Preservation of Multiple Point Structure when Conditioning by Kriging

Weishan Ren, Luciane Cunha and Clayton V. Deutsch

Department of Civil & Environmental Engineering,
University of Alberta

Abstract

The idea of conditioning by kriging is well known in theory and practice. It has been used for conditioning the realizations from unconditional simulation techniques such as the moving average and the turning bands simulation approaches. The basis of the conditioning by kriging approach is to use the same variogram for both the unconditional simulation and the kriging. In this paper, the focus is on using kriging for conditioning of more complex unconditional simulations. Unconditional simulated realizations with multiple point structure are generated for posterior conditioning. Two sets of data are used for kriging. After conditioning, the simulated values at data locations are the real data values, so the local data is honored. Beyond the range of correlation, the simulated values are the unconditional simulated values, which mean the multiple point structure can be preserved.

The results obtained in this work show that conditioning by kriging is a simple, easy and reliable way to account for data with complex multiple point structures. Both continuous and categorical variables are used to show the performance of the conditioning by kriging approach. Moreover, kriging with different sets of data demonstrates that the multiple point structures are well preserved after conditioning.

Introduction

Conditioning by kriging is not a new concept in geostatistics. It has been used for conditioning the moving average and the turning bands simulations. It is an approximation of the conditional simulation. Because the conditional simulation is quite feasible and widely accepted, the conditioning by kriging has little used in practice.

When working on reservoir characterizations, we often need to deal with the complex features. Two-point statistical simulation seems not good in preserving the complex features. So multiple-point statistic is usually needed. However, multiple-point statistic is very complicated and always takes very long time. Object-base simulation can also be used to model the complex features. But conducting conditional simulation with complex features takes much longer time than the unconditional simulation. If conducting the unconditional simulation, then apply the conditioning by kriging to obtain the conditional simulation, the conditional simulation with complex features can be easy and fast. In this report, the focus is on the conditioning by kriging. The unconditional simulation results of Fluvsim are used to show how the multiple point structures can be preserved in the conditioning by kriging. The performance of conditioning by kriging will be shown in both continuous and categorical variables cases.

Theory of Conditioning by Kriging:

Suppose there are N real data, and a p x N data domain need to be simulated. The conditioning by kriging consists of the following successive steps:

1. Carry out an unconditional simulation to obtain the unconditional simulated values $Z_{uc}(x)$.
2. Carry out kriging using the N conditioning real data to obtain $Z_{kr}(x)$.
3. Carry out kriging using the unconditional simulated values at these N data locations to obtain $Z_{kr\text{-}u}(x)$.
4. Calculate the conditional simulation values for each block:

$$Z_{cs}(x) = Z_{uc}(x) - [Z_{kr\text{-}u}(x) - Z_{kr}(x)].$$

This equation implies that at each real data location, the unconditional simulated value is taken out, and the conditioning datum is put in. Near the location, the kriging linear estimators smoothed the change between the real data and the unconditional simulated values outside the range of kriged values. Therefore, after the conditioning, the conditional simulated values at these N data locations will exactly be the real data values. Beyond the range of correction, the conditional simulated values will be the unconditional simulated values. Note that these steps need to be carried out in the Gaussian environment.

The two kriging (steps 2 and 3) can be combined to perform only one kriging using N data differences between the real data and the unconditional simulated values. Then the conditioning can be simply expressed as:

$$Z_{cs}(x)= Z_{uc}(x) + D_{kr}(x)$$

where $D_{kr}(x)$ is the data of the kriging using the residual of the real data values subtracting the unconditional simulated values. It can be interpreted as that the conditioning by kriging is adding correction areas to unconditional simulations based on the differences between real data and unconditional simulated values. This can be easily seen in the Figure 1. These three examples are constructed using the categorical variables. The unconditional simulation realizations and the string of conditioning data are shown together in the left images. The results of conditioning by kriging are shown in the right. The channel is in green, and the overbank is in blue. For the places in overbank where the conditioning data shows there should be a channel, certain size of channel is added by conditioning. In the examples one and two, the channels are added beside the original channels so that the results look like the original channel is dilated. In the contrary, for the places where the conditioning data shows there should be no channel, erosion takes place.

The continuous and categorical variables are used to show the performance of the conditioning by kriging approach. The real data can be obtained from continued cores in a single well or from different wells. Therefore, for each variable, the conditioning data will use a string of data or scattered data.

Continuous Variable Cases:

1. Conditioning with a string of data

A string of data used for the conditioning is shown in Figure 2. The real data actually are only one pixel wide. In order to show them clearly in a plot, the string of data was extended to five pixels. The data domain is 100 x100. The unconditional simulation was implemented using the fluvsim to generate realizations with channels and overbank deposits. Within the channels and overbank deposits, separated sgsim was implemented to generate the realizations for each deposit. The four unconditional simulation realizations are shown in Figure 3. These curvilinear channels are laying out in the vertical direction. The variograms in the vertical and horizontal directions of the first unconditional simulation realization (the top left image in Figure 3) were calculated and plotted in Figure 4. The *varfit* program was used to model the variograms, and the variogram models are used in the kriging. The residual data is calculated by subtracting the unconditional simulated values from the real data values. Therefore, using these residual data, only one kriging is implemented. Then the kriging values are added to the unconditional simulated values to calculate the conditional simulation values. All of the calculations are carried out in the Gaussian space.

The conditional simulation realizations are shown in Figure 5. For comparing, the four unconditional simulation realizations are plotted in the left column of Figure 5. It can be seen that near the centerline of the realizations, there are some difference created by the conditioning. Beyond that, they are exactly same. Apparently, the multiple-point structures are well preserved.

The histograms of the unconditional simulated data, the residual data, the kriging data, and the conditional simulated data are plotted in Figure 6 to check the data distributions before and after the conditioning. The distributions of the unconditional simulated data and the conditional simulated data are nearly identical. The mean, std. dev. and minimum values are almost same. The largest difference is the maximum values changed from 3.05 to 3.52. These data are normal scores. Although their means are not zero and std. dev. is not one, they are stationary in Gaussian space.

2. Conditioning with scattered data

The scattered data used for the conditioning by kriging are shown in Figure 8. The data are taken at ix/iy grid node indices of 10, 30, 50, 70, and 90. The same approach applied for conditioning the four unconditional simulation realizations. Both the conditional simulation results and the unconditional simulation results are shown in Figure 9. It also indicated that the multiple-point structures are well preserved.

Categorical Variable Cases:

3. Conditioning with a string of data

Same as continuous variable case, a string of data is used for the conditioning (Figure 10). The data values are categorical values of 1 and 0. The channel categorical value is 1 and showing in red color. The overbank categorical value is 0 and showing in yellow. The real data is only one pixel wide at index 50 in a 100 by 100 pixel image. And it is also expended to five pixels to show them better. The four unconditional simulation realizations generated from Fluvsim are shown in Figure 11. The data are also categorical values of 1 and 0. These multiple point structures are actually same as the structures in the realizations of continuous variable in Figure 3. These curvilinear channels are laying out in the vertical direction. The variograms in the vertical and horizontal directions of the first unconditional realization were calculated, and then modeled by the *varfit* program. These models are used in the kriging.

Without using the despike and normal transform:

When calculating the residual data from these categorical values, the resulting values can be 0, 1, and –1. But after kriging, the kriged values are not categorical values. So they need to be truncated to categorical values of 0, 1, and –1. The dividing values for the truncation are –0.5 and 0.5. Values between 0.5 and – 0.5 is assigned to be 0, above 0.5 is 1 and below –0.5 is –1. The truncated values are added to the unconditional simulated values to get the conditional simulation data. Both the conditional simulation realizations and the unconditional simulation realizations are shown in Figure 12. The models on the right match the conditioning data, but the added channels do not have the non-linear shape so that the conditional simulation realizations show some artifacts.

The categorical values are not normal scores. So all of the calculations are not in Gaussian space. Because the conditioning should be done in Gaussian space, the despike and normal transform should be used.

Using the despike and normal transform:

The despike is used to change the same categorical values into slightly different values so that the 1 : 1 normal score transformation can be achieved. The categorical values of the unconditional simulated data and the conditioning data are despiked, and transformed into normal scores. So the residual data are calculated in Gaussian space. After kriging, the kriged data are added to the unconditional simulated data to get the conditional simulated data. These values are normal scores but not categorical values. So they need to be truncated into categorical values of 1 and 0. Note the difference here is the truncation is applied to the conditional simulated data not kriging output data. The dividing value is selected to be 0.25 based on the percentage of channels in the gaussian distribution of the conditional simulated data.

The conditional simulation realizations are shown in the Figure 13. There are still some artifacts but looks better than the conditional realizations without using despike and normal transformation. And apparently, the non-linear structures are well preserved.

