The Use of Ranking to Reduce the Required Number of Realizations

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The number of realizations required for rigorous uncertainty assessment is often impractical. Many realizations may be quickly processed through static resource calculations; however, dynamic flow simulation and recovery determination require significant computational effort. Ranking realizations by some simple proxy for the complex flow response and then processing a limited number of realizations is appealing. This note discusses a robust procedure for using the results of ranking.

Introduction and Background

Modern decision-making requires an assessment of this uncertainty. A combination of a scenario-based and classical Monte Carlo sampling is used to assess uncertainty. A large number of realizations may be created quickly with modern geostatistical tools. Due to computer limitations, it is only possible to visualize and perform flow simulation on a limited number of realizations. Techniques must be used to reliably choose realizations for more detailed analysis.

A good ranking measure reduces the number of realizations required to achieve a specified level of precision in the uncertainty assessment. The effectiveness of a ranking measure can be determined when the full performance assessment is available *with* the ranking measure. In general, however, we want to know how the ranking measure helps before all of the full performance calculations. A procedure for dynamic determination of ranking effectiveness is an important element of using ranking to reduce the number of required realizations. The procedure for using a ranking measure is explained before we address this *dynamic determination* problem.

Published applications of ranking from the Ballin paper in 1992 through the Idrobo paper in 2000 consider the ranking measure to be "quantile preserving," for example, the P_{10} of the full performance assessment is determined by running the full assessment on the P_{10} of the ranked realizations. There is no statement of how close the resulting P_{10} is to the true value. This procedure will work when the correlation between the "rank of the ranking measure" and "rank of the full assessment" is large (above 0.9 or so); however, that is not usually the case. When the ranking measure is poorly correlated with the full variable, then the expected behavior is close to the median regardless of the ranking measure.

Figure 1 gives an illustration of ranking with a correlation coefficient of 0.75 with the true ranking, which is realistic. The cross plot of the true rank and estimated rank for 1000 realizations is shown on the left. Histograms of the true rank for estimated rank of 0.05, 0.5, and 0.95 are shown on the right. Note the difference in the actual true rank and the estimated rank, especially for the 0.05 and 0.95 estimated rank; the solid dark vertical line on the histograms is the right value. The estimated rank cannot identify the really low realizations and high realizations – the estimated rank typically chooses realizations closer to the center of the distribution.

