Semi-Automatic Dig Limit Generation

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Abstract

Dig limit selection is a difficult problem in mining. There is imprecise knowledge of the mineral grade and the dig limits must account for the mining equipment. This paper illustrates a technique for semi-automatic selection of dig limits. The technique is called semi-automatic because the user is required to supply initial dig limits. The technique consists of three steps: (1) transformation of a model of grade uncertainty to a map of expected profit, (2) selection of the initial dig limits, and (3) iterative perturbation of the initial dig limits to account for uncertainty and the limitations of mining equipment.

The optimization algorithm, called simulated annealing, is used to iteratively perturb the initial dig limits until they converge to dig limits that maximise profit and account for the mining equipment. A number of implementation details are addressed including: selecting initial dig limits, nested dig limits, and the protocol for dealing with multiple ore types.

Step-by-step examples using real mining data illustrate the technique and its features, and also a comparison between hand drawn dig limits and automatically generated dig limits is provided.

Introduction

This paper is concerned with the problem of selecting optimal dig limits. This problem is difficult due to uncertainty in the precise location of material that can be mined at a profit (ore) and also due to the limitations of the mining equipment. In some mines ore is further classified for stockpiling and blending or to be sent to different treatment facilities. Here the problem of drawing dig limits is just as difficult as with ore/waste dig limits but the aim is to optimally draw boundaries delineating ore types.

Proper dig limits are important because: (1) it is uneconomical to process waste as ore or to mine ore as waste, (2) low quality ore can dilute high quality ore and/or defeat blending practices, (3) large mining equipment cannot efficiently mine tortuous dig limits, and (4) ore is diluted or lost when the dig limits are too smooth.

Background

A mine geologist or planning engineer usually draws the dig limits. The decision is based on assayed blast hole or drill hole information. This assay information rarely provides information at a resolution useful for drawing dig limits. Sometimes the blast hole information is used to draw grade contours which are interpreted as the dig limits. Kriging can be used construct high-resolution models of grade at unsampled locations. The kriged grade model is then used to draw the dig limits.

The dig limits are then transferred to the pit by surveying and marking the interface using stakes. The shovel operator uses the location of the stakes to identify the ore/waste boundary. Whenever possible the shovel operator will use visible ore/waste contacts in addition to the pre-determined ore/waste limits. However, in some mining conditions, particularly in disseminated deposits, visible ore/waste contacts are difficult to discern.

The traditional geostatistical approach to grade mapping involves discretizing the deposit into a grid of blocks. This is the classical "sugar cube" style grade model. Dig limits are usually interpreted as the line that follows the interface between the ore and waste blocks. These types of dig limits are called "blockwise" dig limits. An example of a blockwise dig limit is shown in Figure 1. Blockwise dig limits do not account for the limitations of the mining equipment because they implicitly assume that the mining equipment can perfectly select and mine a specific block of material. This is unrealistic. The dig limits should be calibrated to the mining equipment and should involve an optimization procedure that minimizes equipment induced dilution and maximizes profit. Occasionally blockwise dig limits are smoothed to account for the mining equipment; however, the use of arbitrary



Figure 1: Hypothetical block-by-block dig limit.

smoothers, such as hand contoured maps or erosion-dilation routines, does not account for the value of the material affected by the smoothing [22].

The high-resolution grade model is then used to draw the dig limits. Contour maps and kriging derived dig limits do not account for uncertainty. Geostatistics has tools for constructing and quantifying the uncertainty in the grade between sample locations. Uncertainty is quantified in the form of multiple realizations [3, 12, 16].

Simulated Annealing (SA)

Since its development as an optimization algorithm SA has solved many optimization problems once thought intractable. The problems share three characteristic features: (1) the decision space is very large, (2) the global objective function is not well behaved, and (3) global objective function consists of multiple objectives that sometimes conflict. The classic example of the types of problem that SA has solved is that of the travelling salesman: a salesman must visit n cities only once and return to the starting city.