The effect of the proportion of channel in unconditional realizations on the conditioning:

The proportion of channel facies in unconditional simulation realizations may affect the performance of the conditioning by kriging. The channel percentage in the previous unconditional simulations is 40%. To see the effect of the percentage, three other percentages are also used in Fluvsim to generate the unconditional simulation realizations. They are 20%, 60%, and 80%. All of the steps in conditioning are same as for the 40% channel case.

The conditioning data for these three percentage cases and for the 40% channel case are shown in Figure 14. The unconditional and conditional simulation realizations for the three percentage cases are shown in Figures 15 - 17.

All of the four cases are compared as follows:

Channel 20% (Figure 15):

Near the center line of the realizations, there are obviously some artifacts. Beyond that, they are exactly same. So the multiple-point structure are well preserved.

Channel 40% (Figure 12):

Results look better than 20% one. Although there are still some artifacts, especially in the middle of the second realization, the multiple-point structure are well preserved.

Channel 60% (Figure 16):

Conditioning results look much better. Can not see any artifacts. The multiple-point structure are well preserved.

Channel 80% (Figure 17):

Hard to see any thing. the multiple-point structure are well preserved

4. Conditioning with scattered data

The scattered data used for the conditioning by kriging are shown in Figure 18. The data are taken at ix/iy grid node indices of 10, 30, 50, 70, and 90. The same approach applied for conditioning the four unconditional simulation realizations. Both the conditional simulation results and the unconditional simulation results are shown in Figure 19. It also indicated that the multiple-point structures are well preserved.

Conclusions

Conditioning by kriging is correct when dealing with unconditional realizations based on the simple kriging principle. The conditional realizations respect the data and have no artifacts of the conditioning data. Using kriging to condition realizations that are generated by more complex simulation algorithms has been demonstrated with limited success. The shape/structure of the changes near the conditioning data respects mostly the variogram. The greater the change required, the greater the influence of the variogram and the poorer the complex structure is preserved. The algorithm would work well when the changes are minimal (such as a near solution with annealing) or when the features are reasonably captured by the variogram.

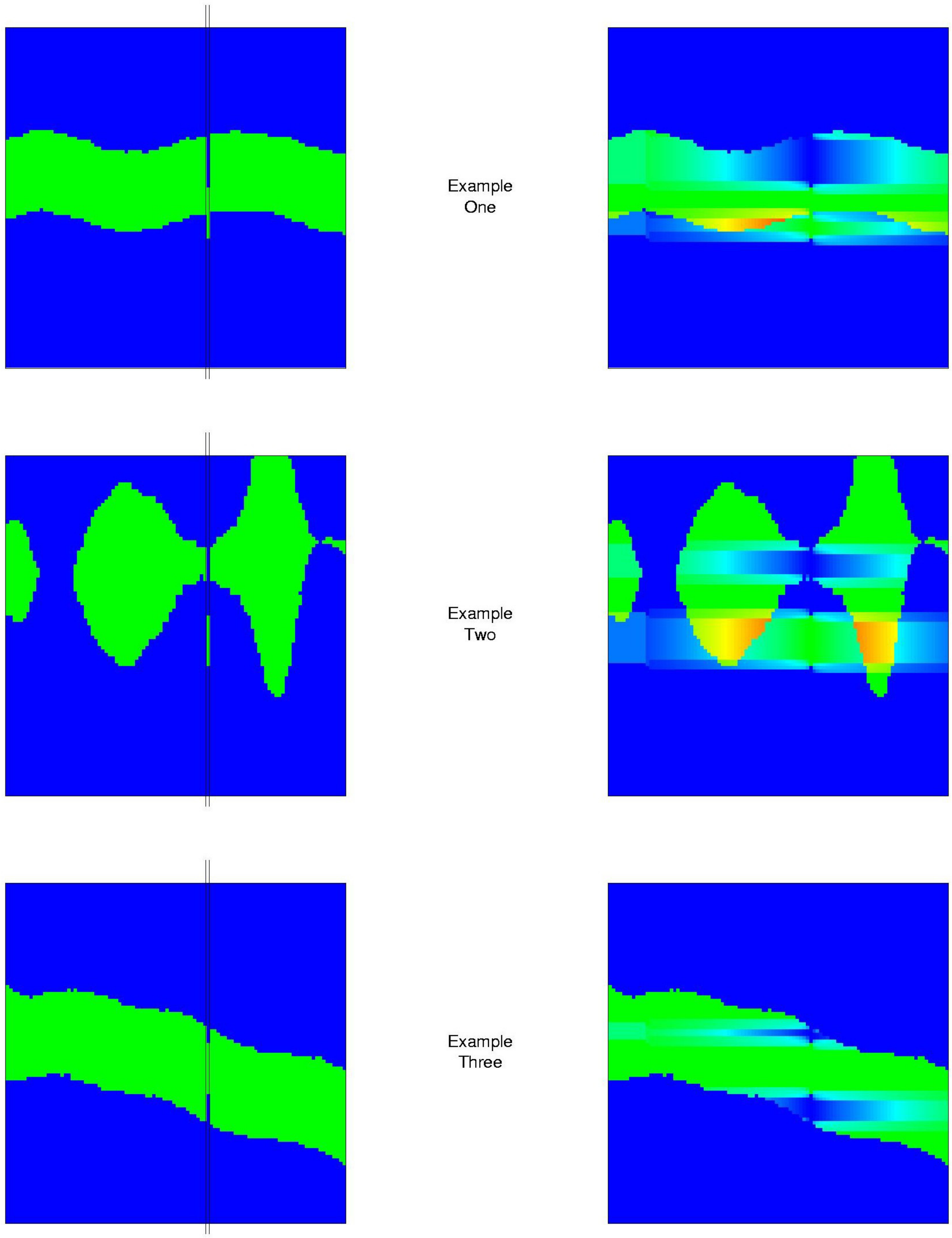

Figure 1: Three examples for the conditioning by kriging. The unconditional simulated data and the string of conditioning data are shown in the left. The results of conditioning are shown in the right. The channel is in green, and the overbank is in blue. See how the erosion and the dilation take place.

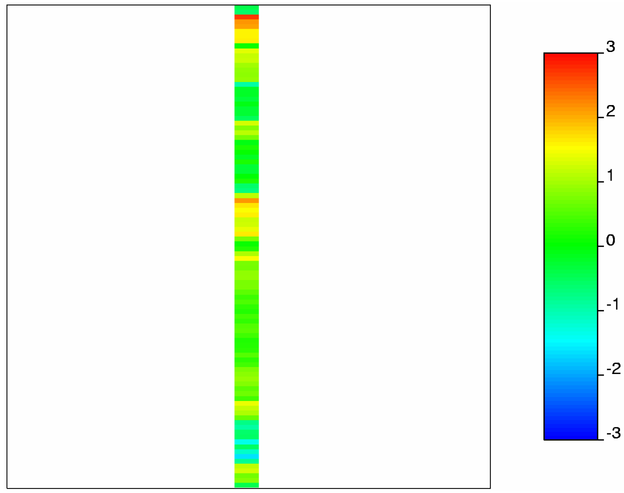

Figure 2: A 1-D string of conditioning data expanded to five pixels. The real data is one pixel wide at index 50 in a 100 by 100 pixel image. The normal scale will be used throughout.

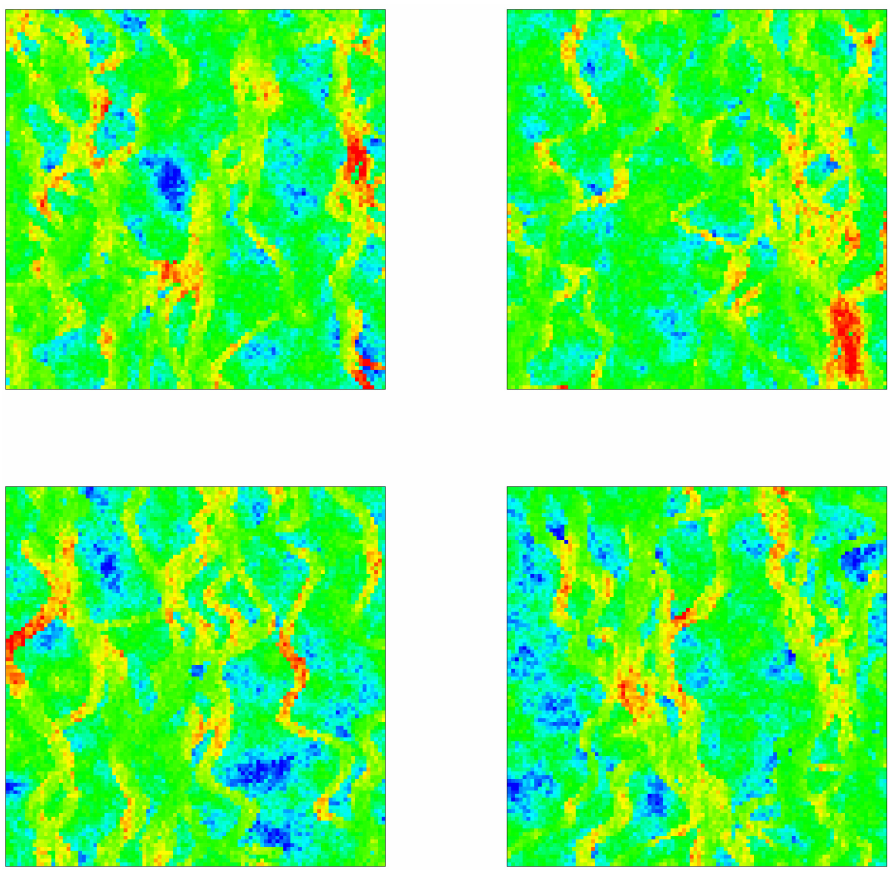

Figure 3: Four unconditional realizations generated by `fluvsim` and then separate `sgsim` runs within the channel and overbank deposits**.**

