A Risk Qualified Approach to Calculate Locally Varying Herbicide Application Rates

T. Faechner[1], K.P. Norrena[2], A.G. Thomas[3], and C.V. Deutsch[4]

Abstract

Weed competition can decrease crop yield and profit. Herbicide is applied to reduce weed populations, minimize crop loss, and maximize profit. Traditional practice is to apply herbicides at a uniform rate over an entire field. Complete knowledge of the weed distribution and appropriate instrumentation on the spraying equipment would allow the farm manager to apply the "correct" locally varying herbicide application rate. The locally variable rate would be greater in areas of high weed density and less where there are few weeds. A locally varying treatment would have both economic and environmental advantages.

A major problem facing farm managers is the unavoidable uncertainty in the spatial distribution of weeds in any particular field. This uncertainty in weed distribution influences the uncertainty in an optimal locally varying herbicide rate. This paper presents mathematical models that establish optimal herbicide application rates using geostatistical models of uncertainty in weed density combined with principles from decision making.

Weed data from a 34 ha field near Saskatoon, Saskatchewan, Canada illustrates the application of these tools. The models achieved a significant reduction in total herbicide use.

Introduction

Weeds reduce crop yield and profit (Thomas *et al.* 1998). Herbicides are important to control weeds and increase yield. In western Canada, herbicides represent up to 30% of the cost of crop production and are applied to more than 60% of cropped acres. Herbicides control weeds but are expensive and can also adversely affect the environment. Spatially selective application of herbicides would increase profitability and reduce environmental impact.

The prospect of increased profit and reduced environmental impact has sparked interest in precision farming techniques. Advances in technologies such as global positioning systems (GPS), computer integrated farm equipment, and numerical modeling including geostatistics offer the potential for site specific and locally varying weed management. The work presented here illustrates a method for risk qualified and optimal locally varying herbicide application rates. The proposed method has the following steps (1) sample the field for weed density, (2) create maps of weed density and related uncertainty over the entire field, (3) generate optimal application rate maps, and (4) download the optimal application rate

[1]Centre for Computational Geostatistics and Alberta Agriculture (ty.faechner@gov.ab.ca)
[2]University of Alberta (knorrena@ualberta.ca)
[3]Agriculture and Agri-Food Canada (thomasag@em.agr.ca)
[4]University of Alberta (cdeutsch@ualberta.ca)

maps to computer integrated farm equipment. The computer integrated farm equipment would apply the optimal herbicide rate throughout the field using GPS.

Weeds do not have a homogeneous spatial distribution; they are often said to be "patchy" (Mortensen *et al.* 1993; Dieleman & Mortensen 1998; Mortensen *et al.* 1998; Clay *et al.* 1999), thus amenable to site specific and locally varying herbicide application rates.

Many farm managers apply herbicides at a "uniform" rate over an entire field, and locations with low weed density receive the same amount of herbicide as those with high weed density. A uniform application rate is usually based on a visual assessment of weed density prior to application, but there is no procedure to balance the risks associated with under- and over-spraying. The result is a subjective assessment of weed density, and uncertainty in both weed density and optimal application rate.

Crop yield losses increase with increasing weed density (Carlson *et al.* 1981; Cousens 1985; O'Donovan *et al.* 1985; Cousens *et al.* 1987; Cousens & Mortimer, 1995). *Avena fatua L.* (wild oat) competition reduced spring wheat (*Triticum aestivum L.*) yields up to 68% compared to weed-free controls (Carlson *et al.* 1981). Similarly, 50 *A. fatua* m^{-2} resulted in a 25% yield loss in spring wheat while 300 *A. fatua* m^{-2} resulted in a 67% yield loss when weed and crop emerged at the same time (Cousens *et al.* 1987). In a survey of 17 farm fields, average yield loss due to *Setaria viridis* (L.) Beav. (green foxtail) competition in wheat and barley (*Hordeum vulgare* L.) crops was about 10% while *Polygonum convolvulus L.* (wild buckwheat) caused a 14% yield reduction (Friesen & Shebeski 1960). Spring wheat yield loss due to *P. convolvulus* and *Thlapsi arvense L.* (stinkweed) was found to be 2.6% when growing degree days were included in a multiple regression equation (Hume 1989). Percentage canola (*Brassica rapa*) yield loss for each *Fagopyrum tataricum (L.)Gaertn.* (tartary buckwheat) plant varied from 0.14 to 0.31 over 3 years (O'Donovan 1994). *Cirsium arvense* (L.) Scop. (Canada thistle) density of 10 shoots m^{-2} is predicted to cause canola yield losses of 11% using a linear model of yield loss.

Herbicide application rate can be adjusted to account for variations in the spatial distribution of weed density. This assumes that weed control is complete, no crop damage occurs and weed density is a satisfactory measure of weed competition (Auld *et al.* 1987). A critical assumption in this research is that the optimal application rate is proportional to weed density, that is, areas with high weed density should receive more herbicide and areas with low weed density should receive less herbicide for effective control (Holm *et al.* 2000; Zhang *et al.* 2000). For example, a 50% reduction in difenzoquat rate controlled 78% of the *A. fatua* compared to the label rate of 1.0 kg a.i. ha^{-1} (Wilson 1979). A seed density in the soil of 15 seeds m^{-2} of *Alopecurus myosuroides* in winter wheat at a half rate of chlorotoluron could result in an increase in weed density (Ulf-Hansen 1989,(see Cousens & Mortimer 1995 p. 205)). Application of 0.1, 0.3 and 1.0 times the recommended field rate of diclofop to a susceptible *A. fatua* biotype resulted in dry matter responses of 20 to 80 % of the control (Seefeldt *et al.* 1994).

Low herbicide doses kill weeds at a low density, however when this same dose is applied on a weed patch at a high density, the herbicide dose received per plant is the same. Two rates of barban, recommended and 50% of recommended and difenzoquat at recommended and 33% of recommended were applied in spring barley to control *A. fatua* (Cussans & Taylor 1976). Both rates resulted in an 80% or higher reduction of *A. fatua* seed suggesting that low doses of herbicide can provide adequate control. Number of spray drops cm^{-2} was calculated for these 2 rates of herbicide. Rate did not affect *A. fatua* control indicating that a sufficient number of drops contacted *A. fatua* leaves at either dose. Although *A. fatua* density was not mentioned, it is likely that densities were different in each of the 5 fields. Thus, herbicide dose is independent of weed density.

Why do we apply a higher herbicide dose to a high density patch? The motivation is to enhance a crop's competitive status with respect to the weeds. For example, when a high, initial weed density occurs in a field, less weed-free crop yield results compared to a low, initial weed density. This is illustrated in Figure 1 where (I^I) represents an area with a initial, low weed density while (II^I) represents a high, initial weed density. When herbicide is applied, the initial density is changed to (I^F) as a result of weed kill resulting in a small change in the percentage of weed-free crop yield (I^F-I^I). For a high weed density, II^F, herbicide application will result in a significant decline in weed population and a large change in weed-free crop yield (II^F-II^I). The economic benefit of this change in crop yield will be small at the low weed density and may not exceed the costs of herbicide and application. However, at a high weed density, the change in crop yield will be substantial and may be economically beneficial to apply herbicide. Weed-free crop yield response will dictate the herbicide rate that is economically beneficial to apply and in areas of high weed density, high rates of herbicide will ensure that the crop is competitive and has the potential to yield more.

