Comparison of Bayesian Updating Methods for Integration of Multiple Secondary Data

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Abstract

This paper focuses on the assessment of an unknown event A using its conditional probability P(A/B,C) given two different events, B and C. This paper shows seven different methods for the integration of multiple secondary data. The methods are different forms of Bayesian updating techniques, based on different assumptions of the relationship between the variable to be estimated and the secondary data, and between the secondary data itself. The seven methods are applied to 73 training images. The relationship between the method of Conditional Independence of secondary data and that of Permanence of Ratios, as well as the relationship between the methods of Permanence of Ratio and Least Squares are discussed.

This paper summarizes the results of a term project conducted by the author for a course on Bayesian updating.

Introduction

Combining information from diverse sources is a recurring challenge in any estimation study. In the oil industry, sparsity of data may result in combining porosity and/or permeability data from core analysis with seismic data and well logs in order to get reliable porosity or permeability model of the reservoir. We need to consider the assessment of an unknown event A (e.g., porosity or permeability) through its conditional probability $P(A/B,C)$ given two data events B (e.g., seismic) and C (e.g., well logs). The probability $P(A/B,C)$ is then used for estimation or simulation of event A.

Consider the joint probability of A and the two data events B and C. The following exact decomposition is the basis of all subsequent derivations and approximations [1]:

$$P(A,B,C) = P(A/B,C) \cdot P(B,C) \tag{1}$$

$$P(A,B,C) = P(A) \cdot P(B/A) \cdot P(C/A,B) = P(A) \cdot P(C/A) \cdot P(B/A,C) \tag{2}$$

Thus, the conditional probability of $P(A/B,C)$ can be calculated by

$$P^*(A \mid B,C) = \frac{P(A,B,C)}{P(B,C)} = \frac{P(A) \cdot P(B/A) \cdot P(C/A,B)}{P(B,C)} \tag{3}$$

where * means estimated value.

The difficulty in formula (3) comes from the determination of joint probability of *B* and *C*.

If the relationship between *B* and *C* can be known, Least Squares method can be used to get the conditional probability and the results are good in many cases, which is shown later in this paper. However, the relationship between *B* and *C* is unknown or difficult to determine in many situations, so the assumption of full independence or conditional independence of *B* and *C* are often used in the calculations. In presence of actual data dependence, the methods provided by the traditional full independence or conditional independence hypotheses are shown to be non-robust leading to various inconsistencies. An alternative method based on Permanence of updating Ratios is developed, which guarantees all limit conditions even in presence of complex data interdependence [2]. Deutsch proposed a kind of modified conditional independence methods by adding two weights to the terms about events *B* and *C* [3]. Two modified Least Squares methods are considered based on the different ways to evaluate the covariance of *B* and *C* when the relationship of *B* and *C* is unknown. These methods can be divided into two classes:

Class I . Some bivariate information between *B* and *C* is taken from data, and

Class II. No information between *B* and *C* is taken from data.

In order to get some information about which method is better in a general sense, seven methods are applied to 73 training images corresponding to 3 templates, 4 ranges, 3 proportions and two simulation methods. The performance of each method is expressed by mean of squared error (MSE) related to the reference value, calculated directly from the training image. The results are analyzed and the relationships between the methods are discussed.

Description of Methods

Method 1: Least Squares (Class I)

The formula of the estimated probability $P^*(A/B,C)$ by Least Square method is [4]:

$$P^*(A \mid B,C) = \overline{P(A)} + \lambda_1\left[P(B) - \overline{P(B)}\right] + \lambda_2\left[P(C) - \overline{P(C)}\right] \tag{4}$$

where $\overline{P(A)}$, $\overline{P(B)}$ and $\overline{P(C)}$ correspond to expected values. $P(B)$ and $P(C)$ correspond to indicator value that is:

$$P(B) = \begin{cases} 1, & \text{if } B \text{ occurs} \\ 0, & \text{if not} \end{cases}$$

$$P(C) = \begin{cases} 1, & \text{if } C \text{ occurs} \\ 0, & \text{if not} \end{cases}$$

λ_1 and λ_2 can be obtained by solving the kriging equations:

$$\begin{bmatrix} C_{BB} & C_{BC} \\ C_{CB} & C_{CC} \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} C_{AB} \\ C_{AC} \end{bmatrix} \tag{5}$$

where,

$$C_{BB} = E\left\{B^2\right\} - (E\{B\})^2 = \overline{P(B)} \cdot (1 - \overline{P(B)})$$

$$C_{CC} = E\left\{C^2\right\} - (E\{C\})^2 = \overline{P(C)} \cdot (1 - \overline{P(C)})$$

$$C_{BC} = C_{BC} = E\{B \cdot C\} - E\{B\} \cdot E\{C\} = \overline{P(B,C)} - \overline{P(B)} \cdot \overline{P(C)}$$

Method 2: Full Independence (Class II)

By assuming that the two secondary data *B* and *C* are independent, i.e., event *B* is independent of event *C*, we have [2]

$$P(B/A,C) = P(B/A)$$

$$P(C/A,B) = P(C/A)$$

$$P(B,C) = P(B) \cdot P(C)$$

Therefore, the Formula (3) becomes:

$$P^*(A \mid B,C) = P(A) \cdot \frac{P(A \mid B)}{P(A)} \cdot \frac{P(A \mid C)}{P(A)} \tag{6}$$

or

$$P^*(A \mid B,C) = P(A) \cdot \frac{P(B/A)}{P(B)} \cdot \frac{P(C/A)}{P(C)} \tag{7}$$

Method 3: Conditional Independence (Class II)

Conditional to event *A*, the two events *B* and *C* are independent if

$$P(B,C/A) = P(B/A) \cdot P(C/A)$$

Therefore, by assuming that secondary data *B* and *C* are conditionally independent, i.e., event *B* given *A* and *C* given *A* are independent, the Formula (3) becomes [5]:

$$P^*(A \mid B,C) = P(A) \cdot \frac{P(B \mid A) \cdot P(C \mid A)}{P(B,C)} \tag{8}$$

Usually the joint probability *P*(*B,C*) is difficult to obtain, so Journel proposed the substitution [2]:

$$P(B,C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A) + P(\overline{A}) \cdot P(B \mid \overline{A}) \cdot P(C \mid \overline{A})$$

It is based on the substitution presented by the author in Equation (7) of his paper [2]:

$$P(B,C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A,B) + P(\overline{A}) \cdot P(B \mid \overline{A}) \cdot P(C \mid \overline{A},B)$$

The terms $P(C \mid A,B)$ and $P(C \mid \overline{A},B)$ can be even more complicated to obtain. Using Bayes Law directly may be more straightforward when applying this method, since conditional probabilities are easier to derive than joint probabilities, that is:

$$P(B,C) = P(C) \cdot P(B \mid C)$$

Method 4: Permanence of Ratios (Class II)

One of the most well-proven paradigms for engineering approximation is the permanence of ratios. Rates or ratios of increments are typically more stable than the increments themselves [2].