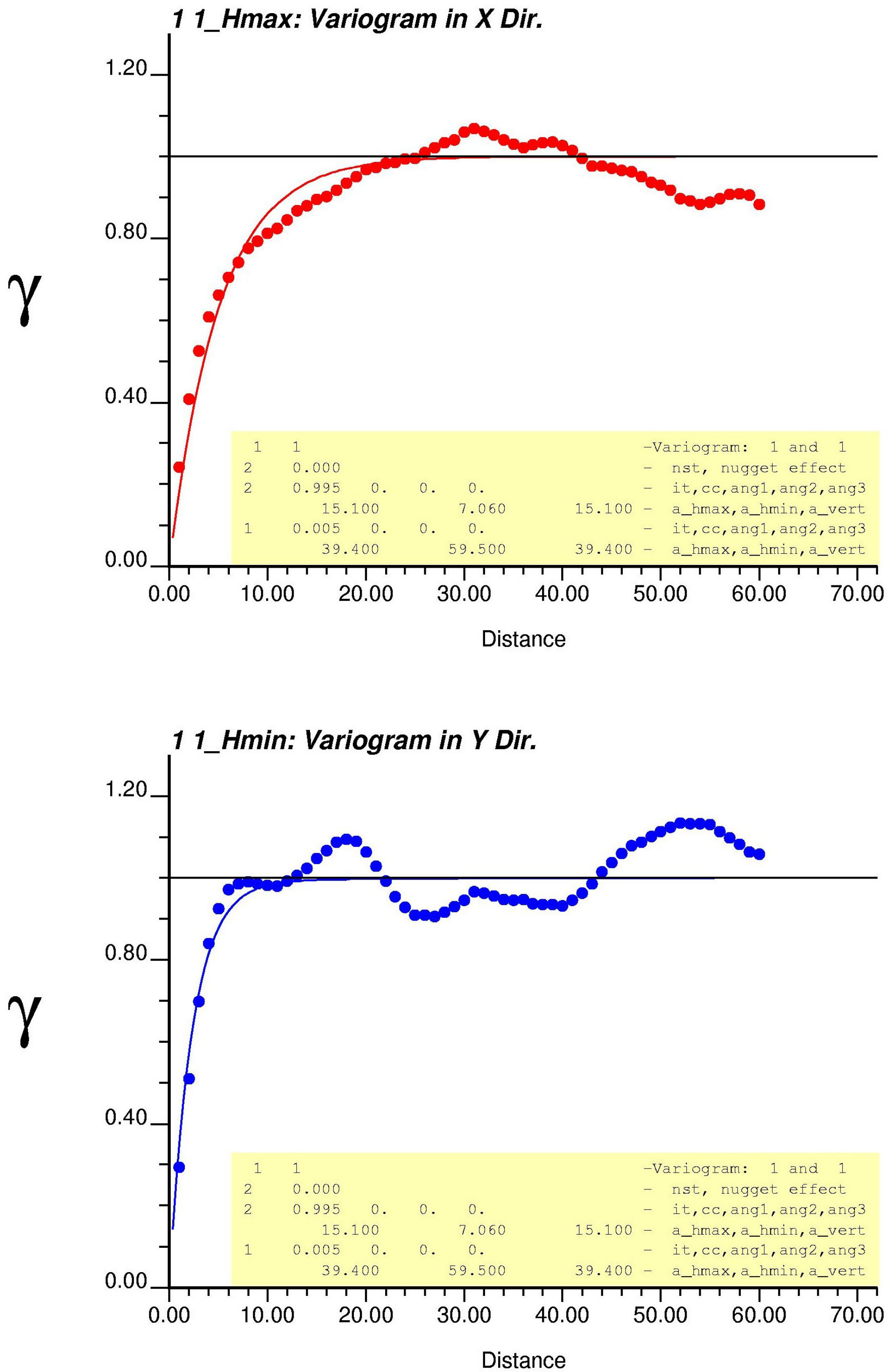

Figure 4: Variograms of the first unconditional realization (Figure 2) in the two main directions. The automatic fit of `varfit` will be used for the kriging.

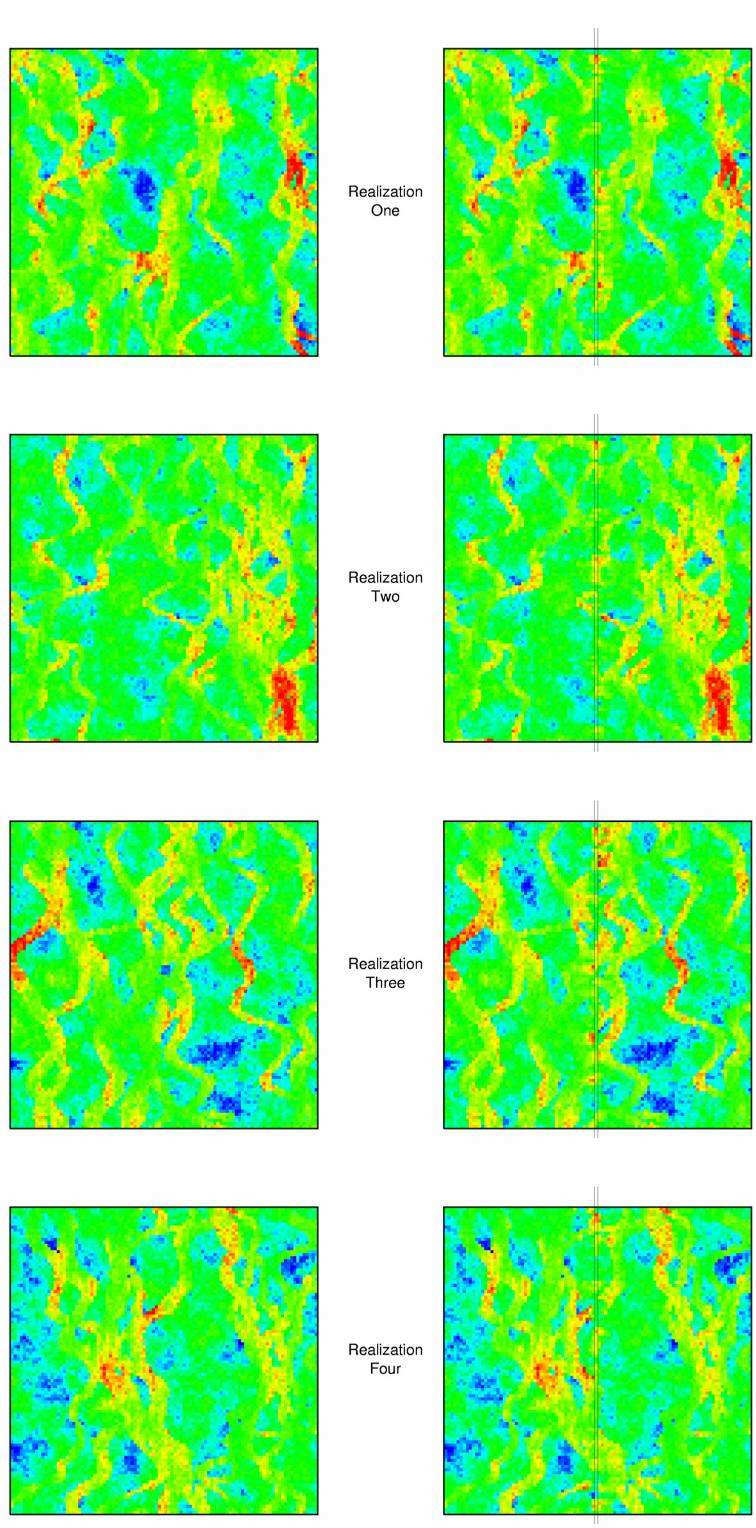

Figure 5: The four unconditional realizations (see Figure 2) conditioned by kriging. Note how the central data location matches exactly the data shown on Figure 1.