Varying the herbicide application rate from the recommended rate is unsupported by manufacturers because there is a legal liability guaranteed by the herbicide label. Herbicide performance testing is conducted on a range of crop varieties, weed densities and species, soil types and weather conditions. The recommended application rate is established for this diversity of conditions; however, the recommended rate could be modified to account for local conditions. The optimal application rate strikes a balance between cost, control and crop yield loss.

The local optimal herbicide application rate scheme requires that the weed density be known prior to application. It is unrealistic to exhaustively sample a field to establish the unique true weed density over the entire field. A more efficient method is to strategically sample the field and construct numerical models of weed density that are based on the sample data and that reflect significant physical and biological features of the weed species. Strategically sampling needs to account for scale and provide a complete picture of the spatial and biological relationships of weeds. Then, a model of uncertainty in weed density addresses the limitations of sparse sampling. Thus, a numerical model's predictive ability is significantly enhanced.

Geostatistics, a branch of applied statistics, has tools for mapping an attribute value and characterizing the uncertainty in weed density at unsampled locations. These techniques are systematically applied in other disciplines such as petroleum reservoirs (Deutsch & Journel 1998), mining (Journel & Huijbregts 1979; Isaaks & Srivastava 1989), and natural resources (Goovaerts 1997). Major decisions are made in the presence of unavoidable uncertainty in these related disciplines. Deciding on locally varying herbicide application rate can be done in light of uncertainty in spatial weed density.

The objective of this paper is to develop a method for mapping locally varying herbicide application rate in presence of uncertainty in weed density. An increased rate will be recommended in areas of high weed density to achieve optimal control and increased yield;

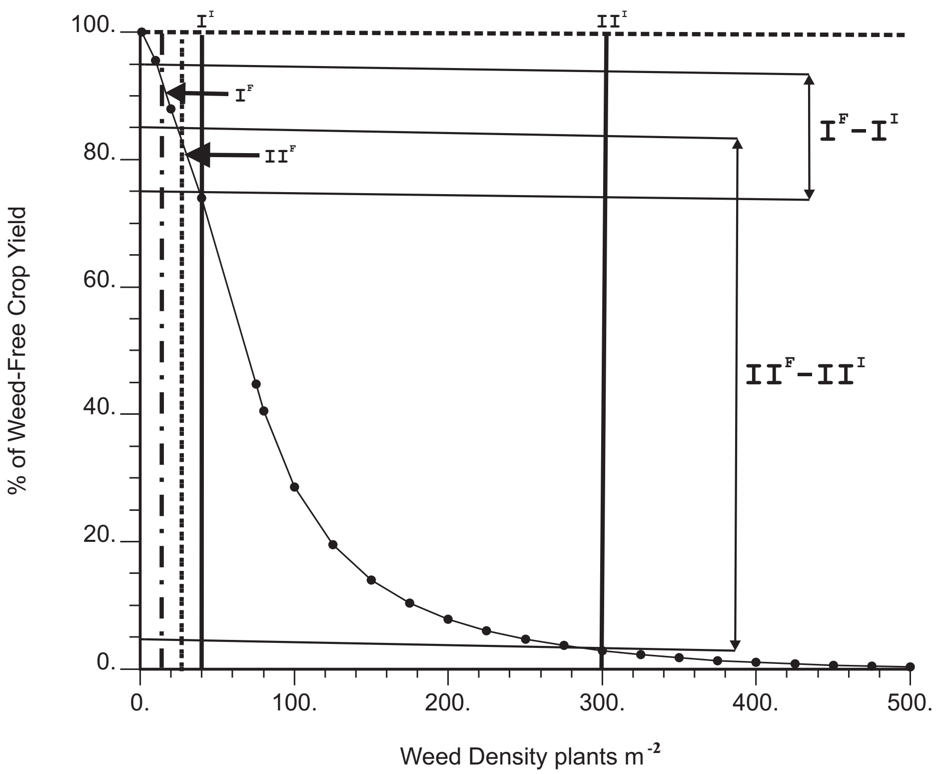


Figure 1: Weed density, plants m^{-2}, decreases % weed-free crop yield at both a low (I^{I} is 40) and high (II^{I} is 300) weed density to 75% and 5% respectively. When herbicide treatment occurs, weed-free crop yield increases to 95% and 85% for low (I^{F} is 15) and high (II^{F} is 25) weed density, respectively. The change in weed-free crop yield for low (I^{F}-I^{I}) and high (II^{F}-II^{I}) weed density can be used to determine whether it is economic to apply herbicide.

a decreased rate will be recommended in areas of low weed density. The optimal locally varying herbicide rate can be determined mathematically with a satisfactory model.

Details of Model

The optimal locally varying herbicide application rate is the rate that leads to maximum profit. The following development assumes knowledge of (1) herbicide efficacy at a given application rate, (2) the competitive relationship between weeds and crops, and (3) the cost of applying herbicide. The notion of optimal locally varying herbicide rates will be derived for known weed density and then uncertainty in weed density will be introduced. Finally, the optimal *risk-qualified* rate is calculated in presence of uncertainty in weed density. Risk-qualified requires multiple realizations of weed density in order to define a space of uncertainty. This uncertainty allows an assessment of risk for decision-making.

Optimal Rate with Deterministic Parameters

To begin, geographic location is denoted by the vector variable $\mathbf{u}$ that consists of east and north coordinates. The herbicide application rate for location $\mathbf{u}$ is denoted by $a(\mathbf{u})$ which is measured in litres ha^{-1} . The optimal local herbicide application rate for location $\mathbf{u}$ is denoted $a_{opt}(\mathbf{u})$.

Weed density $w(\mathbf{u})$ is defined as the number of weeds m^{-2} at location $\mathbf{u}$. This variable is nominally categorical taking values from 0 to some maximum number of weeds that could simultaneously grow in a square meter. In general, there are numerous weed species present in a field, but we only consider those weeds being sprayed for, such as broad-leaved weeds when spraying.

The farm manager cannot spray a different rate on each square meter of the field. We must consider a selective spraying area (SSA) denoted v. This area is probably 20-35 m wide (depending on spray boom length and electronic controls built into the sprayer) and 1-2 m deep because of the potential drift of herbicide. The SSA can be customized for site specific conditions given the fact that sprayer boom sections are being developed that apply herbicide over smaller SSA. The weed density must be averaged from the sampling area, m^{-2}, to the SSA:

$$w_v(\mathbf{u}) = \frac{1}{v(\mathbf{u})} \int_v w(\mathbf{u}')d\mathbf{u}' \quad (1)$$

Weed density is informed by (1) samples of weed density, perhaps over a small area with relatively great spatial detail, and (2) scouting or remotely sensed data, likely over a large area with less spatial detail. The numerical tools of geostatistics are used to model the weed density at the correct SSA areal size (Journel & Huijbregts 1979; Isaaks & Srivastava 1989; Goovaerts 1997; Deutsch & Journel 1998). Next, weed density is averaged from a sampling area to correspond to a SSA that is relevant to the limitations of the application equipment.

The first required input is the *maximum attainable weed-free yield*, or $y_0(\mathbf{u})$. This maximum attainable yield $y_0(\mathbf{u})$ is in units of tonnes ha^{-1} and $y_0(\mathbf{u})$ depends on location $\mathbf{u}$ in the field. Historical information and recent environmental and weather conditions will provide an approximation of $y_0(\mathbf{u})$ over the entire field.