SA has been referred to as "The Algorithm of the Gods" by Scientific American [2]. Simulated annealing was chosen for its ability to solve a large variety of problems and the speed at which it can arrive at the optimal answer. The first application of simulated annealing to the geosciences was by Farmer [9]. Deutsch applied SA to the problem of generating models of uncertainty [4, 6].

Methodology

The proposed technique involves minimal additional effort and can yield profit improvements of 1–2%. The improvement may seem meagre, but over the life of a mine this is a substantial amount of money. The technique could be integrated into an automated grade control program guided by GPS (Global Positioning System). Haul trucks could be automatically dispatched according to the grade control program. A multiple step approach for selecting dig limits is proposed:

- 1. Construct a model of uncertainty in grade using geostatistical simulation.
- 2. Transform the models of grade to models of profit and calculate the expected profit at each location.
- 3. Use the expected profit map to draw initial dig limits and use simulated annealing to optimize them.

The overall program takes very little time to complete for large benches even on a modest PC. Note that the ore/waste classification decision is avoided until the very last step, as this is dependent of the dig limits. The final dig limits account for uncertainty, multiple ore types, and the limitations of the mining equipment.

There are numerous advantages to basing the ore/waste classification on maximum profitability rather than on an estimated grade map. Profitability assessment is improved by explicitly accounting for uncertainty in the grade estimation and the economic consequences of misclassification. Spatial/temporal variations in milling costs, mining costs, plant recoveries, and metal prices can be incorporated into a profit based grade control program.

Most deposits that are mined contain multiple minerals. A profit based grade control program easily handles this problem by transforming each mineral to profit and combining the profit. The effect of contaminants can be transformed to an impact on profit. The final product of the grade to profit transformation is a map of expected profit values for selecting dig limits. Using an expected profit map accounts for uncertainty. This simplistic approach does not account for the complex mineral interactions. The exact process of transforming the models of grade to profit is unique to each mine because each deposit has unique features, interactions between minerals, and milling processes.

The Grade to Profit Transform

Most mining operations understand the unique and complex interactions at their site that affect the quality of the final concentrate or smelter feed. The following grade-to-profit transform is very simplistic and is used for illustrative purposes only. Each mine is imposes their own criteria for a valid profit transformation. The proposed grade to profit transform has four components:

- 1. The grade information $(z^{l}(\mathbf{u}), l = 1, ..., L)$: The grades at the location being considered are represented by a model of uncertainty consisting of L realizations built using geostatistical simulation.
- 2. The cutoff grade (z_c) : The cutoff grade is often defined by management. In general, it is interpreted as the grade at which the mine can operate without loss. Market conditions and the recovery process control the cutoff grade. Other considerations include the mining cost, administration cost, and contract obligations.
- 3. The recovery curve (r(z)): The mineral recovery generally increases with increasing grade, or a constant value may be used.
- 4. The price per unit mineral (price): The selling price of the mineral.

Figure 2 shows a grade to profit transform. The transform function could be a best fit function based on observed recoveries or a continuous function provided by the metallurgical engineer. The vertical line represents the cutoff grade, z_c . The cutoff grade corresponds to the break even point. If the grade is at or below the cutoff grade, the profit will be less than 0. The profit at any location is given by:

$$p = (z \cdot r(z) - z_c \cdot r(z_c)) \cdot price$$



Figure 2: Schematic for transforming the distribution of uncertainty in grade to one in profit. The grade distribution is shown on the abscissa axis, and the profit distribution is shown on the ordinate axis. The transform function is shown as a curved line.

Note that this profit function is assumed to be independent of location.

The grade to profit transform is performed at each location \mathbf{u} in the model over the L realizations:

$$p^{l}(\mathbf{u}) = (z^{l}(\mathbf{u}) \cdot r(z^{l}(\mathbf{u})) - z_{c} \cdot r(z_{c})) \cdot price \quad l = 1, \dots, L$$
(1)

The mapped profit value at location \mathbf{u} is the expected profit:

$$P(\mathbf{u}) = E\left\{p^{l}(\mathbf{u})\right\}, \quad \forall \mathbf{u} \in \mathcal{A}, \uparrow = \infty, \dots, \mathcal{L}$$
(2)

The result of applying this transform is a map of expected profit that accounts for the uncertainty in grade. Thus, selecting dig limits on this map of expected profit accounts for the model of uncertainty.