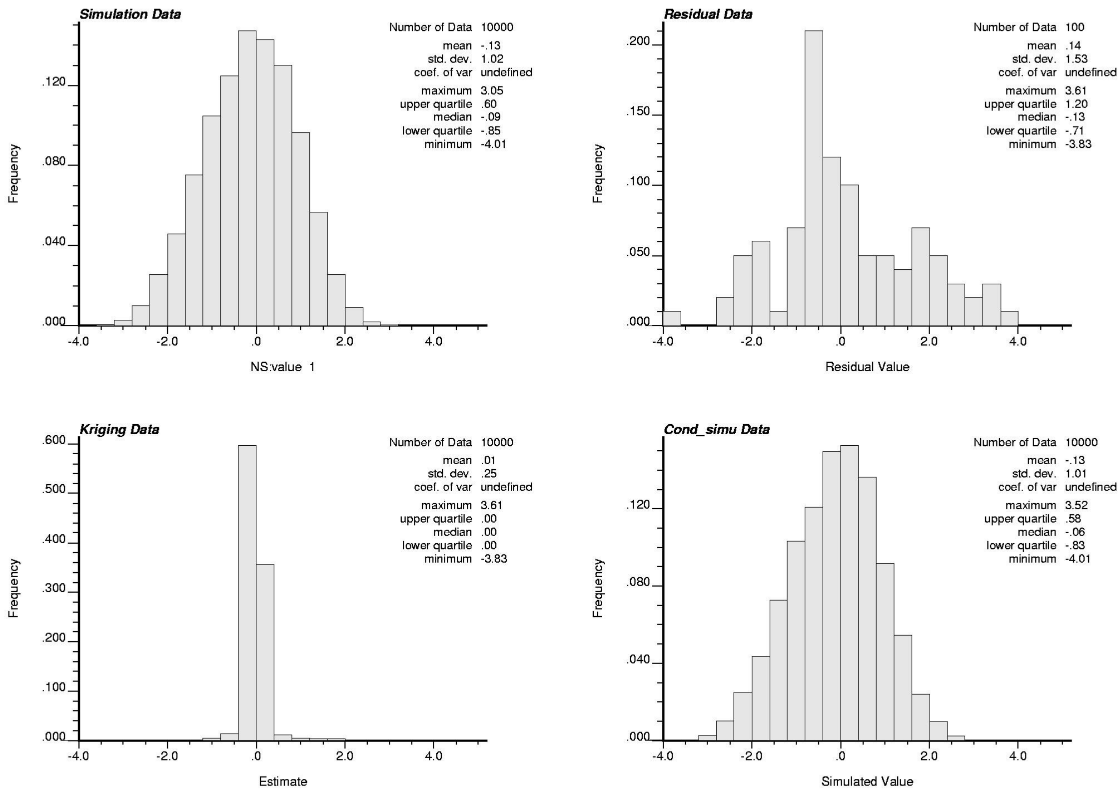

Figure 6: The histograms of the unconditional simulation data, residual data, kriging data and conditional simulation data. The mean, std. dev. and minimum values of simulation data and the cond-simu data are almost same. The only difference is the maximum values change from 3.05 to 3.52.

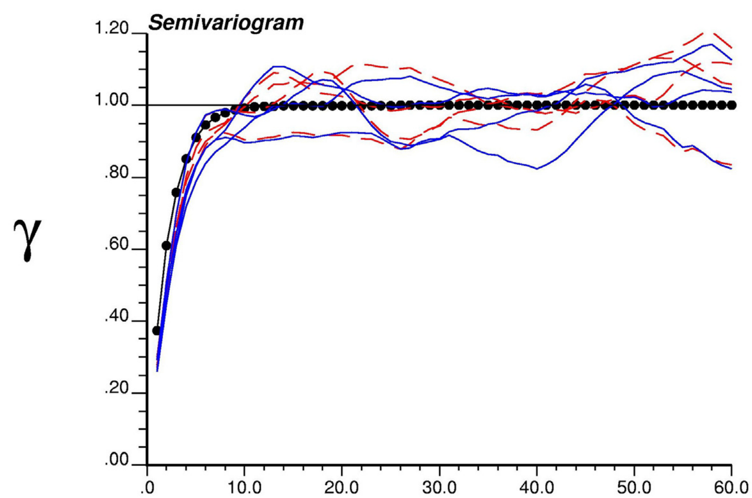

Figure 7: Conditional variograms using the central string of data, that is, the 50th column must come into the variogram calculation. The black line with points in the center is the model variogram. The four dashed (red) lines are the variograms from the unconditional realizations. The four solid (blue) variograms are from the conditional simulations.

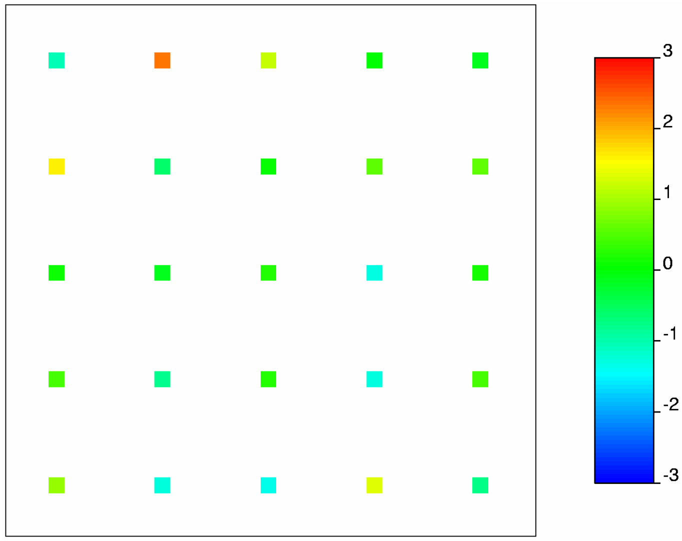

Figure 8: Scattered conditioning data for testing. The data are taken at ix/iy grid node indices of 10, 30, 50, 70, and 90.

