Maximum A Posteriori Selection with Homotopic Constraint

Michael J. Pyrcz and Clayton V. Deutsch

Department of Civil & Environmental Engineering
University of Alberta

Abstract

The addition of homotopic constraint within the MAPS algorithm allows for the preservation of connectivity relationships between geologic categories. The tool is flexible and the results are visually intuitive.

Introduction

Maximum a posteriori selection (MAPS) (Deutsch, 1998) is a valuable tool for correcting categorical stochastic realizations (**1**) to remove short scale variability inherent to realizations calculated with sequential indicator simulation and (**2**) to correct the global probability density function (pdf). Yet, this post processing may change important connectivity relationships. For example, contiguous architectural elements may become discontiguous and genetically interrelated architectural elements such as channel fills, crevasse splay channels and crevasse splays or channel fill and levees may become disconnected. This interruption of contiguous reservoir quality architectural elements with potentially low permeability architectural elements such as overbank fines and low energy channel fills may significantly underestimate reservoir performance after the application of a transfer function such as flow simulation.

These connectivity relations may be preserved with the addition of the homotopic constraint criteria in the MAPS algorithm. The basics of the MAPS algorithm and the homotopic constraint are described. A modified version of MAPS is presented, denoted as HMAPS.

MAPS Algorithm

The MAPS algorithm was developed by Deutsch (1998). The following is a brief description of the algorithm, the reader is referred to the original paper for more detailed discussion.

The algorithm translates a template with user defined dimension and weights through the categorical model and replaces the central node with the most likely category given the surrounding categories. Weights for each category are corrected for the mismatch between the current and target global probability density function.

The procedure is not sequential. The weight template is applied to the initial model and the new local realizations are stored to a second array. This second array is copied over the original model after all locations have been visited. The correction is applied over multiple iterations over the entire categorical model to improve reproduction of the global pdf. After each iteration the scaling factors to improve reproduction of the global pdf are updated. An illustration of the MAPS algorithm on a 2D model is shown in Figure 1.

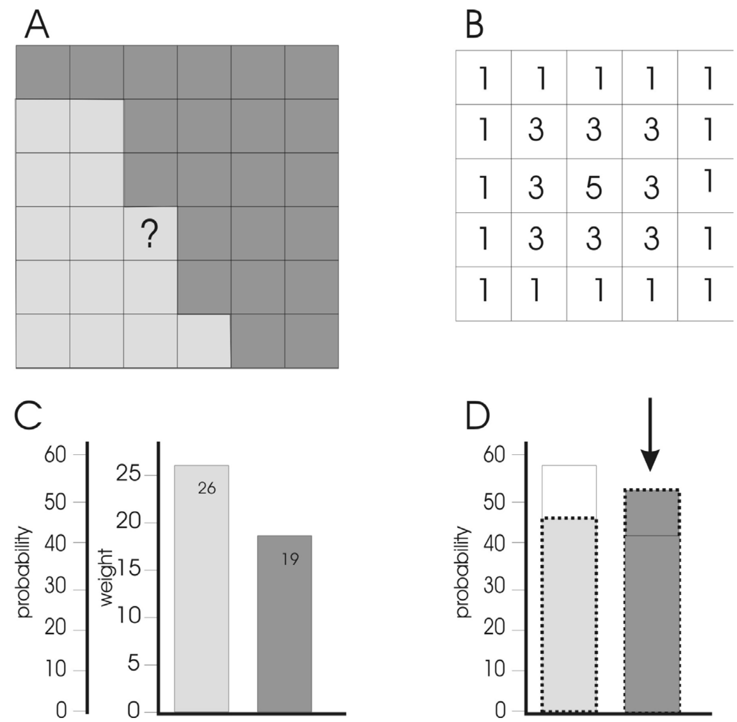

Figure 1 – the MAPS algorithm. Given a 2-D model with two categories (A) and a weight template (B) the probability (%) of each category is calculated (C). The probabilities are scaled by factors (target proportion divided by the proportion after the last iteration) to improve reproduction of the global pdf (D). The category with the maximum probability is selected (indicated by the arrow).