Consider the hypothetical recovery curve shown in Figure 3. A one tonne block of material has an average grade of 0.78% copper, the cutoff grade is 0.8%. The expected grade and profit are calculated using the hypothetical grade distribution shown in Table 1. The probability of high grades, that is, 1.2 and 1.3 with correspondingly high recovery causes the expected profit to be positive and the correct classification to be ore. Uncertainty and a non-linear recovery of mineral result in misclassification when a cutoff grade is used in the above example.

Similar to using a cutoff grade for classification, using the probability of a block to be ore does not account for the non-linear grade to profit transformation. Figure 4 shows distributions of grade for two blocks of material. Both models have the same probability of exceeding the cutoff grade and would be classified as waste. However, the model on the right has a higher probability of having a very high grade, and hence a profit-based classification can change its designation from waste to ore.



Figure 3: A hypothetical recovery curve.

| Realization | Grade $(\%)$ | Recovery $(\%)$ | Profit (\$) |
|-----------------|--------------|------------------|-------------|
| 1 | 0.80 | 0.70 | 0.00 |
| 2 | 0.60 | 0.59 | -306.65 |
| 3 | 0.71 | 0.66 | -140.97 |
| 4 | 0.67 | 0.63 | -202.24 |
| 5 | 1.30 | 0.93 | 973.50 |
| 6 | 0.59 | 0.59 | -321.22 |
| 7 | 1.20 | 0.90 | 780.00 |
| 8 | 0.59 | 0.59 | -321.22 |
| 9 | 0.60 | 0.59 | -306.65 |
| 10 | 0.75 | 0.68 | -78.78 |
| Expected Grade: | 0.78 | Expected Profit: | 7.58 |

Table 1: Summary table of results for grade control experiment using a cutoff grade for classification



Figure 4: Distributions of uncertainty in grade for two blocks of material.

Expected Profit and Risk. We need to select an expected profit value from $P(\mathbf{u})$ for mapping and selecting a dig limit. There are economic methods for incorporating risk into the selection of a value. Our proposed approach does not incorporate risk, we select a risk neutral position and select the expected profit. The ore-waste decision is repeated a large number of times over the life of the mine. Incorporating a risk position in the selection of a value would yield values that systematically under-value or over-value the material and give sub-optimal results.

Expected Profit and Dig Limits: An alternative approach to mapping expected profit for selecting dig limits is to superimpose dig limits over the model of uncertainty in profit. The optimal dig limits would be the dig limits that maximize profit over the model of uncertainty. We can think of no reason that would prevent this from working. The expected profit approach was chosen because it should be more efficient in terms of CPU time.

Accounting for Multiple Ore Types: Many mines have multiple mineralizations that require different treatment facilities, and blending strategies to cope with contaminants. The procedure for selecting dig limits in the presence of multiple ore types is analogous to the problem of selecting ore-waste dig limits; just as misclassifying ore and waste reduces profit, misclassification of ore types reduces profit. The procedure consists of a recursive application of the ore-waste selection procedure for each ore type. The procedure is described in more detail below:

- 1. Construct the N ore type expected profit maps. Do not account for misclassification: calculate the expected profit as though the block at location **u** were classified according to the classification being considered.
- 2. Rank the N ore type expected profit maps according to importance and give the most important ore type the highest rank.
- 3. Let $OTP_i(\mathbf{u}), i = 1, ..., N$ be the profit for the ore type currently being considered. Let $OTP_j(\mathbf{u}), j = 1, ..., N - 1, j \neq i$ be the ore type profit for the classification that maximizes the profit.
- 4. For each location on the i^{th} ore type calculate a new profit that accounts for loss due to misclassification using the decision rule below:

$$P(\mathbf{u}) = \begin{cases} OTP_i(\mathbf{u}), & \text{if } OTP_i(\mathbf{u}) = \max[OTP_i(\mathbf{u}), i = 1, \dots, N] \\ OTP_i(\mathbf{u}) - OTP_j(\mathbf{u}), & \text{otherwise} \end{cases} \quad \forall \mathbf{u} \in \mathcal{A} \end{cases}$$