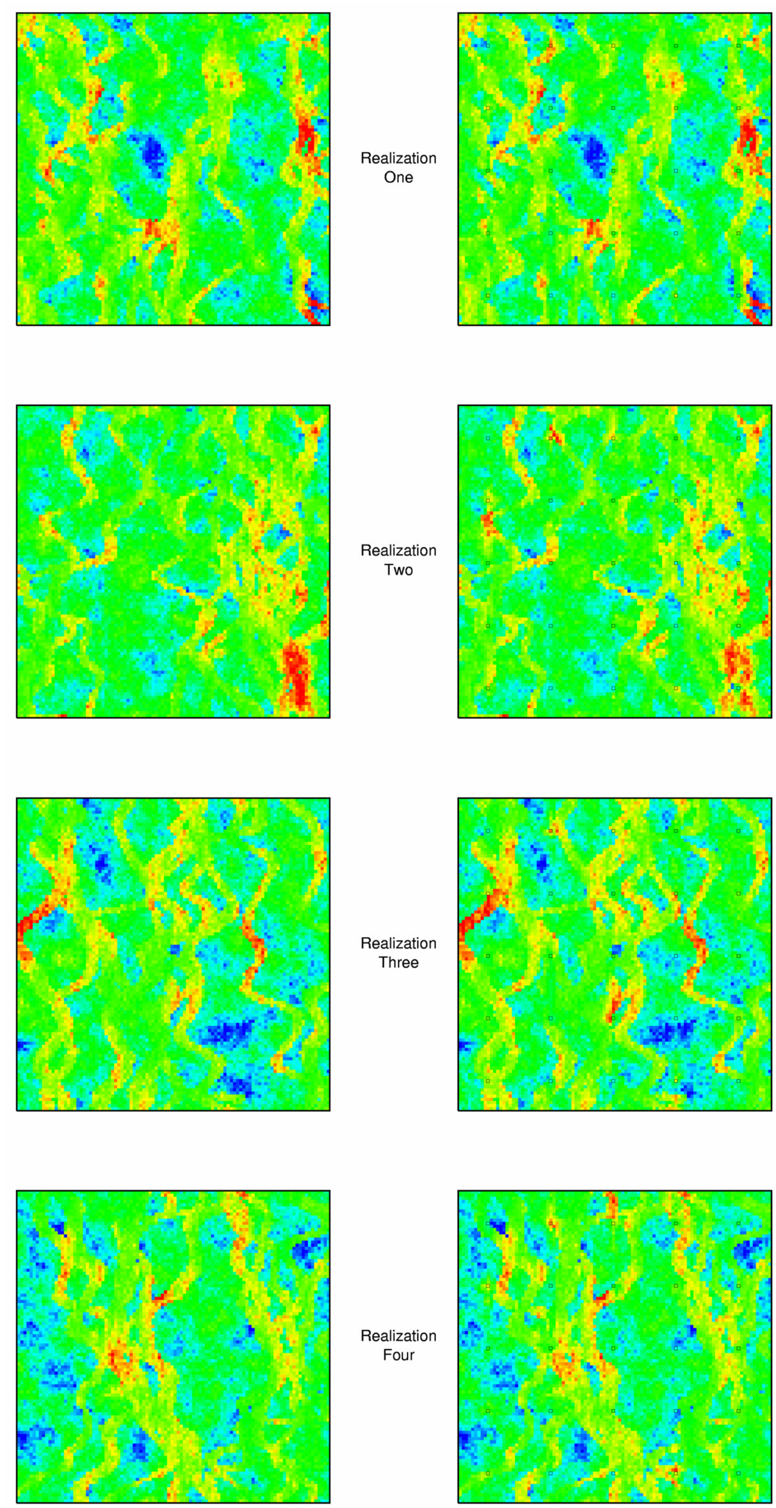

Figure 9: The four unconditional realizations (see Figure 3) conditioned by kriging to 25 scattered data. The models on the right match the conditioning data exactly and the non-linear structure is preserved quite closely.

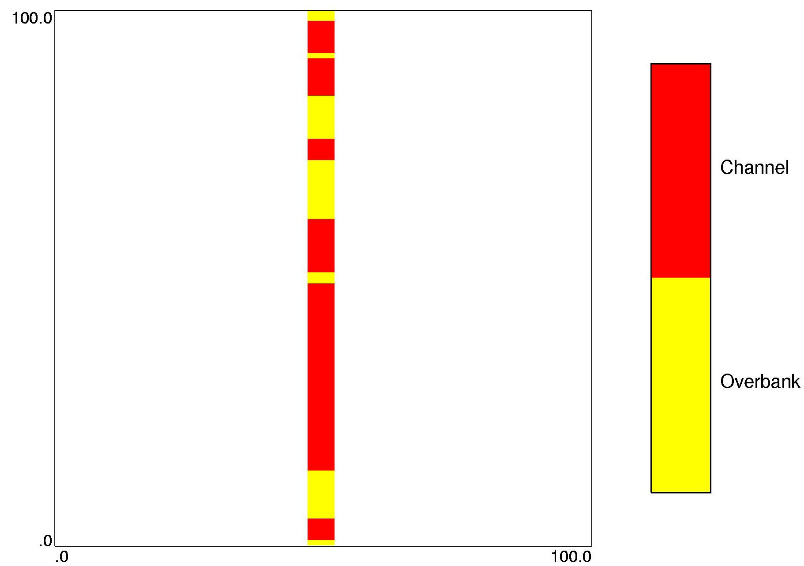

Figure 10: A string of conditioning data for the channel categorical example. Note the data is expanded to five pixels. The real data is one pixel wide at index 50 in a 100 by 100 pixel image.

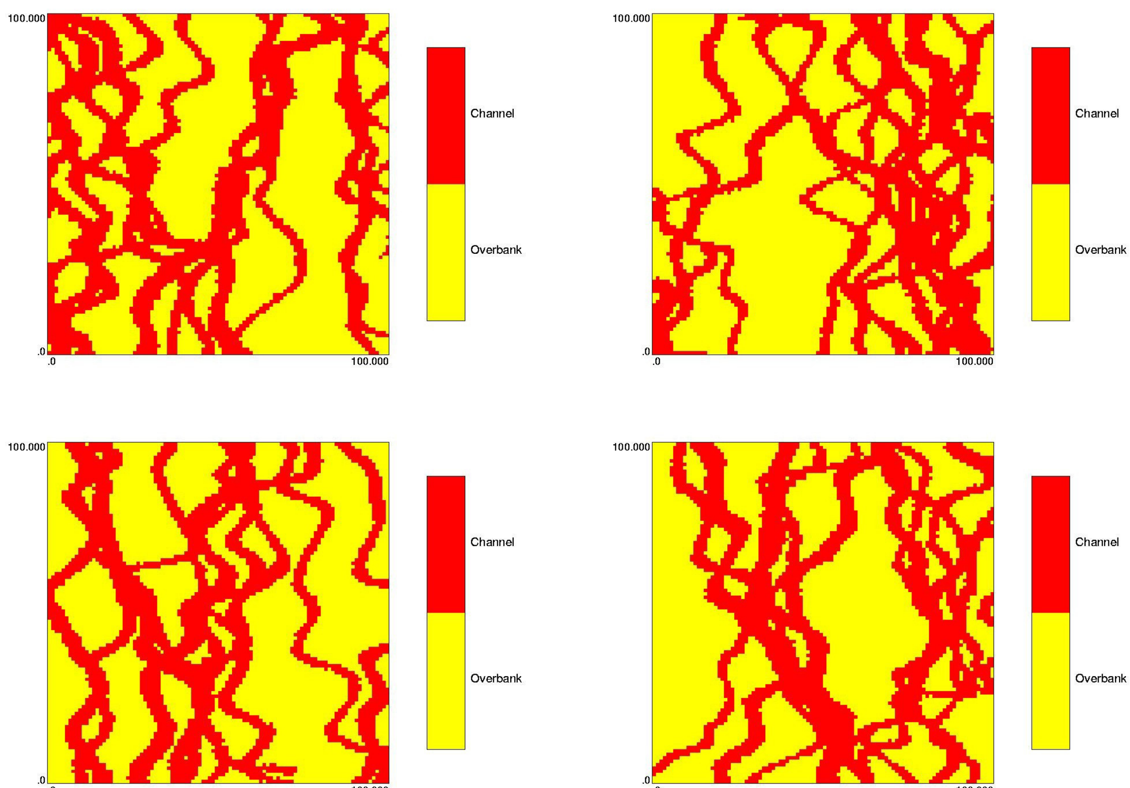

Figure 11: Four unconditional simulation realizations generated by `Fluvsim` for the categorical variables.

