Assignment of Permeability on Unstructured Grids with Reduced Correlation Across Faults

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The direct population of unstructured grids may offer improved computation efficiency. Direct simulation is required since properties do not average linearly after the Gaussian transform and this is implicitly assumed when calculating the volume averaged covariances. Previous CCG reports outline the methodologies to apply direct sequential simulation with histogram reproduction and present the building blocks of FORTRAN code to apply these methods.

This note includes a follow-up case study based on a data set provide by Dave Hale at Landmark Graphics. This work was spurred by interest in direct simulation to Voronoi or atomic grids. The direct population of an atomic grid is demonstrated along with a complicating factor of reduced correlation across mapped faults.

Introduction

In simulation to unstructured grids the areas/volumes of the cells being populated must be accounted for; larger volume cells have less variability. Direct simulation (Journel, 1986) must be employed; variables do not average linearly after Gaussian transformation. Many of these issues have been overcome by the research we have been undertaking at the Centre for Computational Geostatistics (CCG) at the University of Alberta. This has been reported in previous CCG annual reports (Deutsch et al., 2001; Xie et al., 2001; Pyrcz and Deutsch, 2002) and externally (Oz et. al., 2003; Deutsch et. al., 2002). This case study does not address issues related to histogram reproduction and power-law transform of permeability for linear averaging. These topics are covered in the above references.

Case Study

An atomic grid generated to account for geophysical data and engineering constraints including mapped faults is shown in Figure 1. This 2D atomic grid is the basis for this case study. Initially, the problem of populating the grid will be considered, later the faults will be addressed.

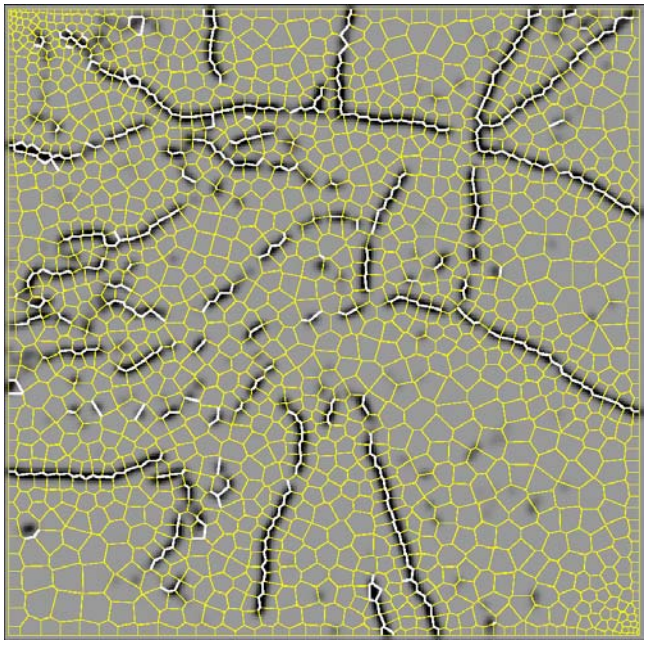


Figure 1 – an atomic grid accounting for geophysical data and engineering constraints with mapped faults (from D. Hale, Landmark Graphics).

A simplified dataset was generated by constructing Voronoi polygons from the provided point coordinates (1603 nodes with locations from 0 to 255 in both X and Y directions). This regridded data set is shown in Figure 2.