The second required input is the *fractional yield loss due to non-zero weed density*, or $L(w)$. This fractional yield loss is a function of weed density. When weed density is high, high yield loss can be expected, while a low yield loss is expected when weed density is low. Yield loss starts at zero, that is, $L(w) = 0$ at $w = 0$ and may increase to its maximum value, 100%, depending on the competitive ability of the weed as its density increases. Experimental data are required to establish this function however yield loss values due to different weed densities were fitted from values provided in the literature (Carlson *et al.* 1981; Cousens 1985; Cousens *et al.* 1987). A family of fitted curves from a lookup table of a hyperbolic type are used to model $L(w)$. The curves represent fractional yield losses due to weeds under different cropping, weed and environmental conditions.

The third critical piece of information is the *fractional weed control as a function of the herbicide application rate*, or $Wc(a)$, where a is the herbicide application rate in litres ha^{-1}. Herbicide manufacturers likely have very good data on this function $Wc(a)$; however, these data may not be publicly available. Model parameters and bounds of this function were based on values from the literature (Anonymous 1998; Cousens 1985; Cousens & Mortimer, 1995). Where these were unavailable, parameters were hypothesized from our understanding of weed control. Nevertheless, much is known about this function: (1) it is bounded between 0 and 100%, (2) there is zero weed control at zero application rate, (3) there will be 80% or more control at the recommended application rate (Anonymous 1998), and (4) full control, $Wc = 100\%$, will be reached asymptotically as a increases (Cousens 1985; Cousens & Mortimer, 1995). Experimental data or a fitted hyperbolic or exponential type function could be used to model $Wc(a)$ (Cousens & Mortimer, 1995; Swanton *et al.* 1999). There may be a different $Wc(a)$ curve for herbicides with different formulations as illustrated by weed response for parallel dose-response curves (Streibig 1984; Streibig 1988).

Other price and cost inputs are required. The net price or net value of the crop, np, in the units of dollars tonne^{-1} must be known. The cost of the herbicide, c, in dollars litre^{-1} must also be known.

Using the input variables described above, it is possible to calculate the incremental revenue for a specific application rate, a:

$$r(a; \mathbf{u}) = \left[L\left(w_v(\mathbf{u})\right) - L\left(w_v(\mathbf{u}) \cdot (1 - Wc(a))\right)\right] \cdot y_0(\mathbf{u}) \cdot np \qquad (2)$$

where the units of $r(a)$ are dollars ha^{-1}. The variable, $r(a; \mathbf{u})$, represents a non-decreasing function of the herbicide application rate a at location $\mathbf{u}$; see the top figure in Figure 2. Of course, the cost of applying herbicide must also be considered.

Yield loss and herbicide application each have associated costs. Increasing herbicide application rate costs money due to increased product consumption. Decreasing herbicide application decreases this consumption but increases yield loss. This is shown in the bottom figure of Figure 2 where the 3 curves represent the sum of the cost of applying herbicide and the cost of crop loss.

There are fixed costs for equipment ownership, depreciation, interest, insurance and so on. These fixed costs are not considered in the equation below since it is assumed that it is economical to spray; the goal is to determine the optimal application rate. Clearly, there are cases of low weed densities where the fixed costs exceed the total revenues and the correct decision is not to spray at all. Given that spraying will occur, the cost of applying

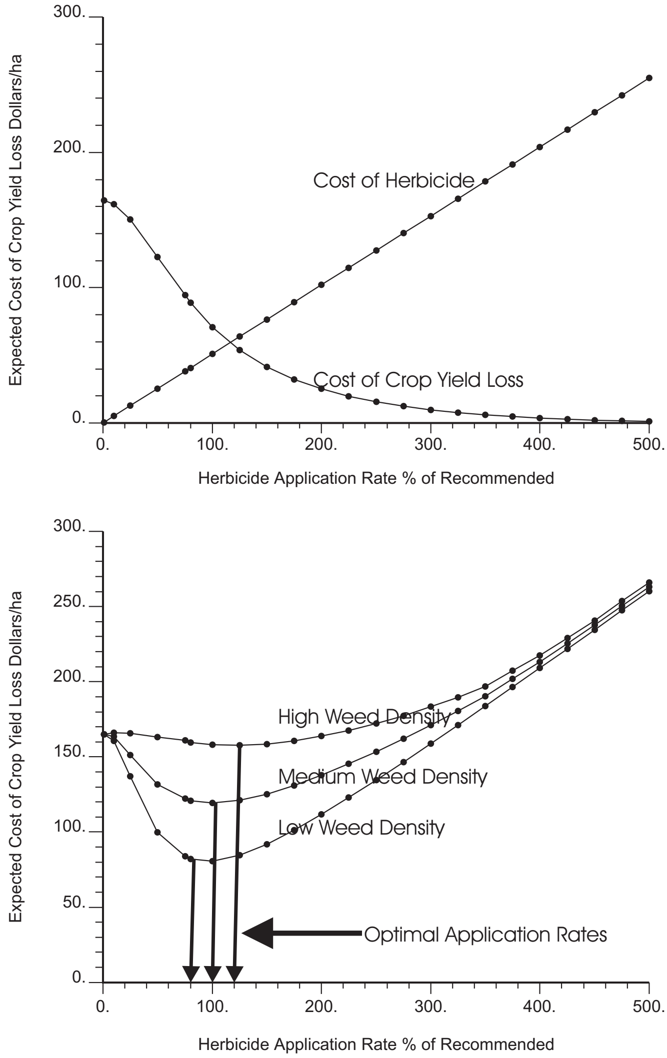


Figure 2: Increasing herbicide as a % of manufacturer's recommended rate, results in increased herbicide costs but decreased crop loss costs in dollars ha^{-1}. Different weed densities, low, medium and high (plants m^{-2}) result in different expected costs of crop yield loss, dollars ha^{-1}, which is represented by the 3 curves. From these curves, 3 different optimal herbicide application rates as a % of the manufacturer's recommended rate are obtained.

herbicide at rate a is given by:

$$c(a) = -c \cdot a \tag{3}$$

where the units of $c(a)$ are in dollars ha^{-1}. A typical approach for optimal application rate is to determine a value function for each decision, then choose the maximum. For a loss function, the idea is to determine the optimal application rate in presence of uncertainty for which the loss is minimized (Goovaerts 1997). This is the reason for the negative c. The incremental profit of spraying at rate a is simply the sum of $r(a)$ and $c(a)$:

$$p(a; \mathbf{u}) = r(a; \mathbf{u}) + c(a) \tag{4}$$

The optimal rate $a_{opt}(\mathbf{u})$ maximizes this incremental profit.

The optimal application rate and profit for a given location, $\mathbf{u}$, will be affected by several factors. Areas having a low weed density will have a low optimal herbicide application rate while areas having a high weed density will have high application rates; see Figure 2. Thus, fields with a patchy weed distribution will be the most amenable to locally varying herbicide application rates. Two additional comments on $p(a; \mathbf{u})$ and determination of the optimal rate $a_{opt}(\mathbf{u})$:

- The incremental revenue $r(a; \mathbf{u})$ curve flattens as a increases since the weed control, $Wc(a)$, and fractional yield loss response, $L(w)$, curves flatten off. The cost of herbicide $c(a)$, on the other hand, continues to decrease linearly since a constant per litre cost is used and as herbicide rate increases, so does its cost. Thus, the optimal application rate $a_{opt}(\mathbf{u})$ is always finite.