This correction may change important features in the original model. Aside from the previously mentioned reduction in short scale noise, connectivity relationships may be modified. For example, a significant reduction in abandoned channel fill architectural elements may break up otherwise contiguous architectural elements (see Figure 2) or a significant reduction in channel fill, levee and crevasse splay architectural element proportion may detach crevasse splays from the source channel and break up channel fill and levee architectural elements (see Figure 3). In addition, this correction may add categories in inappropriate locations. For example, a significant increase in lateral accretion architectural element proportion may result in lateral accretion elements isolated from the source channel fill elements.

Homotopic Constraint

Homotopic erosions and dilations in mathematical morphology may be defined geometrically as modifications of an object that do not punch holes in the object or result in disconnected segments. This constraint has been applied for the construction of hierarchical trend models by Pyrcz (2003 and 2004). In Figure 4 an example object is shown and the nodes that may be subsequently eroded or dilated in the next operation are hatched. Note after this erosion or dilation the object has changed and the hatching would require updating.

In the framework of MAPS with homotopic constraint (HMAPS) the objects are a set of user defined architectural element groups containing one or more architectural elements. For example, channel fill and levee elements may be linked and channel fill and lateral accretion elements may be linked without any relationship between levee and lateral accretion elements. After the maximum a posteriori selection at each location, each association is checked to ensure that the change is consistent with the homotopic constraint. Changes that fail this criterion are rejected.

