A Preliminary Report on Automatic Determination of Optimal Well Paths Subject to Geostatistical and Economic Constraints

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Optimal well placement is extremely important in the petroleum industry. This paper proposes a technique for optimal selection of wells, and well paths. We define optimality as the best balance among multiple objectives. Simulated annealing was chosen for the optimization algorithm because of its ability to handle difficult constraints.

The proposed method requires models of the top surface, zone thickness, porosity, water saturation, lithofacies, and geo-object connectivity. Uncertainty is handled by considering multiple realizations. Vertical and deviated wells are considered. Deviated wells may consist of two or more segments. The proposed method starts with seed well locations and perturbs the locations until the objective is met. The objectives considered here include hydrocarbon pore volume, connectivity, well length, distance from and orientation to the well platform. Results are shown for 2-D and 3-D. Limitations and further work ideas are identified.

Introduction

This paper presents an approach for optimal well location and path selection. The difficulty in selecting these is largely based on heterogeneities and uncertainties in the subsurface reservoir parameters. Various approaches have been proposed to solve the problem including direct optimization using a flow simulator as the evaluation function. While this would be a more accurate approach, it is impractical in most cases due to the number of realizations and computational requirements. These factors contribute to most engineers ignoring the uncertainty quantified with multiple realizations.

A typical reservoir characterization has models of uncertainty for surfaces, zone thickness, water saturation, and lithofacies. It is unclear how to simultaneously consider each of these models for selecting candidate well locations and paths. We show an approach to position both vertical and deviated well paths.

Optimization Procedure

Optimality is often subjective. Our "optimal" solution is one that is simultaneously constrained by a number of objectives. One objective is that of maximum connectivity; an optimal well path/location intersects a maximum of reservoir pay. This combinatorial of possible well paths/locations is very large. The classic combinatorial problem is the travelling salesman, that is, "what is the shortest path through n cities returning to the starting city and visiting each city only once?" Given a large number cities there are an essentially infinite number of combinations to consider and the problem becomes very difficult. Similar to the traveling salesman problem, finding the optimal well paths in a large reservoir model is very difficult.

The optimal solution to combinatorial problems is not probabilistic. In the traveling salesman problem, the distance between each city is known and there exists a single path that is the shortest path through all cities and returning to the starting city. There is no uncertainty. Like the map of distances between cities, each realization of a modeled reservoir parameter is a complete map of modeled parameters, that is, the value of the parameter is known at all locations. Uncertainty in the reservoir parameters is modeled over the suite of realizations. Thus, for each realization of a reservoir parameter, there exists a single optimal solution, and for the suite of realizations there is an equal

Figure 1: An illustration showing good and bad platform location and good and bad well alignment.

number of optimal solutions. There also exists a single optimal solution over all realizations, that is, there is a single path that is jointly optimal over all realizations of the reservoir parameter. The joint optimal is preferred because the solution accounts for uncertainty. In the individual realization optimal solutions must still be reconciled.

The Objective Function

The objective of reservoir development is to maximize recovery, and timely production, at minimum risk. The objective function is ultimately put in terms of profit. Simplisticly:

$$O = P_{revenuefromoil} - P_{costofdrilling} \tag{1}$$

The aim of the optimization is to maximize the profit. There is no need to weight the components of the objective function as the units are identical and of equal importance.

The porosity, fluid saturations, and geo-object connectivity are used to calculate the contained volume of hydrocarbon in connection to a a well. Hydrocarbon pore volume HCPV is the product of porosity and hydrocarbon saturation. Geo-object connectivity identifies blocks connected to a well. The well structure can either be vertical or deviated. Deviated wells may have one or segments.

The considerations that make a well location *good* may be summarized by the following considerations:

- A well with high HCPV connectivity, that is, large volumes of connected oil is good. Blocks high in oil content but unconnected to other blocks offer little revenue.

- Wells that contact other fluids such as water or gas are prone to coning and cusping and are not good. In the worst case such wells may have to be shut in.

- Platform location refers to the proximity of the platform to the well path. Good platform locations have the start of the well path close to the platform. Well geometry refers to the perpendicular distance of the platform to the well path. Good well geometry has short perpendicular distances from the well to the platform. Figure 1 illustrates good and bad platform location on the left and good and bad well alignment on the right, and Figure 2 shows how well alignment and platform location are quantified.

- Drilling wells is expensive. A bulk of the expense is tied up in getting the equipment to the site. The remaining cost is in the length of the well. Except for exploratory drilling, the well path should be kept as short as possible to minimize expense. Moreover, the number of wells should be kept to the minimum required to achieve production targets.

Some of the available techniques for optimization include linear programming, genetic algorithms, and steepest descent. These techniques have varying degrees of success. The objective function is not differentiable with respect to any of the variables and is discontinuous, thus precluding the use many optimization techniques. Genetic algorithms require a "population" of solutions to be maintained, which can become CPU demanding with a large number of variables. Simulated annealing was selected as the approach for optimization because it is easy to understand, can be made to
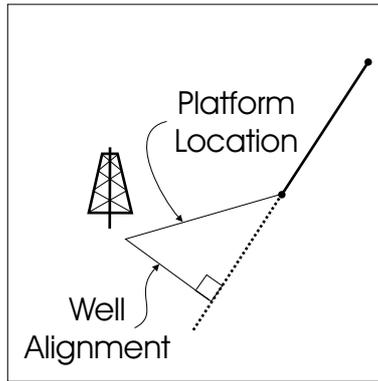
Figure 2: An illustration showing good and bad platform location and good and bad well alignment.

converge quickly, and is straightforward to program. Also, the problem of selecting optimal well locations/paths is not a straightforward problem. Simulated annealing is unique in that it the algorithm does not require a continuous objective function or that the constraints be differentiable.