5. Select the dig limits for the ore type under consideration. The new expected profit map will have profit regions corresponding to the best ore type classification that have a positive profit and negative profit (loss) regions that correspond to the profit lost due to misclassification. The new expected profit map is analogous to the ore-waste dig limit problem.

6. Repeat for the next ore type until all ore types are processed. If this is the N^{th} ore type, the expected profit map will be the ore-waste map.

The set of ore type limits and the ore-waste limits are the dig limits for the area of concern.

Selecting Optimal Dig Limits

Selecting optimal dig limits starts with user supplied initial dig limits that are optimized using simulated annealing (SA). To use SA, we need to formulate the dig limit problem in the form of a mathematical equation that is called a global objective function. The global objective function measures the profit associated with the material contained in the dig limit and the loss associated with the dig limit tortuosity. The idea is to measure the loss, randomly change the dig limits, measure the loss after the change, use the simulated annealing decision rule to accept the change if the loss is reduced or possibly accept the change if the loss increases, and continue perturbing the dig limits until the loss is minimized.

The global objective function is a linear combination of two component objectives: (1) the profit for the dig limits, O_{profit} , and (2) the loss due to the limitations associated to the mining equipment, O_{loss} . The global objective function is:

$$O_{global} = -1 \cdot O_{profit} + \lambda O_{loss} \tag{3}$$

The optimization algorithm used here minimizes a global objective function. Multiplying the profit component objective function by -1 satisfies the convention. In the above formulation dig limits that are too smooth have a large loss due to dilution and low profit. The optimal dig limits minimize equipment induced dilution and maximize profit which minimizes the objective function.

Profit objective function. The profit component objective function, O_{profit} , is calculated by summing the fractional profit of all the blocks that lie inside the dig limits. Figure 5 shows an example dig limit superimposed on a grid that represents a map of expected profit. The shaded area represents the fractional sum of the blocks within the dig limits. The profit for a dig limit polygon is calculated as:

$$O_{profit} = \sum_{ix=1}^{nx} \sum_{iy=1}^{ny} frac_{(ix,iy)} \cdot \bar{p}_{(ix,iy)} \tag{4}$$

where $frac_{ix,iy}$ is the fractional area of the block indexed at location (ix, iy) within the polygon and $\bar{p}_{(ix,iy)}$ is the profit for location (ix, iy). The dig limit might represent an ore polygon or a waste polygon. For either case the procedure is the same.



Figure 5: The fractional blocks of expected profit within the dig limits, shown as shaded fractional areas, are summed to provide the profit from a dig limit polygon.



Figure 6: An example equipment curve: the ordinate axis is the penalty and the abscissa is the angle of operation.

Equipment objective function. The loss incurred by the mining equipment is quantified by accounting for the tortuosity of the dig limit and penalizing the profit accordingly. Loss is calculated using a penalty function, shown in Figure 6. The abscissa axis represents the angle defined by three consecutive vertices on the dig limit polygon and the ordinate axis is the penalty corresponding to each angle. The sum of all the penalties for all angles is a quantification of how difficult a dig limit is to mine:

$$O_{loss} = \sum_{iv=1}^{nv} pen_{iv} \tag{5}$$

where pen_{iv} is the penalty at vertex *iv*. Small angles incur higher penalties than large angles and have a greater impact on profit. We use the term "digability" to summarize how easy a dig limit can be mined. Dig limits with low digability have maximum profit and minimum ore loss but are difficult to mine. High digability dig limits have less than maximum profit due to equipment induced ore loss but are easy to mine.