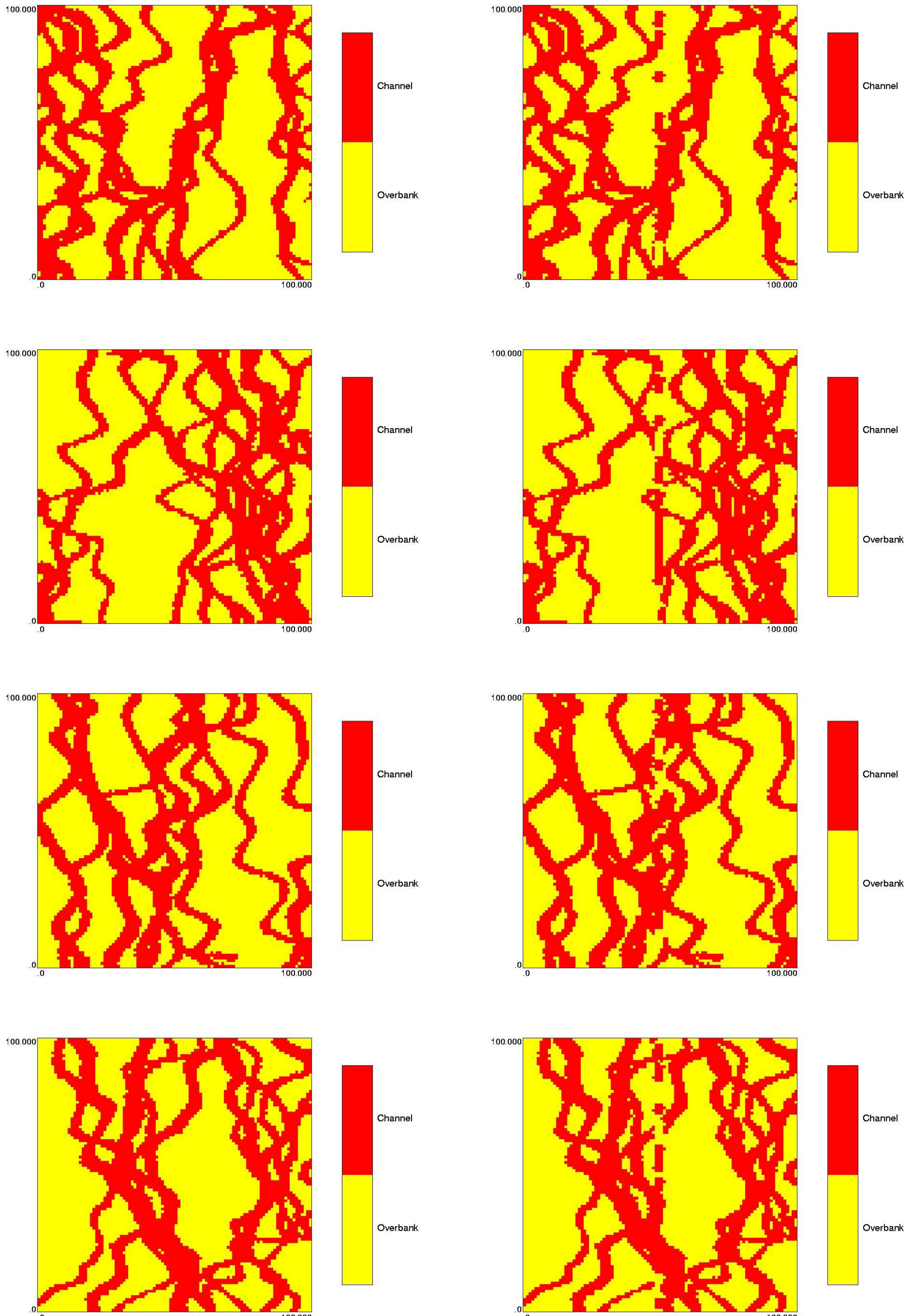

Figure 12: The four unconditional realizations conditioned by kriging with the string of data. The models on the right match the conditioning data with some artifacts, but the non-linear structure is preserved.

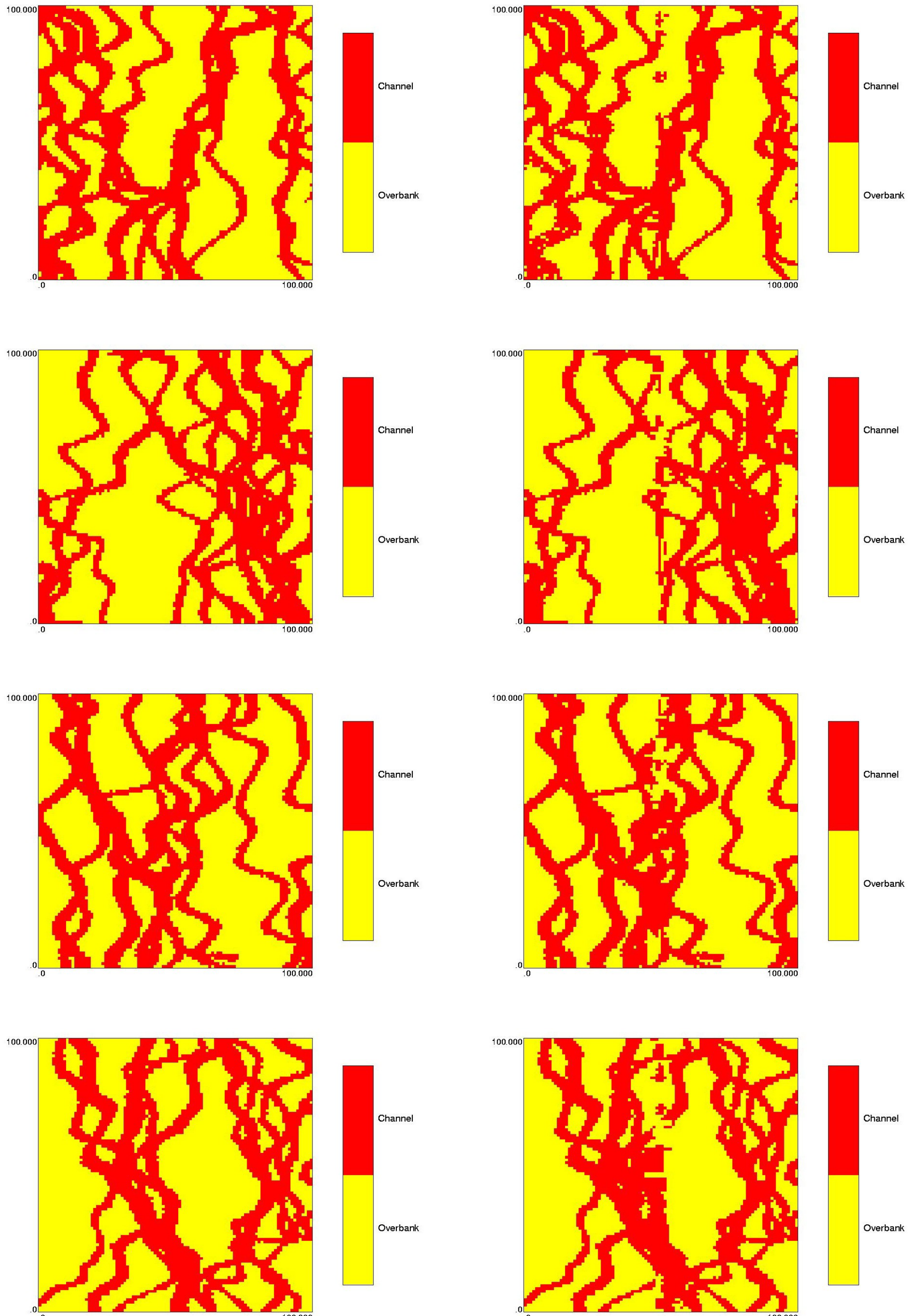

Figure 13: The four unconditional realizations conditioned by kriging with the string of data. Despike and normal transform are used. The models on the right match the conditioning data, and the non-linear structure is preserved.

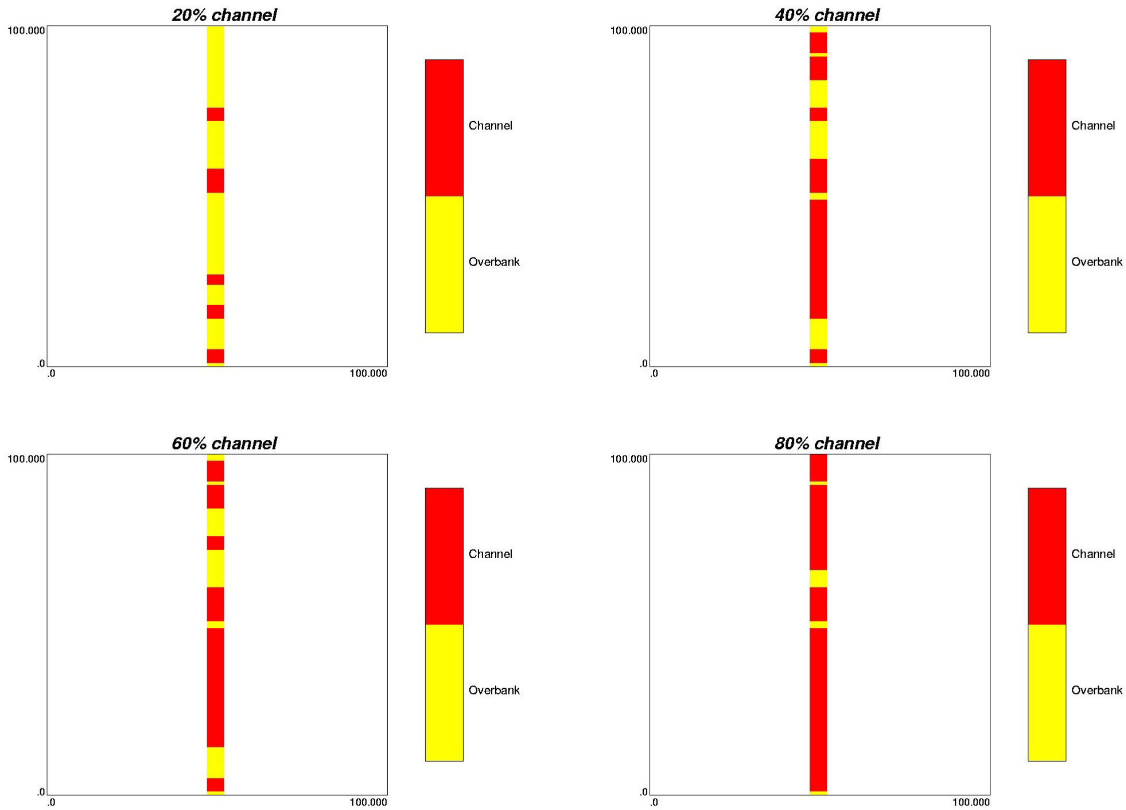

Figure 14: Conditioning data for the four channel categorical cases. The data is expanded to five pixels. The real data is one pixel wide at index 50 in a 100 by 100 pixel image.

