Soil Remediation Decision Making in Presence of Uncertainty in Crop Yield Response [1]

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Compacted soil has a characteristic "hard pan" layer that impedes root penetration resulting in reduced crop yield. Soil remediation measures such as deep ripping would improve yield; however, such remediation is relatively expensive. We show how the economic benefit of remediation can be assessed in the presence of sparse sampling. Geostatistical tools are used for uncertainty characterization. Remediation is recommended when the discounted expected profit due to increased yield exceeds the cost of remediation.

A 320 by 200 m field near Leduc, Alberta, Canada was extensively studied. A penetrometer was used to measure the depth of the compacted soil layer. Elevation and soil characteristics were recorded in 1997 while yield was measured for three consecutive years. The goals of data collection were to (1) predict the yield response of remediation of compact soil, and (2) develop an economic model for cost / benefit analysis of remediation in the presence of sparse data.

Multivariate statistics combined with geostatistical simulation techniques are used to build maps of all needed variables: elevation, depth to compacted soil, and the local soil surface class. The critical relationship between yield response and remediation is established from the historical yield data. This relationship depends on elevation and local soil surface class. Since deep ripping is expensive, determining the precise areas to remediate can result in substantial savings. An economic model is used with the geostatistical maps to quantify risk and the expected benefit of remediation.

KEYWORDS: geostatistical simulation, stochastic modeling, compacted soil, uncertainty analysis

Introduction

Compacting of arable soils due to intensive traffic (Douglas, 1994) is becoming a problem in pasture and agricultural crops. Solonetzic soils may also have a compacted or impermeable "hard pan" layer formed from geological processes. These types of soils share a common characteristic: compacted layers of soil that restrict water penetration, aeration and root growth (Unger, 1980).

Compaction influences physical structure, bulk density, penetration resistance, and aeration of the soil; all of which can affect plant growth (Panayiotopoulos et al., 1994). Root characteristics such as diameter, elongation and morphology are affected when roots are subjected to high soil strengths as a result of soil compaction (Atwell, 1990a). As a consequence, soil compaction is deleterious to crop yields (Thacker and Johnson, 1988; Bateman et al., 1992). Wheat grain yield was 23% less in a compacted soil compared to that in an uncompacted soil (Oussible et al., 1992).

Given that yield is adversely affected by compacted soil, farm managers may choose to remediate such soil provided the yield response warrants the cost. The first step is to determine the extent

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of soil compaction. The penetrometer can be a useful tool to rapidly detect and assess compacted soil conditions over large areas (Fulton et al., 1996; Sanborn, 1991). Soil strength as measured by a penetrometer has been assumed to be equal to the pressure encountered by roots during growth (Bengough and Mullins, 1990). Misra et al., 1986 reported maximum root growth pressures between 0.9 and 1.3 Mpa. But, root elongation in other studies was stopped at penetration resistances between 0.8 and 5 Mpa (Bennie, 1991; Bathke et al., 1992). Thus, penetrometer pressure readings up to 5 Mpa can provide an indication of compacted soils that impede root growth and affect crop yield.

Compacted soils have been remediated with tillage equipment such as deep rippers or subsoilers that fracture the compacted subsoil (Wetter et al., 1987). Canola yield responses due to remediation have averaged 56 kg/ha for the black and thin black soil zones of Alberta, Canada (Lickacz, 1993). Compare this yield response to barley, which has averaged 375 kg/ha. In fact, Bateman et al., 1992 found penetration resistance readings of 0.39 Mpa for a compacted soil that was deep ripped compared to 0.92 Mpa from an unripped, compacted soil. Notwithstanding the benefits of such remediation, deep ripping is time consuming, expensive and requires high-powered tractors to implement (Erdat and Voorhees, 1990), which makes whole field remediation cost-prohibitive.

A map of expected yield response due to remediation is essential for decision making on the economics of remediation. Therefore, an important result of this work is a methodology to delineate areas of compacted soil that can be profitably remediated. Geostatistical tools (Deutsch et al., 1998; Goovaerts, 1997; Journel, 1989; Isaaks et al., 1989) are used to derive the needed maps from limited sample data. An economic model is used to identify those locations that provide the greatest economic return due to remediation.

Global Positioning System (GPS) technology and penetrometer measurements combined with recent geostatistical tools provide farm managers with an ability to precisely locate areas of compact soils and remediate only those areas.

Methodology: In Words

Yield can be improved by timely precipitation throughout the growing season, adequate soil fertility, and optimum growing degree days; however, these environmental factors are not easily changed by man-made activities. Yield at a specific location in a field is also affected by depth of compacted soil, which can be remediated by deep ripping. Deep ripping fractures the compacted soil layer allowing crop roots to penetrate deeper and improve yield. In this study we are concerned about the effect of "depth" on yield. Depth is defined as the thickness of arable soil (in mm) above the compacted soil layer. There will be no change to any other physical soil characteristics such as texture or elevation.

The depth of arable soil can be changed by deep ripping compacted soil and allowing crops the opportunity to penetrate this soil. Yield response attributable to this changing depth may be presented as a *yield nomograph*, see schematic on Figure 1. This chart has been called a nomograph because it represents the typical yield response to improved soil conditions such as depth to compacted soil. In other words, deep ripping breaks up compacted soil increasing the depth of arable soil achieving increased yield. Thus, inference of this yield nomograph is an important step in predicting yield response.