Consider the following ratios

$$a = \frac{1 - P(A)}{P(A)}$$

$$b = \frac{1 - P(A \mid B)}{P(A \mid B)}$$

$$c = \frac{1 - P(A \mid C)}{P(A \mid C)}$$

and

$$x = \frac{1 - P(A \mid B,C)}{P(A \mid B,C)} \geq 0$$

The permanence of ratio amounts to assuming

$$\frac{x}{b} \approx \frac{c}{d}$$

The conditional probability is immediately retrieved as

$$P^*(A \mid B,C) = \frac{a}{a + bc} \tag{9}$$

This method has the advantage that it requires neither derivation of the joint probabilities nor the marginal probabilities of *B* or *C*.

Method 5: Deutsch Proposal (Class II)

The formula of the estimated probability *P*(*A/B,C*) by Least Squares method is[3]:

$$P^*(A \mid B,C) = P(A) \cdot \left(\frac{P(A \mid B)}{P(A)} \right)^{\omega_1} \cdot \left(\frac{P(A \mid C)}{P(A)} \right)^{\omega_2} \tag{10}$$

where ω_1 and ω_2 are weights.

This method is based on Method 2 assuming that secondary data are independent, but the author proposed to weight each update of *P*(*A*) from secondary data, to account for dependence of *B* and *C*. According to the paper, ω_1 and ω_2 are related to the correlation coefficient between *B* and *C*.

Method 6 and method 7: Modified Least Squares Methods (Class II)

Method of Least Squares is the best for many cases, which will be shown later. The drawback of the method is the requirement of the relationship of *B* and *C*.

Can we develop some methods based on the idea of Least Squares but without the need to know the relationship of *B* and *C* or develop some formula to express the relationship between *B* and *C*?

Two methods are proposed in this paper.

Method 6 is described as follows:

When C_{BC} is the known covariance matrix between events B and C, the weights in Formula (4) can be found by solving Kriging equations (5):

$$\lambda_1 = C_{AB}\frac{C_{CC}}{C_{BB}C_{CC} - C_{BC}^2} - C_{AC}\frac{C_{BC}}{C_{BB}C_{CC} - C_{BC}^2}$$

$$\lambda_2 = C_{AC}\frac{C_{BB}}{C_{BB}C_{CC} - C_{BC}^2} - C_{AB}\frac{C_{BC}}{C_{BB}C_{CC} - C_{BC}^2}$$

In many cases C_{BC} is unknown. For the cases with the maximum and/or minimum "statistical" distance between B and C, it can be obtained by the following procedure.

The maximum covariance can be reached when the "statistical" distance between B and C is the minimum. In this case,

$$d_{BA} = 1 - C_{AB}$$

$$d_{CA} = 1 - C_{AC}$$

$$d_{BC} = d_{CA} - d_{BA}$$

$$C_{BC}(\max) = 1 + C_{AC} - C_{AB}$$

The minimum covariance can be obtained when the "statistical" distance between B and C is the maximum. In this case,

$$d_{BC} = d_{CA} + d_{BA}$$

$$C_{BC}(\min) = C_{AC} + C_{AB} - 1$$

Once we have solved Kriging equations for the minimum and maximum weight based on the minimum and maximum of C_{BC}, we can average the resulting weights to find the weights for the Least Squares estimate.

$$\lambda_1 = \frac{\lambda_1(\min) + \lambda_1(\max)}{2}$$

$$\lambda_2 = \frac{\lambda_2(\min) + \lambda_2(\max)}{2}$$

where $\lambda_1(\max)$ and $\lambda_2(\max)$ are the weights when maximum of C_{BC} is used in the calculation, $\lambda_1(\min)$ and $\lambda_2(\min)$ are the weights when minimum of C_{BC} is used. These weights λ_1 and λ_2 can then be used in Formula (4) to estimate the conditional probability.

The basic idea of Method 7 is the same as Method 6 except the calculation of $C_{BC}(\max)$ and $C_{BC}(\min)$. In method 7, they are calculated by:

$$C_{BC}(\max) = 0.618 \cdot \sqrt{C_{BB}} \cdot \sqrt{C_{CC}}$$

$$C_{BC}(\min) = (1 - 0.618) \cdot \sqrt{C_{BB}} \cdot \sqrt{C_{CC}}$$

The two formulas are based on the formula:

$$C_{BC} = \rho_{BC} \cdot \sqrt{C_{BB}} \cdot \sqrt{C_{CC}}$$

0.618 is known as the Fibinacci ratio or the golden ratio/mean. It is used here because " The golden mean appears throughout nature"[6].

Disussions

Relationship between Permanence of Ratios and Conditional Independence Methods

Rewrite the ratios in Formula (9) for method of Permanence of Ratio:

$$a = \frac{1 - P(A)}{P(A)} = \frac{P(\overline{A})}{P(A)}$$

$$b = \frac{1 - P(A \mid B)}{P(A \mid B)} = \frac{P(\overline{A} / B)}{P(A / B)}$$

$$c = \frac{1 - P(A \mid C)}{P(A \mid C)} = \frac{P(\overline{A} / C)}{P(A / C)}$$

Consider that

$$P(\overline{A}, B) = P(\overline{A} / B)P(B) = P(B / \overline{A})P(\overline{A})$$

$$P(\overline{A}, C) = P(\overline{A} / C)P(C) = P(C / \overline{A})P(\overline{A})$$

Substitute a, b and c in Formula (9) and rearrange to get

$$P_{PR} * (A \mid B, C) = \frac{1}{1 + \frac{bc}{a}}$$

$$= \frac{1}{1 + \frac{P(A)}{P(\overline{A})} \frac{P(\overline{A} / B)}{P(A / B)} \frac{P(\overline{A} / C)}{P(A / C)}}$$

$$= \frac{1}{1 + \frac{P(A)}{P(\overline{A})} \frac{P(\overline{A} / B)P(B)}{P(B / A)P(A)} \frac{P(\overline{A} / C)P(C)}{P(C / A)P(A)}}$$

$$= \frac{1}{1 + \frac{P(B)P(C)}{P(\overline{A})P(A)} \frac{P(\overline{A} / B)}{P(B / A)} \frac{P(\overline{A} / C)}{P(C / A)}}$$

$$= \frac{1}{1 + \frac{P(B)P(C)}{P(\overline{A})P(B, C)} \times \frac{P(\overline{A} / B)P(\overline{A} / C)}{\frac{P(A)P(B / A)P(C / A)}{P(B, C)}}}$$

This is compared to the following formula for the method of Conditional Independence:

$$P_{CI}*(A\,|\,B,C) = P(A)\cdot\frac{P(B\,|\,A)\cdot P(C\,|\,A)}{P(B,C)}$$

Therefore

$$P_{PR}^{*}(A/B,C) = \frac{1}{1+\dfrac{P(B)P(C)}{P(\overline{A})P(B,C)}\dfrac{P(\overline{A}/B)P(\overline{A}/C)}{P_{CI}^{*}(A/B,C)}}$$

Relationship between Least Squares and Permanence of Ratio Methods

Methods of Least Squares and Permanence of Ratios are two robust methods in many cases. It is meaningful to study which correlation coefficient for Least Squares corresponds to the result of Permanence of Ratios. This paper tries to get some idea about the correlation coefficients.

