Geostatistical Modeling Across Geological Boundaries with a Global LMC

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Abstract

There are very few instances in nature where hard geological boundaries exist. In most cases, the geological mechanisms that generate a deposit are transitional in nature. Some degree of overlap between geological units can be expected; however, conventional grade estimation usually treats the boundaries between geological units as hard boundaries. This is primarily due to the limitations of current estimation and simulation procedures. The sharing of grade samples across a boundary often has the effect of corrupting the representative statistics of the region of interest, particularly for simulation.

We propose to use a linear model of coregionalization (LMC) to simulate grades using data from adjacent rock types. Although the LMC is traditionally used to characterize the spatial variability of multiple properties in one rock type, we will show that it can be applied to model the spatial variability of one property across the boundary between multiple rock types. Specifically, the cross covariance between two different non-collocated data sets is calculated and the short-scale behavior is extrapolated. This allows inference of the nugget effect of the cross covariance from the nugget effects of the direct covariances. A full model of coregionalization can then be constructed. This model allows the correlation of the grades across the boundaries to be captured through a legitimate spatial model of coregionalization, which can then be used to cokrige or cosimulate grades using data from adjacent rock types. This approach guarantees the correct reproduction of representative statistics of the individual geological units used for resource estimation.

This proposed methodology is applied to a synthetic deposit, and compared to the conventional approach of modeling using hard boundaries. It provides an appealing alternative to capture grade distribution for deposits where complex contacts between different rock types exist. Further, it will improve the resource estimation by reducing the uncertainty in transitional zones around boundaries.

Introduction

Mineral resource and ore reserve estimation requires a critical decision regarding the geological domains that will be used for the grade modeling, as well as the type of boundaries between these domains. The most common geostatistical techniques, such as kriging and sequential simulation, are based on strong assumptions of stationarity of the estimation domains.

Following the definition of the estimation domains, an analysis on how grades change across the boundaries between domains should be done. This validates the proposed units and determines the nature of their boundaries. Domain boundaries are often referred to as either 'hard' or 'soft'. Hard boundaries are found when an abrupt change in average grade or variability occurs at the contact between two domains, such as coal seams or sedimentary zinc deposits. In deposits where the disseminated mineralisation has a gradational nature, such as some porphyry Cu-Au deposits, and grades change transitionally across a boundary, the contact is referred to as a soft boundary.

Soft boundaries are found in several types of deposits due to the transitional nature of the geological mechanisms involved in the formation of a deposit. There is often some degree of overlapping between geological grade controls. Nevertheless conventional grade estimation usually treats the boundaries between geological units as hard boundaries. This is primarily due to the limitations of current estimation and simulation procedures.

Estimation with hard boundaries is straightforward since only the samples within the domain are used. Soft boundaries allow grades from multiple domains to be used in the estimation of each domain. Common practice is to share samples within a given zone of influence of one domain over the other. Samples from different domains are treated equal to those within the domain, that is, the same mean, variance and covariance model from the samples within the domain are assumed. This generally has the effect of changing the representative statistics of the domain of interest. This corruption of the final grades, especially in the transition zones, often dissuades practitioners from using soft boundaries.

Correct representation of soft boundaries should ensure the reproduction of the correlation of the grades across the boundary. Boundaries are of special interest in short term mine planning and improved modelling of boundaries would benefit the design and operation stage in both underground and open pit deposits.

We propose to use a conventional linear model of coregionalization (LMC) to simulate grades using data from adjacent domains. Although the LMC is traditionally used to characterize the spatial variability of multiple properties or metal grades in one domain, we will show that it can be applied to model the spatial variability of one property across the boundary between multiple domains. A full model of coregionalization allows us to capture the spatial correlation of grades across the boundaries through a legitimate spatial model that can later be used to cokrige or cosimulate grades using data from adjacent domains. This approach guarantees the correct reproduction of representative statistics of each geological domain and improves the resource estimation by reducing the uncertainty in transitional zones near boundaries.

The proposed methodology is applicable when the correlation of the variable of interest between two adjacent rock types remains constant within both units and is due to an underlying common factor. One example could be the supergene zone in a porphyry copper deposit; supergene enrichment of these systems began as the portions above the water table are oxidized, transported in solution and precipitated below the ground water table by replacement of pre-existing iron sulfides. The mean and variance in the supergene zone is likely to be higher than the primary zone immediately below, but the spatial correlation structure will remain an underlying common factor because of the original mineralisation.

Theoretical Background

The linear model of coregionalization (LMC) provides a method to model the cross covariance of two or more variables. The LMC model is legitimate, that is, the variance of any possible linear combination of these variables is always positive. Given a set of K second order stationary random variables, $\{Z_k,\ k=1,...,K\}$, the LMC provides a means to model the cross covariance functions, $Cov_{Z_k Z_p}(\mathbf{h})$, $k=1,…,K$, $p=1,…,K$.

Usually Z_k and Z_p represent different properties measured at the same location, for example, gold and copper grades. We consider that each random variable Z_k, $k=1,…,K$ corresponds to the variable of interest in each of the K geological domains or rock types. The model could also be used for multiple grades within multiple rock types.

