Procedures and Guidelines for Assessing and Reporting Uncertainty in Geostatistical Reservoir Modeling

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This preliminary research report was prepared to collect some thoughts on uncertainty quantification. Geostatistics is routinely used for heterogeneity characterization, but uncertainty assessment requires some further assumptions and implementation details. This report will form the conceptual basis for a number of different research/application studies.

Uncertainty assessment requires much more than changing a random number seed and running multiple realizations. The space of uncertainty must be formulated to fairly address all key aspects of uncertainty. This space of uncertainty must be sampled. Classical Monte Carlo simulation amounts to random sampling. Techniques borrowed from experimental design can lead to more efficient sampling of the space of uncertainty. Finally, the results must be analyzed and presented to convey important results and sensitivities. These three aspects of uncertainty management (1) problem formulation, (2) sampling uncertainty, and (3) analysis of results are covered in this report.

1. Introduction

Geostatistical methods have been used historically for heterogeneity modeling to avoid biased flow predictions from too-smooth numerical models. There is increasing interest in applying these techniques to quantify and manage uncertainty.

The general area of uncertainty quantification is rich and varied. "Uncertainty is a measure of the 'goodness' of a result. Without such a measure, it is impossible to judge the fitness of the value as a basis for making decisions relating to health, safety, commerce or scientific excellence.[1]" This succinctly says why we need uncertainty quantification and gives a hint of the diverse areas where uncertainty is required. Many areas of application involve testing physical parameters in a laboratory. They have the luxury of basing uncertainty on repeated testing procedures and can reduce uncertainty by increasing the number of test subjects. This is not the case with geostatistical models that are derived numerically and where hard data collection is extremely expensive.

The application of geostatistical methods to uncertainty management is more complicated than running multiple realizations. The first complication is uncertainty in the input parameters themselves; facies proportions, histograms, variograms, surfaces, and even the most correct geostatistical approach are known with certainty. The ergodic statistical fluctuations that result from running multiple realizations with the same input parameters and a different random number seed do not come near representing a realistic model of uncertainty. Clearly, the uncertainty in input parameters must be limited to first-order variables. It would be fruitless to embark on the "circular quest" for full uncertainty. Nevertheless, an essential component of uncertainty assessment is to formulate the problem and identify those sources of uncertainty that are going to be addressed explicitly. Section 2 is directed toward **Problem Formulation**.

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Problem formulation amounts to define the "space of uncertainty," which is a combination of geological model uncertainty in the form of a decision tree of possible scenarios and geostatistical uncertainty in the form of multiple realizations for the scenarios. This specification of the hierarchical modeling steps and the parameters that are considered uncertain does not actually tell us the uncertainty we want. There is a need to generate or "draw" realizations of the reservoir from this space of uncertainty to pass through a transfer function (resource calculation or flow simulation) to assess output or response variable uncertainty. This aspect of uncertainty assessment can become difficult if we can only afford few realizations. Concepts from experimental design can be applied. Section 3 is directed toward **Sampling the Space of Uncertainty**.

Generating realizations and processing them through some transfer function provides an assessment of the space of uncertainty; however, there is one remaining task. The results must be analyzed and summarized for our understanding and decision making purposes. It is critical to understand the sources of input uncertainty that are the most consequential. Interactions between the input sources of uncertainty are also important. Section 4 is directed toward **Display and Analysis of Results**.

This preliminary report does not definitively spell out the theory and practice of uncertainty assessment. Many details will be addressed over the next year and reported in the Fifth CCG Report. The main aim of this report is to set the stage for future work and provide preliminary guidance. Some references are included at the end, but (once again) a full literature review has not been completed at this time.

2. Problem Formulation

Our quantification of uncertainty is only as good as our problem formation; no amount of experimental design and exhaustive computer runs will overcome a poor formulation in the first place. Our main focus is the first order sources of uncertainty that affect (1) the static petroleum resource and (2) heterogeneity as used by flow simulator. We will not discuss second order uncertainty in data, for example, uncertainty in well log interpretation and acquisition and processing of seismic data. We are concerned mainly with features that can be modeled at the geological modeling scale of roughly, 0.5 to 2m in vertical and from 10 to 250 m horizontal resolution.

The following topics are discussed in this section: (1) A conceptual modeling approach where the main sources of uncertainty are identified, showing, in a very broad sense, how we quantify uncertainty. (2) The procedures to generate the statistical and geostatistical models of the parameters, which were conceptually described in first step. (3) An hierarchical approach for the construction of a decision or scenario tree for reservoir uncertainty. This approach seeks, among other things, to subdivide the reservoir in units, within which the petrophysical properties are stationary. In each step of this subdivision we will identify the first order uncertainties and show how to model them.

Geological Background

These concepts are fundamental when the number of samples is very small. Analogues with other fields are critical. Uncertainty assessment in petroleum exploration and production is unique in the sense that remarkably few *hard* data are available and it is not straightforward to go and collect new data. We must rely on *soft* data and geological principles.

The sediment maturity affects the geometric properties of the grains (sorting and shape), their mineralogical composition and the matrix content. Maturity also directly affects the porosity,

permeability and water saturation. Sediments with high maturity have the tendency to be more predictable than sediments with low maturity, because the latter are more affected by diagenetic processes. Thus, the textural maturity affects variability and the resulting uncertainty.

The geometry of the reservoir is anchored to the correlation between well properties using geological concepts. We will discuss briefly two important aspects that must be taken into account: (i) the way the reservoir was compartmentalized during its deposition, using the concepts gathered from sequence stratigraphy and architectural-elements analysis and (ii) the changes caused by post-depositional movements due to tectonic and compaction processes.

Each sedimentary environment (alluvial, deltaic, turbiditic and so on) has particular stacking patterns typical of its associated hierarchical architectural elements. These patterns are caused by the sedimentary process and by the history of sea-level changes related to the subsidence history of the sedimentary basin and the volume of available sediment – concepts are well formulated by sequence stratigraphy. Sequence stratigraphy is the study of the relationships of sedimentary rocks within a chronostratigraphic framework of genetically related strata, which are limited by erosional or non-deposition surfaces or their correlative surfaces. Reservoirs that have been deposited over a relatively long time period can often benefit from sequence stratigraphic concepts useful to subdivide the reservoir in major units and correlate them between adjacent wells

With exception of stratigraphic traps, most petroleum reservoirs originate due to geometric configurations caused by post-depositional tectonic movement. These tectonic process have a large effect on (i) the configuration of the top, bottom and intermediate surfaces and (ii) the external limits of the reservoir and (iii) horizontal compartmentalization of the reservoir. The best way to model the surfaces of the reservoir is with seismic to get the general configuration and with well logs for local control. These surfaces have one of the most profound impacts on hydrocarbon reserves, and a small shift in the position of these surfaces can dramatically change the volume of hydrocarbon.

Faults can be very important in reservoir modeling. They affect the fluid flow of the reservoir in many ways: (i) decreasing the contact area of reservoir units by vertical displacement. When this displacement is too large, the faults can isolate reservoir units. (ii) Putting units in contact that otherwise would be separated by horizontal barriers. If the fault has no transmissibility then it can isolate reservoir blocks. The isolation due to faulted surfaces can be of two types, related to the timing of fault generation: (i) when the isolation comes together with sedimentation, with the classic examples of those caused by growth faults. (ii) When this isolation is due to later movement (in relation to the time of deposition) of the reservoir section. The difference between these two is that in the second type, the same sedimentary sequence will be found on each side of the fault. Therefore, one can restore the original sequences by compensating the fault movement. On the other hand, restoring the original sequence is very hard when the fault is of growth type. The two sides of the reservoir must be treated separately, because the growth side is much different of the other side: the thickness of each sub—hierarchical elements will be different and so on.