Four types of situations were found for means of squared errors from Least Squares and those from Permanence of ratio versus correlation coefficients between *B* and *C*.

Type 1. Means of squared errors from Permanence of Ratios are lower than those from Least Squares regardless of the correlation coefficients. This is shown in Figure 1.

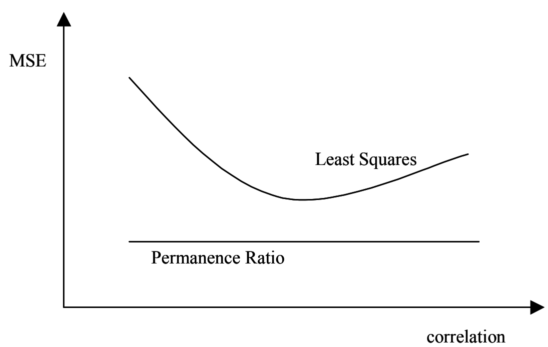

Figure 1 Plot of MSE (mean of squared error) vs. correlation coefficient between *B* and *C* for Type 1

Type 2. Means of squared errors from Permanence of Ratios are higher than those from Least Squares regardless of correlation coefficients. It is shown in Figure 2.

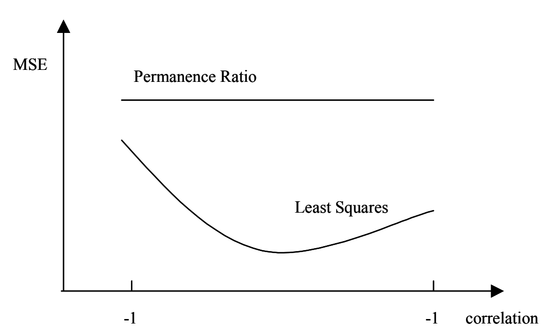

Figure 2 Plot of MSE (mean of squared error) vs. correlation coefficient

between *B* and *C* for Type 2

Type 3. There is one intersecting point of Permanence of ratio and Least Squares curves, as shown in Figure 3. Some of means of squared errors from Least Square are lower than those from Permanence of Ratios for some correlation coefficients.

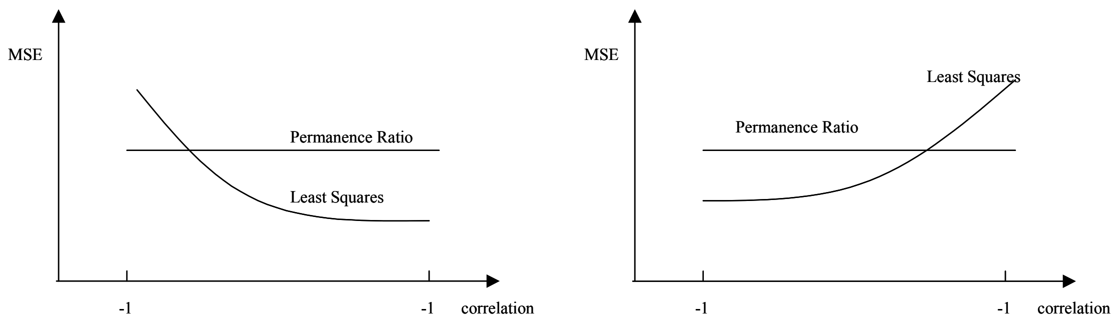

Figure 3 Plot of MSE (mean of squared error) vs. correlation coefficient

between *B* and *C* for Type 3

Type 4. There are two intersecting points of curves from Permanence of Ratios and Least Squares, as shown in Figure 4. The means of squared errors from Least Squares are lower than those from Permanence of Ratios when the correlation coefficients between *B* and *C* fall in the range from Cor1 to Cor2.

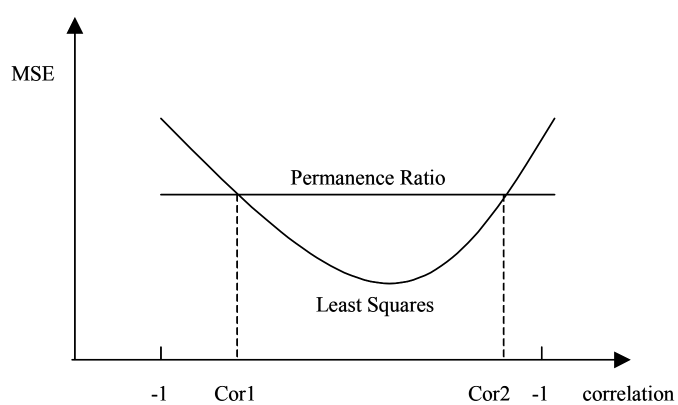

Figure 4 Plot of MSE (mean of squared error) vs. correlation coefficient
between *B* and *C* for Type 4

Case Study

One Image from sisim

A 100×100 grid system shown in Figure 5 is used to check all methods listed above, which comes from SISIM[4] with a spherical variogram of range 10 and nugget effect 0.