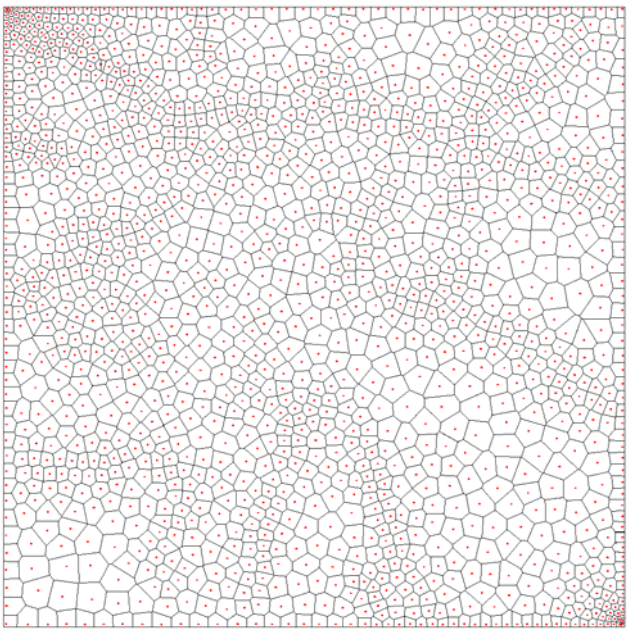
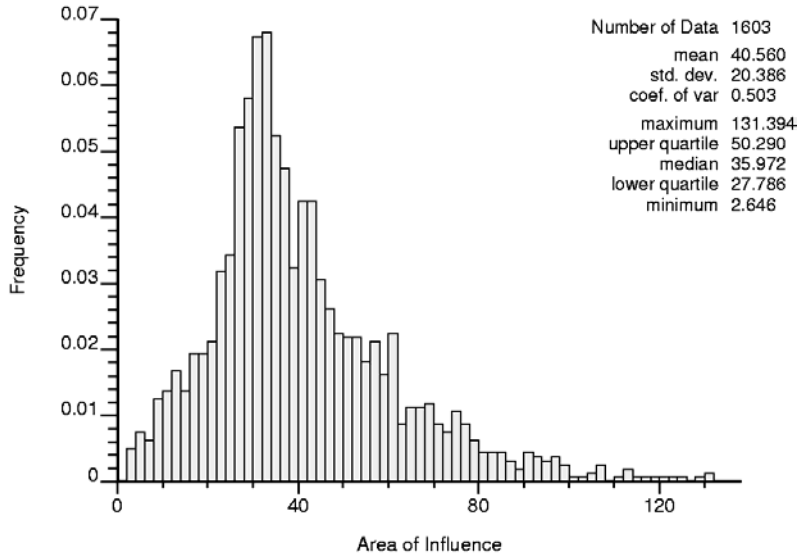


Figure 2 – the regridded Voronoi polygons.

A histogram of the polygon areas is shown below. The average size is 40.56 (close to 256 • 256 / 1603, as it should be).



The largest polygon is about 50 times larger than the smallest polygon. Assuming constant variability within the polygons would be an error. Geostatistical methods cannot simply be applied on an irregular mesh; the volumes of the cells must be accounted for. One alternative would be to simulate a fine scale Cartesian mesh and upscale. A grid of about 10^5 cells[1] would be required to ensure that nine simulated values are in the smallest grid cell. This is two orders of magnitude more than 1603 and it would be inefficient and cumbersome to deal with the fine grids and the required upscaling. A 256 by 256 simulation of a regular Cartesian grid is shown below in Figure 3 for demonstration. A finer grid would be required in practice.

[1] Assume 3x3 discretizations required in the minimum cell size of 2.6, if the cell is nearly square then it is 1.6x1.6 in size. Thus we would need 3 nodes in 1.6 units, or a about a 500 x 500 discretization.