The yield nomograph is constructed to quantify the relationship between yield and depth. This relationship, however, does not apply uniformly across the entire field. Some good soils are productive with compact soils and others respond more significantly to remediation. We group these different soil characteristics into a soil "quality" measure, which can be used to establish a family of yield response curves, see Figure 2.

A unique yield nomgraph is relevant at each location in the field; however, time and economic constraints for data collection preclude such an undertaking. Secondly, for every geographic location in the field there is only one depth measurement and from this measurement, an entire yield nomo-

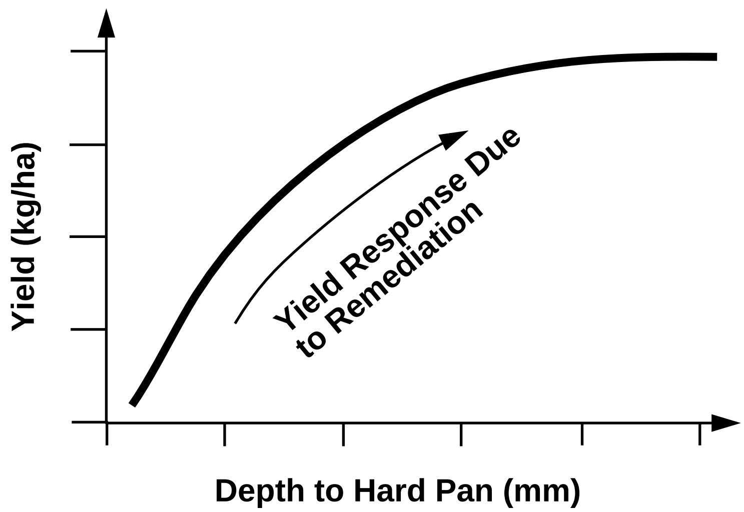

Figure 1: A schematic *yield nomograph* of the yield versus depth to compacted soil. This nomograph can be used to predict the increase in yield as the depth of compacted soil increases due to remediation.

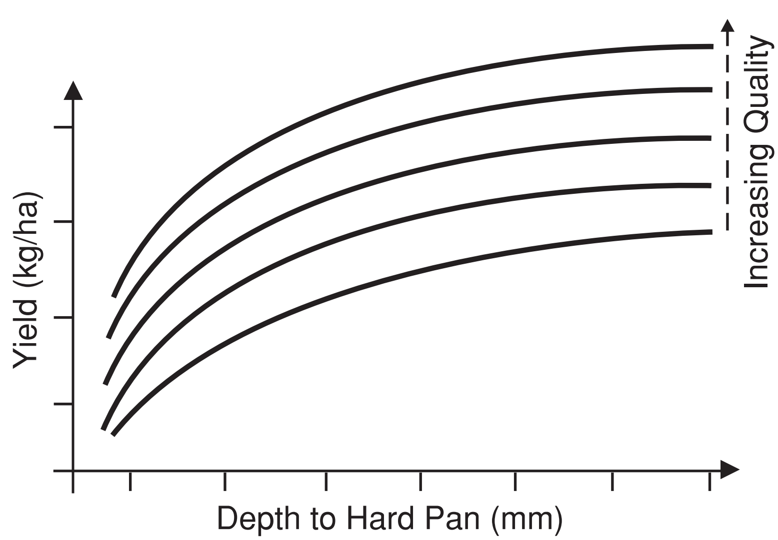

Figure 2: A schematic *yield nomograph* chart of the yield versus depth to compacted soil. This chart shows the idealized quality curve series which are created by grouping similar quality positions and applying regression.

graph could not be extrapolated. The solution is an assumption of statistical “stationarity”. We assume data at similar quality positions behave the same and thus can be pooled into similar quality categories. Each category has sufficient data of varying yield and depth to infer a representative yield nomograph curve. An assumption is made that each location in the field will respond to a change in depth according to it’s category nomograph curve. The number of quality categories are chosen to have sufficient data and such that each category is visually distinguishable from the next category, that is, each nomograph curve must be well defined.

A regression approach is used to develop a quality measure that combines all available data and isolates the effect of a dependent variable, depth, on the independent variable, yield. The resulting regression equation will express the magnitude of influence of each dependent variable, except depth, on yield. Such multivariate regression (Jaluria, 1996) permits the identification of statistically significant dependent variables as well as those that can be rejected due to statistical insignificance.

The goal is to supply the farm manager with a predictive map of yield responses based on local quality values. In order to do this, the significant variables used to develop quality must be mapped over the entire field and not just at sample locations. This can be accomplished through inverse-distance mapping, but this would not utilize all the information about spatial structure. An alternative method would be to use kriging, which provides the best point-wise estimates. Kriging, however, does not offer any assessment of uncertainty and does not lead to a map with the correct spatial structure, that is, it is a smooth interpolation method. Geostatistical simulation techniques such as sequential Gaussian simulation overcome such smoothing and provide a measure of uncertainty.