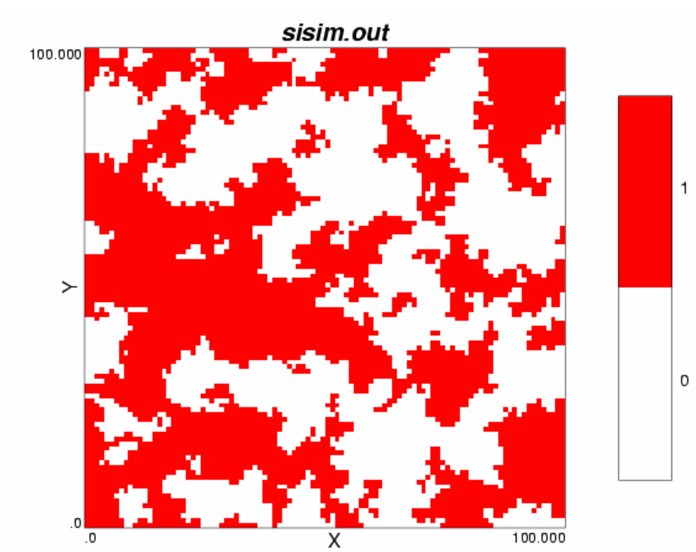

Figure 5 One training image from SISIM

$P^*(A/B,C)$ is calculated for the following three templates:

Template 1

A	B	C

Template 2

B	
A	C

Template 3

B	A	C

The following eight probabilities are calculated by using the seven methods listed above:

$P(A=1/B=1,C=1)$ $P(A=0/B=1,C=1)$

$P(A=1/B=1,C=0)$ $P(A=0/B=1,C=0)$

$P(A=1/B=0,C=1)$ $P(A=0/B=0,C=1)$

$P(A=1/B=0,C=0)$ $P(A=0/B=0,C=0)$

The means of squared errors for this image for all methods are shown in Table 1. The last column in the table shows the results from modified Least Squares method when the true C_{BC}(max) and C_{BC}(min) of the image for the three templates are used in calculating of the weights.

Table 1 Means of squared errors of the image for three templates

Template	Method 1 Least Squares	Method 2 Full Indep.	Method 3 Cond. Indep.	Method 4 Perm. of Ratios	Method 5 Deutsch (ω_1=ω_2=0.5)	Method 6 Modified LS	Method 7 Modified LS	In case of true C_{BC}(max)& C_{BC}(min) for MLS
1	4.1E-06	0.1348	0.2064	0.0208	0.0506	0.1698	0.0173	0.0168
2	1.45E-05	0.1339	0.0132	0.0004	0.0185	0.1550	0.0018	0.00012
3	4E-06	0.1308	0.0005	2.41E-05	0.0205	0.1685	0.0008	0.00022

Note: MLS means Modified Least Squares.

The correlation coefficients of the image for the three different templates are shown in Table 2.

Table 2 Correlation coefficients of the image for three templates

Template	$\rho(A,B)$	$\rho(A,C)$	$\rho(B,C)$	$\rho(B/A,C/A)$
1	0.7703	0.6142	0.7682	0.5833
2	0.7569	0.7709	0.6841	0.2491
3	0.7703	0.7682	0.6142	0.0527

From Table 1 we see that the Least Squares method is the best method and Permanence of Ratios is the second. For template 3 the error for Conditional Independence method is smaller than those for other templates by using this method because B and C are almost conditionally independent in this case. The correlation coefficient of B and C given A is 0.0527, shown in Table 2.

From Table 1, for some cases the results from Method 7 may be better than the results from the other methods except Least Squares. When C_{BC}(max) and C_{BC}(min) are the true values of one image., Modified Least Squares method can get better results than other values of C_{BC}(max) and C_{BC}(min) are used. However, it should be noticed that we can not guarantee the results from Modified Least Squares method are better than those from other methods except Least Squares even though the true values for C_{BC}(max) and C_{BC}(min) for the image are used in calculation.

24 images from SISIM and ELLIPSIM

To test the performance of the seven methods described above, 24 training images generated from ELLIPSIM and SISIM routines in GSLIB[4] with 4 different ranges and 3 different proportions of 1's were used. They are named by sisimrangepropij.ps and ellipsimrangepropij.ps. The ranges in ELLIPSIM and SISIM parameter files are 0, 2, 8 and 32, corresponding to i=1,2,3,4, respectively. The proportions of 1's in ELLIPSIM and SISIM parameter files are 0.05, 0.25 and 0.5, corresponding to j=1,2,3, respectively. The images are shown in Figure 6 and Figure 7.