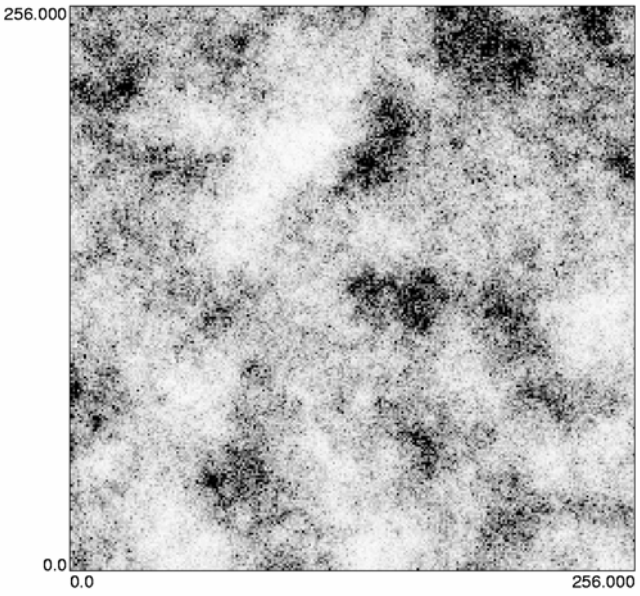


Figure 3 – a fine scale simulation for upscaling to the atomic grid.

This fine mesh realization is averaged within the Voronoi polygons to yield the upscaled realization (see Figure 4).

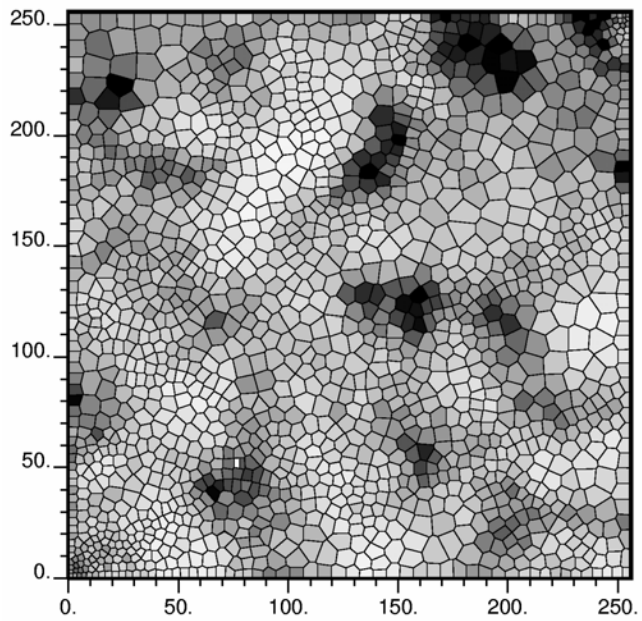


Figure 4 – an upscaled simulated realization.

This realization was generated with a point-scale lognormal distribution of permeability (target mean of 100 mD and standard deviation of 100 mD) and a spherical variogram model with a range of 64 units. The point and block scale histograms are shown below. The minimum block value is twice the minimum point value, the maximum block value

is one half the maximum point value and the variance has decreased by more than 25% (see Figure 5).

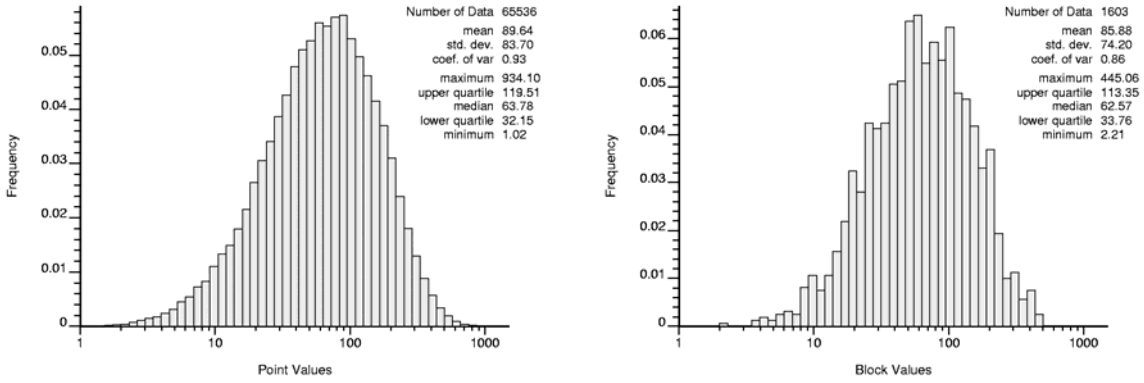


Figure 5 – the distributions at the support size of the fine regular grid (left) and the Voronoi grid (right).

