Hierarchical Trend Models Based on Architectural Elements

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Hierarchies of architectural elements are recognized and characterized in a variety of depositional settings. Their internal and external geometries are often well documented. This information may be applied to construct improved trend models.

A methodology based on established mathematical morphology operations is presented to automatically fit hierarchical trends to indicator, object or surface-based stochastic models. Vertical, lateral and transverse trends are reproduced at each scale.

Large scale areal trends from seismic and well tests and vertical trends may be added a posteriori by a proposed smooth correction algorithm. The resulting trend models may be applied as locally variable mean models for more geologically realistic simulation.

Introduction

The concept of architectural elements has been developed in fluvial settings (Miall, 1986) and extended to deep-sea clastic settings (Ghosh and Lowe, 1996; Pickering et al., 1995) and is widely applied and accepted. The basis of architectural analysis is the identification of groups of process related lithofacies and the characterization of their external and internal geometries and interrelationships. There is a large library of quantitative information available on architectural elements in a variety of depositional settings (Ghosh and Lowe, 1996; Miall, 1996; Pickering et. al, 1995, Stow and Johansson, 1999; Stow and Mayall; 1999).

Trends are an important feature of architectural elements. These trends may vary by direction (vertical, longitudinal and transverse) relative to the axis of transport, and may vary by hierarchical order. A flexible methodology is required to automatically construct trend models based on architectural element models.

Methodology

The proposed algorithm (**1**) constructs trend models that capture features within a hierarchy of architectural elements and then (**2**) corrects this model to honor global features such as global mean, areal and vertical trends that are based on well data and large scale production and seismic information.

Construction of Trend Model

The hierarchical trend model is calculated by (**1**) identifying geo-objects at each hierarchical order, (**2**) calculating streamlines or the curvilinear principal axis, (**3**) calculating the relative location with respect to the streamline within each order of geo-object and (**4**) assigning a trend value as the global average scaled by trend functions that characterize the shape and magnitude of the trend in each direction and for each hierarchical order.

In the case of surface and object based realizations, the geo-objects are generally defined prior to trend construction. In the general case of indicator based realizations, the geo-objects are not defined or the geo-objects may be poorly defined. It is recommended that a smoothing method such as maximum a posteriori (MAP) (Deutsch, 1998) or a mathematical morphological operation such as closing be applied to reduce the noise on the boundary of the geo-objects (Stoyan, Kendall and Mecke, 1987). The geo-objects are identified by an efficient parallel method, and are then indexed (Vincent, 1993).

The flow lines are constructed with mathematical morphology operations. These methods are widely applied to image processing as in automated tool for analyzing large data sets often associated with imagery. Mathematical morphological operations include erosion, dilation, opening, closing, distance functions and skeleton transforms. These operations are performed with specified structural elements. The series of operations and specific structural elements are chosen in order to assess significant image features (Matheron, 1975; Vincent, 1993, p. 255; Stoyan, Kendall and Mecke, 1987, p. 19).

A useful mathematical morphology tool is the skeleton or medial axis transformation. This transform may be applied to automatically calculate the streamline of an architectural element, such as a lobe or channel. This transformation is homotopic; therefore, holes and connected components in the original set of retained in the skeleton (Vincent, 1993, p. 279). To improve interpretability the skeleton holes and unconnected segments are removed. This is accomplished by an opening to remove disconnected fragments (Equation 1) and then closing to remove holes (Equation 2).

$$A \to A_B = (A \ominus \check{B}) \oplus B \qquad \textbf{(1)}$$

$$A_B \to A^C = (A \oplus \check{C}) \ominus C \qquad ; \qquad \textbf{(2)}$$

where $\oplus$ is Minkowski-addition, $\ominus$ is Minkowski-subtraction, B and C are the chosen structural elements for each operation, and A is the original object (Stoyan, Kendall and Mecke, 1987).

A straightforward calibration is required to select the appropriate structural elements, so that the resulting architectural elements do not have holes or disconnected fragments. During the opening operation too large of an element will eliminate geo-objects and remove detail while too small of an element will leave isolated fragments. In the closing operation too large of an element will remove geo-object detail while too small of an element will leave holes in the geo-objects.

