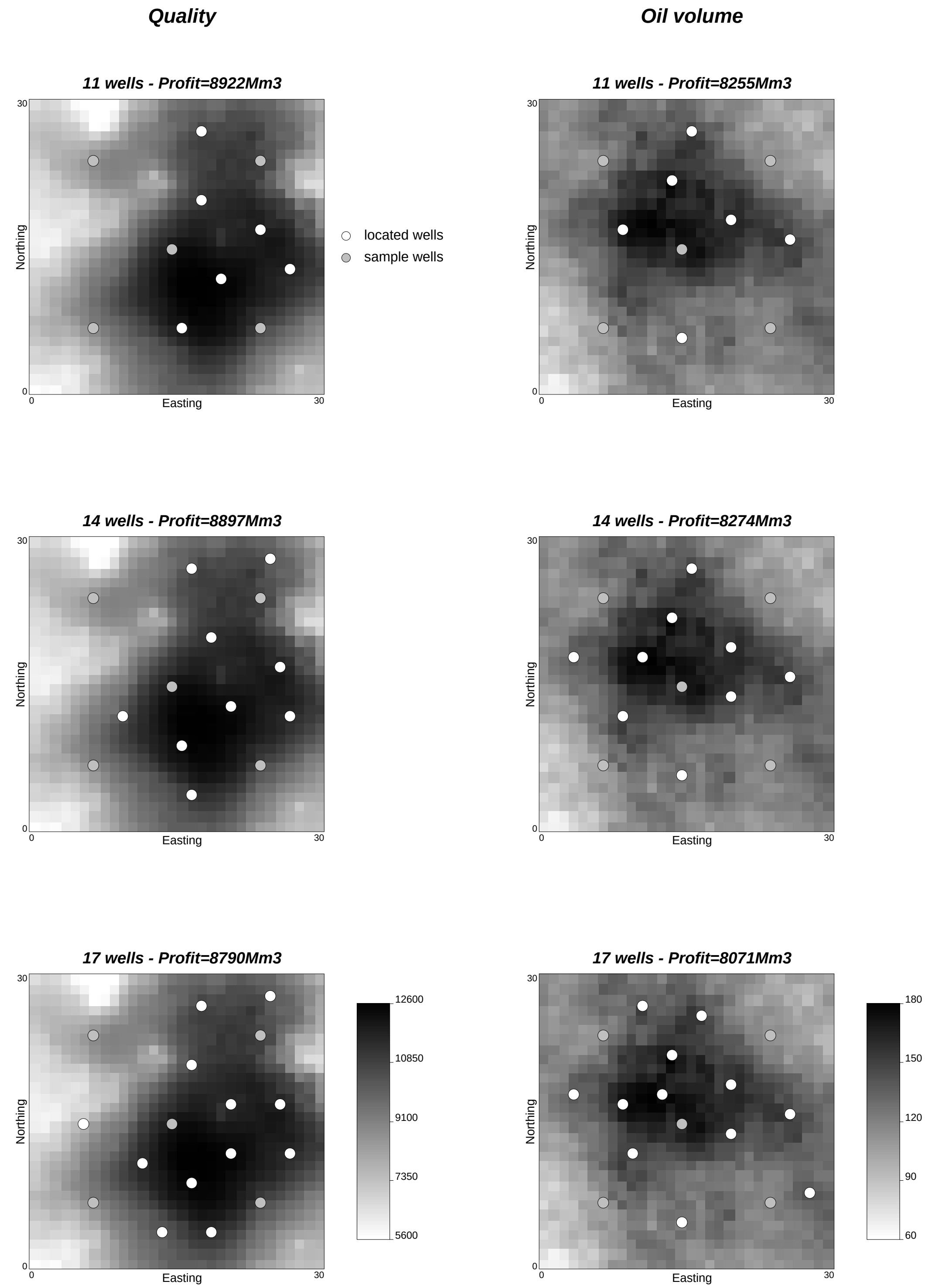

Figure 8: Examples of location of wells and resulting profits using quality map (left) and oil volume map (right) of Realization 1, for 11, 14 and 17 wells. Unit in the maps = Mm^3 oil.