- The optimal rate will be zero if the herbicide is very expensive (c large), there are few weeds ($w_v(\mathbf{u})$ low) and the weeds are poor competitors, and there is moderate response to the herbicide ($Wc(a)$ rises slowly).

The function $p(a; \mathbf{u})$ may be maximized by any classical technique. The $p(a; \mathbf{u})$ function is well behaved and evaluation of $p(a; \mathbf{u})$ is extremely fast; therefore, almost any optimization technique can be considered. For example, simple Newton iterations which is a well known method to optimize a nonlinear function are suitable (Householder 1970).

There are two parameters that depend on location: the weed density $w_v(\mathbf{u})$ and the maximum attainable weed-free yield $y_0(\mathbf{u})$. Knowledge of these two parameters permits calculation of the optimal rate for each location.

An important feature of field-scale weed treatment is that the weed density is not precisely known at each location. There is uncertainty in the weed density for each SSA to be sprayed; see Figure 3. The optimal herbicide application rate must account for this uncertainty.

Accounting for Uncertainty

The consequence of uncertainty is that we have to calculate an *expected* profit instead of the actual profit. In presence of uncertainty, we must calculate the expected profit:

$$\overline{p(a; \mathbf{u})} = E\left\{\left[L\left(w_v(\mathbf{u})\right) - L\left(w_v(\mathbf{u}) \cdot (1 - Wc(a))\right)\right] \cdot y_0(\mathbf{u}) \cdot np - c \cdot a\right\} \tag{5}$$

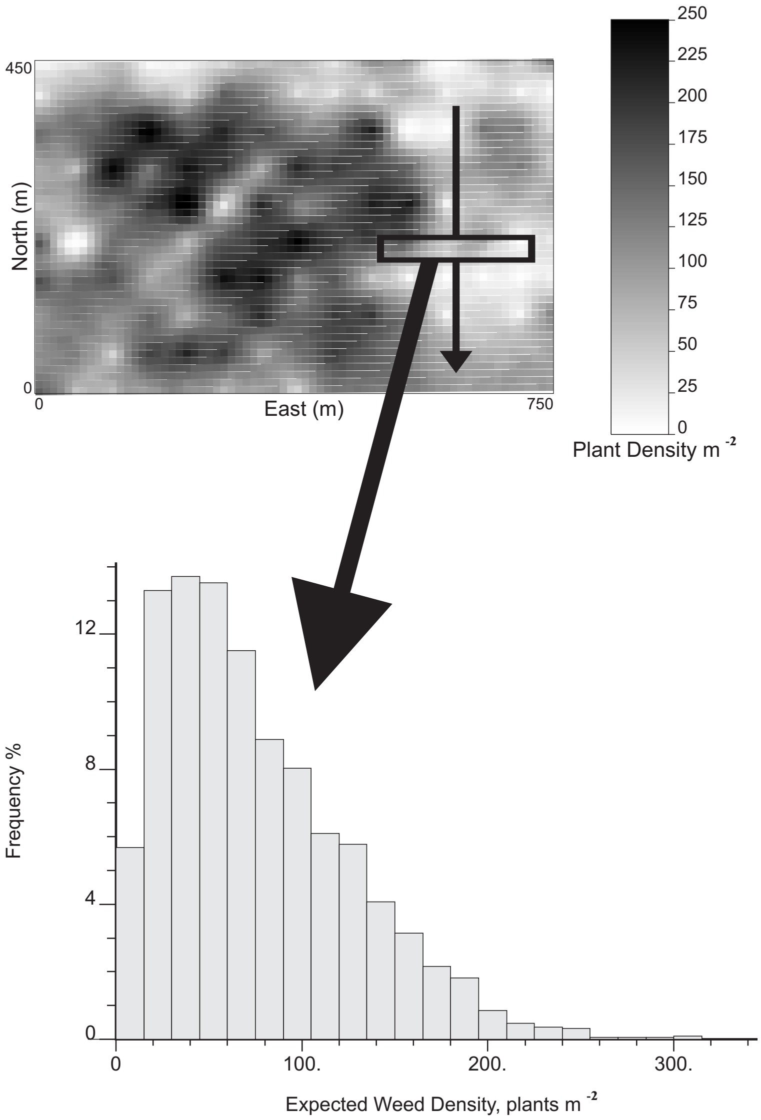


Figure 3: Weed density, sampled at plants m^{-2}, in a 35 ha field can be averaged up from a m^{-2} to a SSA represented by the box in the top figure. For each SSA, there is uncertainty which is depicted in the bottom histogram of expected weed density.

The optimal rate, $a_{opt}(\mathbf{u})$, maximizes the expected incremental profit at location $\mathbf{u}$, that is, $max\{\overline{p(a;\mathbf{u})}\}$. The expected value operator is a probability weighted average $\overline{p} = \int_{-\infty}^{\infty} pf(p)dp$. In practice, this continuous integral is solved by creating a large number, N, of equal probability values. In the context of expected profit, there are N pairs for weed density and maximum weed-free yield, $\{w_v^{(i)}(\mathbf{u}), y_0(\mathbf{u})^{(i)}, i = 1,\ldots,N\}$ and $\{L(w_v^{(i)}(\mathbf{u}))\}$, respectively. The expected value is then approximated as:

$$\overline{p(a;\mathbf{u})} \approx \frac{1}{N}\sum_{i=1}^{N}\{\left[L\left(w_v(\mathbf{u})^{(i)}\right) - L\left(w_v(\mathbf{u})^{(i)}\cdot(1-Wc(a))\right)\right]\cdot y_0(\mathbf{u})^{(i)}\cdot np - c\cdot a\} \qquad (6)$$

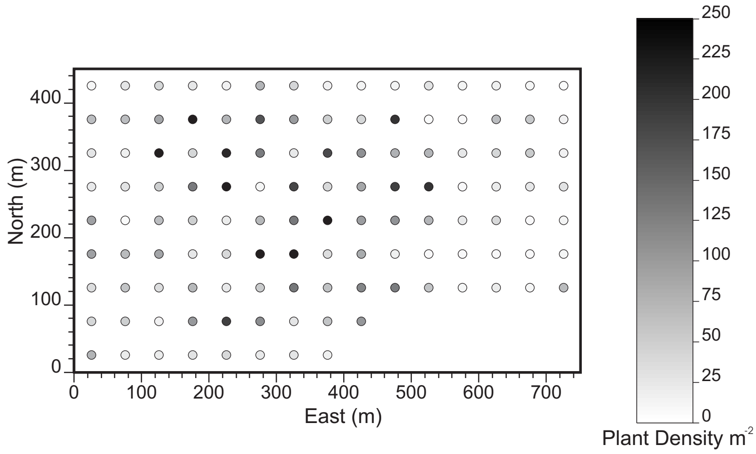
The amount of computer work for this added calculation is reasonable. The result is the same: a map of optimal locally varying herbicide application rate for use in computer integrated, GPS-guided, herbicide application equipment.