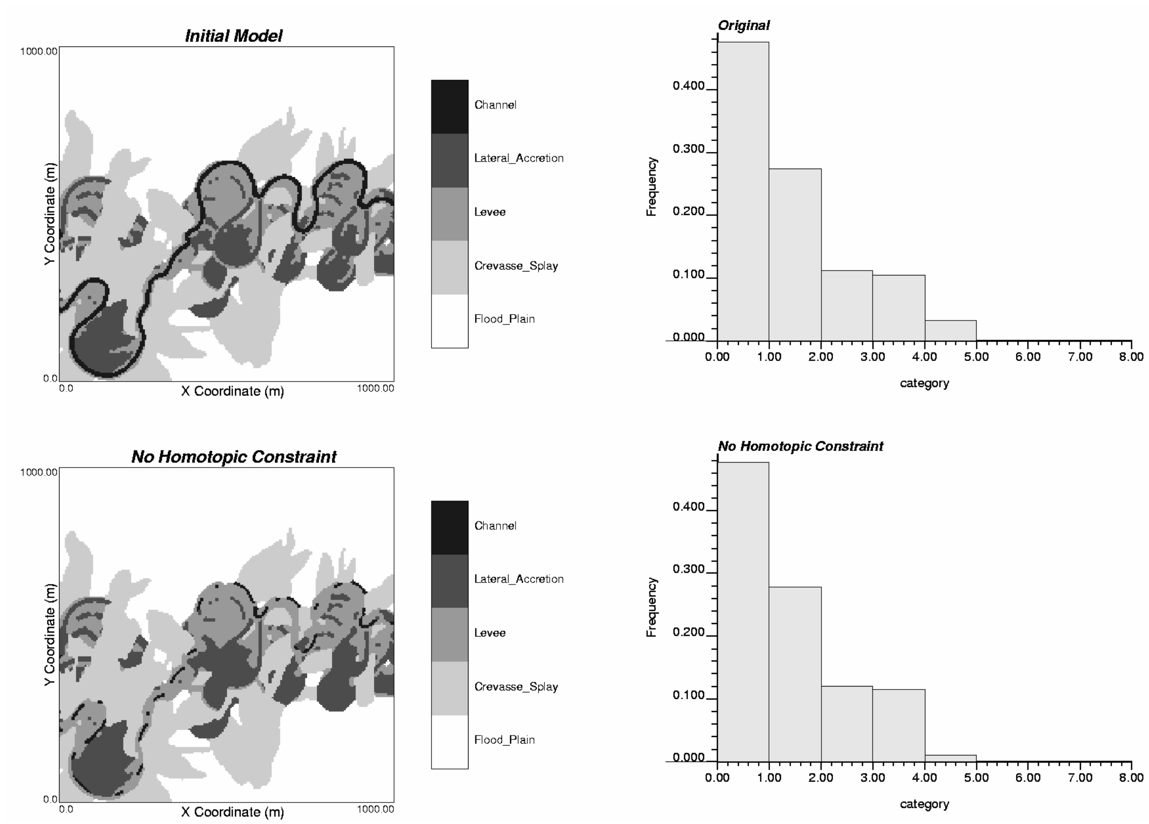

Figure 2 – reduction in the channel fill architectural element results in discontinuous channel fills contrary to the initial model. Architectural elements: 0 = overbank fines, 1 = crevasse splays, 2 = levee, 3 = lateral accretion and 4 = abandoned channel fill.

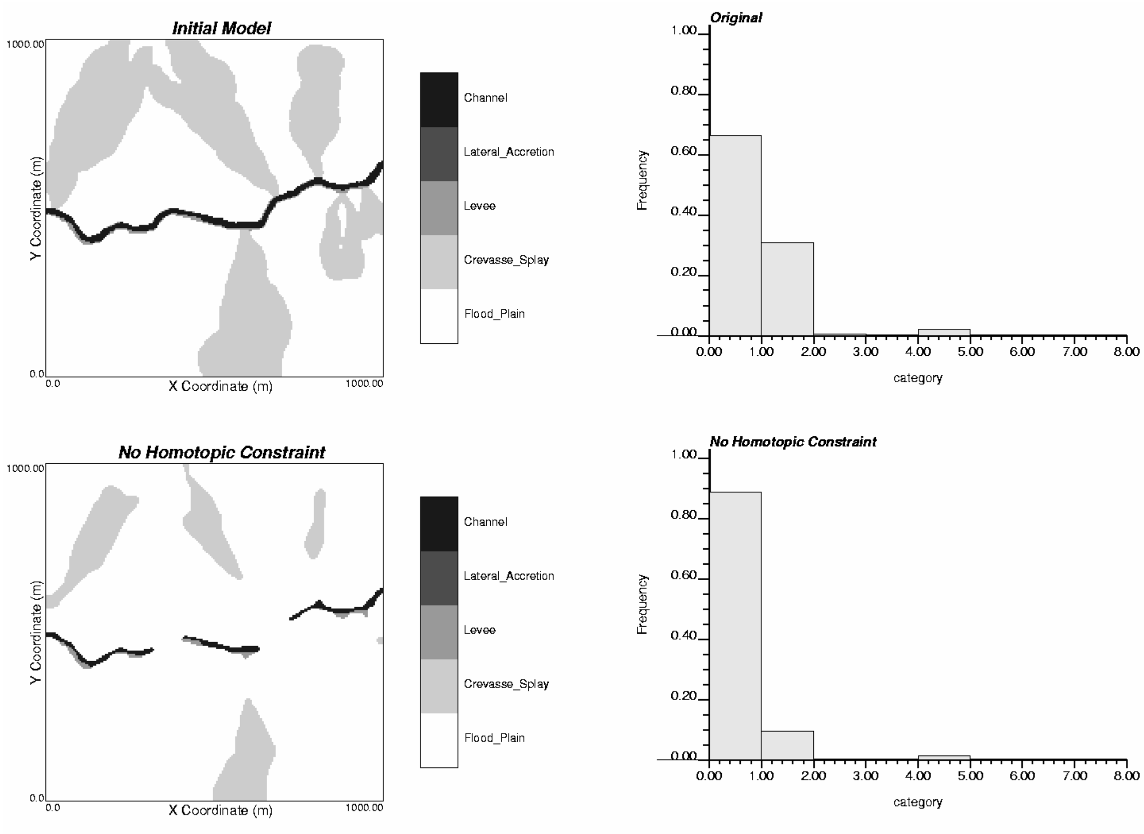

Figure 3 – Reduction in the channel fill, leeve and crevasse splay architectural elements results in distortion of expected connectivity relationships. Architectural elements: 0 = overbank fines, 1 = crevasse splays, 2 = levee, 3 = lateral accretion and 4 = abandoned channel fill.