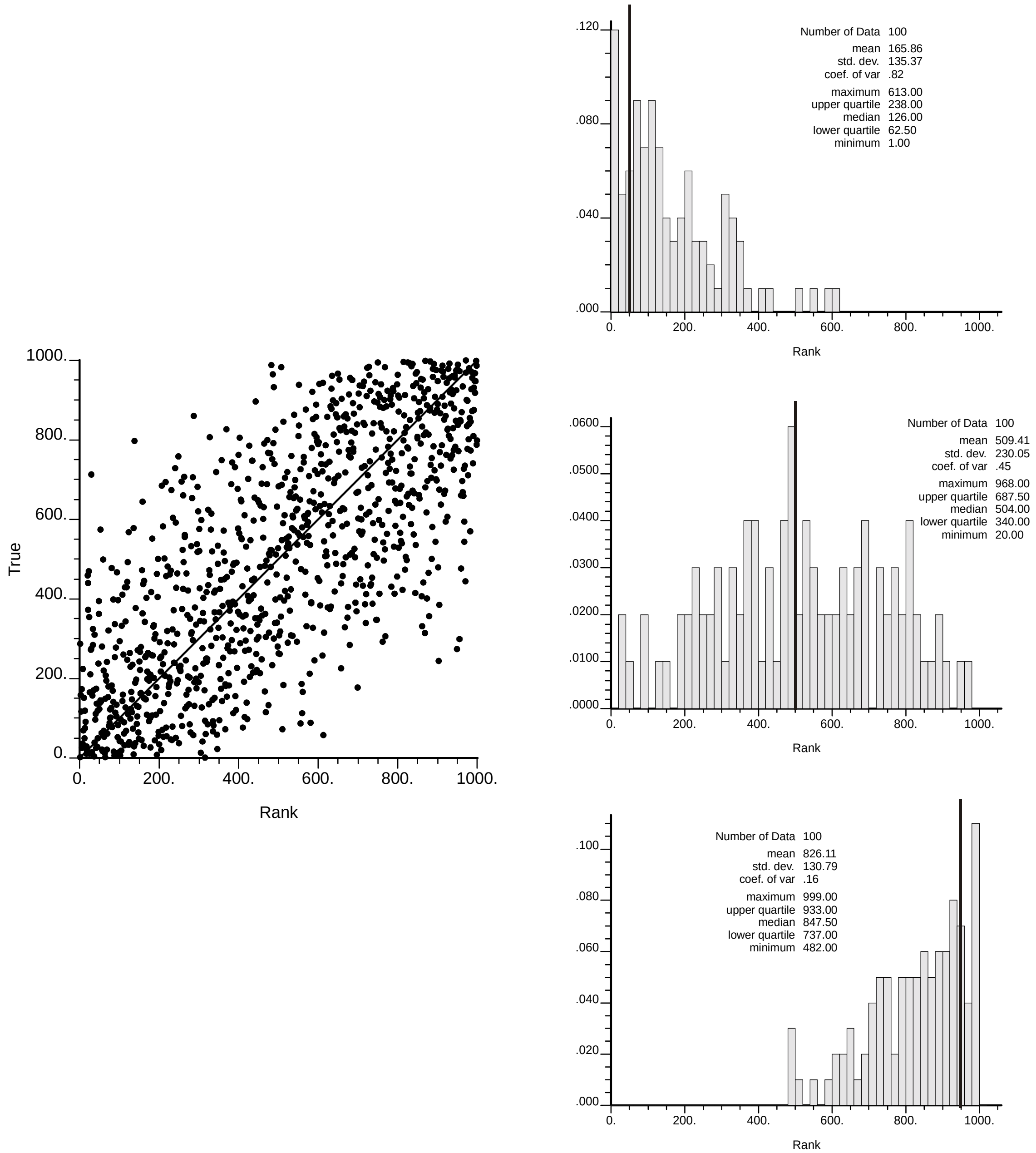

Figure 1: Illustration of ranking with a ranking measure with a correlation coefficient of 0.75 with truth. Cross plot of true rank and rank from ranking measure on left. Histograms of true rank for predicted rank of 0.05, 0.5, and 0.95 are shown on the right. Note the difference in the actual true rank and the estimated rank, especially for the 0.05 and 0.95 estimated rank; the solid dark vertical line on the histograms is the right value.

This reveals a dangerous consequence of the classical approach (recall that the classical approach is to generate L realizations, rank them by a fast performance estimator, and run the full performance estimator on the, say, P_{low}, P_{50}, and P_{high} ranked realizations): the uncertainty as measured by the difference between the P_{low} and P_{high} is almost always too narrow!

Studies to date have not been concerned with this for one of two reasons: (1) the full performance estimate was generated for all realizations and the focus was to identify the ranking measure (streamline or some such fast estimator) that gives the best correlation with the true rank, or (2) the full performance estimate was *not* generated for all realizations and it is not possible to assess any bias in the uncertainty statement. We must propose a reasonable procedure for using ranking to reduce the number of realizations required for uncertainty assessment.

Recommended Procedure for Using Ranking

The recommended procedure for using ranking in realizations is (1) generate more realizations than will be processed M where $M>L$, (2) rank the M realizations, and (3) process L equally spaced realizations. The L realizations generated in this way better represent the distribution of uncertainty than randomly drawn realizations.

The number of realizations M is chosen such that the L realizations are equally spaced in terms of probability. To ensure "equal" spacing, we choose the number of realizations between each of the L realizations, n, where n is an even integer between 4 and 10. Then, $M=(n+1)L$. The realizations $l=1 \ldots L$ are chosen as numbers $1+n/2, 1+n/2+(n+1), \ldots ,1+n/2+(L-1)(n+1)$. This is really quite simple. Consider L=50 and n=4: (1) the total number of realizations needed is M=5x50=250, (2) the 250 realizations are ranked, (3) the 3, 8, 11, … ,248th ranked realizations are kept for full processing.

This procedure leads to realizations that are equally spaced in terms of probability, which leads to an unbiased probability distribution of the full response variable.