The precise relationship between the point and block values depends on the block scale and on the variogram. Two additional example realizations are illustrated with a long range (see Figure 6) and large nugget effect (see Figure 7).

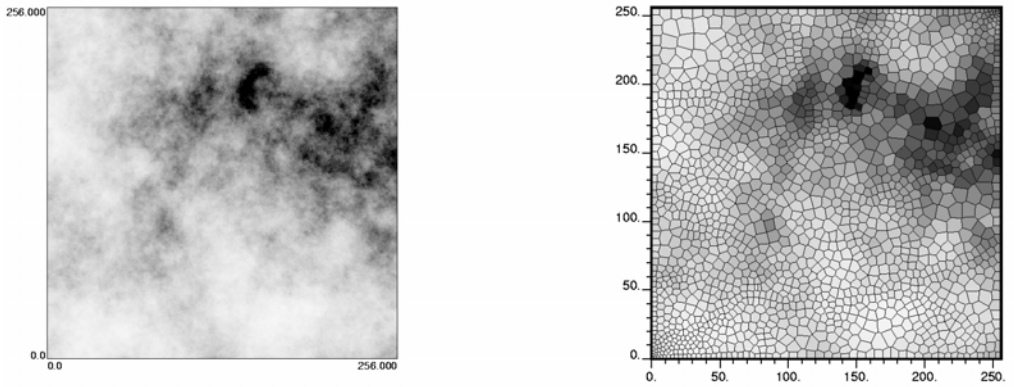


Figure 6 – an example realization fine grid and upscaled to the Voronoi grid with a long range variogram model.

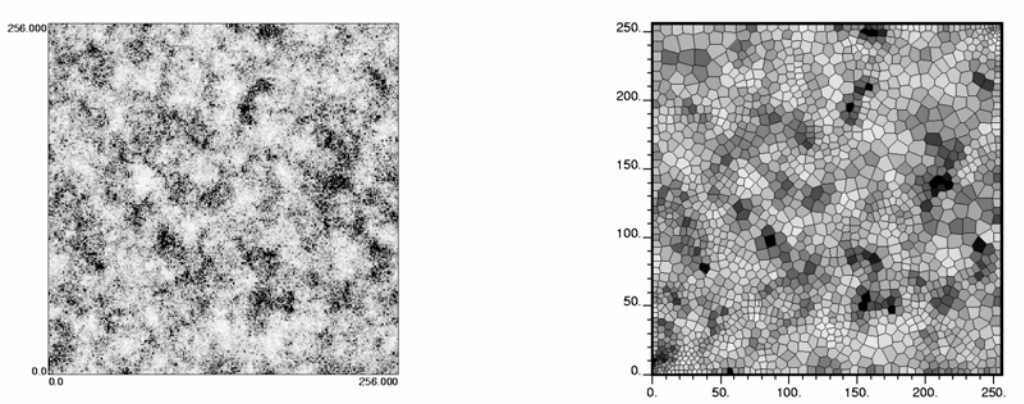


Figure 7 - an example realization fine grid and upscaled to the Voronoi grid with a high nugget variogram model.

The procedure for simulating a fine Cartesian grid and upscaling to an arbitrary grid is classical. Any type of averaging including non-linear averages and flow based averaging could be used. In fact, the script to do the fine scale simulation, the averaging, and all the plotting takes some seconds. We may wonder about the need to directly populate unstructured grids; we could persist with our population of fine Cartesian grids and upscaling. There are, however, two good reasons to be interested in unstructured grids.

1. In general it will be much more efficient to directly populate an unstructured grid. It will take fractions of a second instead of seconds. The CPU time for large 3-D grids will become more significant. Multiple realizations are required for uncertainty analysis and risk assessment; direct population will make that more practical. Current methods to populate unstructured grids generate additional realizations very quickly (Pyrcz and Deutsch, 2002).