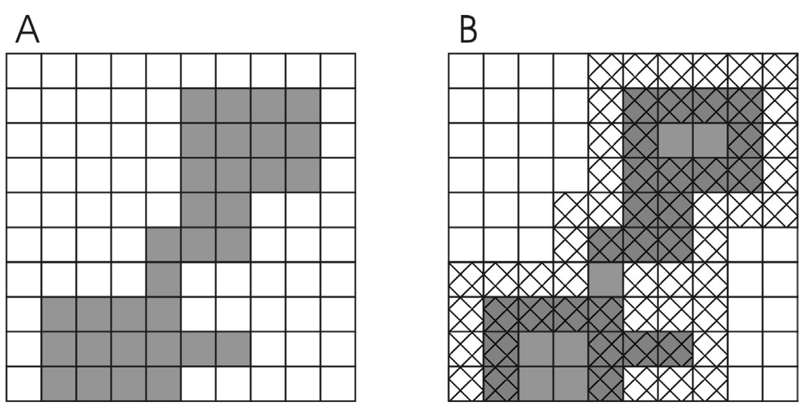

Figure 4 – Homotopic and nonhomotopic erosions and dilations. Given the initial binomial coding (A), the locations that may be switched in the very next erosion or dilation are indicated by cross hatching.

HMAPS Algorithm

The application of homotopic constraint within the MAPS algorithm by Deutsch (1998), requires (1) the addition of a subroutine for checking the homotopy of an candidate erosion/dilations in 1-D, 2-D and 3-D and (2) the addition of connected architectural element groups.

Homotopy Subroutine

The homotopic check is applied for all groups for each MAPS solution that results in a change. For this check all categories within the current group are coded as 1 and categories outside the group are coded as 0. The 1-D case is trivial (see cases in Figure 5 – Part A). For the 2-D case the solution is calculated rapidly by the technique described by Vincent (1993) (see Figure 5 – Part B). For the 3-D cased the homotopy is checked for the combination of 2-D cases illustrated in Figure 5.

Connected Groups

The user supplied list of connected groups is added to the MAPS algorithm. A connected group is a set of architectural elements that should remain in contact with each other. A connected group may include one or more architectural elements. In the case of a single architectural element, this constraint prevents this element from being broken up. For multiple elements, this constraint prevents each element from being detached from other elements in the same group. Any number of groups may be defined. An example architectural element model and three example sets of connected groups are shown in Figure 6. The implication of each set of groups is discussed in the caption.

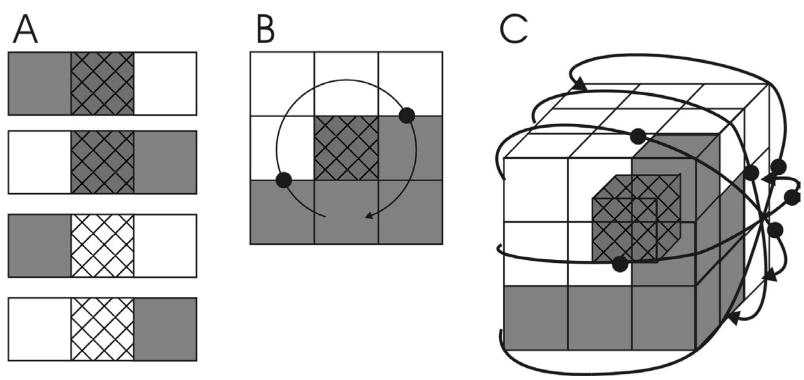

Figure 5: The check for homotopy of a modification. Note the hatched location is switched. A – 1-D cases that preserve homotopy. B – 2-D case for checking homotopy. If two transitions are observed over the path shown, then the switch preserves homotopy. C – 3-D homotopy is checked by applying the 2-D method in the four identified planes. Transitions are identified by black circles.