The idea behind SA is simple: start with an initial configuration and perturb it with random changes until optimality is reached. Perturbations that move the configuration toward optimality are automatically accepted. Perturbations that move the objective function away from optimality are conditionally accepted/rejected. If the perturbation is accepted then the new configuration is kept and the algorithm proceeds. If the perturbation is rejected then the previous configuration is used. The simulated annealing calls for an annealing schedule to control conditional acceptance and rejection.

SA requires an initial configuration from which to proceed. The initial number of wells, the well location and the well geometry, vertical or deviated, are determined by the user. The Perturbation Mechanism is important in SA optimization problems. The perturbations must not be too drastic or most perturbations will not be accepted and convergence will be slow. The perturbations must not be too minor or many perturbations will be required to achieve convergence. Standard practice is to choose a reasonable mechanism and any inefficiencies will be revealed in slow convergence. The algorithm coded here only rarely takes more than a few minutes on a PC for convergence; thus, the algorithm is efficient or the inefficiencies translate to acceptable CPU time.

The perturbation mechanism for vertical wells requires four random numbers: one for a change in the x direction, one for a change in the y direction, one for a change in the z direction, and the last for a change in the z direction at the bottom of the well. The perturbation mechanism for deviated wells differs in that 6 random numbers are drawn for random changes in all directions. Figure refwellcircle illustrase the perturbation mechanism for a vertical well.

The value of the objective function is calculated as the difference of the available oil for the well and the cost of drilling. The volume of available oil is dictated by the drainage radius of the well and the connectivity of the well intersected geo-objects. To determine if a geo- object is intersected the well segments are discretized into an array of x, y, and z coordinates. Each discretized location in space has a corresponding grid index within the geo-object model and the hcpv model. Blocks that fall within the drainage radius and have oil volume and are connected to the same geo-object are considered available for recovery.

Perturbations that move toward optimality are automatically accepted. Perturbations that move optimality are conditionally accepted. Acceptance and rejection of perturbations are provided by simulated annealing and the annealing schedule. Conditional acceptance and rejection of perturbations causing departures from optimality is the key to the success of simulated annealing. There is an extensive literature on this feature of simulated annealing.

At any point after the first perturbation, there are two objective function values and two solutions
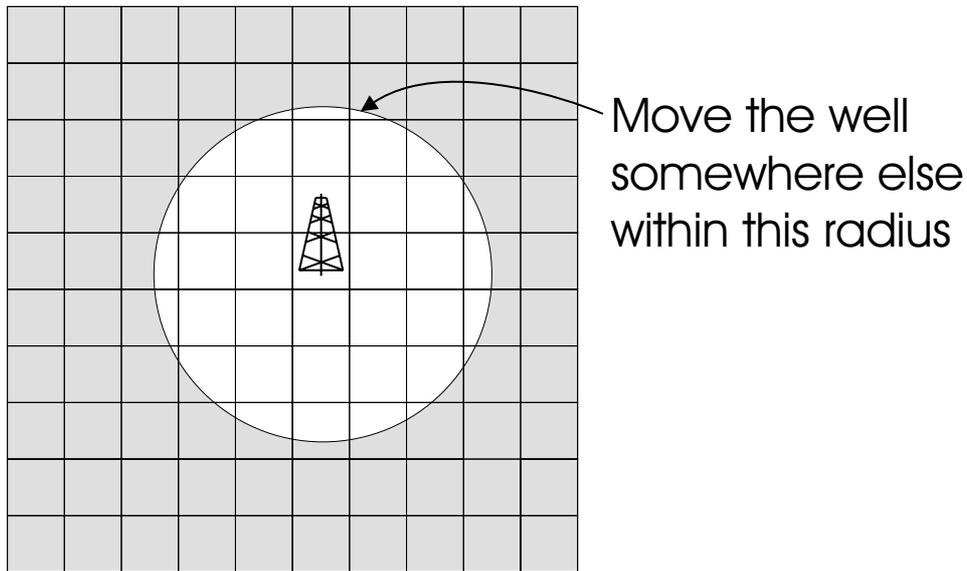
Figure 3: The perturbation mechanism for a vertical well.

to the optimization problem, one prior to a new perturbation, and one calculated as a consequence of a perturbation. When a perturbation is accepted the prior objective function and solution are updated by the new objective function and solution.

The initial number of wells, the well location and the well geometry, vertical or deviated, are determined by the user.

Examples and Future Work

A number of examples are attached. This paper came together so close to press time that a full discussion is not included. Please get a copy of the latest pdf file.

Multiple realizations have not been fully considered yet. Multiple realizations deserve great care because the optimal solution should be robust with respect to geologic uncertainty quantified by multiple realizations.