Model Validation

Weed density data used in this research are taken from a 34 ha field near Saskatoon, Saskatchewan, Canada which was seeded to spring wheat field in 1995 and canola in 1996. All weed species were identified and counted at the 3-4 leaf stage in both years with a 50 m by 50 m grid. In 1996, two 100 point sampling grids with a 10 m by 10 m spacing were established in areas of high weed density. Weeds were counted by species in four (1995) and nine (1996) 50 by 50 cm quadrats at each sampling point in the fields prior to post emergent herbicide application. The various weed species were categorized as either broad-leaved or grass weeds. Fourteen broadleaf species were recorded in 1995 while 15 were identified in 1996. The frequency of occurrence which represents the percentage of total sampling points for which a species appeared for the three most abundant weeds was 51% to 99% for *P. convolvulus*; 54% to 91% for *A. fatua*; and 99% to 100% for *T. arvense*. Other weed species identified at this site with a frequency of occurrence less than 25% included *C. arvense* and *Taraxacum officinale* Weber in Wiggers (dandelion). The results analyzed here are for broad-leaved weeds only which were present at 100% and 93% of the sampling sites in 1995 and 1996, respectively. A histogram for the 137 sample locations in 1995 indicated a broadleaf weed density of 1 to 408 broad-leaved weeds m^{-2} with a mean of 70.8 m^{-2} in Figure 4.

A map showing the sampling points for the weed data in 1995 and 1996 is displayed in Figure 4. Increasing greyscale indicates increasing weed density so white is no weeds present while black represents over 250 broadleaf weeds m^{-2}. Note the significant variability in weed density throughout the field.

The directional, spherical variograms shown in Figure 5 quantify semi-variance versus distance for the 1995 broad-leaved data in Figure 4. The dashed lines show the experimental variogram calculated from normal score transformed data while the solid lines are the spherical model fit to the experimental variogram. The total variability explained by these spherical models was calculated with GSLIB software (Deutsch & Journel 1998) using the nugget and three nested structures for 1995 and 1996 data. The top experimental variogram and model are for the north-south direction ($N0^oE$) while the bottom variogram and model are for the east-west direction ($N90^oE$). Due to limited short scale data from 1995, short



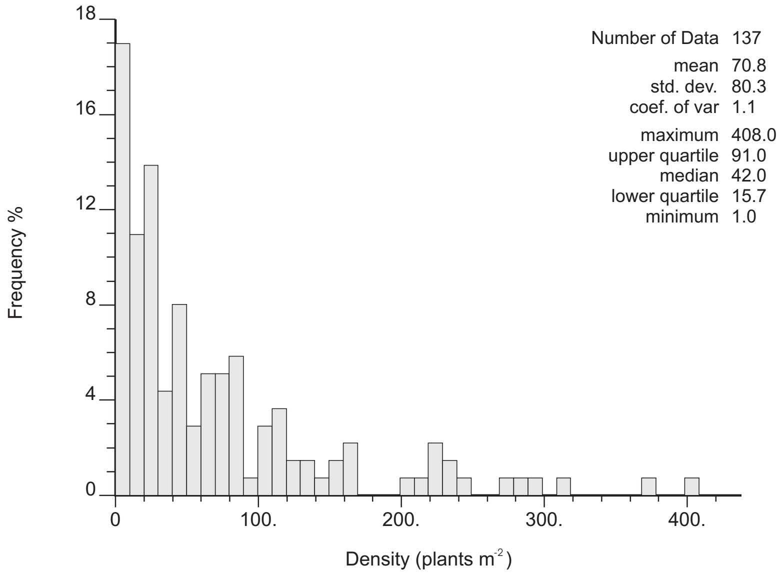


Figure 4: A location map of the 34 ha experimental field for the 137 sampling points of broadleaf weeds in 1995. Increasing greyscale represents increasing plants m^{-2}. Below is a histogram of broadleaf weed density from the same field in 1995.

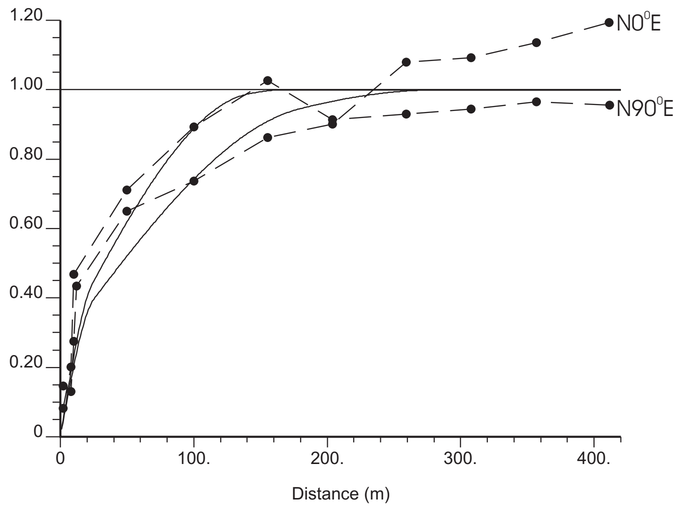


Figure 5: A semivariogram using the 1995 broad-leaved weed density data from the 34 ha experimental field. The dashed lines represent the experimental variograms while the solid lines are the modeled variograms. The top experimental and model variogram are for the $N0^oE$ direction while the bottom variogram and model are for the $N90^oE$ direction.

scale data from 1996 were used to infer the nugget effect. This assumes weed density does not change over time at the short scale. A stationarity decision such as this involves consideration for what data can be averaged over time and space for the experimental field. The model variogram has a moderate nugget effect of 0.05 and a range of 275 m in the direction of maximal continuity ($N90^oE$) and 160 m in the direction of minimal continuity ($N0^oE$). A waterway crosses the $S45^oE$ corner of the field (see bottom right corner of the sampling point map in Figure 4) and it may have influenced the anisotropy of the broadleaf weed distribution.

The variogram shown in Figure 5 was used for kriging a 1 by 1 m^2 grid. Kriging is a classical geostatistical technique for estimation at unsampled locations (Journel & Huijbregts 1979; Isaaks & Srivastava 1989; Goovaerts 1997; Deutsch & Journel 1998). A known limitation of kriging is "smoothing"; low values are typically overestimated, and high values are typically underestimated. A kriged map of weed density data is shown in Figure 6.

Conditional simulation was initially developed to correct the smoothing effect of kriging by creating maps that reproduce the histogram and variogram. It involves creating multiple, equally probable realizations that are conditional if the realization represents the data at their location. Each realization should reproduce the local data at correct scale, the global histogram and variogram. Many techniques can be used to draw these realizations however more recently sequential Gaussian simulation has gained widespread popularity due to its simplicity and flexibility (Deutsch & Journel 1998; Journel & Huijbregts 1979).