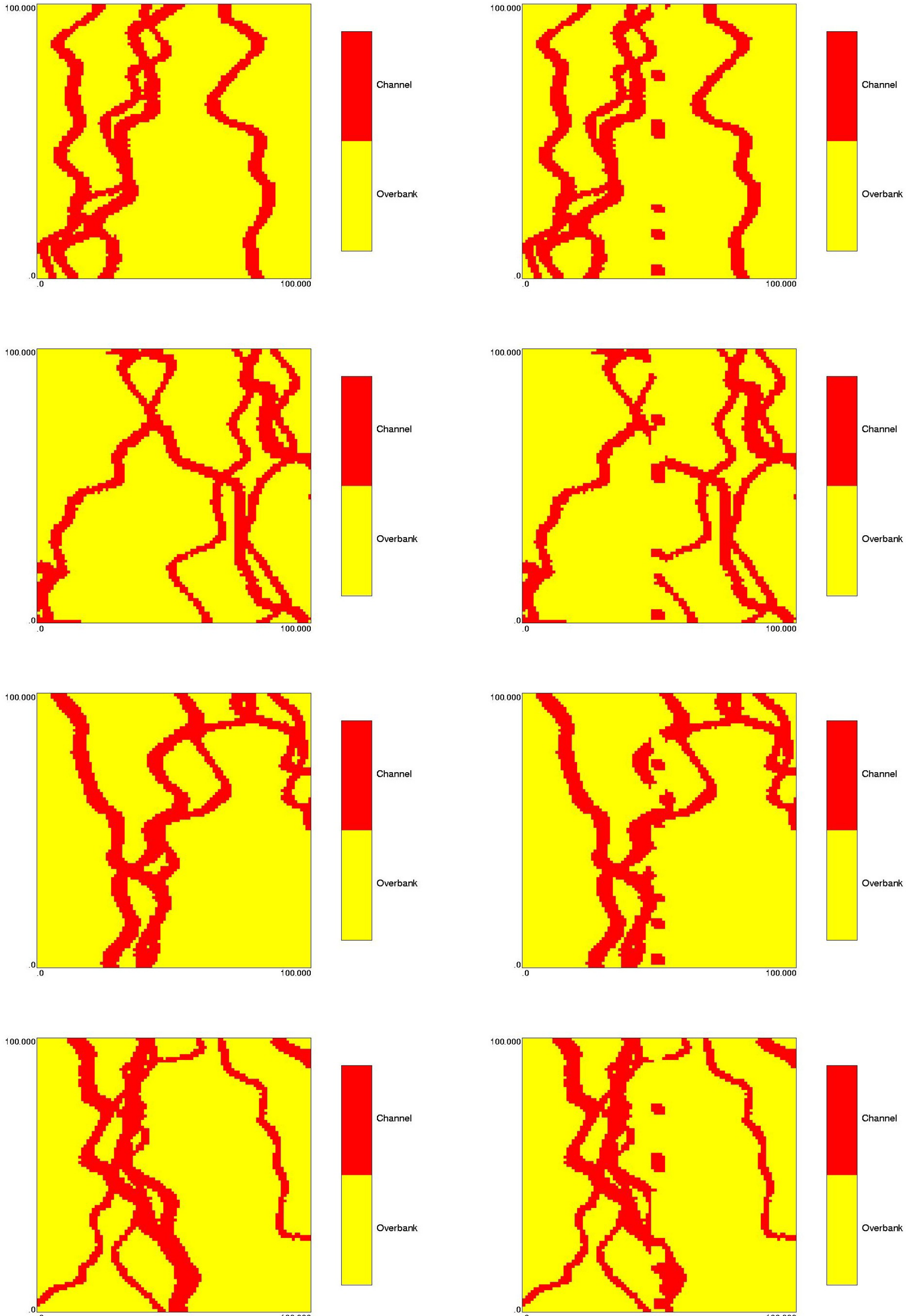

Figure 15: The four unconditional realizations with 20% channel conditioned by kriging with the string of data. The models on the right match the conditioning data but obviously have some artifacts. Beyond the center part, the non-linear structure is preserved.

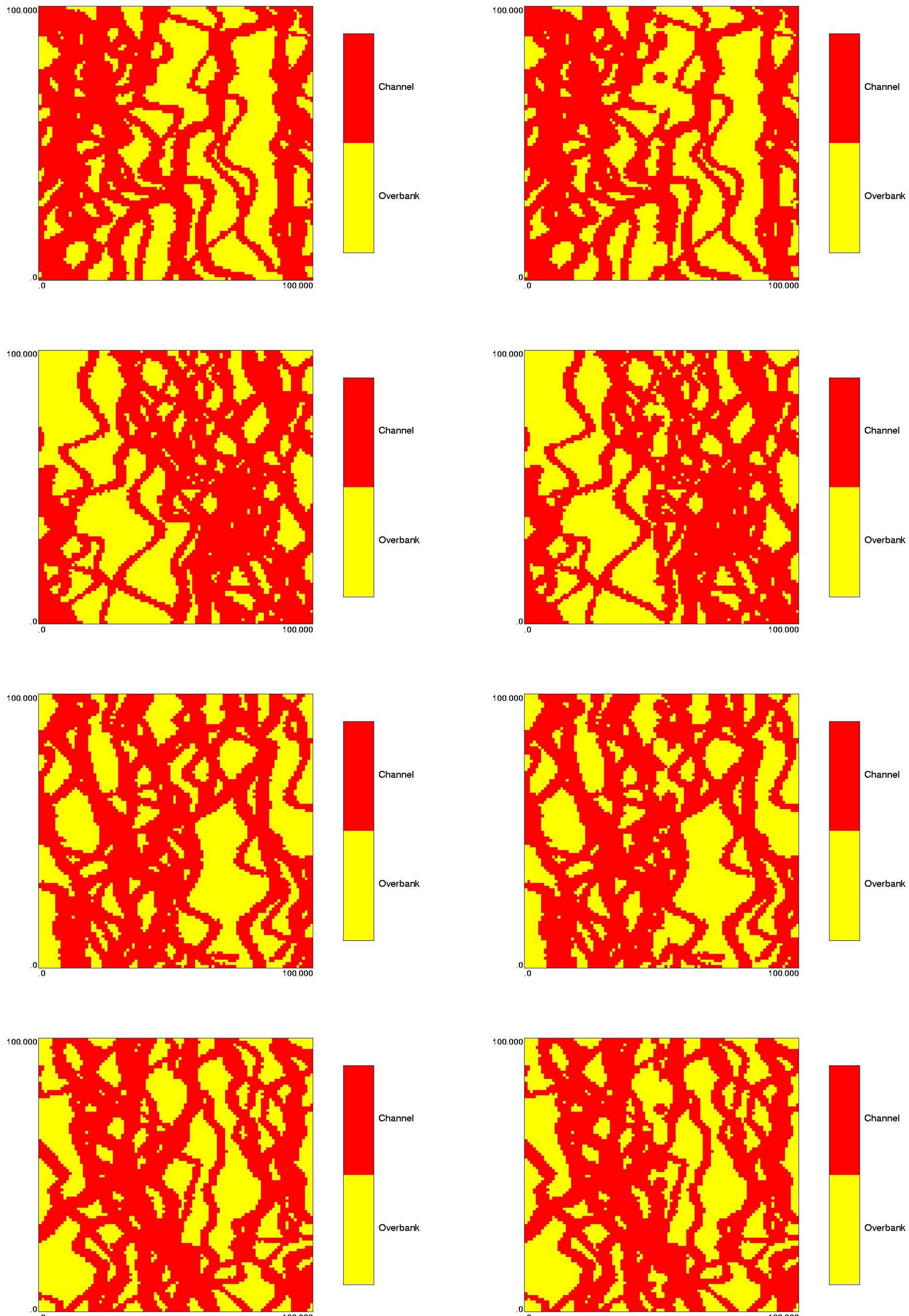

Figure 16: The four unconditional realizations with 60% channel conditioned by kriging with the string of data. The models on the right match the conditioning data, and the non-linear structure is preserved.