The bounding horizontal surfaces and the faults must be treated with care. They delimit large portions of the reservoir with great impact on the overall reservoir behavior. Every possible large-scale barrier must be analyzed and must be included in the uncertainty model.

In general, the reservoir is composed of many facies that form specific patterns. Therefore, there are two main reasons to decompose the reservoir into smaller units: (i) to correlate each part of the entire section between the wells, and (ii) to choose the best grid type (proportional, parallel to top or parallel to bottom) for each unit. The correlation between the wells and the type of grid used has a large effect on uncertainty. The pioneering work of Miall, 1985, established a method

of study called Architectural-Element Analysis for fluvial systems resulting in two basic ideas: (i) the standardization of facies assemblage for modeling purposes at the element level rather than the level of the entire environment, and (ii) much greater emphasis on facies geometries (architecture) than in the past.

Miall divided the four fluvial models into 8 architectural elements, taking into account: (i) the nature of the upper and lower bounding surfaces. (ii) geometry (sheet, lens, wedge, scoop, U-shaped fill). (iii) scale: thickness, lateral extent parallel and perpendicular the flow. (iv) internal geometry: lithofacies assemblage, vertical sequence, presence of secondary erosion surfaces and their orientation, bedform paleoflow directions, and internal bedding. Each architectural element is formed by a facies assembly, with its own geometry and relationships. A given facies can occur in more than a structural element. The eight elements vary in scale and complexity. Smaller elements form stacked "storeys" or complexes within larger elements. The elements therefore form a hierarchy of scales bounded by bedding contacts of variable significance. A hierarchy of three bedding contacts was proposed by Allen (1983): first order contacts bound individual crossbed sets; second order contacts bound cosets or genetically related assemblages of lithofacies. Third-order contacts define groups of elements or complexes, and usually are well-defined erosion surfaces (like the base scour surface of a major channel). Groups of channels, as in paleovalleys, would define an additional fourth order scale. The description of the eight architectural elements is out of the scope of this work. The important point is that this work philosophy has been expanding and today it already includes, for example, the turbiditic models. For uncertainty quantification, we require a geologically - realistic facies model within each reservoir layer or unit. It is important to emphasize that, in the context of reservoir modeling and uncertainty analysis, one does not need to divide the reservoir into the same architectural elements and facies used by the geologist who is interested on the understanding the generation of that rock body. The reservoir geologist must divide the sequence into few facies (we can call them reservoir facies) that are similar and have a greater influence on the flow of fluids.

The analysis of architectural elements and facies give us a powerful tool to understand the overall compartmentalization of the reservoir. However, unfortunately, this analysis does not give us a secure framework to understand *completely* the pattern of porosity and permeability developments. The porosity and permeability depend on the depositional environment. High-energy environments lead to clean facies with low matrix content. Low-energy environments lead to facies with high matrix content. The problems arise with diagenetic processes. Solutions percolating through the sediments can produce cementation, deposition of diagenetic matrix and dissolution. These diagenetic processes can create or destroy the porosity or permeability generated in depositional phase. Therefore, for each reservoir facies, one has to model the petrophysical properties using statistical tools like histograms, variograms, and stochastic simulation *within the reservoir facies framework.* We must perform a complete modeling study for the petrophysical variables for each of the reservoir facies. The stochastic modeling of permeability presents additional problems: it is a tensor, that is, its value depends on the direction of flow and it does not average linearly. In addition, there are few direct measurement techniques. Therefore, in general the approach is to first estimate the porosity and then, using multivariate statistical correlations, estimate the permeability values.

Setup of Uncertainty

To model the uncertainty of surfaces, facies and continuous properties there are three main phases one must follow. The first phase concerns cleaning of the data set. The second phase is related to the sensitivity analysis of every variable that can affect the reservoir. At the end of this phase, important variables are identified and they will be used to construct the scenario tree for the conceptual model of uncertainty (see next section). The third phase is related to the assessment of the uncertainty of each variable identified in the second phase. It is important to emphasize that

the first phase is related to the data set only, and the following phases are related to the behavior and the effect of the variables on the entire reservoir.

At this point, it must be clear that the term "variables", in this context, has a broader connotation than normally used. We are not talking *only* about porosity, permeability, facies and so on. We are talking about these variables *and* every variable related to them. For the assessment of uncertainty, the term variable can be related to, say, porosity or can be related to any parameter of the porosity histogram, like the mean, the maximum value, the minimum value or the shape of the histogram. It can also be related to any parameter of the variogram, like the type of structure, the range of that structure, and so on. Uncertainty is caused by the fact that we do not have enough data or the sampling method was not random. Therefore, these parameters are variables too. Hence, to assess the uncertainty of the entire reservoir we have to consider the range of variation of the parameters, that is, for example, changing the mean value of the porosity, within a reasonable range.

This is important because multiple simulations using the same properties derived from the original dataset will not have the full variability that is expected for the entire reservoir. For example, if one is modeling of the top surface of the reservoir, there are two considerations: (i) many simulations of the surface, using the same parameters, will result in a very small change in volume for each surface. Local high values are compensated by simulated local low values (ii) if the entire surface is shifted up or down (honoring the well data), then the general mean of the depth will be allowed to vary within reasonable limits, and the volume of the reservoir will show greater uncertainty.

In the first phase, the data set must be corrected for clustered data, known bias, outliers, trends and so on. At the end of this step, we must have a cleaned and reliable data set, with the most representative distribution of all variables that can affect reservoir performance. This applies to every variable or parameter that were sampled for that reservoir: porosity, permeability, variogram range, difference of depth from a marker in the well log and the corresponding seismic horizon. Figures 1 and 2 show an example of the changes in the histogram of a continuous variable due to declustering process. Figure 1 shows a histogram without declustering and after declustering. Figure 2 shows another example of the changes in the histogram of a categorical variable due declustering process.

The next phase is construct a model in uncertainty of each variable deemed important. This phase can be divided into three steps. The first step is related to the bias of the mean and any other properties of the histogram of one variable. The second step is related to the correlation parameters of regionalization or coregionalization of the variables. The third step is related to the simulation of the property using parameters drawn from the step 1 (study of the bias) and from the step 2 (study of correlation parameters)

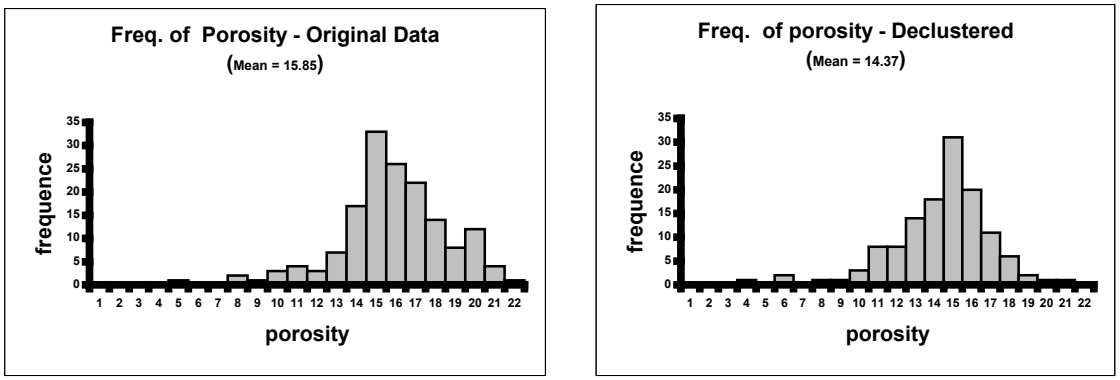

Figure 2.1: Equal weighted porosity histogram (left side) and histogram with declustering weights (right).