Multiple simulated realizations are used to quantify uncertainty in this data using GSLIB

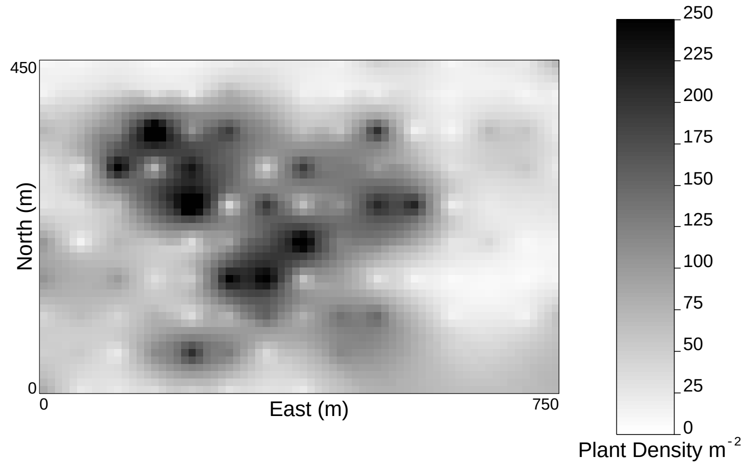


Figure 6: A kriged map of weed density, plants m^{-2}, from the 34 ha experimental field.

software (Deutsch & Journel 1998). One hundred and one realizations were created using sequential Gaussian simulation. Three realizations, and the average map of all 101 realizations are shown in Figure 7. Note that the three realizations are "noisier" than the kriged map. This is a reflection of the true variability in the weed distribution at small scale using the 1996 small grid data. Despite the variation, the simulated maps reflect the histogram and variogram. Also note that the average map of all 101 realizations is nearly identical to the kriged map in Figure 6.

Fractional yield loss as a function of weed density was derived from fitted curves of a hyperbolic or exponential-type for $L(w)$ such as:

$$Y^{(l)}(\mathbf{u}) = \frac{L \cdot w(\mathbf{u}) \cdot W^{(l)}(\mathbf{u})}{1 + \frac{L \cdot w(\mathbf{u}) \cdot W^{(l)}(\mathbf{u})}{A}} \tag{7}$$

where $Y^{(l)}$ is the yield loss in percent for realization l at location $\mathbf{u}$, L is the percent yield loss per unit weed density as density approaches zero, w is the weed density at location $\mathbf{u}$, $W^{(l)}$ is the fraction of weeds controlled by herbicide at location $\mathbf{u}$, and A is the maximum crop loss due to weed competition as weed density approaches infinity. The idea for crop yield loss in a mixed stand of weeds is representing the loss with a family of curves. The experimental field had broadleaf weeds in which 2 species were dominant in each year. Fractional yield loss was determined for each realization and averaged to the SSA over the 101 realizations at a location while the kriged fractional yield loss map was averaged to the SSA for one map. This relationship is an example from the literature (Cousens 1985).

Other assumptions were made when preparing a map of optimal locally varying herbicide application rates:

- $y_0(\mathbf{u})$ grain yield = 3.0 t/ha,

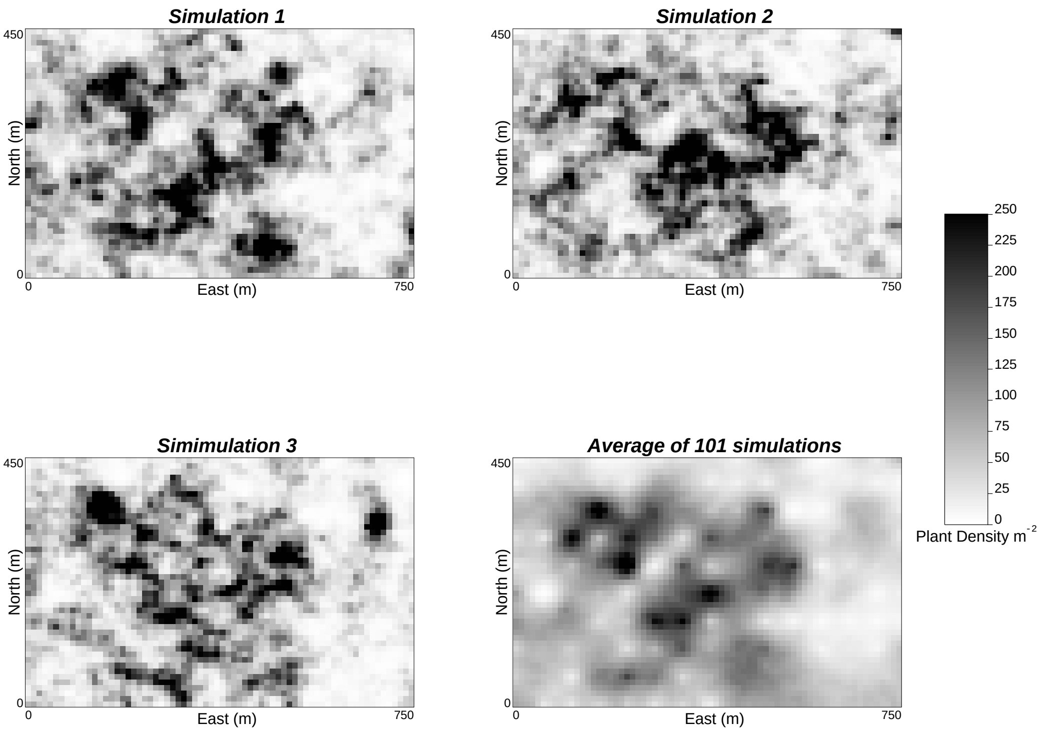


Figure 7: Three of the 101 simulated maps and a map of the average of the 101 maps for weed density, plants m^{-2}, for the 34 ha experimental field. The average map is nearly the same as the kriged map.

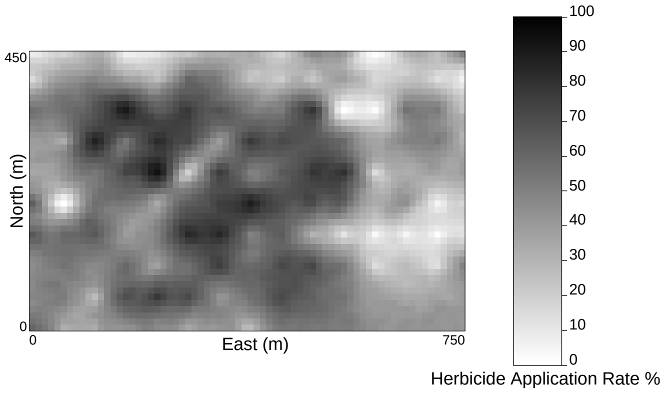
- (np) net selling price of grain = \$100/t,
- $Wc(a)$ herbicide costs = \$50/ha which includes herbicide product at \$40/ha and \$10/ha for application cost at the recommended application rate,
- A maximum crop yield loss = 40%, and
- maximum permissible application rate is 200% of manufacturers' recommended herbicide application rate. The maximum permissible application rate exceeds the manufacturer's recommended rate which is illegal. However, it was allowed in the model to determine if there are areas in the field where the weed density warrants additional control measures at a future time.

Environmental and soil variability influence weeds and this variability is characterized by the uncertainty of weed density at each location. A map of optimal locally varying herbicide application rates is shown in Figure 8. Increasing greyscale indicates increasing herbicide application rate. The distribution of optimal application rates is shown in the bottom figure of Figure 8. We predict greater than 100% of the manufacturers' recommended application rate at some locations, while at other locations the optimal application rate is predicted to be zero. The average optimal rate is 50.4 % per SSA of the recommended rate, with a minimum and maximum rate of 0% and 116%, respectively. In this case, the overall optimal rate is not the manufacturers' recommended application rate and some areas ($< 1\%$) will need extra control treatments. The herbicide cost for this optimal application rate is \$793 for the whole field. A uniform application at the manufacturer's recommended rate would cost \$1575.