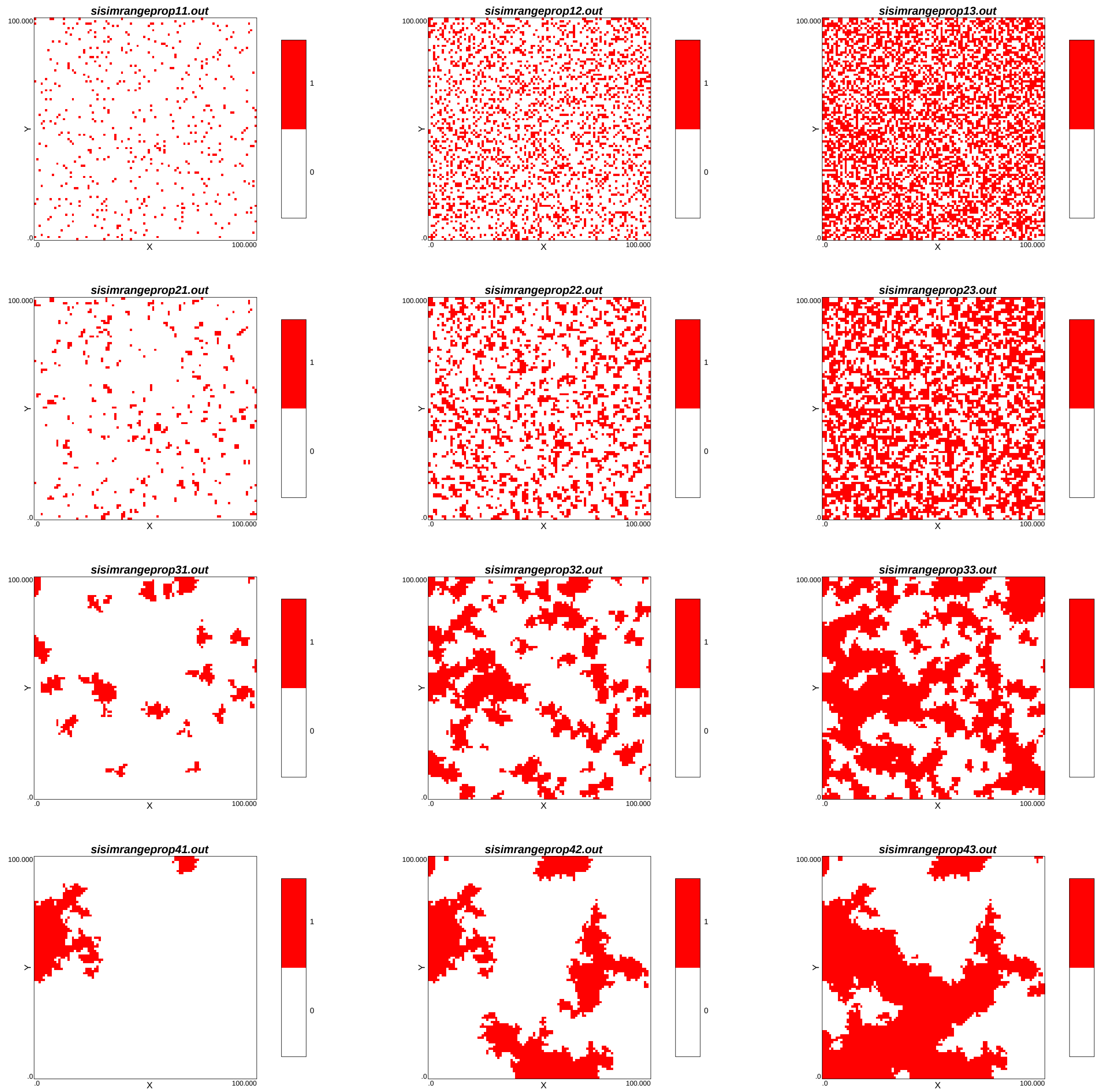

Figure 6 12 images from SISIM

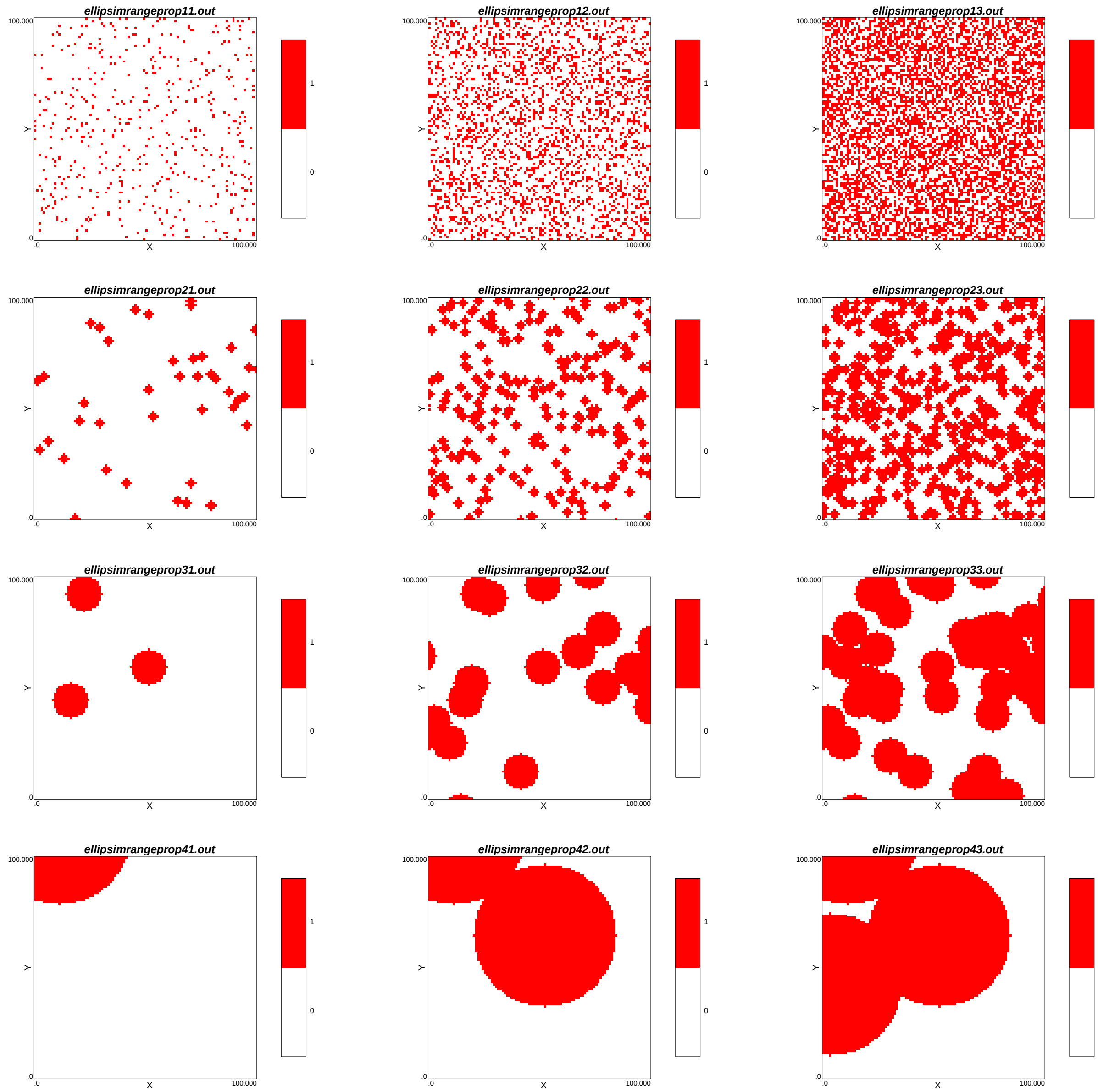

Figure 7 12 images from ELLIPSIM

For each image in the 24 images, the following three templates are considered:

Template 1

A	B	C

Template 2

B	
A	C

Template 3

B	A	C

The means of squared errors of results are shown in Tables 3, 4 and 5. The true correlation coefficients between *B* and *C* are also shown in Table 3, 4 and 5. Cor1 and Cor2 in the tables have the same meaning as those in Figure 4, that is, Cor1 and Cor2 are the correlation coefficient values at left intersection and right intersection between the curve for Least Square method and line for Permanence of Ratios method, respectively. The configurations with valid Cor1's and Cor2's mean that the relationship between the results of Least Square and Permanence of Ratio is Type 4. The results of the method ranks based on means of squared errors are shown in Table 6.

Table 3 Mean of squared errors of 24 images for Template 1.