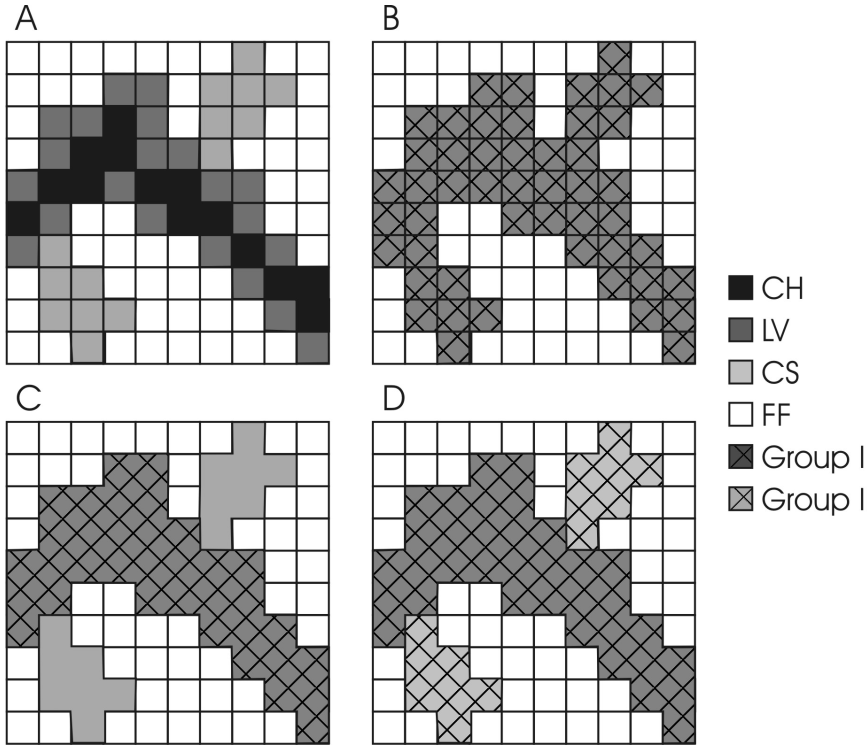

Figure 6: An illustration of connected groups. A - an example fluvial model with architectural elements identified. B – all architectural elements, except for overbank fines (FF) are coded as a single group. Modifications will preserve connectivity of the reservoir quality elements jointly, but not the connectivity of individual architectural elements. C – Channel fill (CH) and levees (LV) are set as a single group. Connectivity of the channel / levee are preserved, but crevasse splays (CS) may not remain attached and the CH and LV elements may become discontinuous. D – two groups are assigned. The difference between D and C is that the CS elements must remain continuous, but may still become detached from the CH and LV group.

HMAPS Examples

The previous examples shown in Figure 2 andFigure 3 are repeated with the inclusion of HMAPS results. For the first example, a single connected group including channel fill elements in defined. The subsequent decrease in the proportion of channel fill elements does not break up the contiguous channel fill element; instead the channel is thinned (see Figure 7).

For the second example a single connected group is defined for all architectural elements with the exception of overbank fines. The reduction of net architectural elements preserves the connected relationships of the fluvial element (see Figure 8).