The distance function (the city block distance from the edge of the geo-object) is calculated for each of the resulting geo-objects. From this distance function crest points are identified as and selected as anchor points. The skeleton transformation is then conducted as outlined by Vincent (1991). For computational efficiency a look up table indexing of all possible neighbourhood configurations is calculated a priori and erosion configurations that result in homotopic modification are flagged. The skeleton is constructed by eroding non-anchor point pixels that do not modify the homotopy. To avoid discretization artifacts, a spline is then fit to the skeleton and is extrapolated to the object edges. These steps are illustrated in Figure 1.

The spline flow lines may be applied to determine the relative lateral and transverse location [0,1] for all locations and for all architectural elements within these locations exist. Where 0 is assigned to locations on the flow line and 1 is assigned to locations on the edge of the architectural element. An example of this calculation for a turbidite 2nd and 3rd order nested lobes (deepwater hierarchy as defined by Ghosh and Lowe, 1992) with the associated flow lines and relative location vectors is shown in Figure 2.

Trend functions are characterized with respect to location defined by relative vertical, longitudinal and transverse location within each scale of architectural element. The closer to 1.0 the less impact the specific trend function will have on the final composite trend value. Calibration and expert judgment is required to set the relative importance of the directional and order trends.

Example trend functions for the previous 2nd and 3rd order lobes are shown in Figure 3 for porosity. Given the relative coordinates, the appropriate trend multiples are retrieved from the trend functions and applied as a multiplier to the global average (see Equation 3).

$$trend(\mathbf{u}) = \bar{\phi} \cdot V2(\mathbf{u}) \cdot V3(\mathbf{u}) \cdot L2(\mathbf{u}) \cdot L3(\mathbf{u}) \cdot T2(\mathbf{u}) \cdot T3(\mathbf{u}) \qquad (3)$$

where u is a location vector, $\bar{\phi}$ is the global average property and $V2(\mathbf{u})$, $V3(\mathbf{u})$, $L2(\mathbf{u})$, $L3(\mathbf{u})$, $T2(\mathbf{u})$ and $T3(\mathbf{u})$ are the local trend multipliers.

There is no unique procedure for combining trend functions. Deutsch (2002) discusses the issues related to merging areal and vertical trends into a full three-dimensional trend model. The method shown in Equation 3 is convenient, but it is based on an assumption of conditional independence. This may not be an acceptable assumption in all settings.

Correction for Global Area and Vertical Trends

The hierarchical trend model may be corrected to honor the global average, areal and vertical trends based on analogue, seismic, well test data, and core and log data.

The proposed method is to randomly visit locations within the trend model, and check the actual and target, vertical and areal trend. If both are consistently low or high then a random smooth multiplier is applied over a specified window centered on the location. The multipliers increase or decrease the local trend values as required. By visiting many locations the trend model is gradually modified to honor the target vertical and areal trend while retaining the small scale features. Potential bias and the level of variability represented by the trend model are fixed by an affine transform of the resulting trend model.

Application

A realization of stochastic surfaces was applied to demonstrate this procedure. The surfaces represent 2^{nd} order lobes within a 3^{rd} order architectural element (see Figure 4). The procedure for calculating a flow line for an example 2^{nd} order lobe is demonstrated in Figure 5.

The trend functions similar to those shown in Figure 3 were applied to calculate a porosity trend model. A long section and cross section of two surface-based simulation realizations and the hierarchical porosity trend models are shown in Figure 6.

The hierarchical trend models reproduce the trends within the architectural elements. For example, the individual flow events, 2^{nd} order lobes show a fining upwards trend, while the 3^{rd} lobe shows a coarsening upward trend. This may be expected in a prograding system of individual Bouma sequence turbidite lobes. These models may be corrected to honor large scale vertical and areal trends.

A target areal and vertical trend was constructed. For the purpose of demonstration these trends were assumed. The initial areal and vertical trends from the hierarchical trend model and the target areal and vertical trends are shown in Figure 7 and Figure 8 respectively.