Gene. routine	Range	Prop. of 1	Method 1 Least Squares	Method 2 Full Indep.	Method 3 Cond. Indep.	Method 4 Perm. of Ratios	Method 5 Deutsch (w=0.5)	Method 6 Modified LS	Method 7 Modified LS	Cor1	Cor2	True Cor. Coeff.
sisim	0	0.05	8.35E-05	8.07E-05	1.79E-05	8.08E-05	5.04E-05	4.06E-05	6.91E-05	0.0204	0.6304	-0.0101
		0.25	3.23E-05	3.51E-05	4.40E-06	3.40E-05	5.23E-05	1.54E-04	1.89E-04	-0.0654	0.1634	-0.0189
		0.5	1.00E-07	1.00E-07	0.00E+00	1.00E-07	5.40E-06	2.41E-05	1.34E-05	-0.0021	0.0001	-0.0010
	2	0.05	3.71E-03	1.77E-02	3.23E-03	1.31E-02	1.55E-02	2.97E-02	7.48E-03	-0.1506	0.6001	0.2712
		0.25	1.51E-04	3.62E-03	1.63E-02	3.26E-03	1.20E-02	3.41E-02	5.01E-03	-0.0046	0.4939	0.3012
		0.5	8.42E-05	2.77E-03	3.34E-02	2.76E-03	1.03E-02	3.37E-02	5.36E-03	0.0014	0.4784	0.3022
	8	0.05	2.69E-03	4.78E+00	9.98E-02	3.48E-02	9.94E-02	2.35E-01	1.17E-02	0.2861	0.8221	0.6975
		0.25	2.25E-04	3.36E-01	1.49E-01	2.11E-02	5.72E-02	1.85E-01	1.08E-02	0.3811	0.8259	0.7186
		0.5	3.46E-05	1.04E-01	2.03E-01	2.05E-02	4.46E-02	1.79E-01	1.24E-02	0.4074	0.8334	0.7328
	32	0.05	5.87E-04	1.97E+01	3.30E-01	1.78E-02	1.13E-01	3.20E-01	3.76E-02	0.6756	0.9156	0.8667
		0.25	5.04E-05	1.16E+00	2.54E-01	2.01E-02	7.98E-02	2.56E-01	3.21E-02	0.6334	0.9037	0.8454
		0.5	1.62E-04	3.14E-01	3.35E-01	1.84E-02	7.23E-02	2.93E-01	3.59E-02	0.7246	0.9314	0.8900
ellipsim	0	0.05	1.51E-04	1.97E-04	3.75E-04	1.96E-04	3.62E-04	6.93E-04	2.81E-04	-0.5904	0.1386	-0.0146
		0.25	1.30E-06	1.20E-06	5.90E-06	1.20E-06	1.00E-05	3.21E-05	6.00E-06	-0.2044	-0.0091	0.0031
		0.5	4.14E-05	4.22E-05	6.00E-07	4.14E-05	5.55E-05	1.07E-04	1.16E-04	**Type 1**		-0.0114
	2	0.05	2.42E-04	6.35E+00	1.03E-01	3.48E-02	6.28E-02	1.77E-01	3.95E-03	0.1786	0.8036	0.6333
		0.25	3.48E-05	1.39E-01	8.61E-02	1.66E-02	3.23E-02	1.15E-01	6.81E-04	0.1729	0.7464	0.5775
		0.5	1.49E-04	2.39E-02	8.77E-02	8.84E-03	1.71E-02	8.33E-02	1.78E-04	0.1696	0.6996	0.5312
	8	0.05	5.58E-05	3.52E+01	3.53E-01	4.26E-02	1.63E-01	3.68E-01	8.82E-02	0.7564	0.9453	0.9098
		0.25	2.56E-04	1.34E+00	3.51E-01	4.03E-02	1.23E-01	3.15E-01	7.33E-02	0.7339	0.9397	0.9011
		0.5	2.86E-04	2.59E-01	3.23E-01	3.32E-02	9.15E-02	3.21E-01	5.74E-02	0.7176	0.9334	0.8922
	32	0.05	4.53E-04	4.03E+01	2.69E+01	1.91E-01	1.97E-01	4.23E-01	1.46E-01	**Type 3**		0.9812
		0.25	1.97E-04	7.12E-01	6.67E-01	5.47E-02	1.65E-01	4.38E-01	1.17E-01	**Type 2**		0.0000
		0.5	8.04E-04	3.38E-01	5.15E+00	1.28E-01	1.42E-01	4.59E-01	1.14E-01	0.3621	0.9879	0.9736

Table 4 Mean of squared errors of 24 images for Template 2.

Gene. routine	Range	Prop. of 1	Method 1 Least Squares	Method 2 Full Indep.	Method 3 Cond. Indep.	Method 4 Perm. of Ratios	Method 5 Deutsch (w=0.5)	Method 6 Modified LS	Method 7 Modified LS	cor1	cor2	True Cor. Coeff.
sisim	0	0.05	2.15E-04	2.41E-04	2.74E-04	2.40E-04	3.72E-04	5.96E-04	3.16E-04	-0.6876	0.1001	-0.0031
		0.25	7.00E-06	5.90E-06	2.40E-06	6.20E-06	3.18E-05	1.59E-04	4.19E-05	0.0209	0.1894	-0.0023
		0.5	4.07E-05	4.06E-05	3.03E-05	4.07E-05	4.16E-05	4.51E-05	4.21E-05	**Type 1**		-0.0017
	2	0.05	2.78E-03	4.11E-01	1.93E-03	5.60E-03	3.95E-02	1.14E-01	1.49E-02	-0.1799	0.2086	0.0944
		0.25	1.11E-04	1.51E-02	1.34E-04	5.73E-04	1.33E-02	6.17E-02	3.59E-03	0.0179	0.2286	0.1339
		0.5	2.31E-05	2.87E-03	1.52E-05	2.90E-05	7.76E-03	4.64E-02	2.47E-03	0.0931	0.1211	0.1072
	8	0.05	5.64E-05	8.75E+00	4.30E-03	3.32E-03	6.02E-02	2.43E-01	1.45E-03	0.4366	0.8101	0.6102
		0.25	1.20E-05	4.64E-01	6.58E-03	9.22E-04	3.00E-02	1.93E-01	9.93E-04	0.4951	0.7621	0.6202
		0.5	2.90E-06	1.08E-01	1.43E-02	5.48E-04	1.50E-02	1.85E-01	1.11E-03	0.5349	0.7794	0.6490
	32	0.05	1.25E-05	2.13E+01	3.07E-02	4.94E-04	7.84E-02	2.94E-01	1.40E-02	0.7514	0.8996	0.8284
		0.25	3.00E-07	1.36E+00	2.59E-02	4.20E-04	4.72E-02	2.41E-01	7.36E-03	0.7149	0.8929	0.8005
		0.5	3.02E-04	3.12E-01	3.46E-02	2.61E-04	4.52E-02	2.84E-01	7.18E-03	**Type 1**		0.8549
ellipsim	0	0.05	3.52E-04	3.51E-04	3.38E-04	3.51E-04	4.73E-04	6.61E-04	4.60E-04	-0.7941	-0.0064	0.0010
		0.25	2.27E-04	2.25E-04	3.02E-04	2.25E-04	2.95E-04	3.95E-04	2.75E-04	-0.7214	-0.0099	0.0056
		0.5	7.14E-05	7.18E-05	1.16E-05	7.14E-05	8.07E-05	1.14E-04	1.10E-04	**Type 1**		-0.0101
	2	0.05	3.07E-04	1.41E+01	2.55E-02	1.15E-02	5.26E-02	2.02E-01	1.50E-03	0.2989	0.9528	0.6199
		0.25	1.95E-04	3.13E-01	3.18E-02	5.53E-03	2.02E-02	1.29E-01	5.26E-04	0.2829	0.9552	0.5864
		0.5	2.22E-04	4.15E-02	3.15E-02	2.47E-03	8.13E-03	9.42E-02	2.66E-04	0.2839	0.8914	0.5309
	8	0.05	2.06E-04	4.30E+01	1.79E-02	2.11E-04	7.97E-02	2.98E-01	2.27E-02	0.8801	0.9242	0.8812
		0.25	1.30E-04	1.50E+00	1.54E-02	2.11E-04	5.34E-02	2.56E-01	1.18E-02	0.8396	0.9273	0.8657
		0.5	3.10E-06	2.34E-01	2.80E-02	2.34E-04	3.76E-02	2.86E-01	7.23E-03	0.7851	0.9462	0.8651
	32	0.05	1.06E-01	2.04E+01	1.25E-01	3.19E-05	1.44E-01	2.41E-01	2.38E-01	**Type 1**		0.9530
		0.25	8.14E-04	6.24E-01	2.87E-02	5.54E-05	6.53E-02	3.36E-01	1.55E-02	**Type 1**		0.9601
		0.5	5.90E-03	3.18E-01	4.29E-02	5.38E-04	1.22E-01	3.67E-01	3.02E-02	**Type 1**		0.9500