Bibliography

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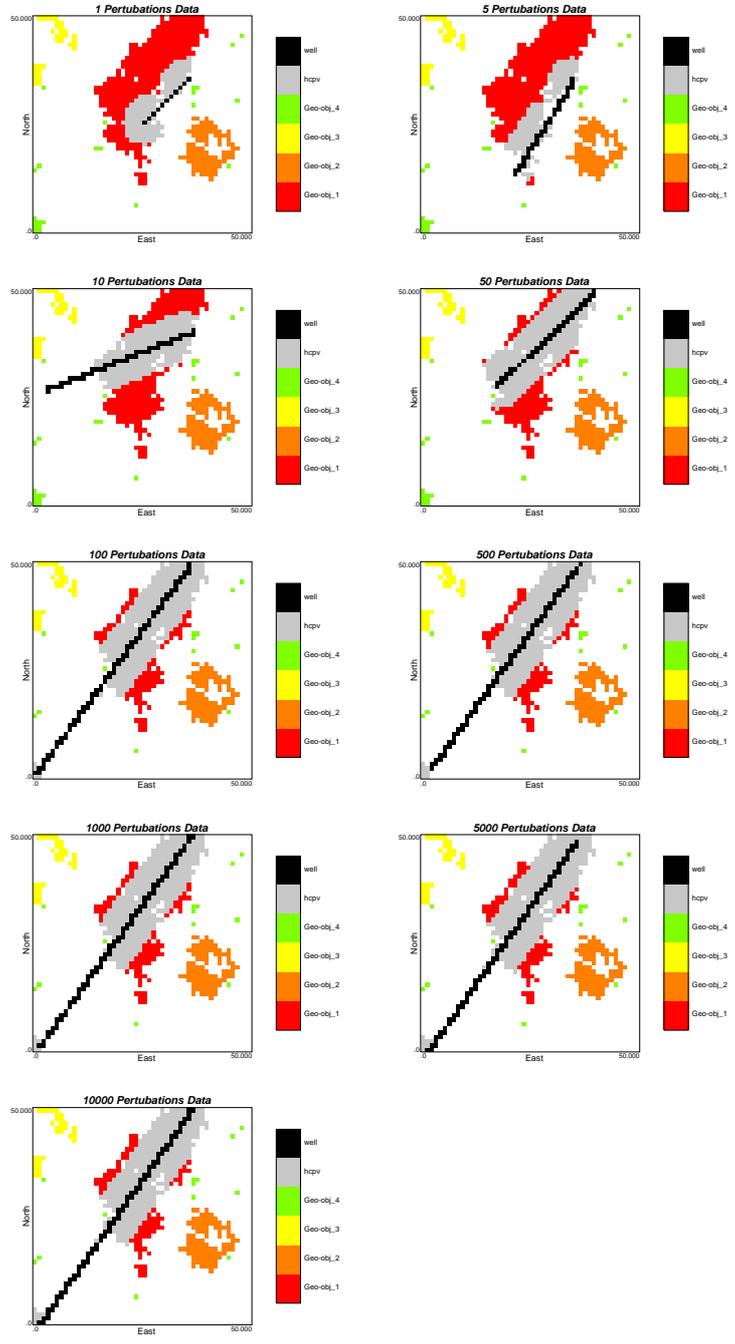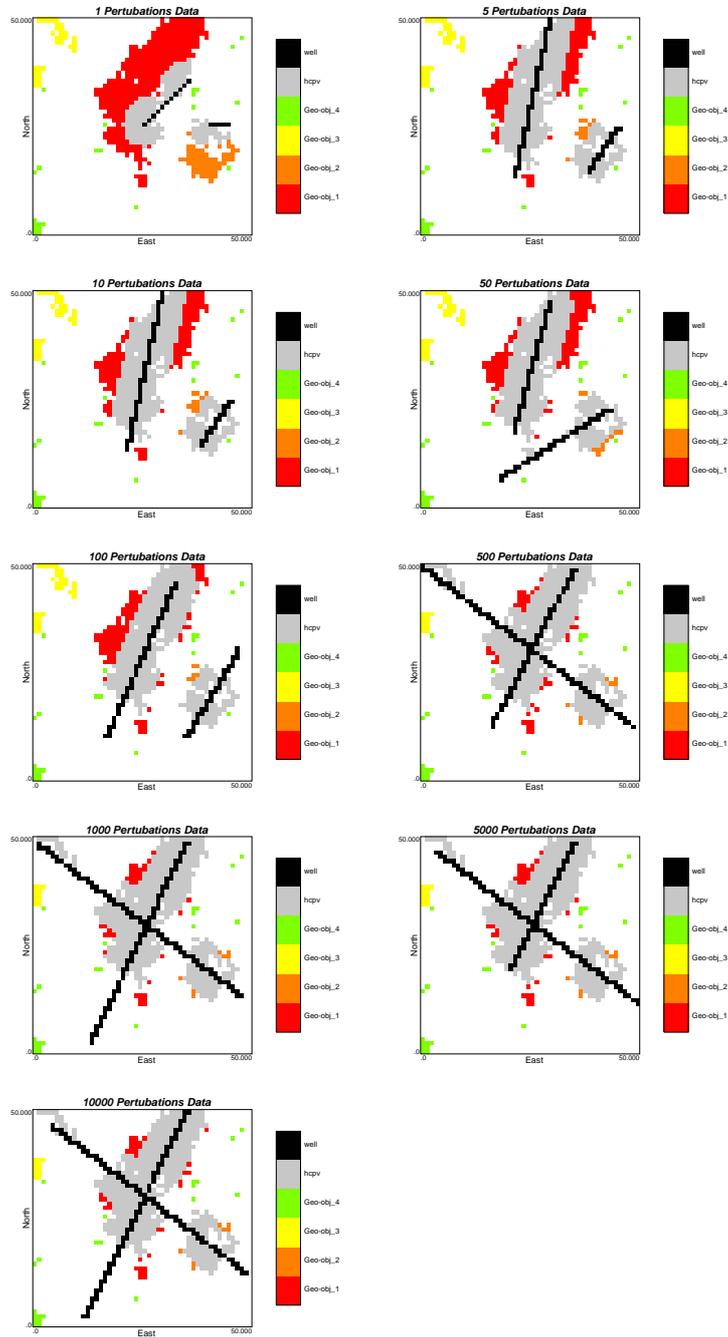