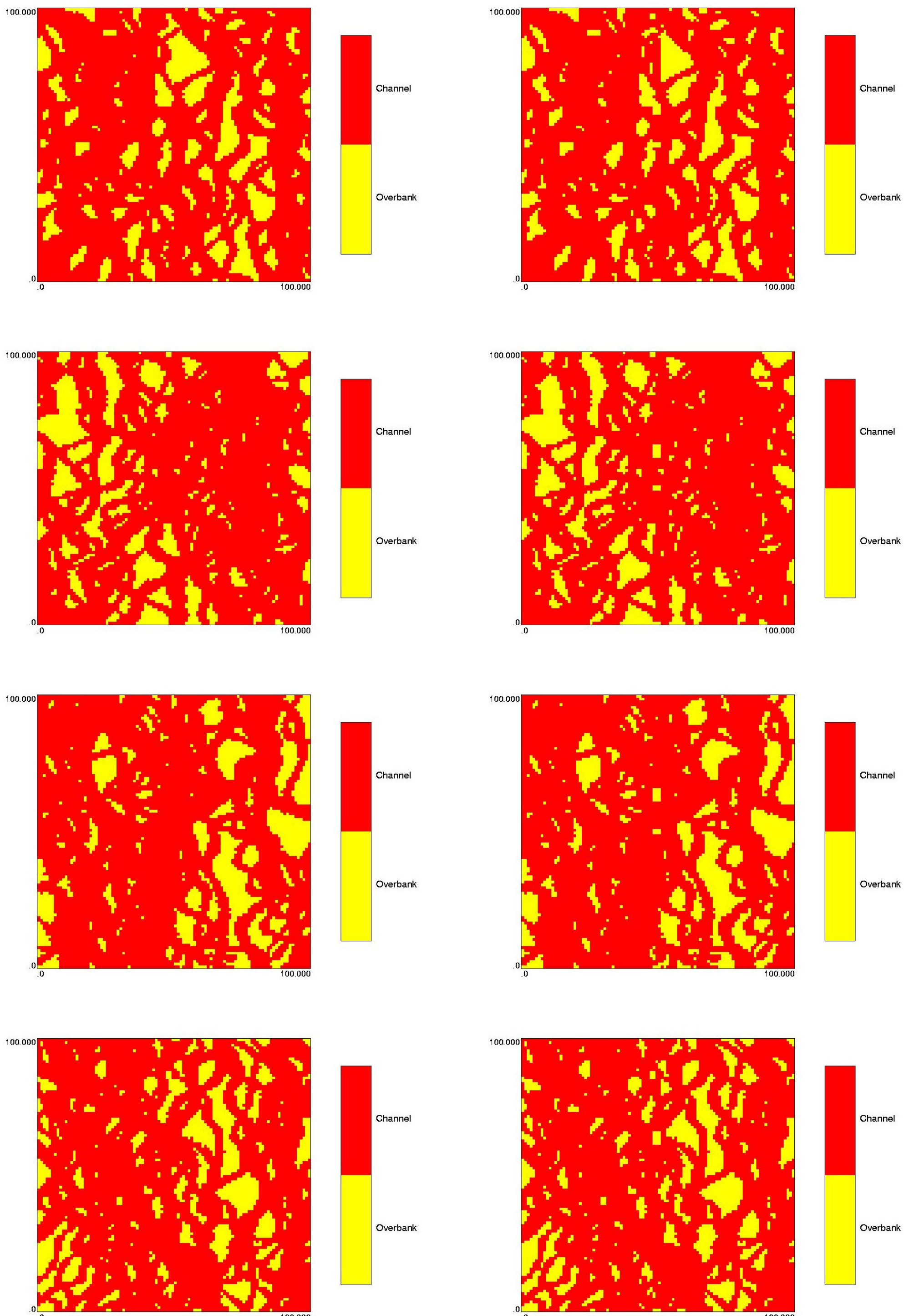

Figure 17: The four unconditional realizations with 80% channel conditioned by kriging with the string of data. The models on the right match the conditioning data. Hard to see any artifacts, and the non-linear structure is preserved.

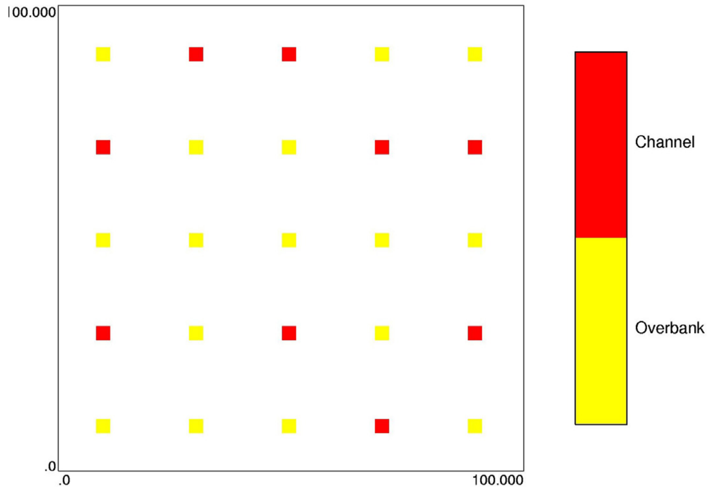

Figure 18: Scattered conditioning data for the channel categorical example. The data are taken at grid node indices of 10, 30, 50, 70, and 90.

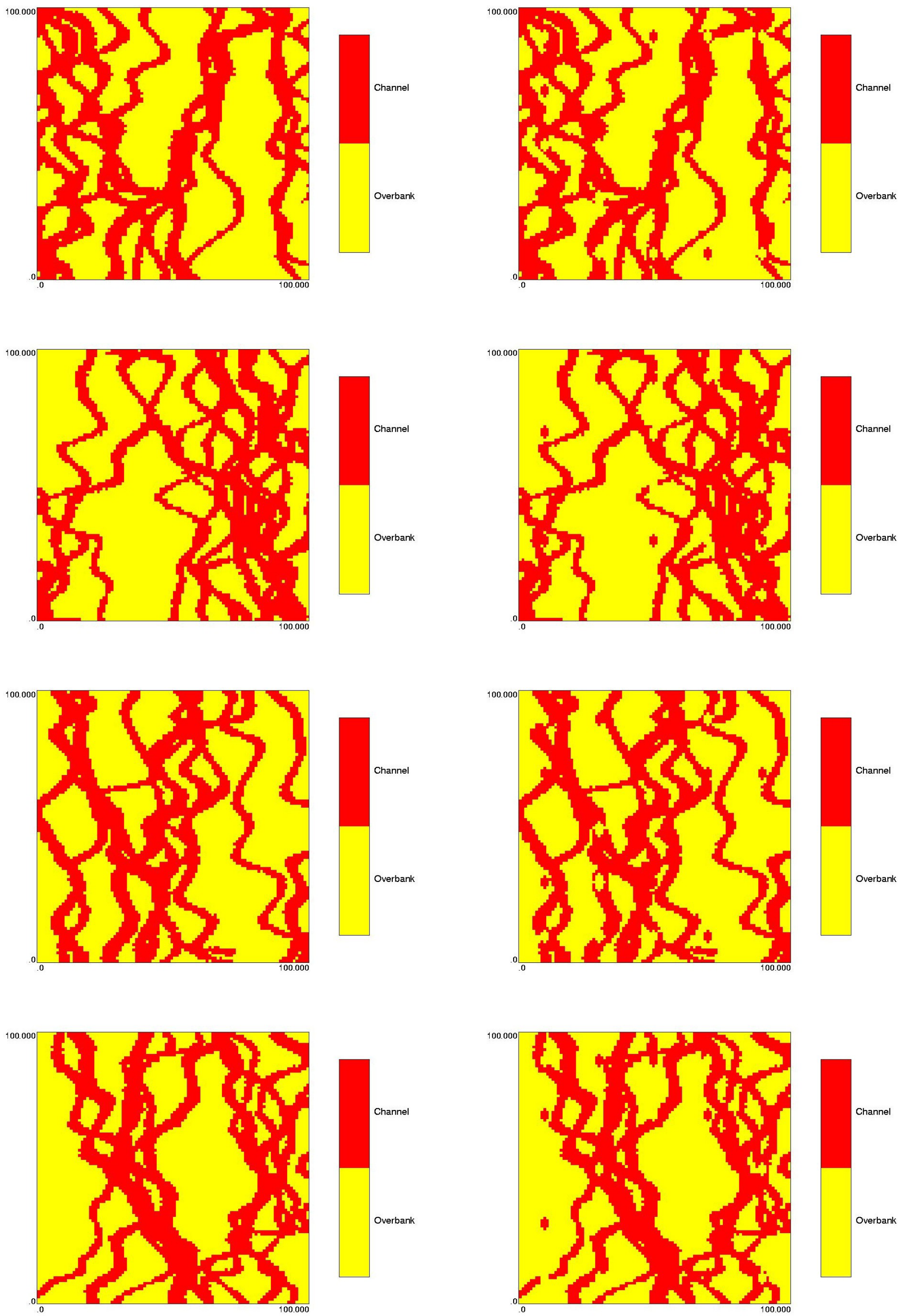

Figure 19: The four unconditional realizations conditioned by kriging with the scatted data. The models on the right match the conditioning data with some artifacts.