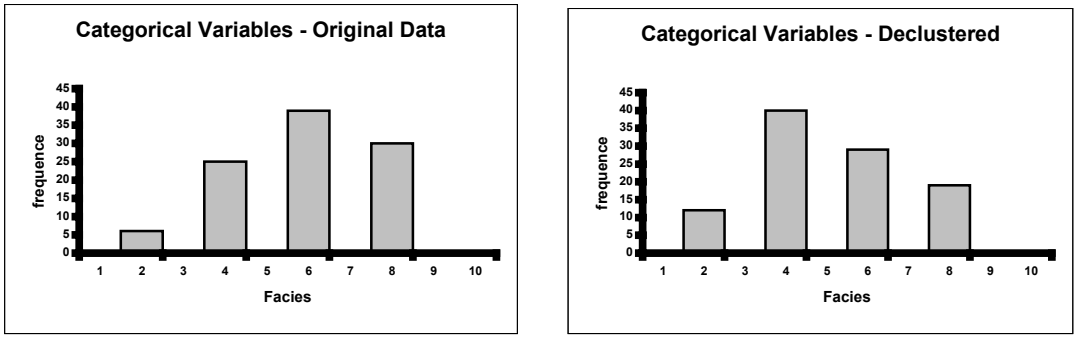

Figure 2.2: Categorical variable histogram before and after declustering.

The first step to assess uncertainty in the mean is done mainly by resampling the histogram. This histogram must be the one that is already cleaned, that is, the histogram obtained at the end of phase 1. It can be the experimental. The idea is to resample this histogram to create many subsets of values. Some care has to be taken to avoid drawing too many samples from the same well.

In the case of continuous variables, the mean is calculated for each subset. One constructs the histogram of the resampled means. The shape of this histogram is related to the uncertainty. If one is studying a mature oil field, with many wells, the histogram obtained at the end of this first step will have essentially zero variance, showing that there is little uncertainty in the original histogram. On the other hand, if one is studying a field with one or two wells, the histogram of the means will have a large variance. Figure 2.3 shows one possible histogram of the mean of porosity, after the bootstrap sampling.

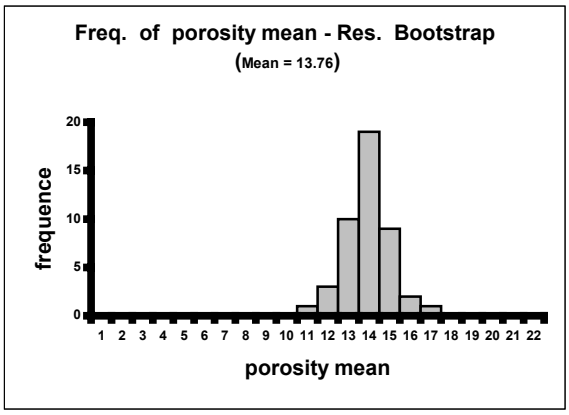

Figure 2.3: Histogram of mean porosity from bootstrap sampling

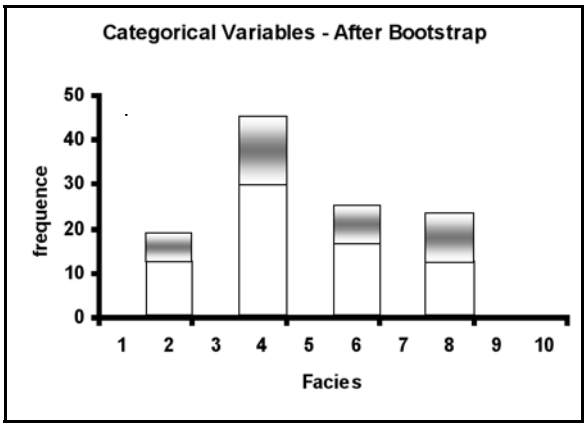

Figure 2.4: Uncertainty in the proportion for each of the categories. The darker the tone, the greater the probability of the frequency value for that category

In the case of categorical variables the goal is to obtain the range of variation for the proportion for each class. The process is similar to the case of continuous variables: the histogram obtained at the end of phase 1 is resampled many times. In each resampling iteration a subset of that data set is created. The subset is used to compute the proportion of each categorical variable. The result of this process is uncertainty in the proportions. Figure 2.4 shows example results of the bootstrap sampling for categorical variables. In this figure, the gray tone is related the probability of the frequency. The darker the tone the greater the probability of that frequency value.

The second step of this phase is the assessment of the uncertainty related to the parameters of the (co) regionalization functions. There is uncertainty related to the variograms; with few wells, it is almost impossible to obtain a clear, well-behaved variogram that can be modeled without any question. This step is complicated by the fact that the variogram is anisotropic and composed of multiple structures. Therefore one has to propose a set of possible parameter values for the variogram that are consistent with the available data.

The third step of this phase is to obtain target distributions of the variable being studied for different values of the mean. The process is to draw a mean value from the histogram generated in step 1. With this mean value, a histogram of the variable is constructed. The histogram shape is preserved in the construction of the continuous variable histogram using the drawn mean. In this case one must take care that the new minimum maximum and values do not become physically inconsistent. For example, porosity cannot be negative, nor above, say, 33 % in well consolidated sands. When the minimum or the maximum values fall below or above these thresholds, one must fix this problem by removing all the implausible values and correct the histogram shape again. This iterative correction is summarized by:

$$\phi_i^{new} = \frac{\overline{\phi}^{l}}{\overline{\phi}^{ref}} \phi_i^{ref}$$

ϕ_i^{new} = new porosity value for the class i,

ϕ_i^{ref} = reference histogram porosity value for the class i,

$\overline{\phi}^{l}$ = mean porosity value drew from the mean porosity histogram,

$\overline{\phi}^{ref}$ = mean porosity value from the porosity histogram of phase 1

Another problem appears when one is drawing values from the proportion curve of one categorical variable, obtained in the first step of phase three. This problem arises when one draws

a frequency value for each category. The sum of those values must reach the correct value that is the unity. Order relations must be the corrected using the equation below:

$$p_i^{new} = \frac{p_i^l}{\sum_{n=1}^{n} p_n^l}$$

p_i^{new} = new proportion value for category i,

p_i^l = proportion drawn for the category i from the histogram

$\sum_{n=1}^{n} p_n^l$ = summation of every value drawn for all categories from the histogram

It is necessary to emphasize that the geostatistical simulation algorithms use the local conditioning data *and* a reference or target histogram that represents the entire reservoir. If there are few data the reference or target histogram is uncertain, but the conditioning data are known. The computations described above create possible reference histograms. The local conditioning data are not changed. Freezing the reference histogram to that obtained from the data would seriously underestimate uncertainty.

In relation to the variogram assessment of uncertainty one must take care when drawing the major and minor range values in the histogram curves. The value for the direction of greater continuity could be smaller than the value for the direction of smaller continuity. An improved approach is to draw a value for the direction of greatest continuity and for the anisotropy ratio.

Hierarchical Aspect of Setup

The uncertainty of variables that affect the reservoir uncertainty must start with those variables that have the most extensive effect and following hierarchically with variables that are more local. This approach is done using a scenario tree, as in Figure 2.5. It is important to emphasize that each branch of the scenario tree does not have to have the same structure. Therefore, the general aspect of the tree is irregular. At each level, each branch is connected to one specific branch of higher level. One must specify the probability of that branch, and the summation of all branch probabilities must be unity.