Figure 4: Two-dimensional example with one well.
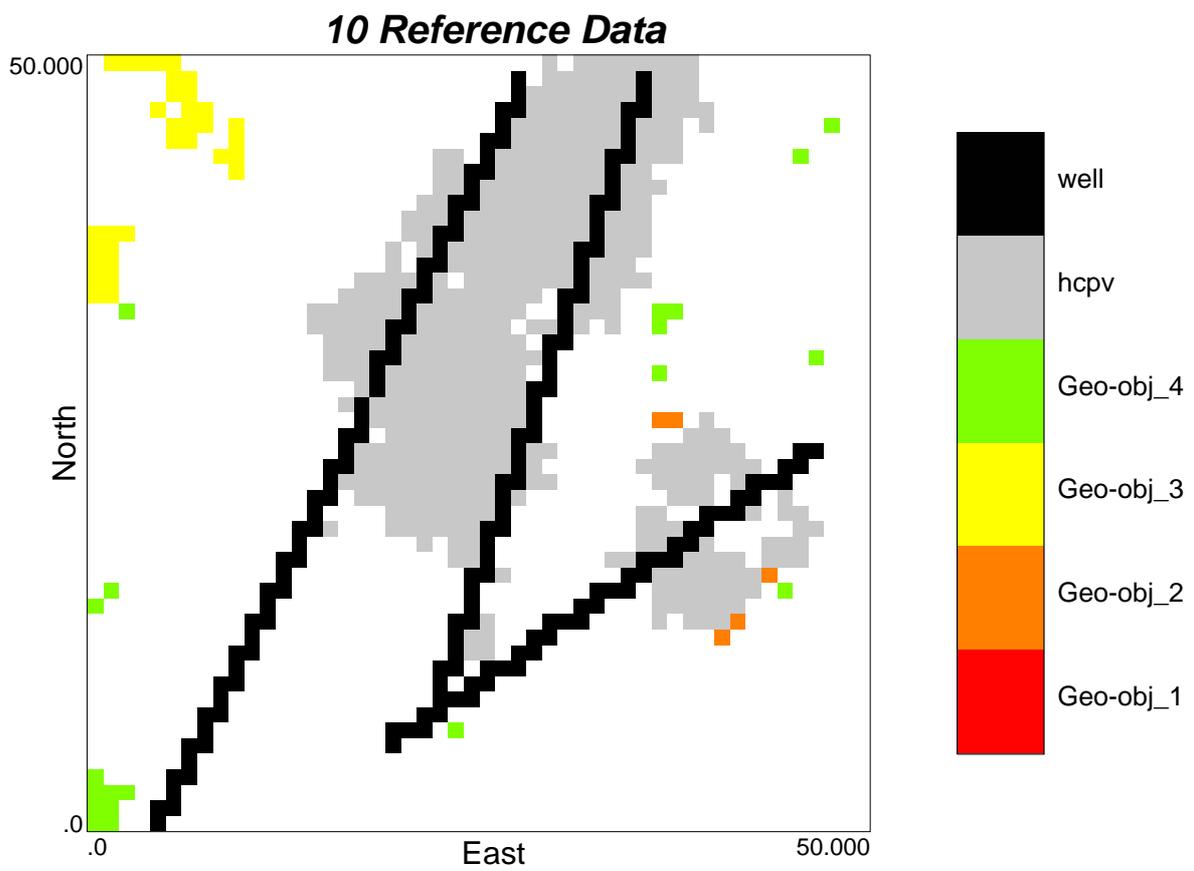
Figure 5: Two-dimensional example with two wells.