2. Consideration of input data at multiple scales requires the same elementary calculations as those required for unstructured grids. Current geostatistical practice is to assume that secondary data such as seismic or inverted production data is at the fine scale of modeling. This is incorrect. When we go to account for the data at their correct scale and precision we might as well populate the ultimate grid directly.

Direct generation of these grids leads to the same basic result as those shown above based on the classical upscaling.

Direct Simulation

The direct simulation of the Voronoi mesh could be undertaken sequentially or all at once with a matrix method. Given the relatively small size of this grid a matrix method is considered. There are n=1603 blocks of different size to assign. The n by n covariance matrix of average covariance between the grid blocks is required:

$$\overline{C}_{i,j} = \frac{1}{|v_i||v_j|}\int_{v_i}\int_{v_j} C(\mathbf{u}-\mathbf{u}')d\mathbf{u}d\mathbf{u}'$$

These can be calculated once and for all time with some numerical discretization and other speed enhancements (Pyrcz and Deutsch, 2002; Xie et al., 2001). Then simulation comes down to inverting this *n* by *n* covariance matrix, generating random vectors, and simple multiplication. Conditioning data (wells, seismic, production, …) add to the size of the matrix, but we can integrate that data in one step. A sequential formalism is often used because it permits very large grids.

The covariance between all grids (1603x1603) was calculated. This was accomplished by applying the same level of discretization applied in the previous upscaled method. The LUSIM matrix simulation algorithm (Deutsch and Journel, 1998) was modified to load the grid node covariance table directly. A single realization was generated in just over a minute (see Figure 8). Additional realizations were generated in a couple seconds each.

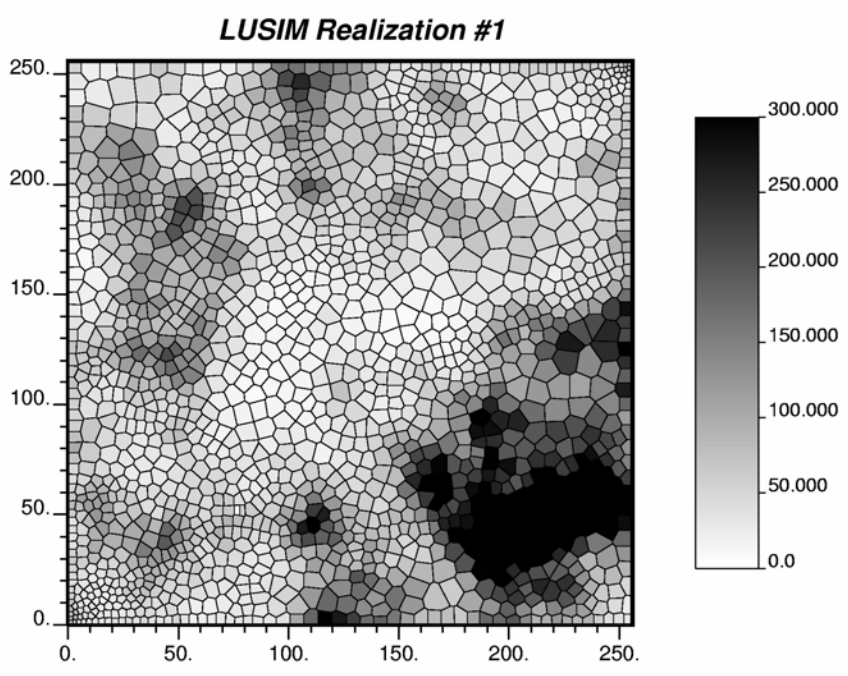


Figure 8 – a direct simulation based on the LUSIM matrix method.