The structural model is related to the configuration of the surfaces and the vertical and horizontal compartmentalization. The sedimentologic model is related to the facies grouping, that is, using the concepts of architectural-element analysis, one has to find the best way to group all the facies found in the reservoir. The term “best” means the minimum number of classes, without ignoring important details and petrophysical proprieties. Finally the saturation model is related to the properties to the rock-fluid interaction, like wettability, relative permeability and so on, and usually it is related to the porosity, permeability and the “kind” of rock.

To assess the uncertainty of the first level of hierarchy, that is the structural component, we must model vertical and horizontal compartmentalization. The vertical compartmentalization results from a sequence of porous layers with some type of discontinuity between them. Sometimes these discontinuities are correlated between adjacent wells. Therefore, it is a good practice to map these discontinuities and divide the reservoir accordingly. The vertical separation may be by depositional systems, for example, reservoir units with on-lap and down-lap, one on top of the other.

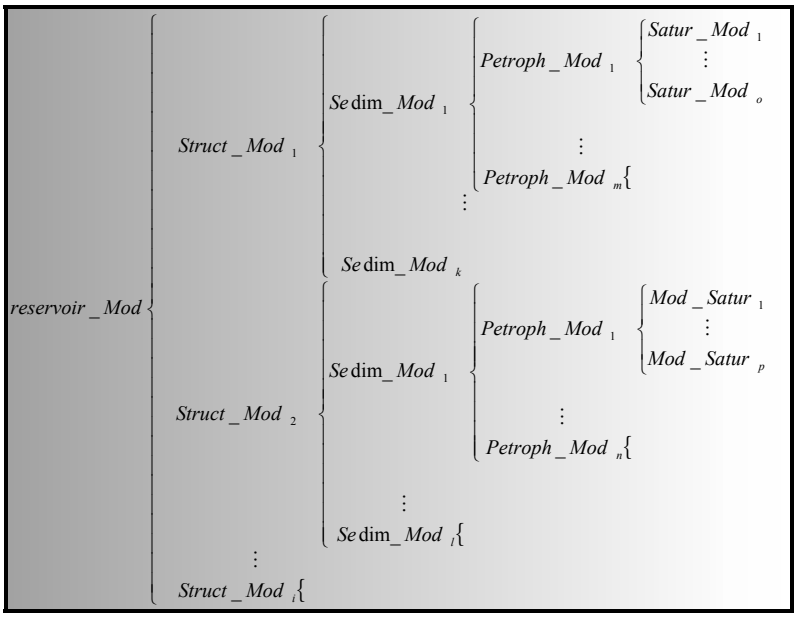

Figure 2.5: Scenario tree for the uncertainty assessment of a reservoir. Note that each branch has a specific number of sub-branch. For example the number of sedimentological models in the structural model 1 can be different from the number of sedimentological models in the structural model 2

Uncertainty in vertical compartmentalization is related to uncertainty in surfaces. Depending on the data set, one can shift the surface up and down while honoring the wells to account for possible bias. One can also use different variogram parameters, and perform stochastic simulations of that surface. The freezing of the reference seismic surface and structural parameters gives very little variation in the final results. Uncertainty may also be assigned to the competence of a surface to act as a barrier for vertical flux.

In the case of horizontal compartmentalization, it is necessary to analyze carefully which faults must be taken into account when modeling the reservoir and the ones that have no impact on fluid transmission. There are some situations where the impact is so big that there are independent fluid contacts in each fault block.

The assessment of uncertainty in the horizontal compartmentalization is similar to the assessment of uncertainty of vertical compartmentalization. Performing many simulations with the same variogram will affect local details of the fault surface, but will *not* have important impact on the final results. So, it is better to consider systematically shifting on the position of the surface, honoring the data set.

In relation to "honoring the data" the horizontal and vertical surfaces have different issues: most horizontal surfaces can be identified in the well logs, core or cuttings. Therefore, for most horizontal surfaces there are hard data or exact depths. However, for the vertical surfaces, like faults, it is very rare for them to be cut by a well. Therefore, there is no hard data to honor. Their uncertainty is determined by seismic data and production data. Besides that, the fault can be better defined using the concept of a fault zone, that is, a semi-vertical zone of many small surfaces, that are roughly parallel, and each responsible for a small offset. We have more " freedom" to model faults, but with more uncertainty.

The change in position of a sub-vertical fault will not be as important as the change in a the horizontal surface, due the difference on size (horizontal surface delimits a huge area of the reservoir, but the vertical surfaces delimit a very small area, comparatively)

The next level of the hierarchy of the scenario tree, is the sedimentological level. This level can

be subdivided in many sublevels depending on the sedimentological knowledge about the reservoir, or the complexity of the reservoir. The worst situation is when there is uncertainty of the sedimentary environment of the reservoir. If one is uncertain whether the sedimentary environment is, say, deltaic or fluvial, then one has to create two complete branches, one for the fluvial hypothesis and one for the deltaic hypothesis. One must create as many hypotheses as the probable sedimentary environments.

The following sub-level of the sedimentological level can be the rock type. The term “rock type” stands for any important sedimentological sublevel that can impact the reservoir performance. Some examples to clarify this term: (i) if one is studying a carbonate reservoir and the most important reservoir rocks is some depositional porous reefal facies, but there are some diagenetic porous lagoonal facies, then one can divide the reservoir rocks into two groups the reefal type of rock and the lagoonal type of rock. (ii) if one is studying a complex system of eolian sands intermixed with ephemeral braided fluvial systems, like in the desertic wadi sediments, one can use the term rock type to differentiate: the eolian kind of rock and the fluvial kind of rock. This subdivision procedure continues until all important sub-levels are modeled.

Reservoir facies should be the last sub-level within the sedimentological level; there is no sub-division below the reservoir facies sub-level. In the first example above, the reefal facies rock type can be sub-divided into the coral reef buildup reservoir facies, the backreef rudstone reservoir facies and the non-reservoir facies (intercalations of lagoon or backreef micrite deposits – here is an example of grouping two sedimentary facies to generate one non-reservoir facies); and the lagoon facies into the non-porous micrite non-reservoir facies and the diagenetic porous micrite reservoir facies.

The last thing one should model is the petrophysical model. The variables that should be modeled are porosity, permeability, fluid saturation and the “rock zone”. In general, the petrophysical data set is composed mainly by porosity measurements, obtained from well log interpretation and a few from direct cores measurements. The measurements of permeability are few, and obtained from core plugs. Finally, the saturation measurements can be obtained from log interpretation (mainly in the transition zone). The term “rock zone” is concerning to rock-fluid interaction and therefore, is dependent on the rock properties and fluid properties. Capillary pressure curve, relative permeability curve and so on, should be set for every reservoir facies or group of reservoir facies. There are many information sources to construct this model. Of course, the best information is the one that comes from the reservoir being studied, but another sources can be considered, as the information that comes from mature analog reservoirs of the same basin and the same prospect, outcrops from analogous formations and so on.

Petrophysical properties are modeled sequentially: porosity, permeability, and residual saturation. A reference histogram and variogram must be drawn for each rock type / facies and, then, conventional geostatistical simulations applied. The relationship between porosity and permeability is reproduced by cosimulation after porosity.