The maximum penalty, on the ordinate axis, is calculated as the average profit of all the blocks in the area of interest that have positive profit. The loss per vertex is calculated



Figure 7: An example equipment curve: the ordinate axis is the penalty and the abscissa is the angle of operation.

as a percentage of the maximum penalty. Using the average profit over the area of interest automatically accounts for the value of the blocks. Consider two regions that are identical in spatial configuration in all respects except that one region has high average grade and the other has low average grade. We would expect the region with high average grade to have more tortuous dig limits than the other region. The more tortuous dig limits account for the higher valued material. Using the average profit ensures that this expectation is satisfied.

The parameter λ is the digability parameter. The digability parameter is proportional to digability: when a dig limit with high digability is desired then high *lambda* values are selected for the digability parameter and vice versa. The digability parameter "translates" the penalty function along the abscissa, see Figure 7. The digability parameter calibrates O_{loss} to the mining equipment. As λ approaches 0, the emphasis is on profit and a high degree of selectivity is assumed. As λ increases the emphasis is on the mining equipment constraints. There is no theory for selecting the parameter λ . It is selected based solely on professional judgment. A suggested technique for selecting the digability parameter is to build a catalogue of dig limits. The catalogue should span the range of digability parameters that stress profit and the mining equipment constraints. The selected digability parameter strikes the best balance between profit and mining constraints based on professional judgment. Selecting low values for the digability factor yields dig limits that cost more to mine because more selective equipment is required. Large equipment accepts more dilution, but it is faster and hence cheaper to mine. The operating strategy of the mine, the skill level of the operators, and other considerations could be accounted for.

The initial dig limit is perturbed under the decision rule in simulated annealing until the dig limits conform to optimal dig limits. A perturbation is a random change in a location for a vertex in the dig limit polygon. The perturbation mechanism is important in SA optimization problems. Perturbations must not be too drastic or most perturbations will not be accepted and convergence will be slow; nor can they be too minor or many perturbations will be required to achieve convergence.

The mechanism chosen is: (1) randomly pick a vertex and (2) choose to move that vertex with uniform probability within a specified distance (up to 20% of the grid block



Figure 8: The algorithm randomly selects a vertex and moves it randomly.

dimension). Figure 8 shows the region for perturbation for one vertex of a polygon. This simple perturbation mechanism must be supplemented by a series of rules including: (1) an additional vertex is added at the midpoint between the distant vertices if the vertices get too far apart, (2) vertices are merged if they get too close, and (3) a candidate perturbation is rejected if the polygon lines cross.

If the random change gives a lower global objective function value, then it is accepted. If the random change increases the objective function then the perturbation is conditionally accepted using the simulated annealing decision rule.

Handling Multiple Polygons

The dig limit algorithm solves for a single ore/waste dig limit at a time. Most benches require multiple ore and waste dig limits. Multiple dig limits can be drawn using a decomposition approach. Figure 9 shows a map of a waste region inside of larger ore body. The first step of the decomposition approach is to select initial dig limits for the ore region as shown in Step 1 in Figure 9. The next step is to select the initial dig limits for the small waste region as shown in Step 2. The total profit of the bench is defined as the sum of profit over all np polygons:

$$O_{profitability} = \sum_{ip=1}^{np} p_{ip}$$

where p_{ip} is the profit of the ip^{th} polygon. The composite dig limits represent the dig limits for the entire bench.

Examples

Two examples are provided to illustrate the application of the dig limit algorithm. The first example is a simple disseminated copper mine, and the second example is a polymetallic zinc/copper mine. The parameters for the program, diglim, are explained in the appendix.



Figure 9: A decomposition approach is used to obtain the dig limits.

Copper Mine Example

There are 372 blasthole data with an average copper grade of 1.015% and a standard deviation of 1.19%. A kriged map of the grades, generated from the data, is shown in Figure 10 for reference only. A high resolution realization from the simulation is shown in Figure 11. Sequential Gaussian simulation was used to construct 100 realizations. A block averaged realization is shown in Figure 12 (the same shown in Figure 11). We considered a cutoff grade of 0.5% copper. The transform function used has been shown in the previous discussion. Figure 13 shows the expected profit map that was used to select dig limits.