Sequential Gaussian simulation reproduces the correct covariance structure and provides a measure of pointwise and global uncertainty, which can aid in decision making. Uncertainty is captured in the form of multiple realizations or distributions of possible results for every estimated point. A farm manager can use these distributions to quantify the risk associated with making a decision to deep rip or not. Figure 3 gives an illustration of how uncertainty could be considered at one particular location in a field. There are L realizations of depth D and quality Q. Remediation increases the depth from D to some remediated depth (say, 450 mm). Thus, using the yield nomograph, an increase in yield ΔYield can be estimates for each pair D/Q. The expected yield response is the average of all yield responses.

Geostatistical simulation allows us to estimate an accurate yield response map. At a single location, multiple realizations of the dependent variables are generated. By using these multiple realizations of the dependent variables, multiple realizations of quality can be generated, and each response categorized. For each category, the independent variable is assumed to mimic the regression curve of the group, and this results in multiple realizations of yield response. Then an economic model of expected profit per year as a result of remediation with deep ripping can be generated from the expected yield response and the value of the crop.

A cost map must be constructed to create the net profit map. For the cost map, it is assumed that deep ripping has a constant unit area cost over the entire field. The cost can be calculated as the cost of deep ripping a unit area amortized over a number of annual payments with the prevailing interest rate. Subtracting the values of the uniform cost map from the expected values of the net benefit realizations equals a net profit map. Profitable areas of remediation can be determined by averaging a large number of these net profit realizations to provide an expected or predicted value of profit. Therefore, geostatistical simulation accounts for uncertainty due to limited sampling and our imperfect knowledge of the spatial distribution of soil quality variables.

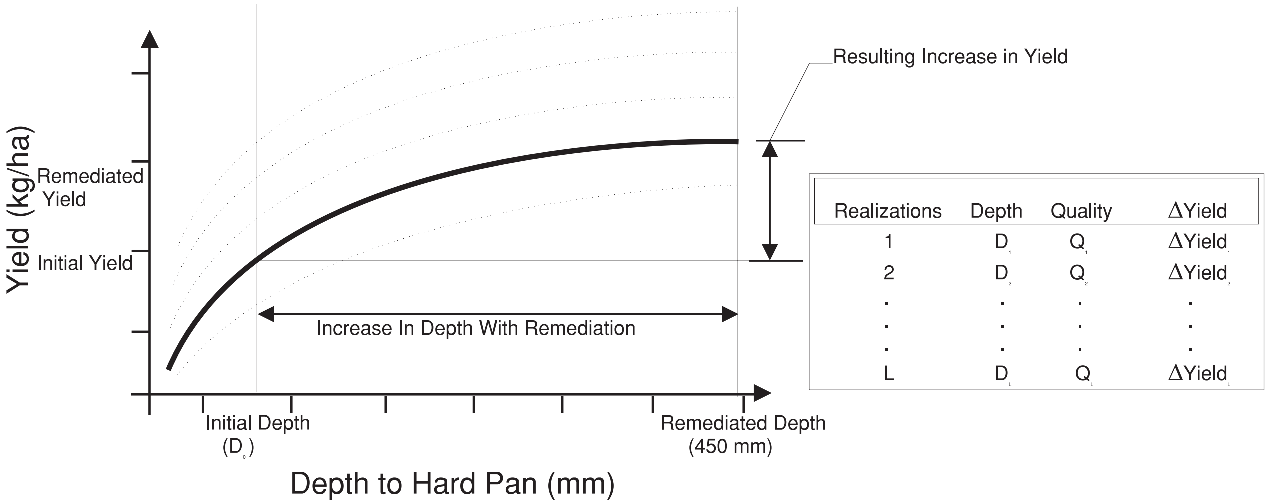

Figure 3: An illustration of the application of multiple realizations to calculate multiple realizations of yield response.

Methodology: In Equations

Consider the following regionalized variables: Y - yield, D - depth to compacted soil, F_1 - first factor affecting quality, F_2 - second factor, ... and F_n - n^{th} factor affecting quality. The soil quality at a location $\mathbf{u}$ (corresponding to a particular vector location in the field) is a linear function of the n factors:

$$q(\mathbf{u}) = a_0 + \sum_{i=1}^{n} a_i f_i(\mathbf{u}), \quad (\mathbf{u}) \in \textit{Area of Interest}$$

The parameters $a_i, i = 1, \ldots n$ are coefficients that explain how each factor contributes to yield. The factors (possibly correlated) $F_i, \ \ 1, \ldots, n$ are uncertain and are modeled by L-multiple realizations: $l = 1, \ldots, L$. Thus, there are L-quality values at each location:

$$q^{(\ell)}(\mathbf{u}) = a_0 + \sum_{i=1}^{n} a_i f_i^{(\ell)}(\mathbf{u}), \quad (\mathbf{u}) \in A, \ \ \ell = 1, \ldots, L$$

The L realizations are created by geostatistical methods (Deutsch and Journel, 1998; Goovaerts, 1997). The $a_i, i = 1, \ldots, n$ values are obtained by linear regression of the measured yield by the measured factors, that is, a model of the following form:

$$y = a_0 + \sum_{i=1}^{n} a_i f_i$$

The quality at a location $\mathbf{u}'$ is therefore the estimation of yield at $\mathbf{u}$. This ensures that the quality is built with the correct contribution from each factor.