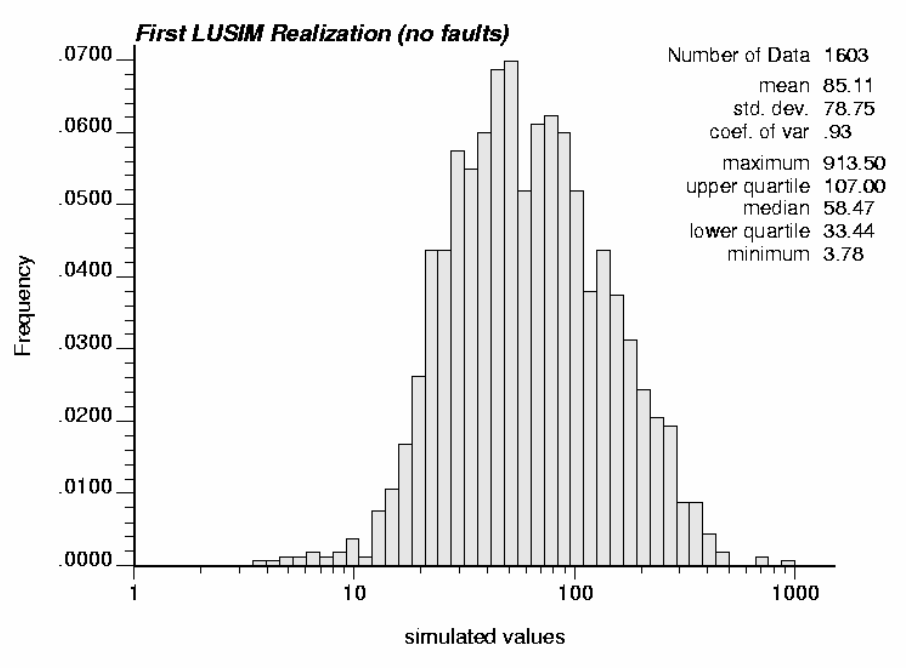


Figure 9 – the distribution of the directly simulated realization of permeability.

Accounting for Faults

This first realization does not account for the faults. Of course, that is most often the correct approach. The faults are post-depositional and affect the transmissibility across block-block faces, but do not affect the block properties nor their spatial correlation. The transmissibility in the region of the fault can be enhanced by associated fractures or reduced by cement filled fractures, shale smear, or fault gauge. Nevertheless, the boundaries (that we currently call faults) could represent a geological boundary where it makes sense to reduce the correlation across the boundary.

Statistical covariances are not like block-block transmissibilities – the entire matrix must be positive definite and each block "communicates" much farther than its immediate neighbors. Flow simulation matrices (aside from wells and other such non-neighbor connections) are quite sparse because they are limited to their nearest neighbors. This is relevant to us because of faults that do not completely enclose a particular region.

Consider the schematic example below with four grid blocks (see Figure 10). The thicker line represents a fault. Consider a variogram (covariance) range larger than the size of the picture. We cannot have 1 correlated to 2, 2 to 3, 3 to 4, and 1 **not** correlated to 4. We can put those numbers in a covariance matrix, but it will not be positive definite and geostatistical calculations will be awkward. The matrix *may* be non-singular and solvable, but the solution may be unreliable.

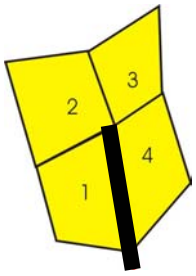


Figure 10 – a schematic of 4 Voronoi polygons and a fault segment indicated as a thicker line.

The matrix methods are very sensitive since all locations are solved simultaneously. The matrix of all covariances between all combinations of grid nodes must be positive definite. This practically precludes application of matrix methods for generating stochastic realizations in this application. This was demonstrated by attempts to discount covariances across faults in the 1603x1603 covariance matrix. Even the slightest discount factor resulted in a covariance matrix that was not positive definite and resulted in unreliable solutions with LUSIM.