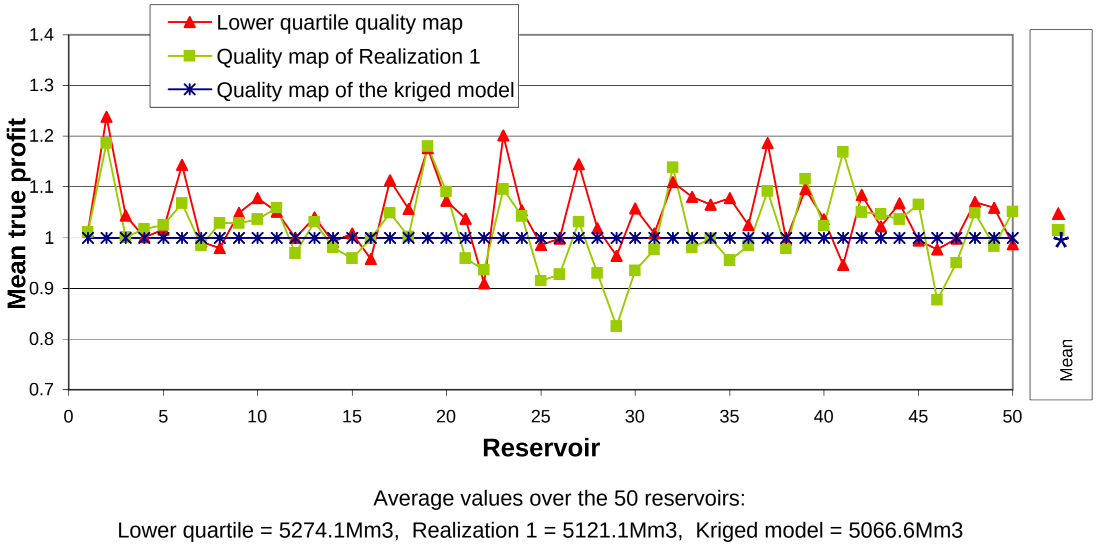

Figure 9: Comparison between the location of wells using three different quality maps for 50 reservoirs. The results in the figure are divided by the result using the quality map of the kriged model.

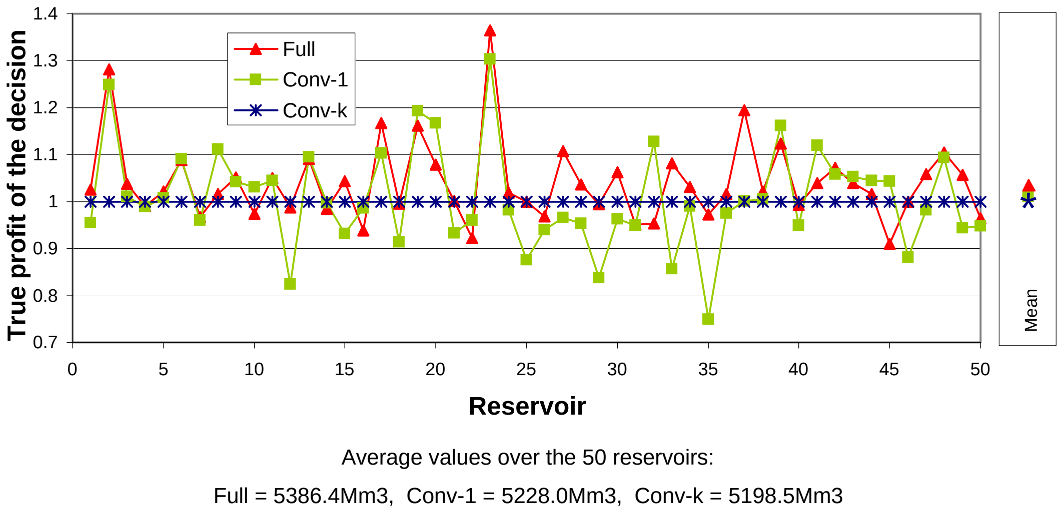

Figure 10: Comparison between the results of the decisions with *Conv-1*, *Conv-k* and *Full* for 50 reservoirs. The results in the figure are divided by the result of *Conv-k*.

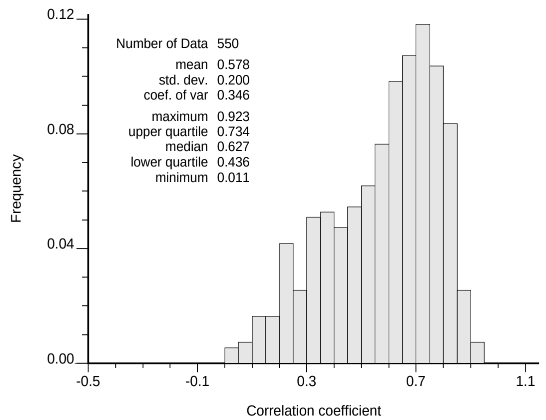

Figure 11: Correlation coefficient between the rank of 20 realizations obtained with profit and with total quality