A yield nomograph curve is expressed as a function of quality and depth

$$Y_R(d \mid q)$$

where Y_R is yield response, d is depth, and q is quality. The quality values are binned into classes $q_c, c = 1, \ldots, N_c$ such that there is a clear difference between the classes. The yield values may be plotted against depth for quality values in a given class, i.e. y vs. d given $q \epsilon q_c \ + / - \ \Delta q$. The nomograph is the smoothed yield response expressed as a regression equation.

$$Y_R(d \mid q) = b_0 + \sum_{i=1}^{n'} b_i g_i(d)$$

where $g_i(d) i = 1, \ldots, n'$ are functionals of the depth such as linear, logarithmic, or other functions of depth. The yield is an increasing function of depth, that is, $Y_R(d \mid q) > Y_R(d'|q)$ when $d > d'$.

Remediation (deep ripping) increases depth from present depth $d(\mathbf{u})$ to some maximum d_{max}. Therefore, ripping is expressed as:

$$\Delta Y(\mathbf{u}) = Y_R(d(\mathbf{u}) \mid q(\mathbf{u})) - Y_R(d_{max} \mid q(\mathbf{u}))$$

Since the depth and quality at any unsampled location is uncertain, we calculate the expected increase in yield as:

$$\overline{\Delta Y}(\mathbf{u}) = \frac{1}{L} \sum_{\ell=1}^{L} \left[Y_R(d^{(\ell)}(\mathbf{u}) \mid q^{(\ell)}(\mathbf{u}) - Y_R(d_{max} \mid q^{(\ell)}(\mathbf{u}) \right]$$

This is balanced against the cost of remediation for decision making and building a final remediation map.

There are a variety of geostatistical simulation methods that could be used to generate realizations of the needed variables. The widely used sequential Gaussian simulation algorithm has been considered here. The public domain software program, SGSIM was used (Deutsch and Journel, 1998).

The essential feature of sequential Gaussian simulation is the assignment of values by following a sequential path through the grid. At each node, the kriging estimate and kriging variance are determined with the nearby data (and previously visited grid node values) and the correct covariance / semivariogram model of spatial correlation. To create simulated values without the smoothing effect of kriging, a value is simulated from this local distribution of uncertainty. Then, the simulated value is added to the data set and the next node is visited. The simulated Gaussian values are back-transformed into the original data units. Other realizations can be produced by repeating the procedure with a different random number. These simulations honor the data, reproduce the histogram of the sampled data and reproduce the correct pattern of spatial variability. Differences among these realizations provides a measure of spatial uncertainty. This uncertainty at a location can be used in targeting areas for remediation with the aid of a decision making model.

Case Study

Data was collected for penetration resistance (PR), yield, elevation, and surface class on a 320 by 200 m field near Leduc, Alberta, Canada. PR measurements were collected from this area every 10 meters on 21 transects. Yield data were recorded every second with a GPS controlled yield monitor in 1996, 1997 and 1998 which led to a very dense grid of data. Redundant data were removed by retaining only the nearest sample to each node on the 10m by 10m grid. Elevation data was collected with a GPS rover and base station at 5 meter intervals in 1997. This elevation data was transformed to the 10m by 10m grid.

Digital elevation models (DEM) incorporate measures of relative and absolute landform position to define management units for precision farming (MacMillan et al., 1998a, 1998b). Since compacted soils have a tendency to accumulate moisture which impacts crop growth and yield, it is important to use landform position or surface classes. A model using derivatives calculated from elevation and a fuzzy rule identified 15 different defined landform units or surface classes. These 15 surface classes serve as a classification for the DEM of this field.

Depth of compacted soil was calculated from penetrometer readings using a threshold soil strength of 4.0 MPa. It is assumed that root growth and hence crop yield will be affected by penetrometer readings above 4.0 MPa. This assumption was verified after multiple runs showed the correlation between yield and depth was highest at the 4.0 Mpa threshold. Penetration resistance was measured using a manually operated digital penetrometer in October, 1998. This penetrometer is preset to measure and record penetration resistance at 15 mm increments to a depth of 45 cm. The recording penetrometer was equipped with a 30^o cone having a 0.95 cm^2 basal area (11 mm diameter) and a 45-cm long extension rod (Model CP-10, RIMIK PTY LTD, Toowoomba, Australia). If a rock was hit when inserting the penetrometer, the reading was redone. It was not always possible to get a complete set of readings to 45 cm. If the instrument stopped recording due to extremely hard soil or a rock, the last measurement was discarded and an average of the previous readings was used. The last recorded measurement was discarded for two reasons: first, the reading could be low because the operator stopped pushing, and second, the reading could be unrealistically high in the event of hitting a rock.