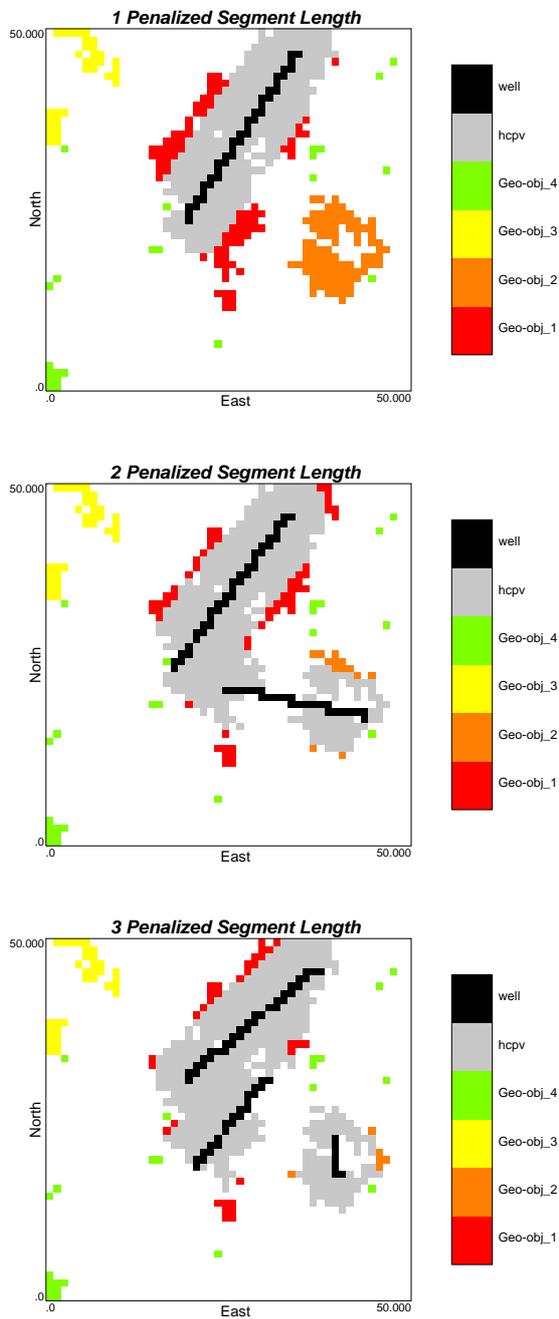**10 Reference Data**

Figure 6: Two-dimensional example with three wells.
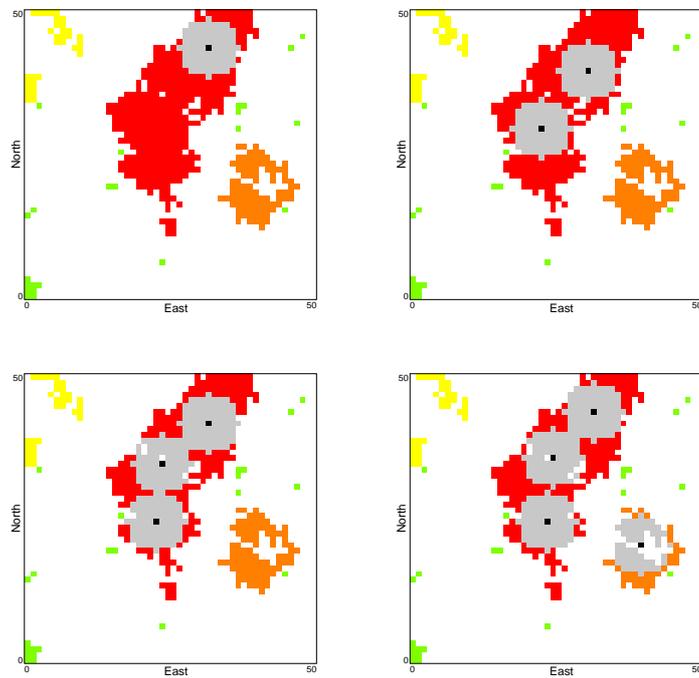
Figure 7: Example where length of well is penalized.

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Figure 8: Optimal well locations for 2-D example with 1, 2, 3, and 4 wells.