The iterative smooth correction method was applied. A smoothing volume of $^1/_5$ the model volume and 5,000 iterations were applied. The algorithm required a couple minutes on a PIV 2.5 GHz PC. The areal and vertical trends of the corrected trend model are shown in Figure 9.

Comparison of corrected trend model and the target areal and vertical trends reveals that the large scale features are imported into the hierarchical trend model and that the global mean is reproduced. The mismatch (corrected-target) of the areal and vertical trends was calculated (see Figure 10).

A series of long sections (see Figure 11) and fence diagrams of the surface-based model, the initial trend model and the correction trend model (see Figure 12) of the corrected trend model are included.

Conclusions

- Hierarchical trend models reproduce internal trends within architectural elements over multiple orders of scale.

- These trend models may be automatically constructed from indicator, surface or object based stochastic models and applied as locally variable mean models for more realistic stochastic property models.

- These models may be corrected to honor areal and vertical trends based on analogue, seismic, well test data, and core and log data.

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Proletariat

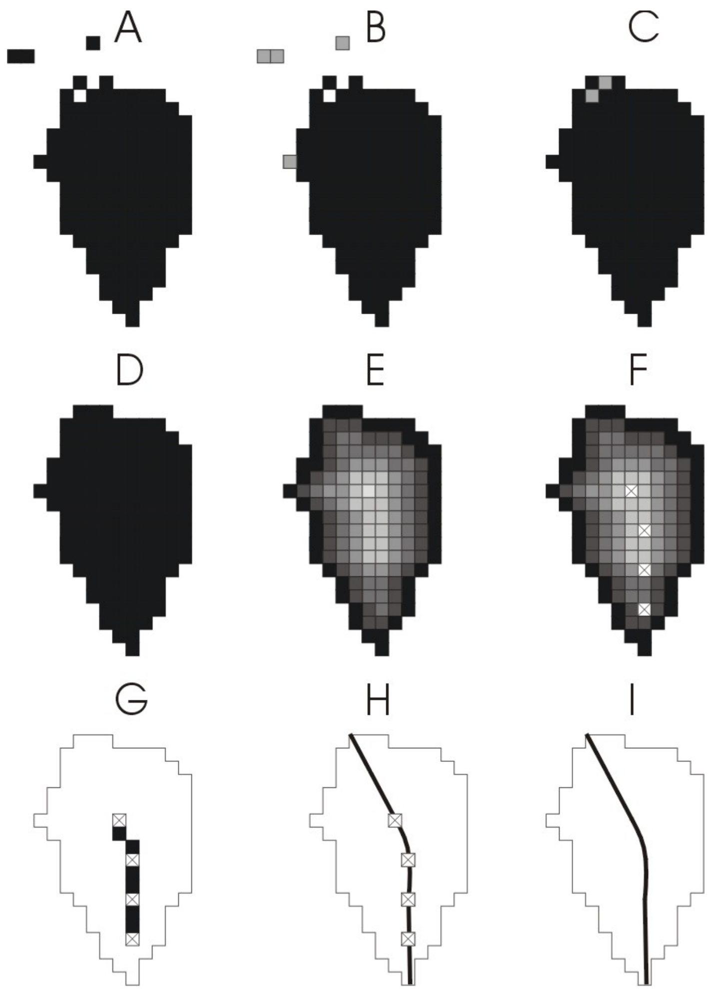

Figure 1 – schematic illustration of operations to construct a flow line: A – original object, B – opened to remove isolated blocks, C – closed to remove voids within the object, D – corrected object, E – distance function, F – anchor points selected, G – homotopic erosion to calculated skeleton, H – spline fit to skeleton and extrapolated to the object boundaries, and I – object and spline skeleton. Figure is an approximate representation.