To understand the relationship between depth of compacted soil and expected yield response, a quality nomogram was developed for the grid area. The nomogram was created by rating every geographic location within the sample space based on the measured variables. A linear regression equation was developed which describes quality as a function of elevation and surface at a geographic location i and j :

$$Quality_{i,j} = f(Elevation_{i,j} + Surface_{i,j})$$

The result at the Leduc site was:

$$Quality_{i,j} = -74.965 \cdot surface + 716.13 \cdot elevation - 533978$$

Quality was divided into categories. The histogram of quality values was examined for natural thresholds in the data. In some cases, natural thresholds did not exist and groups were assigned to contain sufficient data. Using the points within each group on a yield response versus depth of compacted soil plot, regressive curves were fit for each quality category to form a family of quality curves to represent the benefit of remediating the soil. By adjusting the thresholds in the quality categories and observing the resulting series of regression curves, the categories were fine tuned. When the curves crossed, this indicated that the categories had too few data points. By increasing the bin size of the categories, the curves became smoother due to averaging. The objective of these iterations is to produce descriptive quality curves, which illustrate meaningful yield response differences with respect to quality categories. Five quality logarithmic regression curves were derived from this data set.

The quality curves (see Figure 4) show flat yield response in the ultra high quality category. This suggests that in certain field locations, yield is non-responsive or independent of depth. Consequently, there is less need for deep root development in these locations since soil and environmental conditions are more favorable. Compare this to the ultra low quality curve, where depth affects crop yield (220 kg/ha yield response). This can be expected since soil characteristics appear to be limiting yield response. Between these soil quality curves, there is a range of yield responses and soil qualities.

The graph on the top left of Figure 4 shows the quality groups plotted with their accompanying regression curves. It is assumed that all locations within each quality group will have a similar response to increased depth due to remediation of compacted soil. Maps of quality and depth at all geographic locations in the field are needed to estimate yield response. To produce the best maps possible, geostatistical tools are used. These tools create stochastic maps that reproduce spatial correlation and honor the sample data.

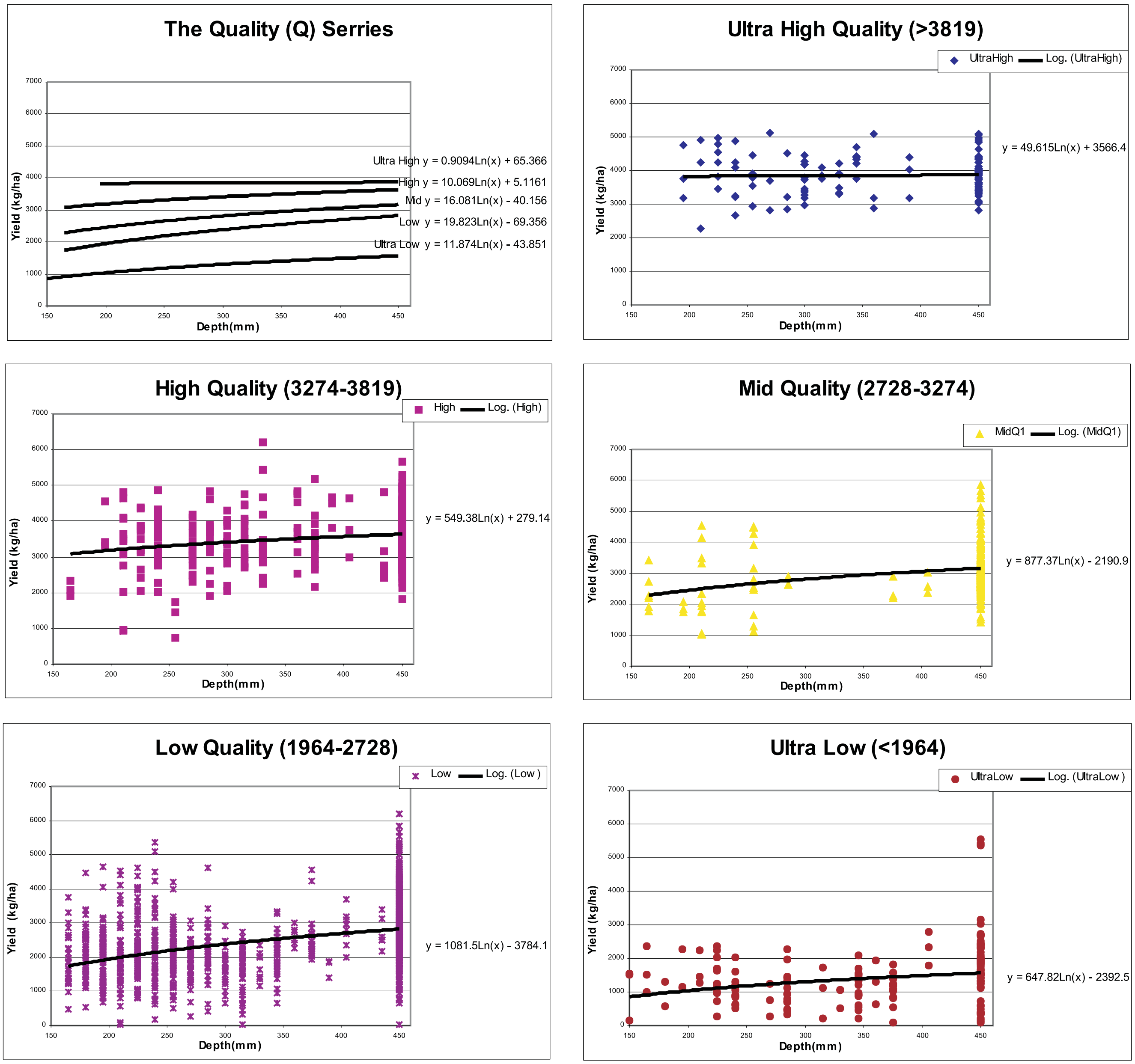

Figure 4: The resulting quality curve series and the quality categories plotted separately with their regression curves.