A map of the expected cost of crop yield loss using our calculated optimal locally varying herbicide application rate is compared to the expected cost when a uniform herbicide application rate of 50% is applied to the experimental field in Figure 9. Since the average optimal herbicide rate was 50.4%, a uniform rate of 50% was chosen as a comparison to it. The cost histogram for the 50% application rate illustrates a wider, flatter distribution of costs compared to the optimal cost histogram. Cost of herbicide consumption is the same for either application rate; however, some areas receive too much and others too little with the 50% rate. Expected cost of the yield loss is more than 4% greater with a uniform application rate compared to the optimal herbicide application rate.

Discussion

Spatial distribution of weeds can be characterized using weed density data collected at locations in a field. Different weed-mapping techniques may be used to build different maps of weed density; however what technique should be applied under what conditions? The chosen technique will depend on the goals of the weed study. Here we have utilized discrete sampling with quadrats. Our goal was to assess and provide a local measure of uncertainty conditioned to neighboring data values for decision making. Other goals may include quantifying the weed density over an entire field for a global average to direct sampling efforts or visualize the variability. Alternatively, the sampling strategy may be utilized to show large-scale trends and provide a visual overview of a weed distribution on a



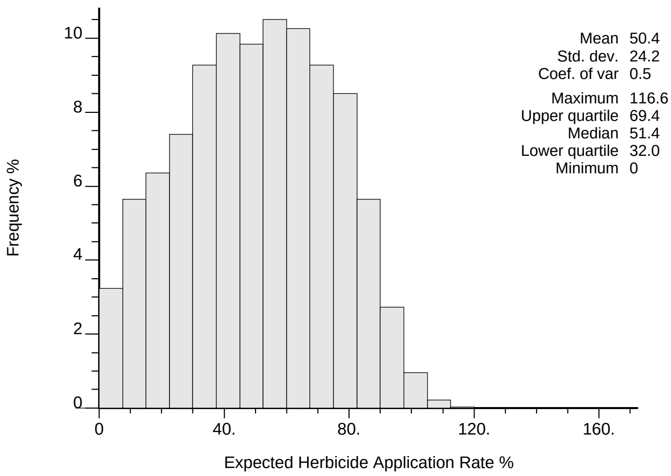


Figure 8: Locally varying optimal herbicide application rate map compared to manufacturers' recommended application rate at 100% with a histogram of the locally varying optimal herbicide application rates. The locally optimal herbicide rate mean is 50.4%.

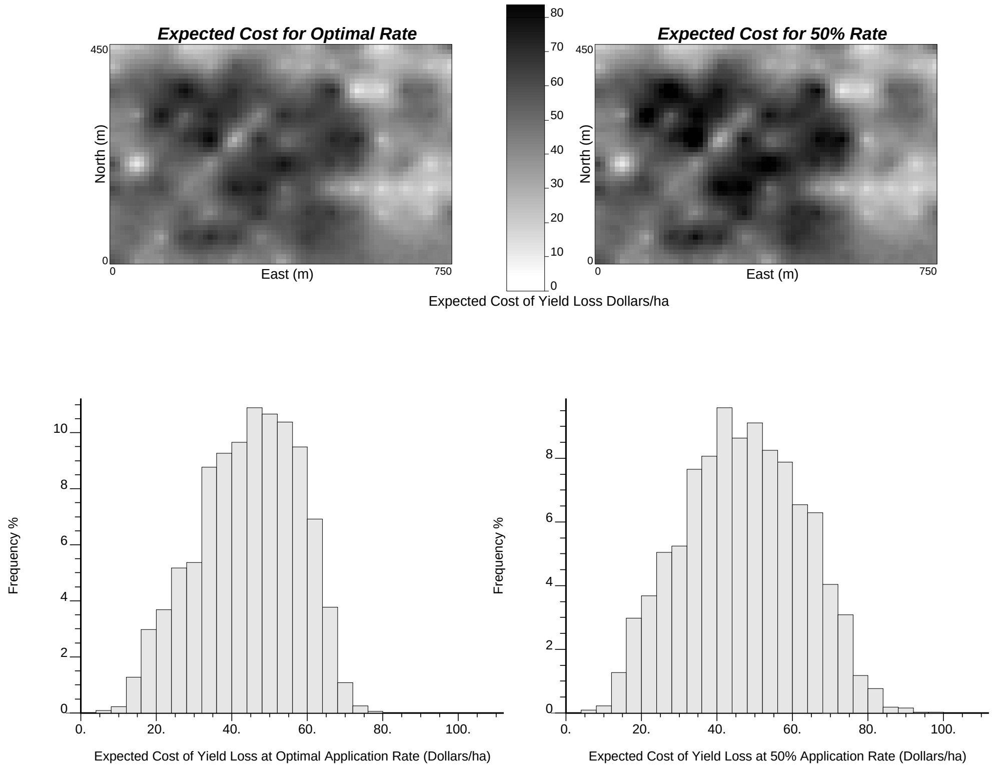


Figure 9: Map and histogram (left) of the expected cost of crop yield loss in dollars ha^{-1} for the optimal herbicide rate applied on the experimental field. Map and histogram (right) of the expected cost of crop yield loss in dollars ha^{-1} when 50% of the manufacturers' recommended herbicide rate is applied to the experimental field.

field basis for exploratory purposes, or to integrate multiple sources of data. For each goal to be implemented, the question of appropriate scale for cost effectiveness and usefulness of information is a significant consideration (Rew & Cousens 2001).

An outstanding question from this study is the appropriate sample spacing that balances time and labour costs with required local precision in the description of the spatial distribution of weeds. Discrete data are generated from quadrats and estimates are interpolated between sampling points with spatial statistics. The kriged map in this study illustrated its limitation where low weed numbers are typically overestimated, and high weed numbers are typically underestimated. This can be a problem in optimizing herbicide application rates.

Other techniques can be used to collect weed data on a continuous basis such as weed surveying using an all terrain vehicle and digital elevation maps. A visual rating of weed numbers using an all terrain vehicle provides a qualitative description for analysis (Hall & Faechner 1999). The time required to collect data and its accuracy are challenges facing it's application. Similarly, digital elevation maps are extremely useful if they have a high conditional expectation with weed density (Faechner *et al.* 2000). A review of sampling strategies for arable crops highlighted some of the challenges facing weed scientists in site specific weed management (Rew & Cousens 2001).

The effects of a mixed weed species infestation on crop yield poses a challenge for being incorporated into crop yield loss equations. Two species models of crop and weed where the yield and density per plant are a function of both species have been developed. They have not been extended to mixtures of more than two species (Doyle 1991). However, a competitive index has been established for weeds in soybean based on a ranking according to their degree of competitiveness in relation to the crop (Wilkerson *et al.* 1991). This competitiveness index could be expanded to include some of the major weeds which reduce canola and spring wheat yields.

Dose response curves that quantify a herbicide's effect on weeds and crop have been described by Streibig 1988. Optimizing herbicide doses depends on knowledge of those response curves. There are studies from the literature that provide data for some grass and broadleaf herbicides however it is limited. More research effort needs to be devoted to this area to increase our understanding of how weeds react to herbicide rates.

A quadrat used for counting weeds is scaled up to a SSA. The SSA represents the volume over which herbicide equipment can apply a spray. As electronics advance for spraying equipment, the SSA can decrease in volume. Consequently, the change in spatial resolution with decreasing SSA will result in further optimization of herbicide use. Wallinga *et al.* 1998 found that herbicide use could be reduced by 26% when changing spatial resolution from 4 to 2 m. Alternatively, since we are changing spatial resolution from 1 m^{-2} to 100 m^{-2}, there is a possibility that savings in herbicide use may be minimal. This is due to a reduction in variance as spatial resolution of weed density data are increased.