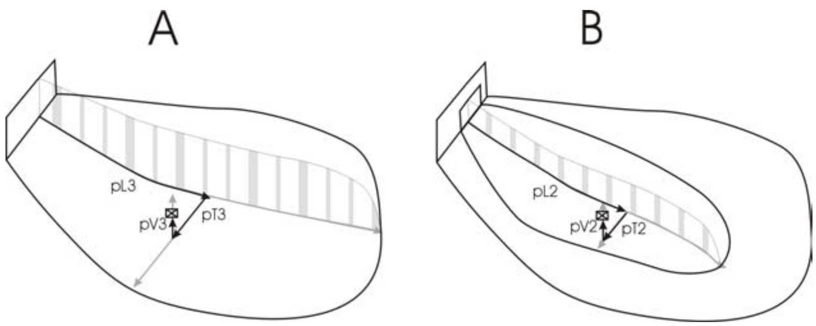

Figure 2 – The calculation of proportional vertical, lateral and transverse locations within hierarchical architectural elements. A – the 3rd order lobe, with label pL3, pV3 and pT3 indicating the relative locations in the longitudinal, transverse and vertical directions. B – the 2nd order lobe within the 3rd order lobe, with label pL2, pV2 and pT2 indicating the relative locations in the longitudinal, transverse and vertical directions

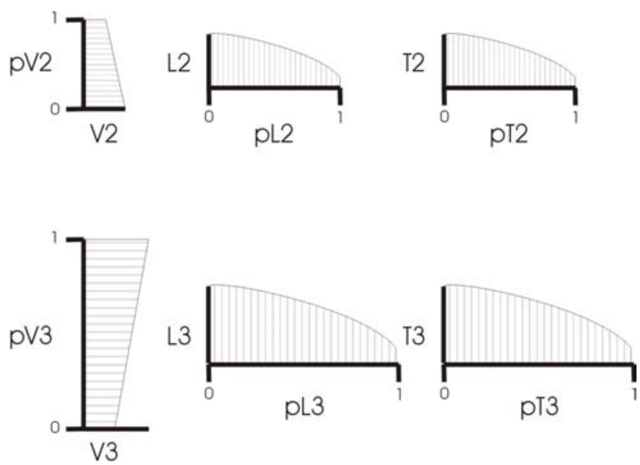

Figure 3 – a potential suite of porosity trends that may be applied in the vertical, longitudinal and transverse directions for 2nd and 3rd order architectural elements. Individual flow events (2nd order) may be characterized by fining upward sequences while 3rd groups may coarsen upwards due to progradation. Fining may occur in both basin ward (longitudinal) and away from the flow line (transverse). The pV2, pV3 etc. axes are the relative locations, and the V2, V3 etc. axes are the trend functions centered on 1.0.

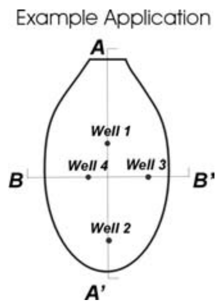

Figure 4 – plan view of the 3rd order lobe and the 4 vertical wells.