Table 5 Mean of squared errors of 24 images for Templates 3.

Gene. routine	range	prop of 1	Method 1 Least Squares	Method 2 Full Indep.	Method 3 Cond. Indep.	Method 4 Perm. of Ratios	Method 5 CVD (w=0.5)	Method 6 Modified LS	Method 7 Modified LS	cor1	cor2	True Cor. Coeff.
sisim	0	0.05	6.88E-05	5.41E-05	4.91E-05	5.45E-05	2.53E-05	4.95E-05	3.38E-05	0.1169	0.9685	-0.0011
		0.25	3.36E-05	2.93E-05	8.98E-05	3.09E-05	4.76E-05	2.28E-04	2.83E-05	0.0379	0.5589	0.0111
		0.5	1.00E-07	1.00E-07	1.84E-05	1.00E-07	2.00E-07	4.00E-07	2.00E-07	**Type 1**		-0.0092
	2	0.05	5.71E-03	2.79E-01	8.60E-02	1.86E-03	5.32E-02	1.32E-01	2.57E-02	**Type 1**		0.0214
		0.25	1.21E-03	6.43E-03	3.28E-03	5.48E-04	2.42E-02	8.44E-02	1.09E-02	**Type 1**		0.0183
		0.5	5.63E-05	1.43E-03	1.25E-03	3.41E-04	1.13E-02	5.56E-02	4.84E-03	-0.0689	0.0919	0.0049
	8	0.05	2.02E-05	8.39E+00	9.82E-04	1.32E-03	6.95E-02	2.74E-01	2.91E-05	0.4014	0.6119	0.4989
		0.25	4.48E-05	4.40E-01	3.94E-05	1.48E-04	3.23E-02	2.09E-01	2.24E-04	**Type 2**		0.5399
		0.5	1.49E-05	1.01E-01	3.56E-04	3.08E-05	1.71E-02	2.03E-01	2.29E-04	**Type 2**		0.5563
	32	0.05	1.20E-04	2.20E+01	4.32E-03	1.47E-04	7.45E-02	2.93E-01	1.31E-02	0.7861	0.8331	0.7938
		0.25	5.87E-05	1.32E+00	1.73E-03	6.44E-05	4.75E-02	2.49E-01	5.74E-03	**Type 2**		0.7479
		0.5	5.00E-07	3.00E-01	3.31E-03	3.53E-05	4.10E-02	2.90E-01	5.95E-03	0.7909	0.8526	0.8215
ellipsim	0	0.05	1.26E-04	1.77E-04	2.99E-04	1.75E-04	3.48E-04	7.14E-04	2.60E-04	-0.5521	0.1536	-0.0122
		0.25	1.20E-06	1.20E-06	1.34E-05	1.20E-06	6.50E-06	1.90E-05	4.40E-06	-0.2559	-0.0066	0.0061
		0.5	4.14E-05	4.06E-05	1.32E-04	4.14E-05	5.71E-05	1.15E-04	4.82E-05	-0.0056	.0.211	0.0211
	2	0.05	4.03E-03	1.38E+01	3.94E-04	8.27E-05	6.63E-02	2.79E-01	5.23E-03	**Type 1**		0.4121
		0.25	9.45E-04	2.32E-01	1.97E-04	2.91E-05	2.84E-02	1.72E-01	2.41E-03	**Type 1**		0.3500
		0.5	8.24E-05	2.61E-02	8.36E-04	1.38E-04	1.17E-02	1.19E-01	1.18E-03	0.2841	0.3529	0.3176
	8	0.05	2.97E-04	4.18E+01	3.28E-05	1.55E-05	8.31E-02	3.15E-01	1.84E-02	**Type 1**		0.8287
		0.25	3.15E-04	1.48E+00	1.23E-04	7.02E-05	5.29E-02	2.68E-01	9.87E-03	**Type 1**		0.8125
		0.5	8.55E-05	2.22E-01	5.02E-04	7.72E-05	3.49E-02	2.98E-01	5.46E-03	**Type 1**		0.8035
	32	0.05	6.92E-03	4.03E+01	2.45E-02	6.37E-03	7.38E-02	2.89E-01	5.09E-02	**Type 3**		0.9647
		0.25	7.70E-05	6.30E-01	1.34E-04	6.06E-05	7.42E-02	3.45E-01	1.32E-02	**Type 1**		0.9467
		0.5	9.69E-04	2.45E-01	3.20E-03	1.00E-03	6.18E-02	3.70E-01	1.24E-02	0.8976		0.9654

Table 6 Rank of the methods(from 1 to 7, best to worst).