Spatial Correlation

The location maps of the data are shown in Figure 5. The yield and elevation data have been cleaned by accepting data at each node on the PR (10 X 10 m) grid and the yield data for 1996, 1997 and 1998 have been normalized to a mean of 2728 kg/ha and a standard deviation of 1091 kg/ha, see Figure 5.

The spatial variability of the depth, elevation and surface were expressed through semivariograms (see Figure 6). The semivariograms showed zonal anisotropy in the major continuity direction and a trend in the minor continuity direction. The range of continuity in the minor directions was greater for depth, surface and elevation than 1996, 1997 or 1998 yields. As expected the elevation semivariogram shows good short scale correlation. There is zonal anisotropy in the west - east direction and a trend in the north-south direction. The surface and depth semivariograms have higher nugget effects compared to elevation and the zonal anisotropy is in a slightly different direction. For depth, elevation and surface, three nested structures were required to model the semivariograms. These semivariograms were Gaussian in shape while the second and third nested structures were spherical for elevation and Gaussian for surface and depth.

Simulation

Kriging provides the best pointwise estimates but does not reproduce the correct spatial correlation between estimated points. However, sequential Gaussian simulation reproduces the correct semivariogram and is used to build appropriate maps for each variable. Simulation provides a distribution of possible values for every point being estimated.

In this study 60 realizations were created for the net benefit model and then these realizations were averaged. The factors $F_i(\mathbf{u}), \ \ 1, \ldots, n$ are uncertain and are modeled by L-multiple realizations: $l = 1, \ldots, L$. Thus, there are L-quality values at each location:

$$q^{\ell}(\mathbf{u}) = a_0 + \sum_{i=1}^{n} a_i f_i^{\ell}(\mathbf{u}), \quad (\mathbf{u}) \in A, \ \ \ell = 1, \ldots, L$$

The $a_i, i = 1, \ldots, n$ values were obtained by linear regression of the measured yield by the measured factors, that is, a model of the following form:

$$y = a_0 + \sum_{i=1}^{n} a_i f_i$$

The quality of a location $\mathbf{u}'$ is directly related to the modeled yield. Using these realizations, an expected crop yield response map was developed.

Elevation was simulated first since it is the most continuous variable and therefore the easiest to simulate. Sixty simulations were arbitrarily used because the spreadsheet software limited the amount of data that could be conveniently processed (independent studies show that 20 realizations are often sufficient). Next, surface was simulated with elevation as a covariate. A correlation coefficient of -0.342 between surface and elevation was used. The output from this cosimulation of surface was used as secondary data to simulate with depth. The correlation coefficient between depth and surface was -0.341. After this last cosimulation, the 60 realizations were used to calculate ΔY (see Figure 3) and were averaged to create the net benefit map.

Yield Calculation

The geostatistical realizations of surface and elevation were used to calculate quality and categorize each location and each realization. The yield increase of each realization and the average over all realizations was calculated.