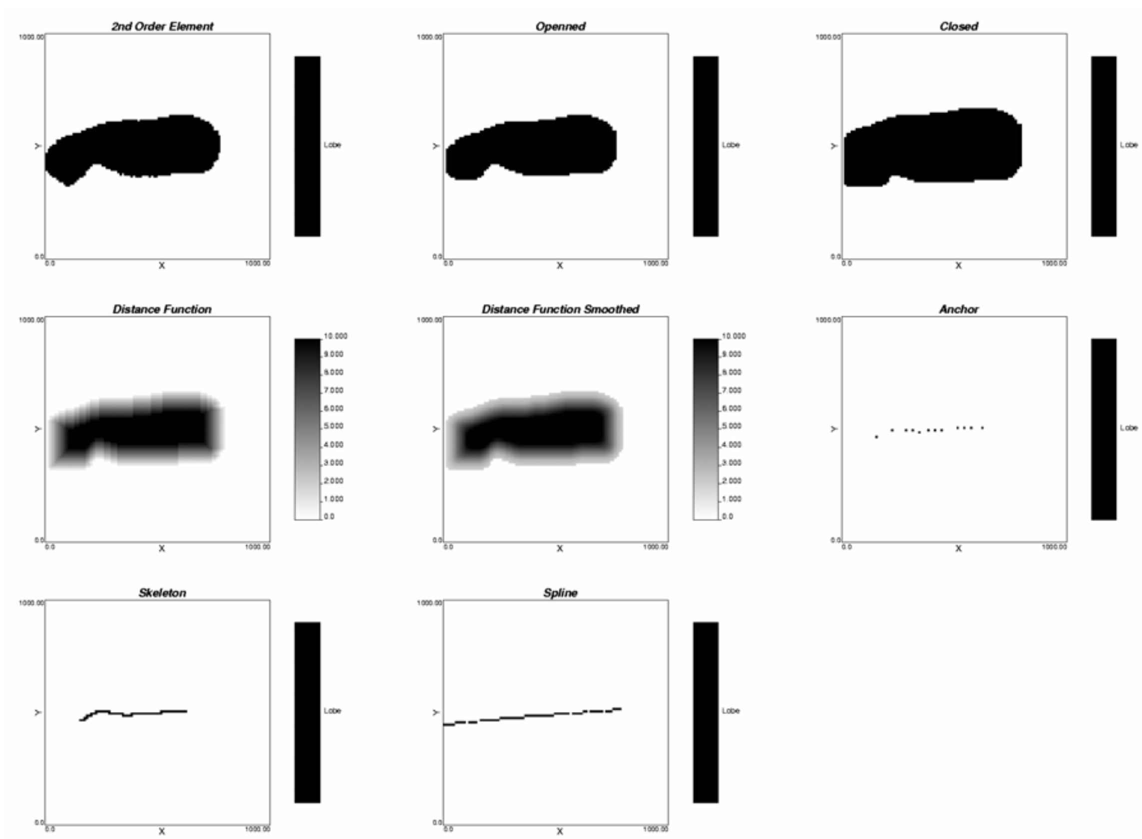

Figure 5 – the construction of a flow line for an example lobe from a surface based stochastic realization. These steps are explained in the methodology section.