A ranking measure will reduce the number of realizations required to achieve a specified precision. The better the ranking measure the more we reduce the number of required realizations. It was possible to work out this relationship analytically with L random realizations. The intermediate step of ranking M realizations prior to the selection of L realizations significantly complicates an analytical derivation. A numerical approach can be used. Consider the following procedure to calculate the value t for specified ranking measure, M, L, Δ_F, and F values:

1. Draw M realizations or values from a uniform distribution between 0 and 1 (the cdf values on the vertical axes of Figures 2 and 3).
2. Rank the M values with a related ranking measure and select the L values to be used to build cdf.
3. iSort the L values in ascending order and create a sample cdf. Determine the F-quantile of the sample distribution, F^*. See if the sample quantile value F^* is within the required tolerance, i.e., $F^* \in [F-\Delta_F,F+\Delta_F[$.
4. Repeat steps 1 through 3 many times (say, N=10000) and calculate t as the proportion of times that the sample quantile meets the precision criterion.

This procedure can be repeated for many L, Δ_F, and F values to build a family of curves that tell us how ranking reduces the number of realizations required for a precision specification. A small program `get_r` was written for this purpose.

An important step in the following derivation is the creation of a ranking measure. The ranking measure is calculated as the true rank with some Gaussian error: $rank = true + fac \cdot y$, where y is a standard normal random value. The correlation between the true rank and the estimated rank decreases as the standard deviation of the ranking measure, *fac*, increases, see Figure 2.

Figure 3 shows the first numerical experiment verifying that using a ranking measure improves specification of uncertainty. The value of ranking is greater for larger correlation coefficients. These curves are all for the same level of refinement, i.e., n=4, which means that 5 L realizations are generated, ranked, and then L are extracted for building the distribution of uncertainty.

Figure 4 shows that the results are not very sensitive to the number n chosen. A value of n=4, which amounts to $5 \cdot L$ realizations for the initial ranking is deemed adequate. This figure applies to ρ=0.75. The noisiness of the lines on Figure 4 is due to the relatively few (500) cases evaluated at each L. Another check was done for the higher correlation coefficient of 0.95, see Figure 5. At greater CPU expense, 1000 cases were used for each point on these curves; hence, less noise. The difference from $n=0$ to $n>0$ is more significant but, once again, we see that

We adopt a fairly simple model for the value of ranking. The effective number of realizations is increased by a factor $1/f(\rho)$:

$$L_{eff} = \frac{1}{f(\rho)} \cdot L \quad \text{or} \quad L = f(\rho) \cdot L_{eff}$$

where $f(\rho)$ is a factor that is 1 when the correlation of the ranking measure to the true rank is ρ=0 and decreases to a value near 0 when the correlation increases, e.g., $\rho \rightarrow 1$. The factor is defined in this manner because it is standardized between 0 and 1 and decreases approximately by $1-\rho$. We get the idea from Figure 10 that the relationship drops more slowly than a linear relationship. Figure 6 shows a schematic illustration of what the relation could be like.

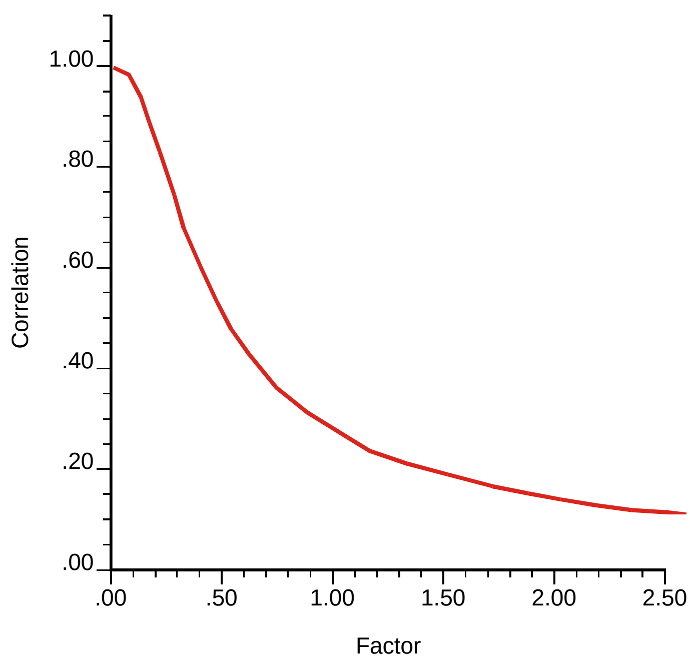

Figure 2: Relationship between the correlation of a ranking measure and the noise "factor" used to get it in the first place.