A linear model of coregionalization assumes that each variable Z_k is a linear combination of n second order stationary independent random variables Y_i with mean m_i and variance σ_i^2, with $i=1,…n$. These n random variables are independent, that is, their cross covariances are zero: $Cov_{Y_i Y_j}(\mathbf{h}) = 0, \forall \mathbf{h}$ and $i \neq j$. Each Z_k variable is assumed to be a linear sum of the independent factors:

$$Z_k = \sum_{i=1}^{n} a_{ki} Y_i \quad k = 1,...,K$$

The coefficients a_{ki} can be positive, negative or zero.

The mean of the kth stationary variable Z_k is:

$$E\{Z_k\} = m_k = \sum_{i=1}^{n} a_{ki} m_i$$

$$E\{(Z_k - m_k)^2\} = \sigma_k^2 = \sum_{i=1}^{n} a_{ki}^2 \sigma_i^2$$

The covariance of Z_k at a vectorial distance $\mathbf{h}$, can be calculated as an expression of the coefficients a_{ki} and the covariances of Y_i for $i=1,…n$:

$$Cov_{Z_k}(\mathbf{h}) = \sum_{i=1}^{n} a_{ki}^2 Cov_{Y_i}(\mathbf{h})$$

The cross-covariance of Z_k and Z_p, $\forall k \neq p$, with $k,p=1,…,K$ can also be derived as a linear combination of Y_i covariances and coefficients a_{ki} for Z_k, and a_{pj} for Z_p, $i,j=1,…n$,

$$Cov_{Z_k Z_p}(\mathbf{h}) = E\left\{\sum_{i=1}^{n} a_{ki} Y_i(\mathbf{u}) \sum_{j=1}^{n} a_{pj} Y_j(\mathbf{u}+\mathbf{h})\right\} - E\left\{\sum_{i=1}^{n} a_{ki} Y_i(\mathbf{u})\right\} \cdot E\left\{\sum_{j=1}^{n} a_{pj} Y_j(\mathbf{u}+\mathbf{h})\right\}$$

$$= \left(\sum_{i=1}^{n} a_{ki} a_{pi} E\left\{Y_i(\mathbf{u}) \cdot Y_i(\mathbf{u}+\mathbf{h})\right\} + \sum_{i=1}^{n} \sum_{j \neq i}^{n} a_{ki} a_{pj} E\left\{Y_i(\mathbf{u}) \cdot Y_j(\mathbf{u}+\mathbf{h})\right\}\right)$$

$$- \left(\sum_{i=1}^{n} a_{ki} a_{pi} E\left\{Y_i(\mathbf{u})\right\} \cdot E\left\{Y_i(\mathbf{u}+\mathbf{h})\right\} + \sum_{i=1}^{n} \sum_{j \neq i}^{n} a_{ki} a_{pj} E\left\{Y_i(\mathbf{u})\right\} \cdot E\left\{Y_j(\mathbf{u}+\mathbf{h})\right\}\right)$$

But since Y_i is independent of Y_j there is no cross spatial correlation between $Y_i(\mathbf{u})$ and $Y_j(\mathbf{u}+\mathbf{h})$, that is, $E\left\{Y_i(\mathbf{u}) \cdot Y_j(\mathbf{u}+\mathbf{h})\right\} = E\left\{Y_i(\mathbf{u})\right\} \cdot E\left\{Y_j(\mathbf{u}+\mathbf{h})\right\}$, $\forall i \neq j$. Then,

$$Cov_{Z_k Z_p}(\mathbf{h}) = \sum_{i=1}^{n} a_{ki} a_{pi} E\left\{Y_i(\mathbf{u}) \cdot Y_i(\mathbf{u}+\mathbf{h})\right\} - \sum_{i=1}^{n} a_{ki} a_{pi} E\left\{Y_i(\mathbf{u})\right\} \cdot E\left\{Y_i(\mathbf{u}+\mathbf{h})\right\}$$

$$= \sum_{i=1}^{n} a_{ki} a_{pi} Cov_{Y_i}(\mathbf{h})$$

A linear model of coregionalization, with Z_k and Z_p representing the distribution of the variable of interest in rock type k and rock type p, respectively, is a legitimate spatial correlation model that yields the correct statistics at unsampled locations near the boundary where samples from both domains are used for the estimation or simulation. The calculated LMC spatial model can be used in cokriging or cosimulation to model locations near geological boundaries using samples from adjacent domains. This is a more consistent alternative to the estimation of domains with soft boundaries than assuming the grades are independent or from the same domain.