TOP

BOTTOM
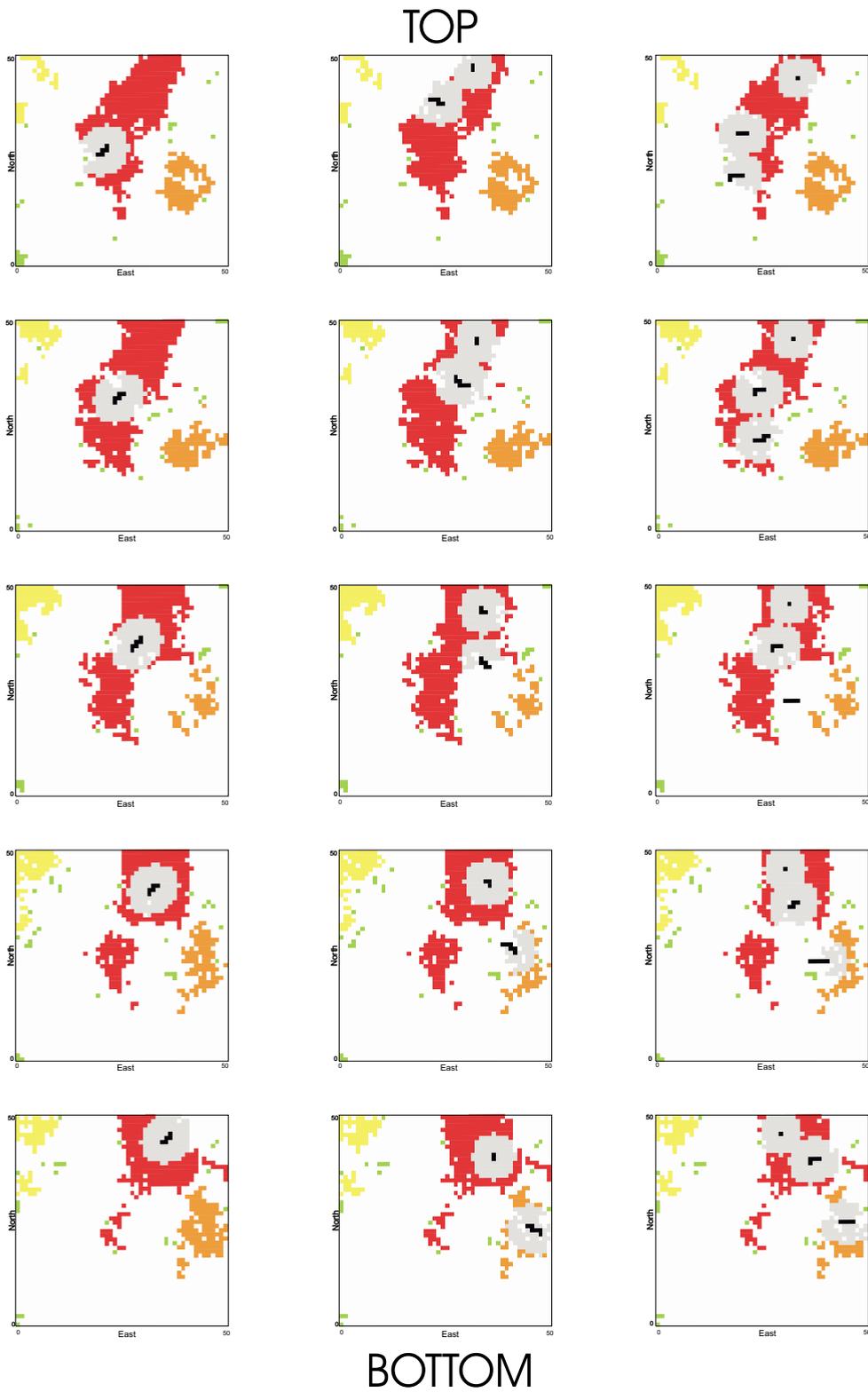
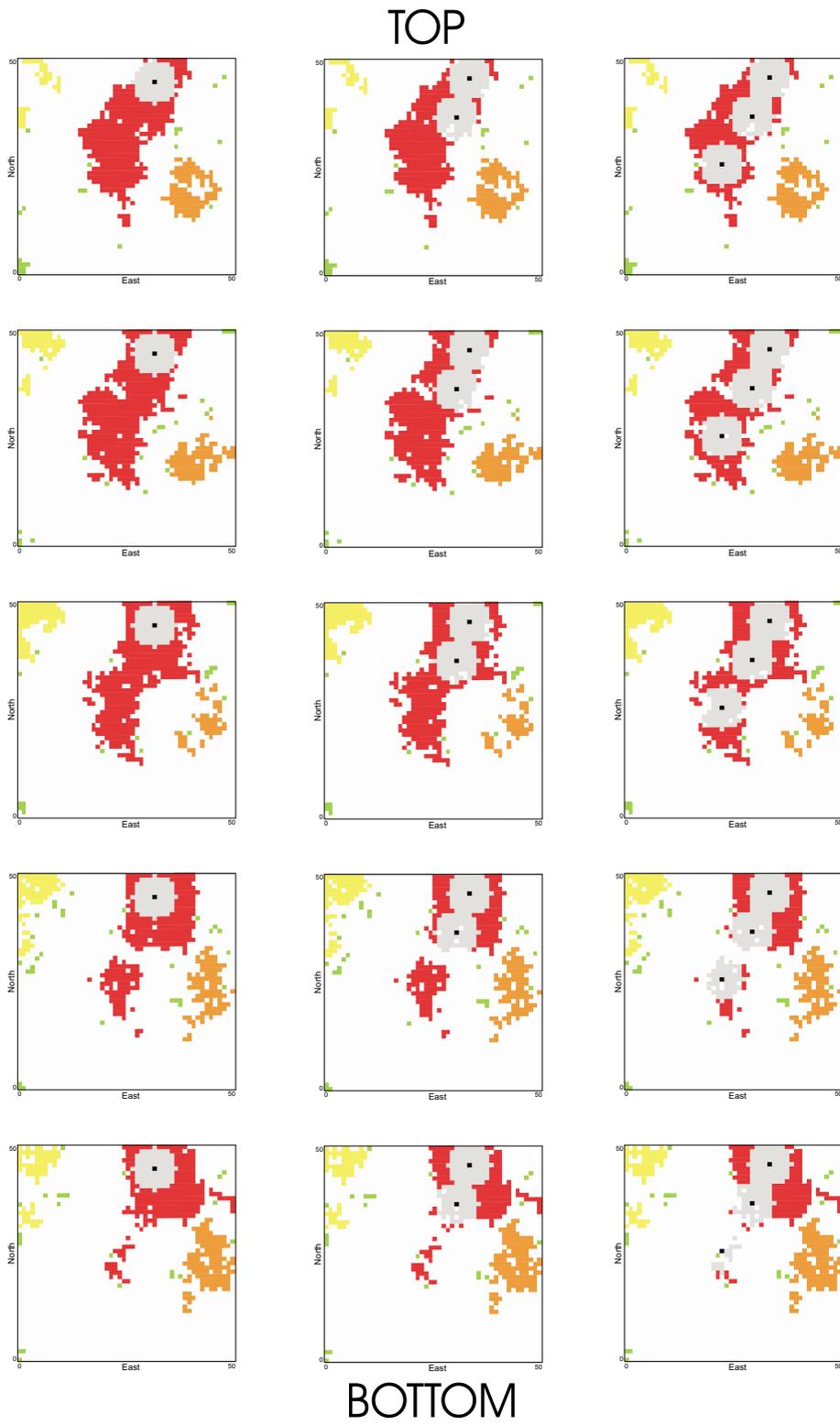Figure 9: 3-D examples with 1, 2, and 3 deviated wells.