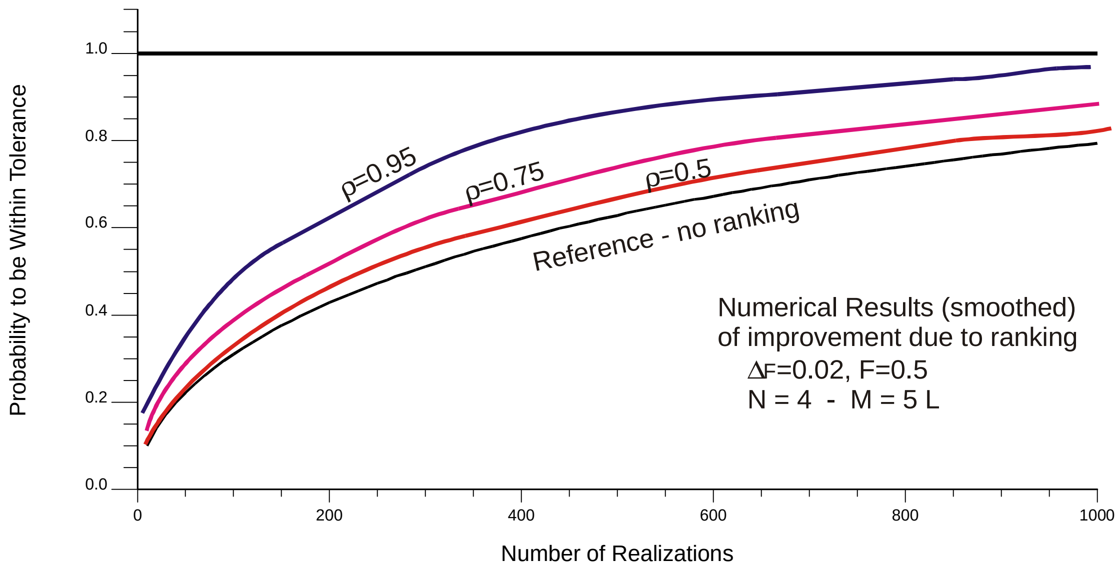

Figure 3: The value of ranking is shown for three different correlation coefficients (0.95, 0.75, and 0.50). This example considers Δ_F=0.02, F=0.5, and n=4. The irregularity in the curves is due to a limited number of samples at each L (500 in this case).