Alternatively, a sequential method may be applied. The sequential methods require only that the covariances within the local search neighborhood are positive definite. The local search was modified to ignore previously simulated nodes from across faults. This has the practical impact of removing correlation across faults. This restricted search resulted in stable solutions. A single realization accounting for the faults is shown in Figure 11.

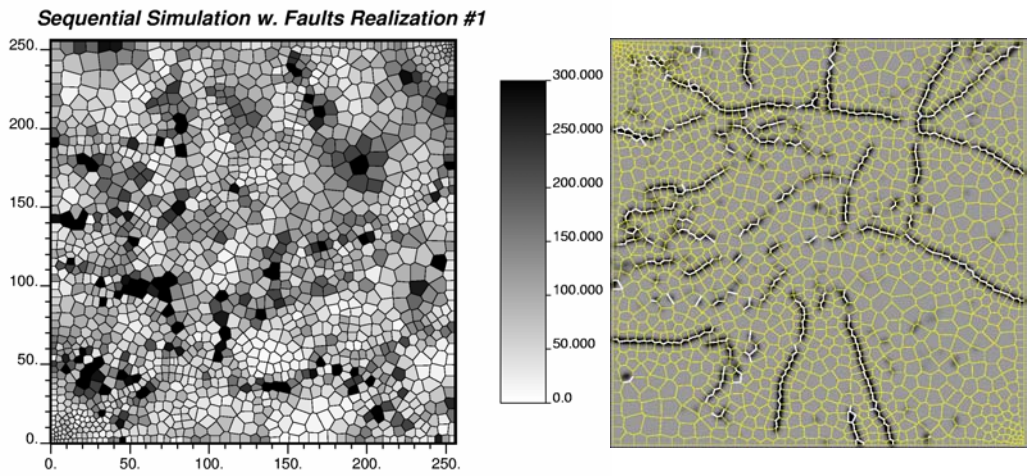


Figure 11 – a sample realization with limited search (left) and the map of faults (right).

Conclusions

- Simulation to unstructured grids is generally more computationally efficient than upscaling from fine / regular grids and allows for improved integration of data of different support sizes.

- Prior calculation of all covariances within an unstructured grid allows for the utilization of currently available simulation algorithms. Only minor modifications were required to allow GSLIB algorithms such as LUSIM to load in a grid node covariance table.
- Accounting for reduced correlation across faults is possible with a restricted search with sequential techniques.

Future Research

Subsequent research will tackle variety of areas in order to improve the application of direct sequential simulation to unstructured grids. The following are some of these areas.

Construction of Unstructured Grids From Simulation

Unstructured grids are often based on engineering and geologic constraints. Engineering constraints are the need to minimize discretization effects and improve CPU efficiency of flow simulation. Geologic constraints are a result the distribution of the subsurface heterogeneity. Unstructured grids may be fit to subsurface heterogeneity to avoid averaging out important subsurface features and to improve CPU efficiency.

Important subsurface features may be characterized stochastically. It may be necessary to construct multiple realizations of unstructured grids matched to subsurface property realizations.

Facies Control and Mixing Within Unstructured Grids

The simulation of categorical variables to unstructured grids is problematic. Change of support size may require the consideration of mixtures of facies. Implementation details and workflows will be explored.

Multivariate Simulation to Unstructured Grids

Direct sequential multivariate simulation will allow for the ability to account for support size and to reproduce the multivariate distribution. Implementation details are being investigated for the development of CPU efficient algorithms.

Cokriging with Seismic and Production Data

Direct sequential simulation has the advantage of explicit accounting of support size. This results in improved and more intuitive integration of multiple scales of data, such as large scale seismic and production data. Implementation details and workflows will be demonstrated.

Tensor Permeabilities

Unstructured grids require simulation of permeabilities in non-principal directions, and requires the full permeability tensor. The associated procedures will demonstrated.

Computational Comparison

Multiple case studies will be constructed to assess the relative computational cost of direct sequential simulation compared with the traditional fine scale simulation and scale up.

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