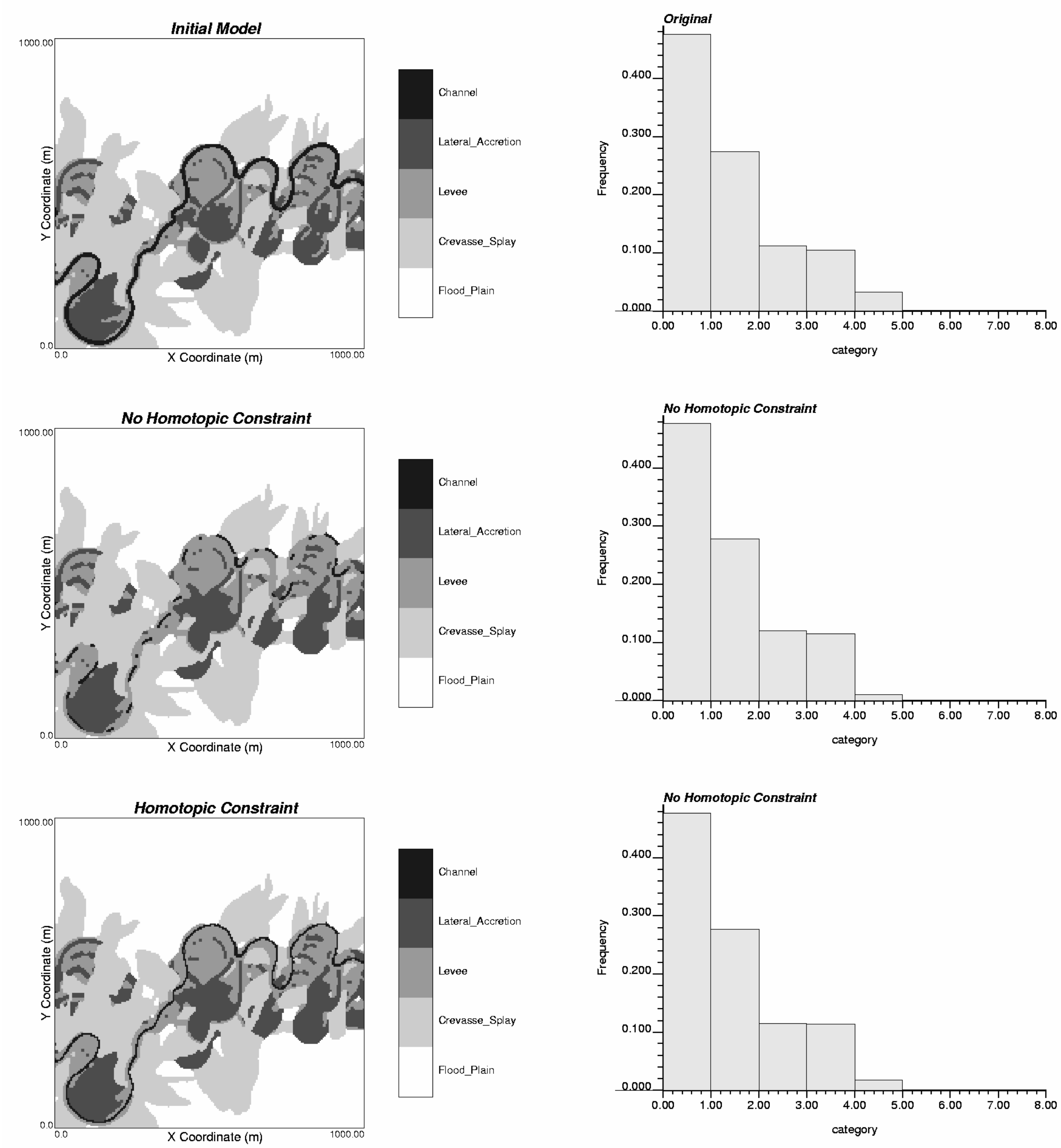

Figure 7 – reduction in CH proportion with and without homotopic constraint on the CH architectural element alone. Architectural elements: 0 = overbank fines, 1 = crevasse splays, 2 = levee, 3 = lateral accretion and 4 = abandoned channel fill.