The initial dig limits are shown in Figure 14. During the implementation of the program a portion of the line was perturbed. The dashed portion of the line was not perturbed while the solid portion of the line was perturbed. The optimized dig limit, shown in Figure 15, represent the dig limit that maximize profit and accounts for the limitations of the mining equipment for a digability of 0.5.



Figure 10: The map of kriged estimates in copper grade.



Figure 11: One realization of 100 at a fine blast hole scale.



Figure 12: A block averaged realization of grades.



Figure 13: Final expected profit map.



Figure 14: The seed dig limit. The solid portion of the dig limit will be perturbed, while the dashed portion of the line will not be.



Figure 15: The final optimal dig limits for a specified digability parameter of 0.5.



Figure 16: A digability catalogue for the copper mine example.

A digability catalogue is shown in Figure 16. The top left map has a digability factor of 0.0. The catalogue increments by 0.1 from left to right and down to the bottom of the figure. We can see that as the digability factor increases, the smoother the dig limits become. This is consistent with dig limits required for increasingly larger equipment.

Polymetallic Mine Example

In the presence of multiple mineralizations the calculation of expected profit and the optimal dig limits is almost as easy as singular mineralization. Two types of mineralization are considered in this example: the primary mineral of interest is nickel with copper taking a secondary role.

Similar to the previous example, blast hole data were sampled, simulation was performed at a fine scale resolution, and then block averaged to a larger size. Figures 17 and 18 show the nickel and copper simulations respectively. The expected profit is the sum of the profit from the nickel and copper minus the cost of mining the ore and treating it in the plant, or the cost of mining it as waste. Shown in Figure 19 is the expected profit map and in Figure 20 is the classification of the material to be mined. Material is called marginal ore if it does not make money when processed but costs less to process than to treat it as waste.



Figure 17: A single Nickel realization.



Figure 19: The expected profit that is calculated as the sum of the nickel and copper profits.



Figure 18: A single Copper simulation.



Figure 20: The blocks are classified as ore, marginal ore, and waste based on the expected profit map. The final dig limit is shown as a solid line. The dashed line constrained the dig limit to the upper portion of the expected profit map.

Figure 20 also shows the final dig limit as a solid line. All of the vertices in the dig limit were allowed to perturb and the dig limit was optimized to the ore/waste boundary. The marginal ore was mined as ore because it could not be left in place the during mining sequence.

Comparison with Hand Drawn Dig Limits

The Dig Limit Challenge was an experiment, conducted in 2001, designed to show that semi-automatic dig limits are at least equal to or better than hand drawn dig limits. Four mining professionals were given the map of profit shown in Figure 21 and asked to select the best dig limits. The hand drawn dig limits were digitized. From the digitized coordinates the minimum, maximum, and average line segments were calculated. A penalty curve was constructed from the cumulative distribution function of the angles. The penalty is calculated as $100 * (1 - q_{bin})$. The minimum and maximum line segment and the penalty function for each professional were used as parameters for the diglim program. The hand drawn dig limits were used as final dig limits in all cases. Two comparisons were made: (1) do not constrain the dig limits to the users overall penalty, and (2) constrain the dig limits to the users overall penalty, and the first because it



Figure 21: The map of profit used in the Dig Limit Challenge.

 Table 2: The numerical results for the first dig limit challenge comparison.

| | Proposed Limits | | diglim Limits | | Difference | |
|--------|-----------------|------------|---------------|------------|------------|----------|
| Expert | Profit | Penalty | Profit | Penalty | Profit | Penalty |
| 1 | $1\ 196\ 888$ | 14 529 | $1\ 210\ 401$ | $11 \ 372$ | $13\ 513$ | $3\ 157$ |
| 2 | $1\ 204\ 202$ | $16 \ 347$ | $1\ 210\ 873$ | 10 801 | $6\ 671$ | 5546 |
| 3 | $1 \ 197 \ 635$ | $14 \ 487$ | $1\ 212\ 118$ | 7 991 | $14\ 483$ | $6\ 496$ |
| 4 | $1\ 168\ 998$ | $12 \ 111$ | $1\ 209\ 424$ | $12 \ 432$ | $40 \ 426$ | 321 |