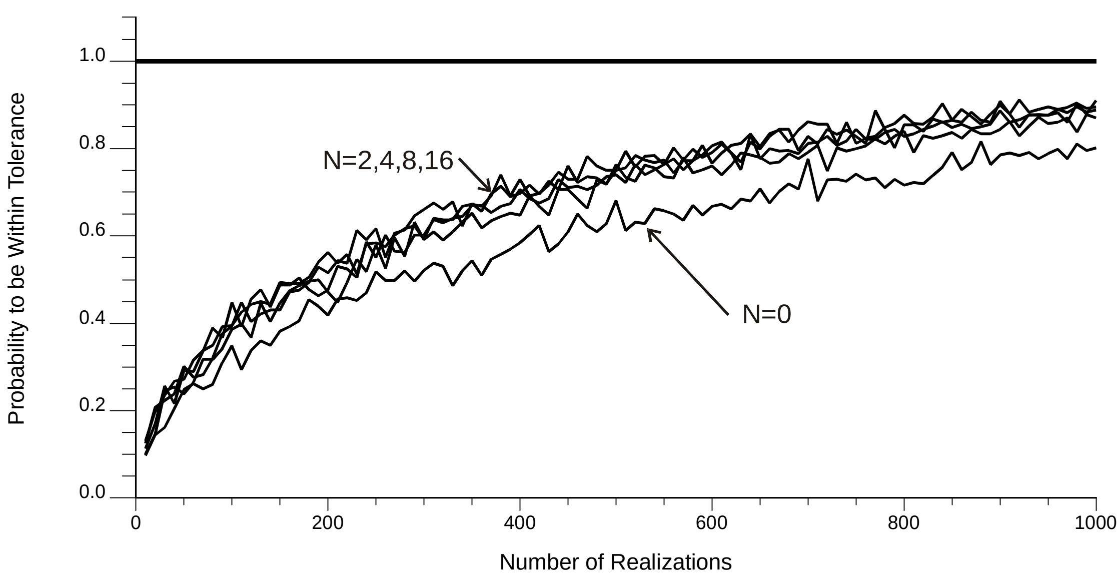

Figure 4: Increasing the pool of realizations for ranking (increasing n and $M=(n+1)L$) has little effect as long as it is greater than 2. Note that n=2 amounts to M equal to three times L, which is already a fair number of realizations. The noisiness of these lines is the relatively few (500) cases evaluated at each L.

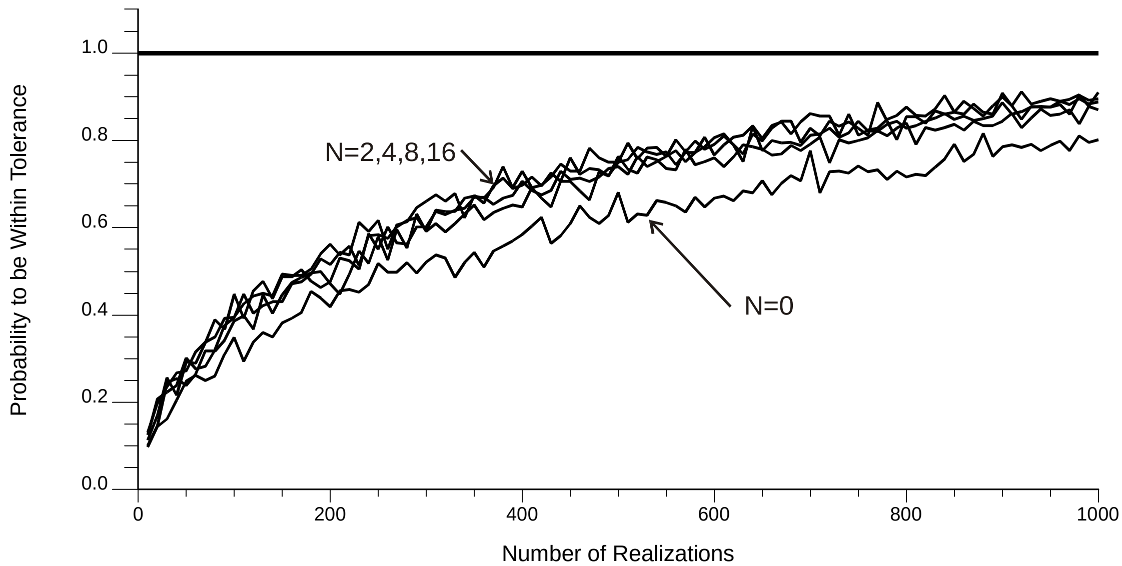

Figure 5: The effect of increasing the pool of realizations for ranking (increasing n and $M=(n+1)L$) with a higher correlation coefficient (ρ=0.95).

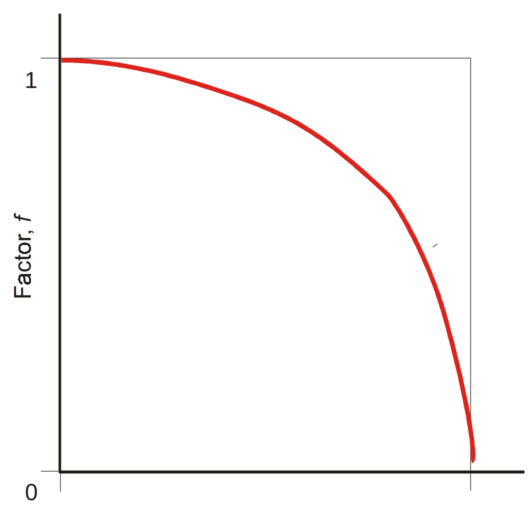

Figure 6: schematic illustration of the factor $f(\rho)$ versus ρ. The relation is anticipated to be a power-law relation.

References

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