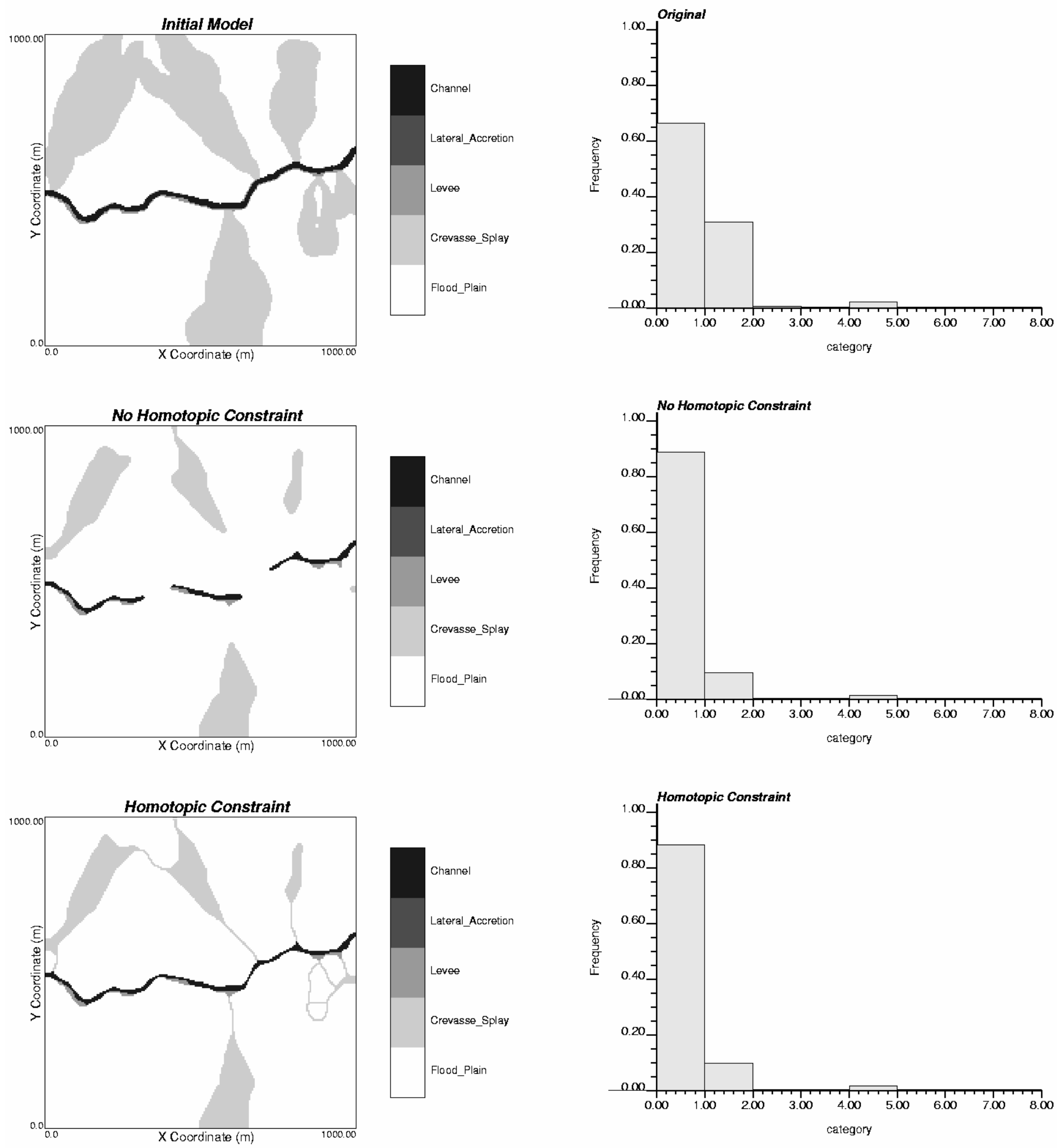

Figure 8 – reduction of channel and crevasse splay proportions with and without a homotopic constraint on the architectural element group (CH, LV and CS). Architectural elements: 0 = overbank fines, 1 = crevasse splays, 2 = levee, 3 = lateral accretion and 4 = abandoned channel fill.

Future Work

Further examples will be developed to demonstrate the flexibility and application of the HMAPS algorithm. In addition, the homotopic constraint may be applied in gradual deformation for updating categorical realizations to honor conditioning data. This requires demonstration to illustrate feasibility and the maintenance of the important geometries of the architectural elements.

Conclusions

The addition of homotopic constraint in the MAPS algorithm allows for the preservation of connectivity relationships observed in the original model. A user supplied list of connect architectural element groups defines the important connectivity relationships. These relationships may include the contiguity of architectural elements and the connection between genetically related architectural elements. These connectivity relationships may have a significant response on reservoir response.

References

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