We describe a method for establishing optimal locally varying herbicide application rates. The method requires geostatistical models of uncertainty in the weed density and a model of the weed response for different application rates. This method has the potential to reduce weed control costs. Practical application requires calibration to a particular crop, weed, and herbicide.

Our example considered studies published in the literature for the required weed response to herbicide rate. This must be verified under specific environmental and cropping conditions. Such variable rate information is limited.

Spatial statistics are useful to characterize the heterogeneity of weed distributions as well as quantify the uncertainty due to incomplete data. The proposed methodology accounts for risk along with uncertainty in the spatial distribution of weeds. Such local precision and optimality is a worthy goal in light of economic and environmental concerns related to herbicide application.

References

ANONYMOUS (1998) *Registration handbook* Pest Management Regulatory Agency, Government of Canada, Ottawa.

AULD BA, MENZ KM & TISDELL CA (1987) *Weed Control Economics.* Academic Press Inc, London.

CARLSON E, HILL J & BAGHOTT K (1981) Wild oat competition in spring wheat. In: *Proceedings Annual California Weed Conference* **33**, 13–24.

CLAY SA, LEMS GJ, CLAY DE, FORCELLA F, ELLSBURY MM & CARLSON CG (1999) Sampling weed spatial variability on a fieldwide scale. *Weed Science* **39**, 674–681.

COUSENS R (1985) A simple model relating yield loss to weed density. *Annals Applied Biology* **107**, 239–252.

COUSENS R, BRAIN P, O'DONOVAN JT & O'SULLIVAN PA (1987) The use of biologically realistic equations to describe the effects of weed density and relative time of emergence on crop yield. *Weed Science* **35**, 720–725.

COUSENS R & MORTIMER M (1995) *Dynamics of Weed Populations.* Cambridge University Press, Cambridge, U.K.

CUSSANS GW & TAYLOR WA (1976) A review of research on controlled drop application at the ARC weed research organization. In: *Proceedings 1976 British Crop Protection Conference - Weeds,* Brighton, UK, 885–894.

DEUTSCH CV & JOURNEL AG (1998) *GSLIB: Geostatistical Software Library and User's Guide.* 2nd ed, Oxford University Press, New York.

DIELEMAN JA & MORTENSEN DA (1998) Influence of weed biology and ecology on development of reduced dose strategies for integrated weed management systems. In: *Integrated Weed and Soil Management.* (eds JL Hatfield, DD Buhler & BA Stewart), Sleeping Bear Press, Michigan, pp. 333–362.

FAECHNER T, HALL LM & MacMillan R (2000) Landscape influence on wild oat (*Avena fatua*) distribution. *Proceedings of Weed Science Society of America* **40**, 101–102.

FRIESEN G & SHEBESKI LH (1960) Economic losses caused by weed competition in

Manitoba grain fields. I. Weed species, their relative abundance and their effect on crop yields. *Canadian Journal of Plant Science* **40**, 457–467.

GOOVAERTS P (1997) *Geostatistics for Natural Resources Evaluation.* Oxford University Press, New York.

HALL LM & FAECHNER T (1999) Operational Site Specific Spraying. *Proceedings of Weed Science Society of America* **39**, 34–35.

HOLM FA, KIRKLAND KJ & STEVENSON FC (2000) Defining optimum herbicide rates and timing for wild oat (*Avena fatua*) control in spring wheat (*Triticum aestivum*). *Weed Technology* **14**, 167–175.

HOUSEHOLDER AS (1970) *The numerical treatment of a single nonlinear equation.* McGraw-Hill, New York.

HUME L (1989) Yield losses in wheat due to weed communities dominated by green foxtail (*Setaria viridis*) L. Beauv.:a multispecies approach. *Canadian Journal of Plant Science* **69**, 521–529.

ISAAKS EH & SRIVASTAVA RM (1989) *An Introduction to Applied Geostatistics.* Oxford University Press, New York.

JOURNEL AG & HUIJBREGTS CJ (1978) *Mining Geostatistics.* Academic Press, New York.

MORTENSEN DA, DIELEMAN JA & JOHNSON GA (1998) Weed spatial variation and weed management. In: *Integrated Weed and Soil Management* (eds JL Hatfield, DD Buhler & BA Stewart), Sleeping Bear Press, Michigan, pp. 293–309.

MORTENSEN DA, JOHNSON GA & YOUNG LJ (1993) Weed distribution in agricultural fields. In: *Soil Specific Crop Management* (eds PC Robert, RH Rust & WE Larson), American Society of Agronomy, Madison, WI, pp. 113–124.

O'DONOVAN JT (1994) Canola (*Brassica rapa*) plant density influences tartary buckwheat (*Fagopyrum tataricum*) interference, biomass, and seed yield. *Weed Science* **42**, 385–389.

O'DONOVAN JT, DE ST. REMY EA, O'SULLIVAN PA, DEW DA & SHARMA AK (1985) Influence of the relative time of emergence of wild oat (*Avena fatua*) on yield loss of barley (*Hordeum vulgare*) and wheat (*Triticum aestivum*). *Weed Science* **33**, 498–503.

O'SULLIVAN PA, WEISS GM & KOSSATZ VC (1985) Indices of competition for esti mating rapeseed yield loss due to Canada thistle. *Canadian Journal of Plant Science* **65**, 145–149.

REW LJ & COUSENS RD (2001) Spatial distribution of weeds in arable crops: are current sampling and analytical methods appropriate? *Weed Research* **41**, 1–18.

SEEFELDT SS, GEALY DR, BREWSTER BD & FUERST EP (1994) Cross-resistance of several diclofop-resistant wild oat (*Avena fatua*) biotypes from the Willamette Valley of Oregon. *Weed Science* **42**, 430–437.

STREIBIG JC (1984) Measurement of phytotoxicity of commercial and unformulated soil-applied herbicides. *Weed Research* **24**, 327–331.

STREIBIG JC (1988) Herbicide bioassay. *Weed Research* **28**, 479–484.

SWANTON CJ, WEAVER S, COWAN P, VAN ACKER R, DEEN W & SHRESHTA A (1999) Weed thresholds: theory and applicability. In:*Expanding the Context of Weed Management.* (ed DD Buhler), Haworth Press, Inc., (Food Products Press), pp. 9–29.

THOMAS AG, FRICK BL & HALL LM (1998) *Alberta weed survey of cereal and oilseed crops in 1997*, Vol 98-2. Agriculture and Agri-Food Canada, Saskatoon, Saskatchewan.

WALLINGA J, GROENEVELD RMW & LOTZ LAP (1998) Measures that describe weed spatial patterns at different levels of resolution and their applications for patch spraying of weeds *Weed Research* **38**, 351–359.

WILKERSON GG, MODENA SA & COBLE HD (1991) HERB: Decision model for post-emergence weed control in soybean *Agronomy Journal* **83**, 413–417.

WILSON BJ (1979) The effect of controlling *Alopecurus myosuroides* Huds. and *Avena fatua* L. individually and together, in mixed infestations on the yield of wheat. *Weed Research* **19**, 193–199.

ZHANG J, WEAVER SE & HAMILL AS (2000) Risk and reliability of using herbicides at below-labeled rates. *Weed Technology* **14**, 106–115.