Template	Generate routine	Range	Prop. of 1	Method 1 Least Squares	Method 2 Full Indep.	Method 3 Cond. Indep.	Method 4 Perm. of Ratios	Method 5 Deutsch (w=0.5)	Method 6 Modified LS	Method 7 Modified LS
1	sisim	0	0.05	7	5	1	6	3	2	4
			0.25	2	4	1	3	5	6	7
			0.5	2	4	1	3	5	7	6
		2	0.05	2	6	1	4	5	7	3
			0.25	1	3	6	2	5	7	4
			0.5	1	3	6	2	5	7	4
		8	0.05	1	7	5	3	4	6	2
			0.25	1	7	5	3	4	6	2
			0.5	1	5	7	3	4	6	2
		32	0.05	1	7	6	2	4	5	3
			0.25	1	7	5	2	4	6	3
			0.5	1	6	7	2	4	5	3
	ellipsim	0	0.05	1	3	6	2	5	7	4
			0.25	3	2	4	1	6	7	5
			0.5	3	4	1	2	5	6	7
		2	0.05	1	7	5	3	4	6	2
			0.25	1	7	5	3	4	6	2
			0.5	1	5	7	3	4	6	2
		8	0.05	1	7	5	2	4	6	3
			0.25	1	7	6	2	4	5	3
			0.5	1	5	7	2	4	6	3
		32	0.05	1	7	6	3	4	5	2
			0.25	1	7	6	2	4	5	3
			0.5	1	5	7	3	4	6	2

2	sisim	0	0.05	1	3	4	2	6	7	5
			0.25	4	2	1	3	5	7	6
			0.5	4	2	1	3	5	7	6
		2	0.05	2	7	1	3	5	6	4
			0.25	1	6	2	3	5	7	4
			0.5	2	5	1	3	6	7	4
		8	0.05	1	7	4	3	5	6	2
			0.25	1	7	4	2	5	6	3
			0.5	1	6	4	2	5	7	3
		32	0.05	1	7	4	2	5	6	3
			0.25	1	7	4	2	5	6	3
			0.5	2	7	4	1	5	6	3
	ellipsim	0	0.05	4	2	1	3	6	7	5
			0.25	3	1	6	2	5	7	4
			0.5	3	4	1	2	5	7	6
		2	0.05	1	7	4	3	5	6	2
			0.25	1	7	5	3	4	6	2
			0.5	1	6	5	3	4	7	2
		8	0.05	1	7	3	2	5	6	4
			0.25	1	7	4	2	5	6	3
			0.5	1	6	4	2	5	7	3
		32	0.05	2	7	3	1	4	6	5
			0.25	2	7	4	1	5	6	3
			0.5	2	6	4	1	5	7	3
3	sisim	0	0.05	7	5	3	6	1	4	2
			0.25	4	2	6	3	5	7	1
			0.5	3	1	7	2	5	6	4
		2	0.05	2	7	5	1	4	6	3
			0.25	2	4	3	1	6	7	5
			0.5	1	4	3	2	6	7	5
		8	0.05	1	7	3	4	5	6	2
			0.25	2	7	1	3	5	6	4
			0.5	1	6	4	2	5	7	3
		32	0.05	1	7	3	2	5	6	4
			0.25	1	7	3	2	5	6	4
			0.5	1	7	3	2	5	6	4
	ellipsim	0	0.05	1	3	5	2	6	7	4
			0.25	1	3	6	2	5	7	4
			0.5	2	1	7	3	5	6	4
		2	0.05	3	7	2	1	5	6	4
			0.25	3	7	2	1	5	6	4
			0.5	1	6	3	2	5	7	4
		8	0.05	3	7	2	1	5	6	4
			0.25	3	7	2	1	5	6	4
			0.5	2	6	3	1	5	7	4
		32	0.05	2	7	3	1	5	6	4
			0.25	2	7	3	1	5	6	4
			0.5	1	6	3	2	5	7	4

Tables 3, 4, 5 and 6 show that the method of Least Squares is the best based on the means of squared error with respect to the reference values ("true" probabilities extracted from the training images), but it requires that the relationship between B and C is known . For the methods in Class II, method of Permanence of Ratios is the best.

According to the sum of MSE for different templates and different parameters, we rank the methods as shown in Table 7.

Table 7 Ranks of the methods for different templates and parameters.

Generate routine	Range	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7
		Least Squares	Full Indep.	Cond. Indep.	Perm. of Ratios	Deutsch (w=0.5)	Modified LS	Modified LS
Template	1	1	7	6	2	4	5	3
	2	2	7	4	1	5	6	3
	3	2	7	3	1	5	6	4
Generate rountine	SISIM	1	7	5	2	4	6	3
	ELLIPSIM	1	7	6	2	4	5	3
Range	0	1	3	4	2	6	7	5
	2	1	7	5	3	4	6	2
	8	1	7	5	2	4	6	3
	32	1	7	6	2	4	5	3
Probability of 1's	0.05	1	7	6	2	4	5	3
	0.25	1	7	5	2	4	6	3
	0.5	1	5	7	2	4	6	3
Total		1	7	5	2	4	6	3

From the tables, we see:

· Method of Full Independence is the worst for all templates because B and C are not fully independent in many cases, shown in Table 2. For template 3, the method assuming conditional independence of B and C gets better results than using the method in other templates. This is because B and C are close to conditionally independent in Template 3.

· As we see in Tables 3, 4 and 5, based on the mean squared error (MSE) with respect to the reference values, the performance of the Least Squares method is the best in 44 of 75 configurations, the method of Permanence of Ratio is the best in 14 of 75 configurations, Conditional Independence method is the best in 12 of 75 configurations, Full independence is the best in 3 of 75 and Method 7 is the best in 1 of 75. Deutsch's method is the best in 1 of 75 when ω_1 and ω_2 are set to 0.5's, which may get better results if we select other ω_1 and ω_2.

· For ranges equal to zero the best method varies widely between the methods, shown in Table 6 and the method assuming full independence of B and C gets better results than for other ranges.

· For Template 1, the predominantly best method is Least Square. For Template 2 and Template 3, Permanence of Ratios is the best but Least Squares almost has the same probability as Permanence of Ratios to be the best method.

· From Tables 3, 4 and 5, we also know that in case of valid Cor1 and Cor2 values, the true correlation coefficients of B and C are often between the range of cor1 and cor2.

In general, the best method seems to be Least Squares method in a very consistent way. The predominantly best method in Class II is permanence of ratios.

Conclusions

1 For the same image with different templates, the correlation of *B* and *C*, *B*/*A* and *C*/*A* are different:

—When the correlation coefficients between event *B* and *C* are close to zeros, method of Full Independence and Conditional Independence can often get good results.

—Method of Least Square is the best for many cases. Its drawback is that it needs to know the relationship between the two secondary data.

2 Method of Permanence of Ratios is the best when the relationship between the two secondary data is unknown. Its advantage is that does not require joint probabilities to be derived, nor does it require the marginal probabilities of *B* or *C*.

3 Method of Conditional Independence and method of Permanence Ratios are different. However, there is a relationship between the estimated conditional probabilities from the two methods.

4 There are four types of relationship between means of squared errors from Least Square and those from Permanence of Ratios.

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