In the porosity modeling, if there is availability of post-processed seismic data, like seismic inversion and so on, the procedure starts with the calibration of porosity from seismic: (i) extraction of seismic data at well location. (ii) Upscale the well porosity values to the seismic scale. (iii) Transformation of the two data set (average porosity and seismic at well location) to the normal space (normal score transformation). (iv) Study the relation between the two data set using cross-plots, variograms and so on. If there is a (good) correlation between these two variables, one can simulate the porosity, in a 3dimentional grid, using the seismic as soft data.

If the collocated seismic data are correlated with the porosity data, there are some possible ways to obtain the porosity value using seismic as the secondary variable.. The Locally Varying Mean (LVM) method starts with the conversion of the seismic data to the mean porosity values using a

nonlinear correlation curve, obtained from a cross plot of the seismic attribute versus the mean porosity. Then sequential Gaussian simulation is performed using simple kriging with the mean porosity obtained in the previous step. Another way is the collocated cokriging that avoid the calculation of the cross variograms or covariances, the fitting a coregionalization model and handling with the large-scale nature of the seismic data. Block cokriging is another algorithm that relates porosity and seismic data, but in this case all the computations avoided with collocated cokriging must be done. All the computation described above must be done in a facies dependent type simulation.

Permeability is modeled after the simulation of porosity values for all cells in the grid. As in the case of porosity simulation, the permeability simulation must be done in a facies dependent manner. The simplest algorithm uses the regression of the logarithm of the permeability versus porosity. The regression curve can be used to obtain permeability for each porosity value. The advantages are simplicity and approximate reproduction of important features. The disadvantages are the smoothing of low and high permeability values and no accounting for uncertainty specific to permeability, beyond that of porosity.

Another way to obtain permeability from porosity is using a Monte Carlo simulation. In this case, the porosity is divided in classes and for each class is created the histogram of the permeability values of the pairs porosity-permeability in which the porosity falls in that class. That is, the porosity – permeability pairs are classified according the porosity value. A permeability histogram is constructed for each class of porosity. The second step is to perform permeability Monte Carlo simulation for each cell in the grid using the value of porosity in that cell.

These two algorithms should not be used because they disregard the spatial characteristics of high and low permeability values, which are important to flow simulations.

Gaussian simulation techniques can be used for permeability. In this case all data must be transformed to normal space. The correlation coefficient between the two normal variables and, perhaps, the cross-variogram are calculated and modeled. Gaussian simulation proceeds in standard fashion. The problem with this method is that Gaussian simulation has the characteristic feature of disconnected high and low permeability values, with large impact on flow simulation. If there are good facies models, then this procedure can be used.

The indicator method for permeability prediction has high computational and technical cost, but can better preserve the continuity of extreme values. The permeability distribution is divided into 5 to 11 thresholds. The porosity data is transformed into prior probabilities of permeability at those thresholds. Indicator (co)kriging is used to merge the soft porosity-derived data with the hard permeability data.

This section provides a starting point to define the conceptual space of uncertainty related to a petroleum reservoir. There remain important issues related to sampling this space of uncertainty and analyzing the results.

3. Sampling the Space of Uncertainty

The preceding section described how to formulate the problem and define uncertainty. This space of uncertainty must be sampled and processed through the transfer function to quantify uncertainty in the output variables.

A major consideration in this aspect of uncertainty assessment is the time required to generate realizations and process realizations. Great care and planning is required when these times are significant. A range of procedures will be discussed below and then guidelines will be summarized at the end for different types of problems.

A Realization

Sampling a space of uncertainty requires generating a *realization*. For reservoir modeling, consider a realization to be a complete specification of a reservoir, for example, the structure, facies, porosity, and permeability. Questions arise about how to combine the different sources of uncertainty: should multiple porosity models be considered for each facies model? Should multiple permeability models be considered for each porosity model? The short answer is "no." True Monte Carlo simulation requires that each realization be constructed as a new random drawing from the "space of uncertainty." Generating multiple porosity realizations for a single facies realization would impart unwanted correlations between the "realizations."

Base Case and "Vary One at a Time"

A base case scenario where all input variables are set at their expected values is required. This is straightforward in conventional uncertainty analysis; however, there may be multiple runs required for the base case in geostatistical assessments of uncertainty. For example, taking the mean porosity at each location in the reservoir is not a plausible base case. A geostatistical realization of porosity would be more realistic. The parameters could all be set at their base case values. Of course, there would be more than one base case by simply changing the random number seed inside the geostatistical-modeling algorithm. Considering a few (up to 10) base case models may be required.

Then, each input variable can be varied one at a time. The response variables can be compared to the base case(s). This method gives the sensitivity of the response function to independent variations in the input parameters; the method does not give a full assessment of uncertainty. Nevertheless, this approach is robust and useful to establish the sensitivity of the response variables to each input variable.

Monte Carlo Simulation

Unlike the "vary one at a time" approach, a proper Monte Carlo Simulation MCS involves varying all variables simultaneously. The idea is to randomly draw values from each input distribution and then to evaluate the response variables of interest with the full set of simulated parameters. The drawing of each variable or parameter is done sequentially, that is, the variables are ordered and each *j* variable that is drawn is conditioned to the *j-1* previous simulated values. This ensures that the simulated realization respects correlations between variables and is a plausible realization. Many realizations are generated and the response variables calculated for each to construct histograms of each response variable. These histograms inform the full space of uncertainty of the response variables. This very simple and powerful Monte Carlo Simulation works just fine if the problem is simple and there are few sources of uncertainty.

Depending on the objective of a study and the time required generating and processing realizations; certain experimental design techniques may be useful. To increase efficiency in sampling the uncertainty space, a directed sampling technique such as latin hypercube sampling

may be used. To screen out variables that have significant effect on the response variable, Plackett-Burman's multifactorial design approach may be adopted.

Latin Hypercube Sampling

A latin cube is an $n \times n \times n$ three-dimensional matrix consisting of n distinct elements each appearing n^2 times arranged in such a manner that the n elements appear in each layer and is repeated n times in that layer (Dénes and Keedwell, 1974) (See Figure 3.1). A latin hypercube is simply an extension of the latin cube to more than three dimensions.

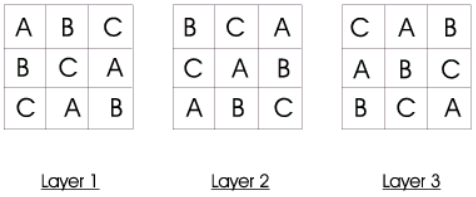

Figure 3.1 Latin cube with three distinct elements: A, B, C. Three layers of the cube are shown to illustrate the repetition of the three elements in each layer (occurring exactly three times), and a total of exactly nine times across all three layers.

The use of this concept in data sampling for uncertainty analysis is promoted by the inherent efficiency in the sampling procedure. Basically, the idea is to partition each of the univariate distributions (corresponding to the input variables) into equally probable classes or discretizations. The initial sampling pass involves randomly drawing from each univariate distribution and calculating the response function, while keeping track of the probability class from which the drawn numbers were obtained. The second pass of this process allows for Monte Carlo simulation to be performed for other probability intervals not previously drawn. Subsequent sampling passes are further constrained by all previously drawn classes. Figure 3. shows a schematic of the procedure for Latin hypercube sampling using three variables and six classes (wherein only the first four passes are shown).

The number of probability discretizations then gives the number of passes required until all intervals have been sampled. Defining too few partitions gives too few realizations, from which an uncertainty assessment may be unreliable. Given the case where the number of intervals is adequately specified, defining the space of uncertainty only requires very few sampling passes and as a result this method is much more efficient than the classical MCS approach.