TOP

BOTTOM

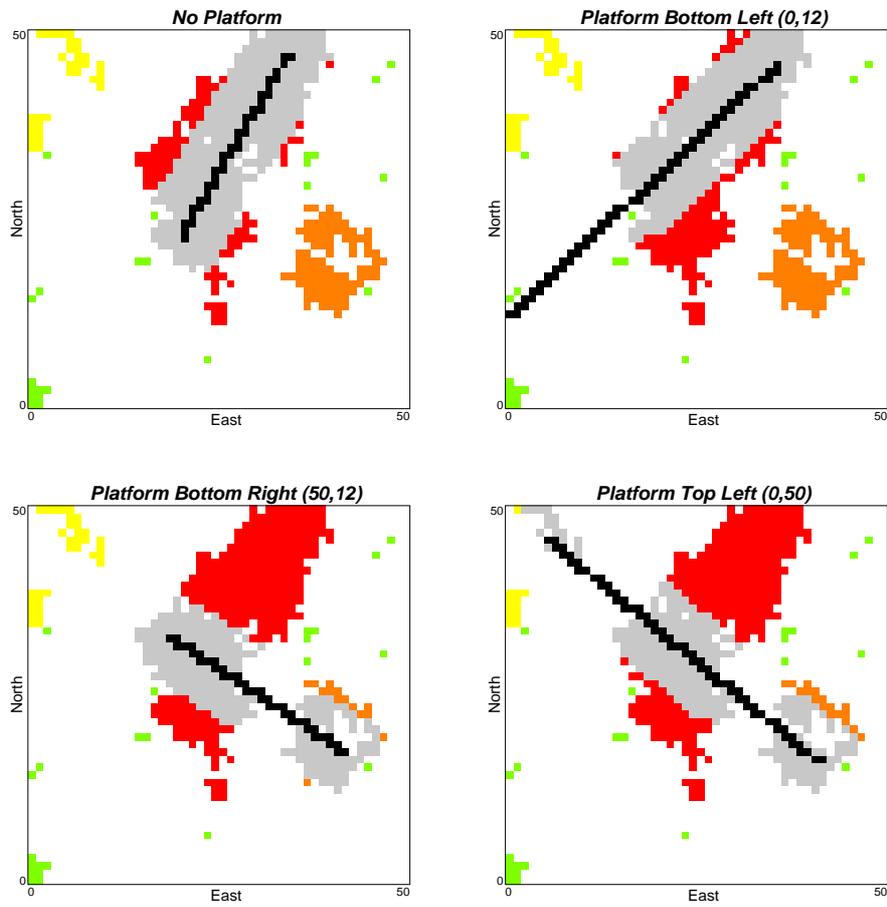Figure 10: 3-D examples with 1, 2, and 3 deviated wells.

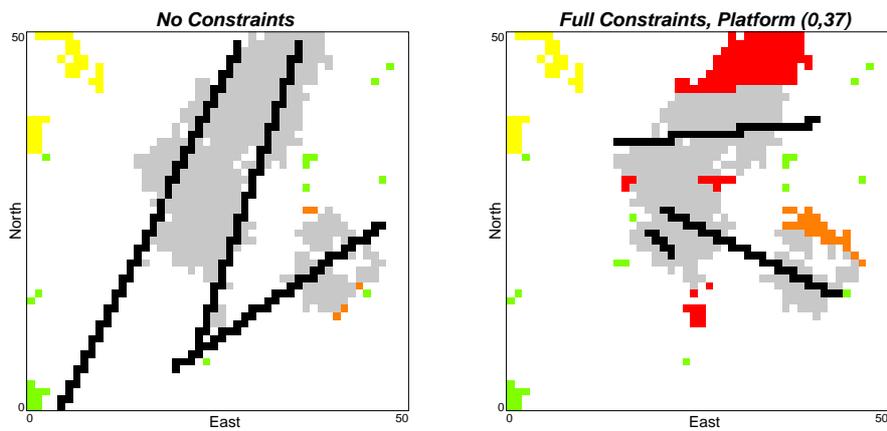Figure 11: Illustration of the platform constraint.

Figure 12: Illustration of the 3 well platform constraint.

APPENDIX: Program Description

Inputs

The program requires the following inputs:

Surface Map The program requires a surface model from which the thicknesses will be mapped from. The surface map must use the corner point grid system.

Thickness Model The program requires a thickness model with a corner point grid system rather than a centre point grid system. Thicknesses are projected down from the surface map. The average thickness is used to calculate the volume of a particular block. The program can operate in 2 or 3-D, but both require a thickness model to calculate hydrocarbon volumes. For example, vertically averaged thickness could be used for 2-D.

Porosity and Water Saturation Models Hydrocarbon volume is calculated using the porosity and water saturation. The porosity and water saturation models should be on a centre point grid system.

Geo-object Model This model is used to calculate the connected volume available to the well. The geo-object model must use a centre point grid system.

Outputs

The program provides the following outputs:

Debug File This file contains minimal debugging information: the perturbation (at the reporting interval), the available hydrocarbon, the number of blocks in the draw down radius, and the temperature. The final well coordinates, the final available hydrocarbon, and final number of blocks can be found at the end of the file.

Output File This file contains the nx, ny, nz reservoir model, the proposed well paths, and the blocks available to the well. The output is presented as indicators. The well path is written as indicator 900, blocks within the draw don radius are coded as 800, and the geo-object codes are left intact.

Algorithm Description

The program reads the input data in the following order: the well segments, the surface map, the thickness model, the porosity model, and the geo-object file. At the time of reading the porosity and water saturation models, the algorithm populates an array, of dimension nx,ny,nz, with the volume of hydrocarbon calculated as the product of the average cell thickness, porosity, and water saturation. The model limits are calculated, and the array of drainage template indexes is established. Three nx,ny,nz arrays are initialised (1) the cell array holds the number of well path discretizations for intersected cells, (2) a logical array for accounting hydrocarbon volume, and (3) an array for updating intersected cell information.

Using the well segment data, the algorithm determines which cells are intersected by calling the dtrace subroutine. The empty array of intersected cells, the thickness and surface models, the starting and ending coordinates for each segment, a write or erase flag, and a logical flag indicating if the drill trace leaves the model are passed. The idea of the subroutine is to convert the segment into a number of discretized points and keep track of which cells hold discretized points and how many discretized points the occupied cells hold by starting at one end of a drill segment proceeding to the end of the segment via the discretized points. The subroutine first finds the segment length and the number of discretizations. Then the increment for each coordinate is calculated. The first discretization is at the start of the segment. The x and y index are found. The z index is

more difficult to determine because the cells are not uniform in size; the surface model defines the elevation and the thickness is defined by the thickness model. Also, the thickness and surface models are is on a corner point grid system thus there are nx+1, ny+1, and nz points in the thickness and nx+1, ny+1 points in the surface array. To determine the z index the subroutine first determines which face, the top, bottom left or right, the discretization is closest to. The point with the highest elevation on the closest face and the associated x and y index are saved as the start of vector and the lowest elevation point and associated x and y index are saved as the end of the vector. Using the x and y coordinate of the discretization and the previously discussed elevation vector the test elevation is calculated. If the test elevation is lower then the elevation of the discretization then the discretization falls in the associated z index. If it is below then another thickness is subtracted and the test elevation is recalculated and compared to the elevation of the discretization. This is repeated until either a z index is found. If no z index is found then the discretization is assumed to be outside the model. The comparison is repeated over the nx, ny, nz array. On completion the cell array and the logical indicator indicating if the drill trace in inside or our side the model are passed back to the main program.