aims to reproduce the users "sense" of what the mining equipment can handle. The results for comparison 1 are in Table 2. The units are simply profit units and not really monetary values, but they are good yardsticks for comparison. Table 2 shows that semi-automatic dig limits improve profit and digability. The results for the second comparison are shown in Figure 22 and Table 3. The digability is well honoured and the semi-automatic dig limits outperform the hand drawn dig limits in producing higher profit.

Table 3: The numerical results for the second dig limit challenge comparison.

| | Proposed Limits | | diglim Limits | | Difference | |
|--------|-----------------|------------|---------------|------------|------------|---------|
| Expert | Profit | Penalty | Profit | Penalty | Profit | Penalty |
| 1 | $1\ 196\ 888$ | 14 529 | $1\ 214\ 297$ | 14 556 | $17\ 409$ | 27 |
| 2 | $1\ 204\ 202$ | 16 347 | $1\ 216\ 690$ | 16 342 | $12\ 488$ | 5 |
| 3 | $1 \ 197 \ 635$ | $14 \ 487$ | $1\ 216\ 118$ | $14 \ 494$ | $19\ 383$ | 7 |
| 4 | $1\ 168\ 998$ | $12 \ 111$ | $1\ 210\ 911$ | $12 \ 123$ | $41 \ 913$ | 13 |



Figure 22: The hand drawn dig limits are on the left and the semi-automatic dig limits are on the right.

Discussion

The following remarks discuss algorithm-based selection of initial dig limits, an alternate approach to using an expected profit map, and CPU time.

Remark 1 - Automatic Selection of Initial Dig Limits: Automatically selecting initial dig limits is not trivial. Several approaches were attempted, but none performed well enough to be incorporated into the dig limit algorithm. The initial dig limits rely on a complex interaction of issues: the direction of mining, availability of equipment, mining volume requirements, and geology. The complexity is unique to each mine and thus it is extremely difficult to devise an algorithm for selecting initial dig limits.

Remark 2 — Alternative Approach for Dig Limit Selection: An alternative to selecting dig limits on a map of expected profit is to select dig limits on the full model of profit uncertainty. The idea is presented in Figure 23 and explained below. The figure shows the same dig limits superimposed on three profit realizations. The procedure would be



Figure 23: The dig limit polygon is superimposed on the L realizations. Optimization would proceed by calculating profit over the L realizations.

similar to optimizing on an expected profit map except that the objective function would be evaluated on the L realizations instead of the expected profit map.

This approach has not been implemented yet, but the following advantages are apparent: (1) the step requirement of an expected profit map would be avoided, and (2) the effects of averaging would be avoided. The obvious disadvantage is that of increased CPU requirements since the objective function would have to be evaluated L-1 times more than using an expected profit map alone.

Remark 3 - CPU time: The processing time required by the program for a moderate sized bench of 200 x 200m is approximately 2–3 minutes. This is quite a long time especially if the user is going to run the program several times with small changes between each run. Some ideas have been proposed that should shorten the processing time substantially. These changes will be implemented and test as soon as possible.

Conclusion

The algorithm for selecting dig limits gives ore/waste limits that account for grade uncertainty and the limitations of the mining equipment. The first step is to construct a model of uncertainty in grade. This is then transformed to a model of uncertainty in profit, from which an expected profit map can be obtained. Initial dig limits are then proposed and the dig limit algorithm is used for optimization.

The final dig limit is optimal in the sense that the best balance is found between equipment induced dilution and profit for a particular digability factor. The appropriate balance is selected by the user and can be fine tuned to each individual operation. In addition, the steps taken to determine the optimal dig limits can be documented and reproduced.