For instance, consider a petroleum example where the Oil in Place (OIP) is the response variable (see Figure 3.2). This response is a function of three variables: porosity (ϕ), water saturation (S_w) and the net to gross ratio (NTG). We are interested in the deciles, and so the initial pass will cover 10% of the entire probability range. The first pass allows unconstrained MCS on all four variable distributions while keeping track of the deciles from which each value was drawn, and OIP is calculated. The second pass allows for constrained MCS to be performed, that is, constrained in the sense that random numbers can be drawn from any other decile that has yet to be drawn. After all probability intervals have been sampled, the uncertainty in the statistic of interest is completely spanned by only drawing ten times (relative to the hundreds or thousands of realizations required for conventional MCS).

The use of the latin hypercube concept to data sampling is quite ingenious. The efficiency of sampling from multiple distributions and spanning the range of uncertainty in the statistic of interest makes it an attractive approach to uncertainty quantification.

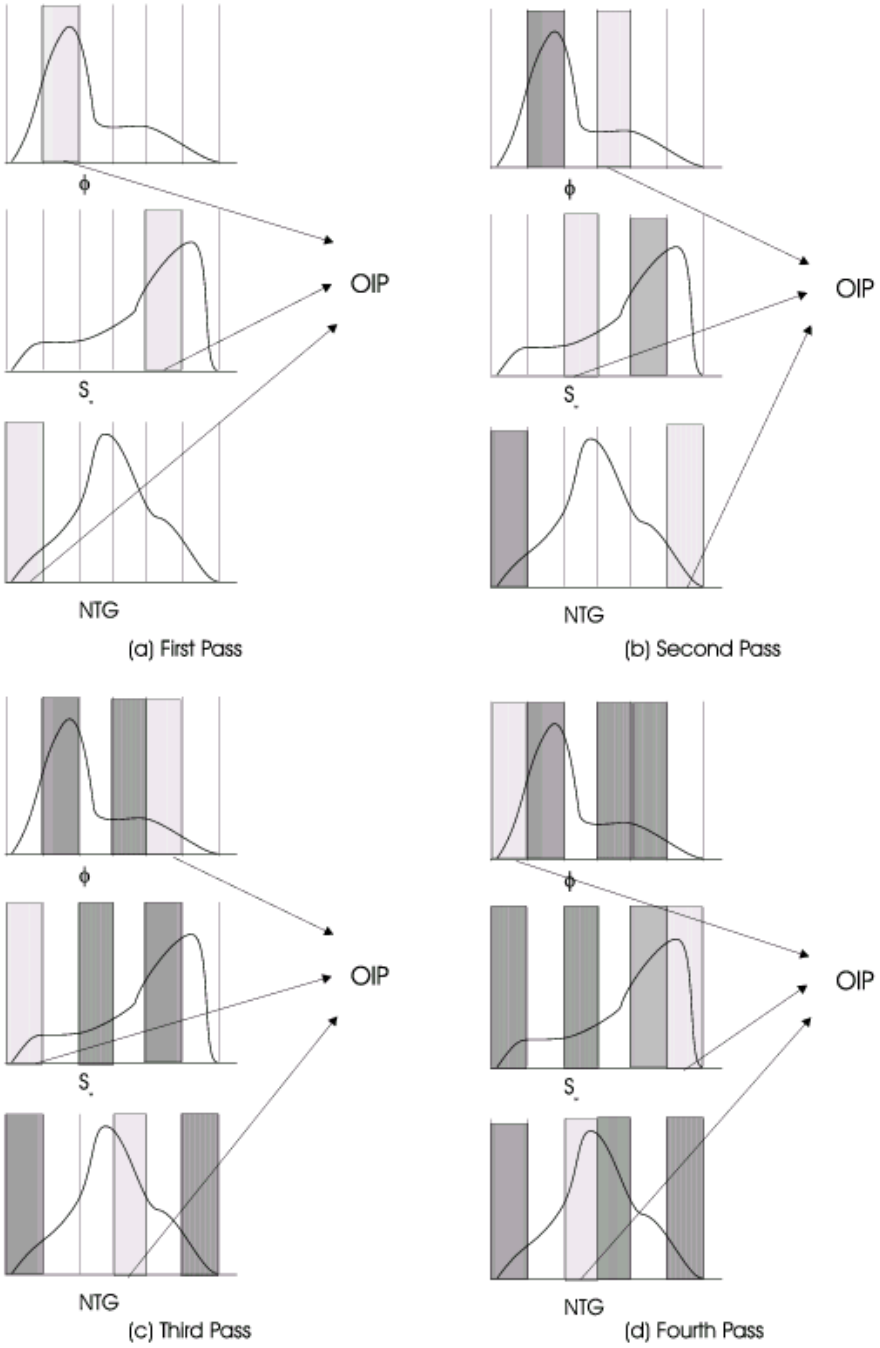

Figure 3.2 Latin hyper cube sampling schematic illustration using three variables. (a) First pass - Random drawing from any probability interval (light grey shading); (b), (c) and (d) Subsequent passes - Random drawing from any probability interval not previously sampled. Dark grey areas are previously sampled intervals, and unavailable for future sampling. Light grey areas are the sampled interval for that particular pass.

Plackett- Burman's Multifactorial Design

Suppose there are n predictor variables, all of which can take L possible outcomes. For $L=2$, the space of all combinations is 2^n, the exploration of which quickly becomes impractical for large n. Plackett and Burman proposed a multifactorial experiment in order to efficiently determine the main effect of a predictor variable on the response variable (Plackett and Burman, 1946).

The multifactorial design consists of determining a rectangular matrix of N rows and n columns containing L different integers corresponding to L different values that each predictor variable can take. This matrix is referred to as the design matrix, where N is the number of combinations and n is the number of variables. The matrix is designed such that (1) each variable is replicated at each of its L values the same number of times, (2) each pair of components occurs at every combination of values the same number of times; and (3) the number of assemblies N is divisible by the number of possible outcomes squared, i.e. N/n^2. If K is the number of times each L value is replicated and $N=KL^2$, then the maximum number of columns is

$$n = \frac{KL^2 - 1}{L - 1}$$

Plackett-Burman gives two approaches to determining the design matrix. The first method is to identify a pair of Latin squares that form an orthogonal set, successive columns accounting for interaction between components are then generated by performing simple arithmetic operations of preceding columns. The second method relies on the theory of Galois fields in order to specify the initial column, from which successive columns are obtained by cyclic permutation. The main difference between the two approaches is that the second method of Galois fields assumes that the interactions between components are negligible.

The determination of a design matrix is not trivial. Design tables have been obtained for a certain number of combinations N when $L= 2, 3, 4, 5$ and 7. A cyclic solution is obtainable when $N=L^m$. For practical purposes, the practitioner should refer to design tables published in Plackett and Burman (1946).

For illustrative purposes, the design table is given below for three possible outcomes (in a continuous case, the three values may correspond to minimum, mean, and maximum values), and 9 combinations, i.e. $L=3$ and $N=9$ with $N=L^m$, so $m=2$. Each column corresponds to a component or variable and each row corresponds to an assembly or combination. The last row of 0s is added in the design to denote the base case, where all variables are at their expected value.

A \ C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0	1	1	2	0	2	2	1	0
2	1	0	1	1	2	0	2	2	1
3	2	1	0	1	1	2	0	2	2
4	2	2	1	0	1	1	2	0	2
5	0	2	2	1	0	1	1	2	0
6	2	0	2	2	1	0	1	1	2
7	1	2	0	2	2	1	0	1	1
8	1	1	2	0	2	2	1	0	1
9	0	0	0	0	0	0	0	0	0

Table 3.1. Design table for 8 components or variables and 9 assemblies or combinations.