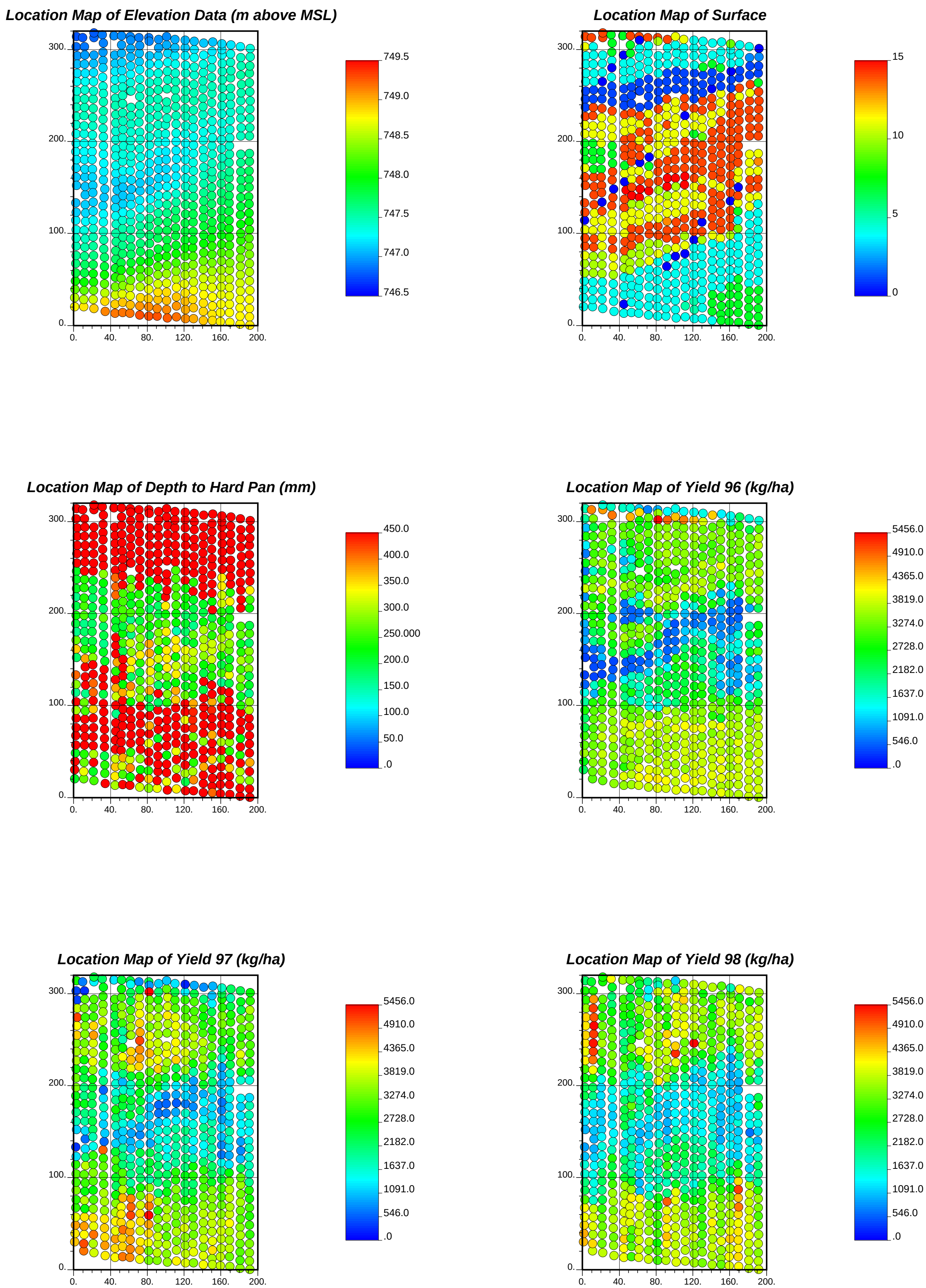

Figure 5: The cleaned data from the test plot.

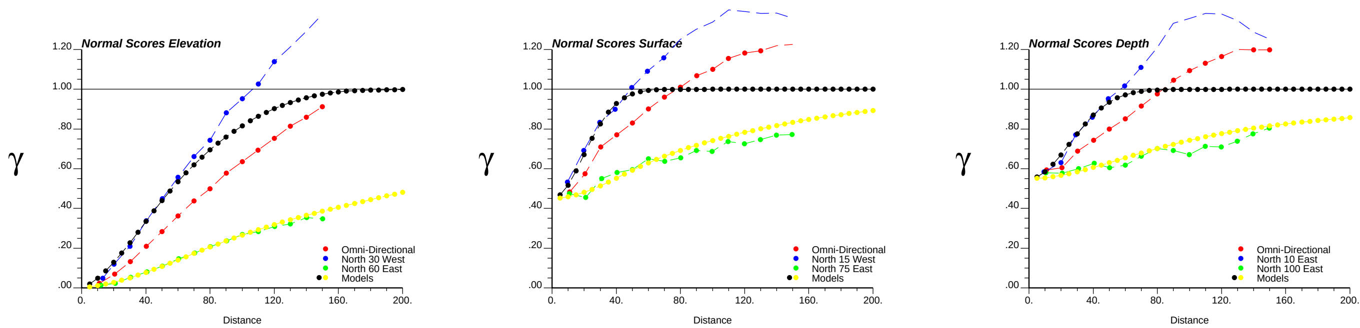

Figure 6: The experimentally derived semivariograms and the fitted models.

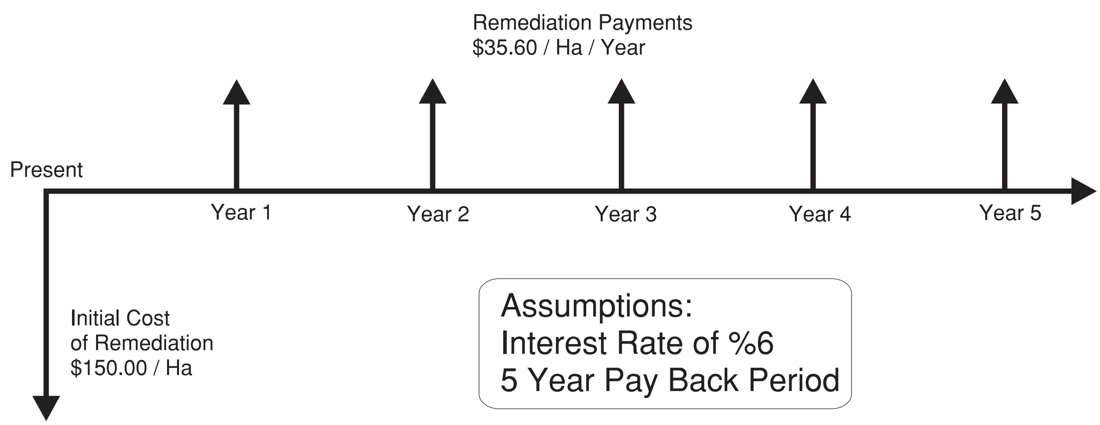

Figure 7: The net present value method used to define cost of remediation.