The program now accumulates the initial hydrocarbon volume. A loop visits each node in the cell array and the geo-body array. If there exist both an intersected cell and a geo-body the geo-body identifier is saved and the program loops over the draw down radius. Any block having the same geo-body identifier in the draw down radius is accumulated. The taken array accounts for already accumulated cells so that even if two wells cross the contained hydrocarbon is only accumulated once. The update cell and taken arrays are populated with the initial intersected cells and accumulated hydrocarbon.

The program now starts to perturb the well locations. A well is randomly drawn. The candidate well is removed from the cell array using the dtrace subroutine and setting the add / remove flag to remove. A segment end is randomly selected and its coordinates perturbed. If the perturbation yields coordinates that fall outside the model then another coordinate is drawn. The dtrace subroutine is called to determine which cells are now intersected. The taken array is cleared and the new hydrocarbon volume is accumulated. The new volume is compared to the old. Perturbations that increase accumulated volume are automatically accepted. Perturbations decreasing hydrocarbon volume are conditionally accepted. If a perturbation is accepted, the update cell is updated, the taken array is updated, the number of accepted perturbations is incremented and the number of attempted perturbations is reset to zero. If a perturbation is rejected then the cell array is refreshed to the update array and the segment coordinates are returned to the old coordinates.

Now the stopping criteria are checked. If any of the stopping criteria are met then the program writes the output files. If none are, then the program continues and updates the annealing schedule. The program goes back for another change. The loop continues until one of the stopping criteria are met.

The Parameter File

The parameters required for the program are listed below and shown in Figure 13:

- **topsfl:** the file containing the surface model.

- **thickfl:** the file containing the thickness model in corner point format.

- **outfl:** the name of the output file that will contain the indicator output.

- **porfl:** the name of the file containing the porosity file in centre point grid format.

- **swfl:** the name of the file containing the water saturation model.

- **geofl:** the name of the file containing the connected geo- objects.

- **temprad, tempdeep:** the X-Y drainage radius, and the drainage depth. If these values are set to 0 then only those cells intersected by the drill trace are considered.

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- **discr:** the discretization interval for the drill trace.

- **vert:** the indicator for a vertical or other drill path

- **nx, xmn, xsiz:** definition of the grid cell system (x axis). The program automatically accounts for the differences in grid system, and requires the user inputs the number of cells in the centre point sytem.

- **ny, ymn, ysiz:** definition of the grid cell system (y axis). The program automatically accounts for the differences in grid system, and requires the user inputs the number of cells in the centre point system.

- **nz, zmn, zsiz:** definition of the grid cell system (z axis). The program automatically accounts for the differences in grid system, and requires the user inputs the number of cells in the centre point system.

- **ixv:** random number seed (a large odd integer)

- **mdx,mdy,mdz:** the maximum allowable random change in the x, y, and z directions.

- **maxpert:** the maximum allowable number of perturbations.

- **nrepo:** the reporting interval. After a fixed number of perturbations the number of perturbations, the profit, the penalty, and the temperature are written to the screen and the output file.

- **t0, tfactr:** the finial temperature, and the reduction factor are part of the annelaing schedule.

- **kmax:** the maximum number of perturbations per temperature. When exceeded, **t0** is multiplied by tfactr.

- **kaccept:** the target number of perturbations. When exceeded t0 is multiplied by tfactr.

- **ksas:** the maximum number of times ka can provoke a temperature change.

- **maxnochange:** the maximum number of perturbations allowed without a change.

- **nwells:** the number of well seed locations to read in.

- **dmax:** the maximum perturbation distance.

- **datafl(i):** the file containing well *i coordinates.*

- ***nseg(i): the number of segments used in well (i).***

```
                  Parameters for DTRACE
                  ********************

START OF PARAMETERS:
10top.out                      -file with the tops
10thick.out                    -file with the thicknesses
dtrace.out                     -output file
pert10000.out                  -output file for perts
por.dat                        -porosity file
sw.dat                         -sw file
geo-obj.mod                    -geo-object file
5 0                            -drainage radius,drainage depth
0.1                            -segment discretization
0                              -vertical well? (1=yes,0=no)
50 0.5 1.0                     -nx,xmn,xsiz
50 0.5 1.0                     -ny,ymn,ysiz
10  0.5 1.0                    -nz,zmn,zsiz
69069                          -random number seed
40 40   0                      -max change in x,y,z
10000                          -perturbations
100                            -reporting interval
0.5 0.25                       -t0, lambda
500 50 100 10000               -kmax, ktarget, ksas, max nochange
1                              -number of wells
seg1.dat                       -#1 file with the drill segments
1                              -segments for well #1
seg2.dat                       -#2 file with the drill segments
1                              -segments for well #2
seg3.dat                       -#2 file with the drill segments
1                              -segments for well #2
```

Figure 13: An example parameter file for dtrace