The concept of the Plackett-Burman design is good. Setting up a design for an arbitrary number of variables that can take an arbitrary number of outcomes is non-trivial. In certain cases the few standard design tables can be extended.

Plackett-Burman or multifactorial designs in general do not provide an assessment of the space of uncertainty in the same way as MCS and LHS. The aim of these experimental design techniques is to determine the effect of each variable on the transfer function, but account for the variations in more than one variable at a time. The "vary one at a time" approach presented above is a primitive approach to such sensitivity analysis.

Let's see some more details of the Plackett-Burman approach. The determination of the $N \times n$ elements of the rectangular matrix relies on a field of finite order known as Galois fields. Consider a Galois field of order L^m, denoted as $GF(L^m)$, represented by a set of m ordered samples and L is prime. For $m \geq 2$, this field is called an extension field. The elements of $GF(L^m)$ can be represented by integers ranging from 0 to $L-1$. A cyclic group of nonzero elements can be generated by determining the primitive elements of $GF(L^m)$ and raising it to some integer power c, $c=0, \ldots, L^m-2$. Plackett-Burman uses the theory of Galois fields to obtain a cyclic solution for a certain number of assemblies N when $L= 2, 3, 4, 5$ and 7.

Suppose one value of variable Z_i, $i=1, \ldots,p$, is the nominal or average value and the other possible outcome is an extreme value (for an $L=2$ design). In an industrial setting, the variables may be the components of a machine and the response variable may be the performance of the machine. The nominal value that each component can take may be the normal operations of a component, while the extreme case may be failure of a component.

For $L=2$, the primitive elements of the corresponding $GF(2^m)$, $m \geq 2$ consist of ± 1. The precision at which the main effect of a component is desired determines the number of assemblies required. For example, if the user wishes to calculate the main effect at four times the precision that an assembly can measure, then the number of assemblies is 16 (i.e. $N=L^m$, where $m=4$). The first component of the elements for $GF(2^4)$ are:

+1,+1,+1,+1,-1,+1,-1,+1,+1,-1,-1,+1,-1,-1,-1

This set of elements forms the first column of the required rectangular matrix. A row of -1's are added to form the base case, where –1 indicates the nominal value of a component and $+1$ indicates the extreme value of the component. Permutation of the column matrix consisting of the 15 elements of $GF(2)$ in a cyclic manner yields the complete multifactorial design:

A \ C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1
2	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1
3	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1
4	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1
5	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1
6	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1	−1
7	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1	+1
8	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1	+1
9	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1	−1
10	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1	−1
11	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1	+1
12	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1	−1
13	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1	−1
14	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1	−1
15	−1	−1	−1	+1	−1	−1	+1	+1	−1	+1	−1	+1	+1	+1	+1
16	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1

where each column corresponds to a component and each row corresponds to one of the 16 assemblies.

For each assembly, evaluate the response variable, r_i, $i=1, \ldots, N$. The main effect of a component is given by the linear combination of the response value, with the coefficients given by the

column vector corresponding to that particular component. For example, the main effect of component *1* is :

$$m_1 = \frac{r_1 + r_2 + r_3 + r_4 - r_5 + r_6 - r_7 + r_8 + r_9 - r_{10} - r_{11} + r_{12} - r_{13} - r_{14} - r_{15} - r_{16}}{N}$$

Comparing the main effect of each component allows for identification of the components with the greatest effect on the response variable. If only *9* components are considered (i.e. $n=9$), then the main effect of components $n+1, ..., 4K-1$ should be zero. Due to experimental error, however, this may not be so. The variance due to error is approximated by the sum of squares of residuals divided by the degrees of freedom.

$$s^2 = 4K\left(\frac{m_{n+1}^2 + \ldots + m_{4K-1}^2}{4K - n - 1}\right)$$

The error variance of each n component is $m_i = s^2{}_i = s^2/4K$.

Designs can also be worked out for $L=3$; in a continuous case, the three values may correspond to the minimum, mean, and maximum values, and *9* assemblies, i.e. $L=3$ and $N=9$ with $N=L^m$, so $m=2$. The elements of this $GF(3^2)$ are generated by raising a primitive root to c:

[0,0],[0,1],[1,1],[2,0],[2,1],[0,2],[2,2],[1,0],[1,2]

The first column of the design rectangle is given by the first component of each element: 0, 0, 1, 2, 2, 0, 2, 1, 1. Cycling through this $(n-1)$ times gives the rectangular matrix that describes the multifactorial design. For $n=9$:

A \ C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0	1	1	2	0	2	2	1	0
2	1	0	1	1	2	0	2	2	1
3	2	1	0	1	1	2	0	2	2
4	2	2	1	0	1	1	2	0	2
5	0	2	2	1	0	1	1	2	0
6	2	0	2	2	1	0	1	1	2
7	1	2	0	2	2	1	0	1	1
8	1	1	2	0	2	2	1	0	1
9	0	0	0	0	0	0	0	0	0

where each column corresponds to a component and each row corresponds to one of the *9* assemblies. For $L= 3,5$, and *7*, the complete design is given by cyclically permuting the first column $(N-1)/(L-1) -1$ times and adding a row of zeros.

Experimental design is important when the cost of the transfer function is high. Basic MCS is perfectly reasonable when it is fast to process realizations.

Guidelines for Sampling Uncertainty

The following guidelines are driven by a desire for practicality and simplicity. A base case (or set of base cases) must always be established. There are two choices for sensitivity analysis: (1) the vary one parameter at a time approach for basic results, or (2) Plackett-Burman and other multifactorial experimental design techniques for a more complete assessment when the uncertainty setup corresponds to a known design. There are two choices for uncertainty analysis: (1) MCS when the transfer function is fast and a basic result is required, or LHS when the transfer function is costly.

4. Display and Analysis of Results

The results of our uncertainty sampling must be summarized and presented to decision makers and technical staff. There are two aspects to the display and analysis of results: (1) sensitivity and (2) uncertainty. Although closely related ideas, there is a distinct difference between sensitivity and uncertainty analysis. Sensitivity analysis evaluates the contribution of the input variables to the total uncertainty in the output variable; while uncertainty analysis quantifies the uncertainty in the output variable resulting from uncertainty in the input variables.

Displaying Uncertainty

A histogram and cumulative distribution of each response variable are standard means of displaying the uncertainty in each response variable. The conventional definition is the probability to be lesser than or equal to the threshold, but some companies consider the probability to be above the threshold.

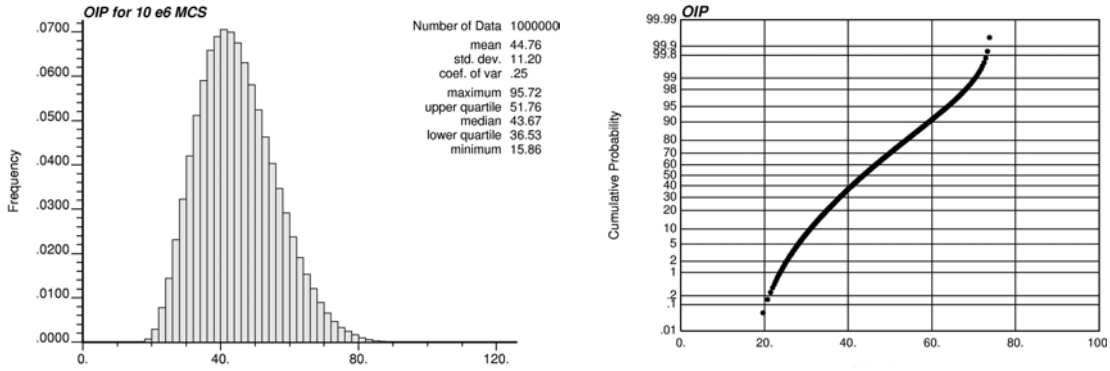

Figure 4.1 histogram and cumulative distribution (with normal or Gaussian scale) for the oil in place for an example problem.