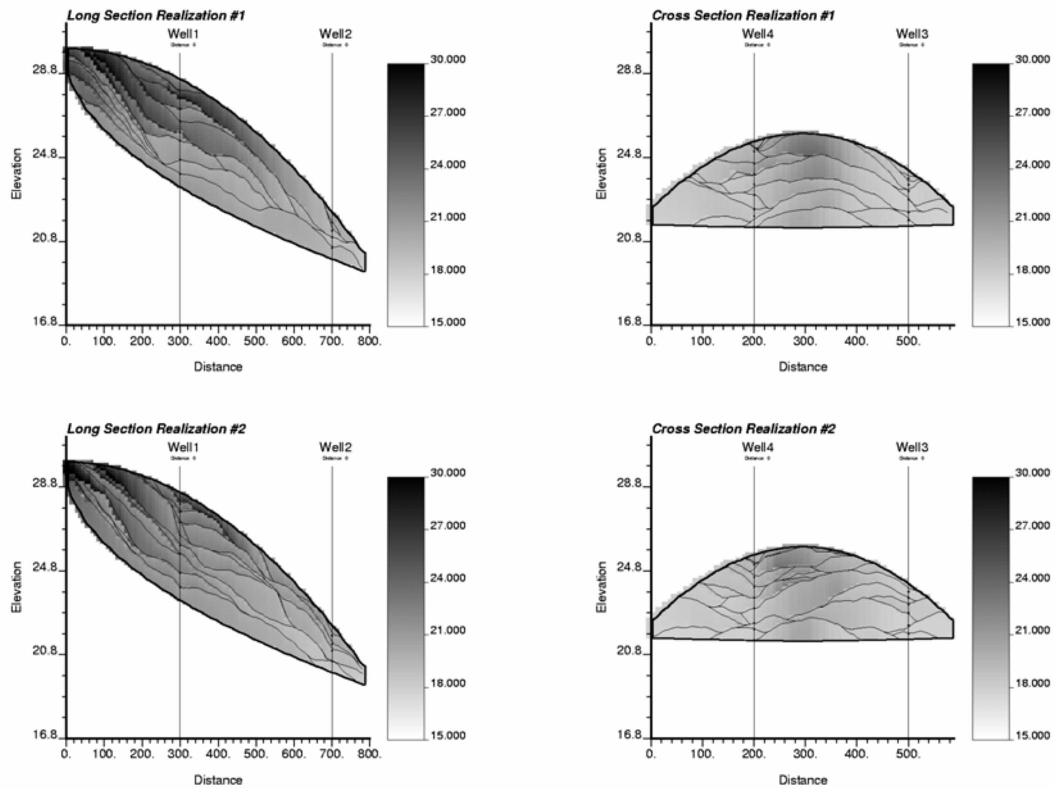

Figure 6 – long and cross sections for two surface based realizations and the associated porosity trend model.

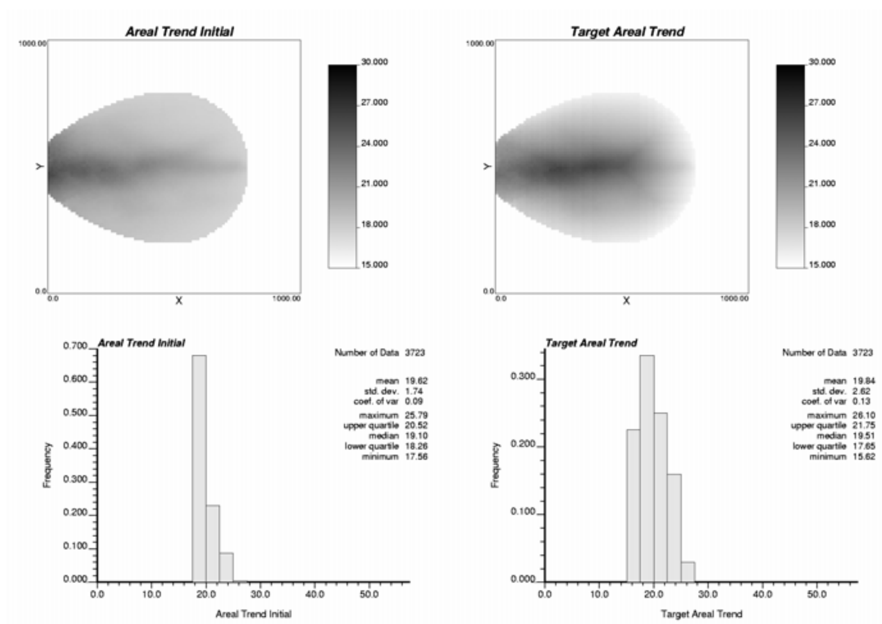

Figure 7 – the initial areal trend model from the hierarchical trend model and the target areal trend based on conditioning.

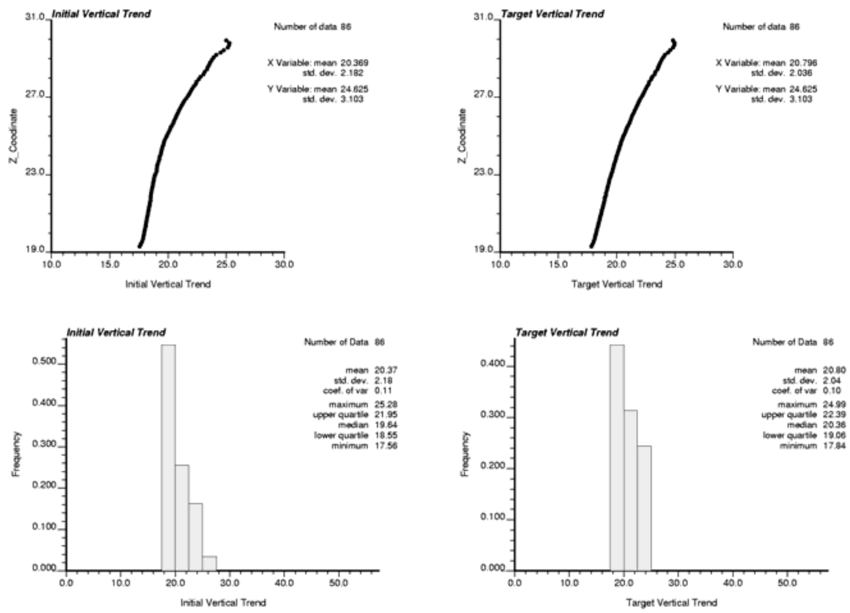

Figure 8 - the initial vertical trend model from the hierarchical trend model and the target areal trend based on conditioning.