An economic model was used to calculate a net profit map as a result of remediation with deep ripping (see Figure 7). Gross benefit maps were created assuming crop prices of 9.5 cents/kg and 30 cents/kg, respectively for barley and canola. Net benefit maps were created by subtracting the amortized cost of deep ripping ($150 / ha) over 5 years ($35.60 per year per ha) from the gross benefit. Deep ripping was assumed to cost $150/ha and interest over the 5 year payback period was assumed to be 6% (see Figure 7). The net benefit map was smoothed using a 7 X 7 grid to weight each sample point based on its surrounding neighbors. This was done in recognition that remediation could not take place on isolated 10m by 10m plots - larger areas would be considered.

Limitations and Future Work

Quality is a parameter that changes from field to field depending on soil attributes. For example, pH or organic matter may significantly influence yield in other fields. Consequently, the appropriate variables must be investigated with each new growing region. The regression equation of quality to describe yield response would also need to be redone for each new region.

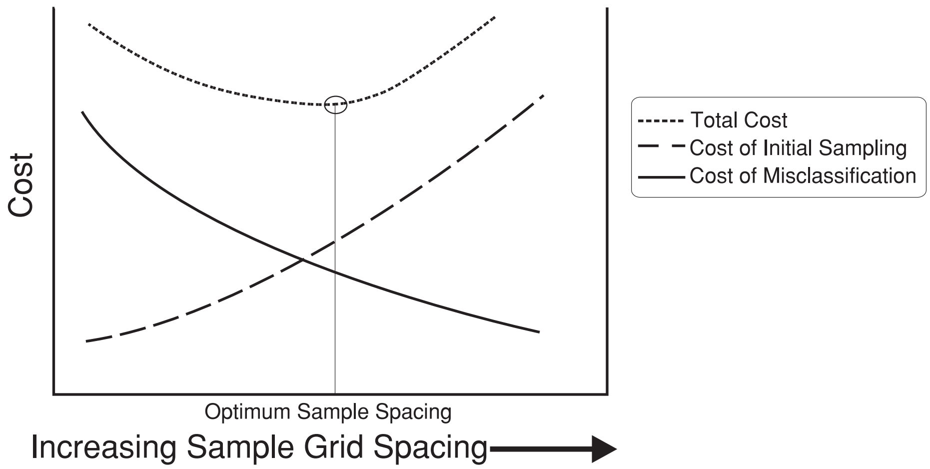

Figure 8: The experimentally derived cost of sampling and cost of misclassification curves could be used to determine the optimum sample spacing in a specific setting.

There is uncertainty in depth due to soil variability and sampling error in penetration resistance. This can be inferred from the nugget effect of 0.55 in the depth semivariogram. Thus, there may be some features occurring at a scale smaller than the 10 m sample size. This may require a more intensive, consequently more expensive, sampling scheme to capture this shorter scale variability.

Optimum sample spacing has not been investigated. Sample spacing which optimizes time and labor costs compared to gains in accuracy needs further study. This project had a 10 m grid sample spacing, however, this may be too expensive to implement on a field scale. Local calibration of an ideal sample spacing could be validated in several fields. A small data set could be evaluated with various data spacing, imposed by leaving some data out. Then the cost of sampling could be calculated for each grid and compared to the cost of misclassification. Several trials would approximate a curve which would be similar to the above. The minimum total cost would tell us the optimal data spacing (see Figure 8).

There is an inherent assumption in the study that an increase in depth will result in an increase in yield. This is based on observed trends in the data. Although the trends do exist, cause and effect have not been scientifically proven with this data. Thus, this cause and effect relationship needs to be confirmed and the quality model validated by remediating the field and comparing the actual yield responses to those predicted by the model.

Conclusion

We have presented a new method to delineate areas of profitable remediation in a field with compacted soil. The steps are to (1) build a soil quality map, (2) map the depth to compacted soil, (3) calculate the expected benefit of remediation, and (4) lastly, use this map with other economic and practical constraints to make remediation decisions. The farm manager can use this methodology to understand the expected benefit of remediation at every position in the field.

The quality nomograph is built by isolating the significant variables which effect yield and combining all variables, with the exception of depth, into a quality value. This can be simply a regression model of the variables with respect to yield. By judicially assigning unique quality categories, the yield response is quantified at every location.

The regression step requires a data set relevant to the field under consideration. Sequential Gaussian simulation, a tool of geostatistics, is used to infer between the data samples in the field. The simulation requires the construction of a semivariogram, which captures the spatial behavior of the data. With these steps, remediation can be performed in a predictive manner, with optimized returns, rather than as is currently done on a whole field basis.

Other areas of agriculture where this technique may prove useful are site specific application of fertilizers and herbicides, or mapping insect infestations in a field for therapeutic treatments using insecticides.

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