Displaying Sensitivity

Tornado charts and spider diagrams are commonly used to show sensitivity of the output variables to the input sources of uncertainty. These may be constructed by directly calculating the change in the response variables for changes in each of the input variables. Often, however, the covariance or correlation between each of the input variables and the response variables is calculated. The magnitude of the covariance provides a measure of sensitivity.

Ideally, we would be able to directly assess the sensitivity of each response variable to each input variable, that is, the full set of sensitivity coefficients:

$$\frac{\partial Z_k}{\partial Y_j},\ j = 1,...,n; k = 1,...,K$$

where n is the number of input variables Y_j, $j=1,...,n$ and K is the number of response variables Z_k, $k=1,...,K$. This requires that the relation between the response and predictor variables is known, and that this function is differentiable. Further, evaluation of the partial derivative is sensitive to the magnitude of the values of the predictor variables. This scale effect can by mitigated by multiplying the partial derivatives above by the standard deviation of the specific predictor variable, that is,

$$\frac{\partial Z_k}{\partial Y_j} \cdot \sigma_j, \; j = 1,...,n; k = 1,...,K$$

Removing the scale effect of the variables provides for more reasonable sensitivity comparisons to be made.

Spider diagrams show the sensitivity to a response variable to the suite of input variables. The standard approach is to consider the change in the input variable as a percentage change from the base case value. See Figure 4.2 below. These figures are not a good way to communicate sensitivity. Certain variables may be known within 1% while others may be uncertain within a wide range of uncertainty. All sources of uncertainty are shown +/1 the same percentages. Although we have not fully developed the idea, it makes good sense to consider *p-spiderdiagrams* where the X axis is changed to represent the quantile on the distribution of uncertainty corresponding to each variable, see Figure 4.3 below.

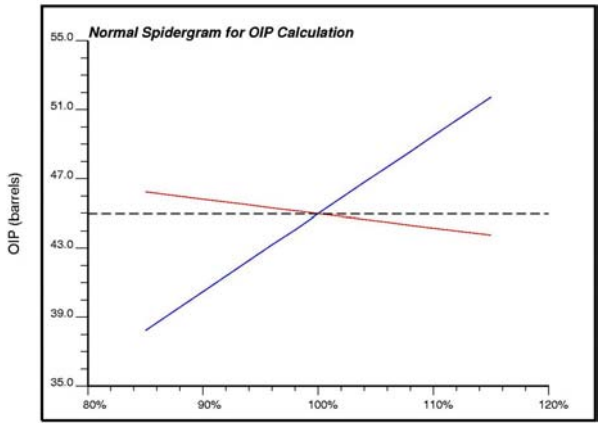

Figure 4.2 conventional spider diagram where the change in the response variable (OIP in this case) is shown on the vertical axis relative to percentage changes in the variable. The horizontal dashed line is the base case, the blue line is porosity, and the red line is water saturation. The gross reservoir volume and the net to gross ratio would also fall on the red line because these variables go into the calculation in the same direct multiplicative manner.

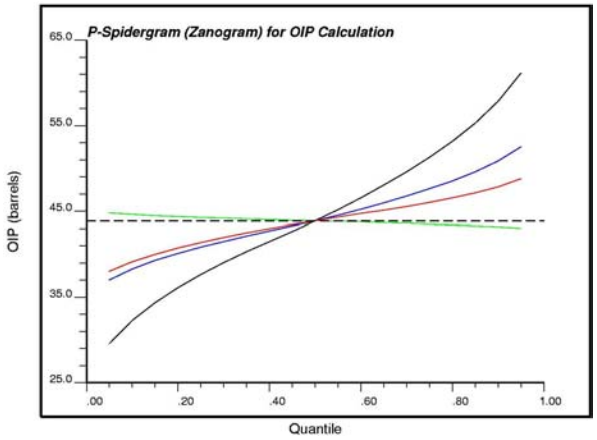

Figure 4.3 an improved *p-spider diagram* where the change in the response variable (OIP in this case) is shown on the vertical axis relative to specific percentile values in the variable. This reflects the importance of each variable and the influence of the distribution of uncertainty. In this case the lines are: porosity (black), Sw (green), NTG (red), and GRV(blue). Note the richer information content.

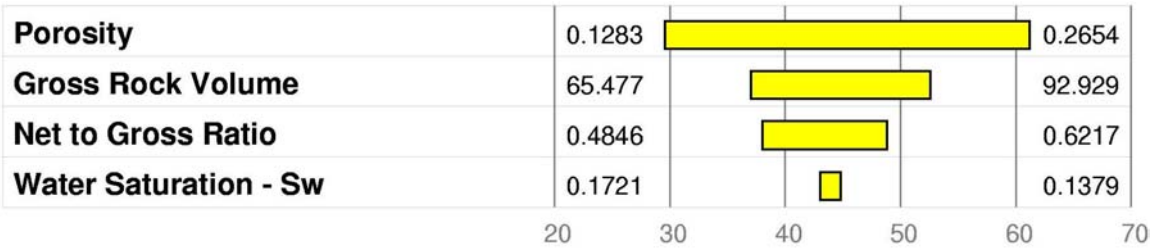

Figure 4.4 an example tornado chart where the sources of uncertainty are explained by showing how each variable affects the final result. The numbers on the side show the P10 and P90 values that led to each result. Note that the effect of water saturation is an inverse relation.

Tornado charts show the input variables ranked according to sensitivity to an output variable. Figure 4.4 shows an example where the sensitivity of four variables is shown relative to the calculation of oil in place. The limits to the right and left are the calculated OIP for P10 and P90 values of each input variable. Those values are shown beside each shaded bar.

The Tornado chart does not convey correlation between the input variables. For example a positive change in one variable can be related to a positive change in another variable and the effect can either be "compounded" or "mitigated." One can perform a full Monte Carlo Simulation and calculate correlation coefficients between each of the input variables and output variables. These correlation coefficients can be used for a tornado chart; however, these values will suffer the same problem as the conventional spider diagrams, that is, the units and uncertainty in each factor is not accounted for. Nevertheless, the full correlation matrix between the set of input parameters and all output parameters should be constructed and presented. The correlations between the input variables should be known ahead of time because they are required input.

Some Final Thoughts

Geostatistical tools are commonly used to model heterogeneity. Applying them to uncertainty evaluation is becoming increasingly common. A reasonable assessment of uncertainty is not as simple as changing the random number seed and running software to create multiple realizations of reservoir properties.

This is a preliminary report. There is scope in this area for much research and documentation. There are numerous subjects that are worthy of additional attention (1) documentation and reporting standards for uncertainty, (2) ranking geostatistical realizations for "approximate directed sampling" schemes and to reduce computational effort, (3) a practical assessment of the number of realizations required, and (4) numerous well-documented case studies.

The clever combination of programs and scripts in a scripting environment (UNIX or bash on a PC) is very powerful for uncertainty calculation; however, companies are somewhat uncomfortable with this ad-hoc approach to uncertainty quantification. Statistical programs such as @risk and crystal ball are fine for simple modeling; however, there is no geostatistical or complex multivariate analysis. Commercial software such as Jacta® has limited market penetration at this time. There is a need for healthy competition and openness in this area.

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