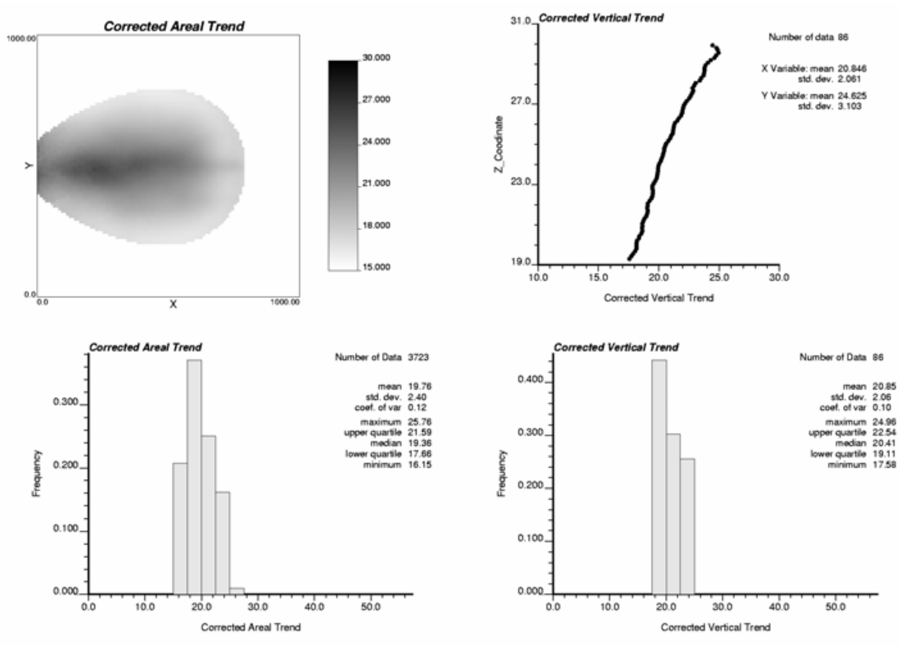

Figure 9 – the areal and vertical trend of the corrected trend model.

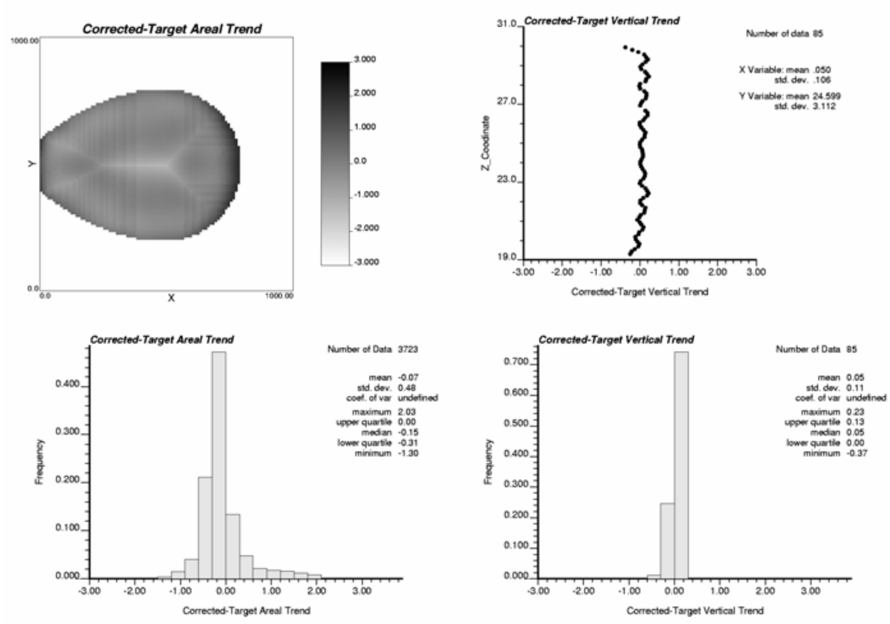

Figure 10 – the mismatch between the corrected and target areal and vertical trends.