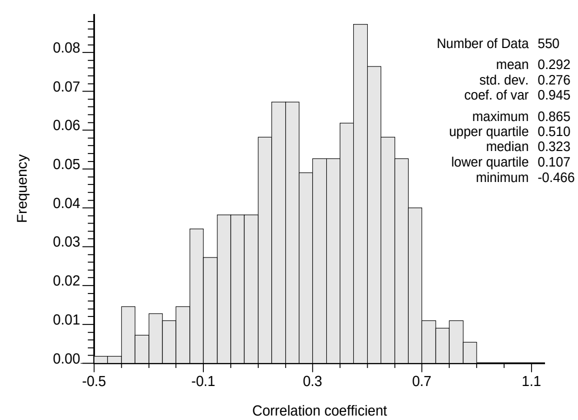

Figure 12: Correlation coefficient between the rank of 20 realizations obtained with profit and with oil volume.

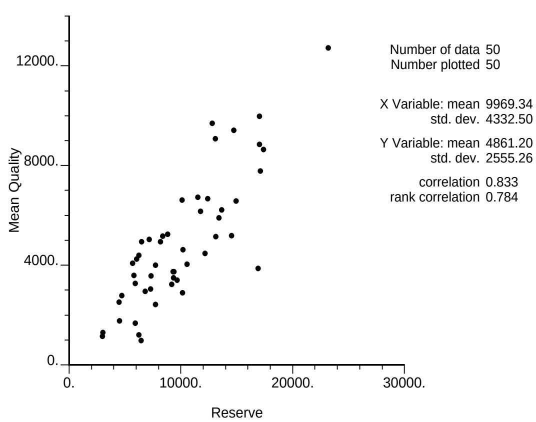

Figure 13: Reserve versus average value of the mean quality map.

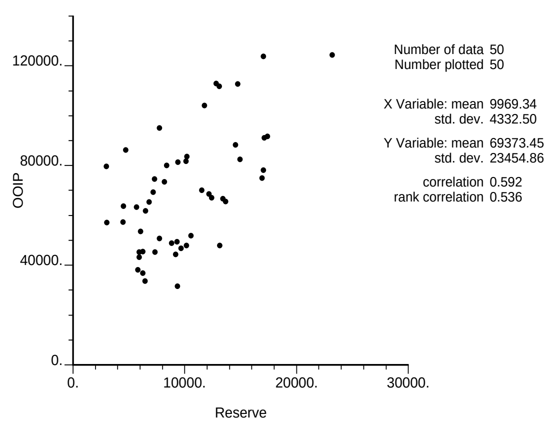

Figure 14: Reserve versus original oil in place.

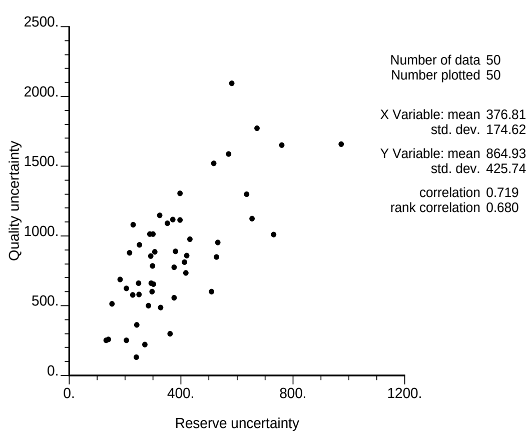

Figure 15: Reserve uncertainty versus quality uncertainty.

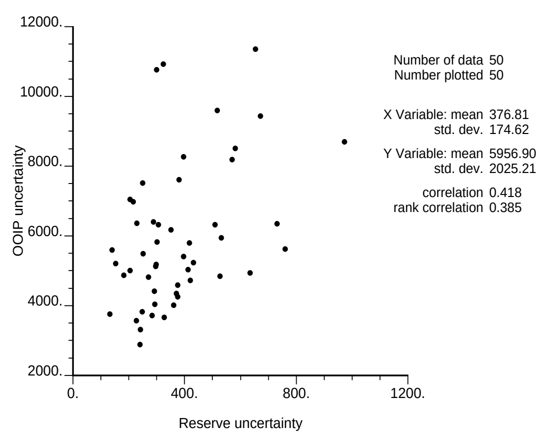

Figure 16: Reserve uncertainty versus original oil in place uncertainty.