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Appendix: diglim Inputs, Outputs, and Parameters

The program used to determine the optimal dig limits is diglim. The program requires a 2-D map of expected profit as input data. The outputs include:

- **Dig Limit Output:** The true x and y coordinates for each vertex in the dig limit polygon. These are in Geo-EAS format.
- **Debugging File:** This file includes the information reported to the screen while the program is running. The file is in Geo-EAS format. Plotting the objective function versus the number of perturbations is useful for evaluating convergence: there should be no change in the objective function near the end of the run. If the plot does not "flatten out," then more perturbations should be run. The temperature should be zero for a large number of perturbations.
- Gridded Output: The gridded output gives the fraction of each block included in the dig limits. Blocks that are not included in the maximum dig limit window are written as -1.0. Blocks falling within the maximum dig limit window range from 0 to 1 according to the fraction of the block within the dig limits. These fractions are appended to the input file and written to the new file as output.

Listed below and shown in Figure 24 are the parameters for the program:

- datafl: the input data file with the expected profit.
- icol: column number for the expected profit.
- **polyfl:** file with the initial polygon (the *x* coordinate must be in column one and the *y* coordinate in column two).
- **outfl:** the output file for the dig limit polygon (the x coordinate will be in column one and the y coordinate in column two).
- **dbgfl:** the debugging file. The information written to the debugging file is the same as what appears on the screen during the program execution.
- **grdfl:** the gridded output file that contains the input file and the appended block fractions.
- nx, xmn, and xsiz: definition of the grid system (x-axis).
- ny, ymn, and ysiz: definition of the grid system (y-axis).
- xmin, xmax, ymin and ymax: definition of the dig limit boundary window. Minimum and maximum x and y values.
- iorelim: indicator for the material type contained in the dig limit (0=waste, 1=ore).
- **dig:** the digability or equipment factor (0 to 1).
- **freezeflag:** indicator for freezing a portion of the dig limit during the optimization (0=no, 1=yes).
- **xs**, **ys**, **xe**, and **ye**: start and end *x* and *y* coordinates of the vertices to freeze. The two vertices closest to the points are frozen and all the vertices between are frozen in a clockwise direction.
- schedflag: indicator for an automatic or user specified simulated annealing schedule (0=automatic, 1=user specified). With an automatic schedule the remaining lines in the parameter file are ignored.
- seed: random number seed (a large odd integer).
- t_0 , redfac, ka, k, and num: the initial temperature, reduction factor, number of perturbation attempts at one temperature, target number of perturbations at one temperature, and the number of times that ka can be reached before the program exits.

Parameters for DIGLIM ****

```
START OF PARAMETERS:
eprofit.out
                              -file with the input data
2
                               - column for expected profit
initial polygon.dat
                              -file with initial polygon
diglim.out
                              -file for dig limit output
                              -file for debugging output
diglim.dbg
grid.out
                              -file for gridded output
           1.0
50
     0.5
                              -nx, xmn, xsiz
                              -ny, ymn, ysiz
-dig limit window: xmin, xmax, ymin, ymax
        5 1.0
5 35
50
     0.5
20
    50
                              -type of dig limit (1=ore, 0=waste)
1
                              -equipment factor/digability
0.3
0
                              -use fixed vertices? (1=yes, 0=no)
0.0 0.0 10.0 10.0
                              -fixed vertices xs, ys(start), xe, ye(end)
                              -automatic schedule=1, user schedule=0
1
69069
                                    random number seed
                               -
2.0 0.5 1000 500 100
                                     SA schedule: t0, redfac, ka, k, num
                               -
50000 100
                               -
                                     maximum perturbations, reporting
0.5 1.0
                               -
                                     minimum and maximum interval length
                               _
0.4
                                     maximum perturbation distance
```

Figure 24: An example diglim parameter file.

- **maxpert** and **nrepo:** total number of perturbations to be carried out and the interval for reporting.
- dismin and dismax: the minimum and maximum distance between nodes. If the distance is smaller than dismin a node is removed and if the distance is greater than dismax a node is added.
- **dmax:** the maximum distance that a node can be moved during a single perturbation. The amount a node moves is drawn randomly from 0 to the maximum.