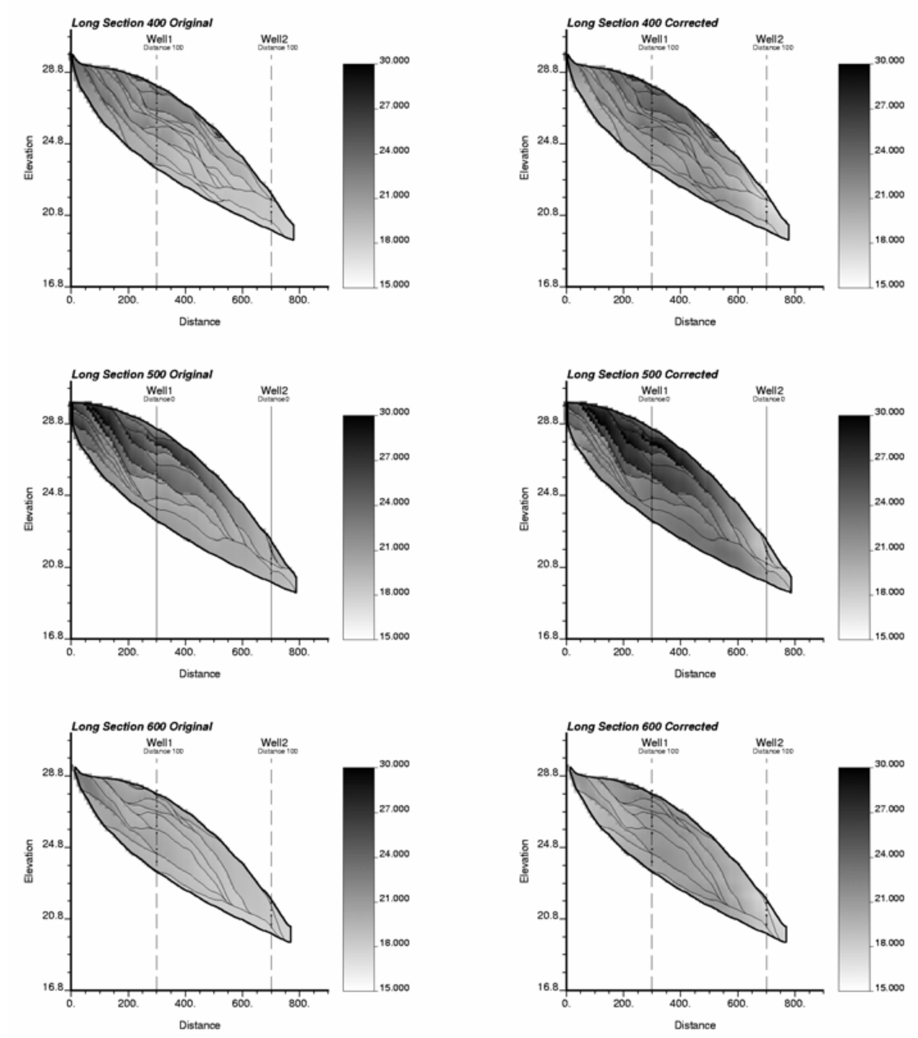

Figure 11 – long sections of the hierarchical trend model before and after correction to honor areal and vertical trend model.

Figure 12 – fence diagrams of the initial surfaced-based model (left), the initial trend model (center) and the corrected trend model (right). This demonstrates that the correction to honor areal and vertical trends preserves the small scale features imparted by the construction of the hierarchical trend model.