Illustration of Theory

To illustrate how a linear model of coregionalization can be used to characterize the spatial variability of multiple rock types, consider a 2D example with two domains. The corresponding random variables Z_1 and Z_2 were constructed as a linear combination of three underlying non-standard normal random variables:

$$Z_1 = \sqrt{0.5} \cdot Y_1 + \sqrt{0.5} \cdot Y_2 + 0.0 \cdot Y_3$$
$$Z_2 = \sqrt{0.5} \cdot Y_1 + 0.0 \cdot Y_2 + \sqrt{0.5} \cdot Y_3$$

where,

$$Y_1 \sim N(0.5, 0.5) \text{ with } \gamma_{Y_1}(\mathbf{h}) = 0.05 + 0.45 \cdot Sph_{\left(\substack{h\max=200 \\ h\min=200}\right)}(\mathbf{h})$$

$$Y_2 \sim N(2.0, 1.0) \text{ with } \gamma_{Y_2}(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\left(\substack{h\max=50 \\ h\min=300}\right)}(\mathbf{h})$$

$$Y_3 \sim N(1.0, 0.5) \text{ with } \gamma_{Y_3}(\mathbf{h}) = 0.05 + 0.45 \cdot Exp_{\left(\substack{h\max=400 \\ h\min=100}\right)}(\mathbf{h})$$

The random variables Y_i were obtained by unconditional Gaussian simulation for a grid of 1000 by 2000 meters. Ten realizations were simulated.

The cross covariance between Z_1 and Z_2 was calculated and checked against its analytical model,

$$Cov_{Z_1,Z_2}(\mathbf{h}) = 0.5Cov_{Y_1}(\mathbf{h})$$
$$= 0.25 - 0.025 - 0.225 \cdot Sph_{\binom{h\max=200}{h\min=200}}(\mathbf{h}) \qquad (1)$$

three different spatial arrangement of Z_1 and Z_2 were considered: (1) collocated (just as a check), (2) the two domains adjacent to each other (Figure 1A), and (3) the two domains merged (Figure 1B) using a categorical binary model obtained via a Boolean simulation program, `ellipsim`, that generate a 2D map of ellipsoids of variable size and anisotropies for a given target proportion (Deutsch and Journel, 1998).
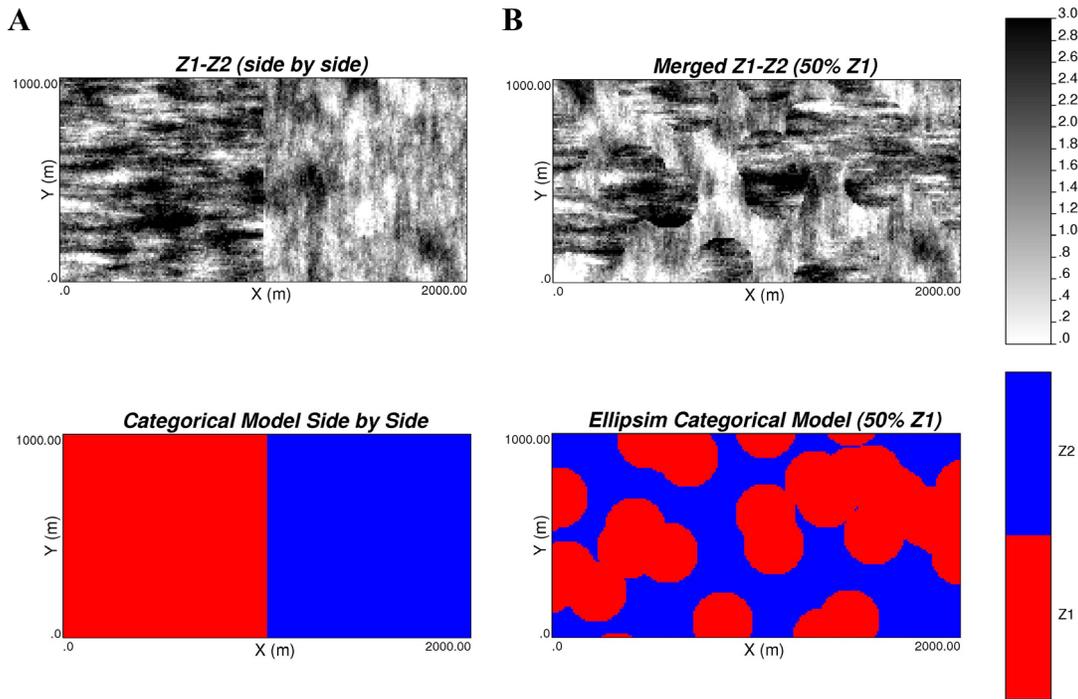
Figure 1: Example of two domains and the corresponding categorical models.

As a check of our derivations we compare the cross-covariance between Z_1 and Z_2 when both variables are collocated with the analytical derived model. As shown in Figure 2, the average variogram over all realizations is very close to the analytical model (Equation 1). The ergodic fluctuations associated with the different realization are very small.

In the case where the two domains are side by side, the covariances correspond to the analytical model fairly well (Figure 3A), although configurations where the boundary is parallel to the major anisotropy of one of the domains (Z_2 in this case), showed a systematically lower covariance at shorter lag distances than the analytical model (Equation

1), and the dispersion of the ergodic fluctuations is greater at lag distances near zero. Inference of the nugget effect of the cross covariance is more uncertain in geometrical configurations similar to this one.
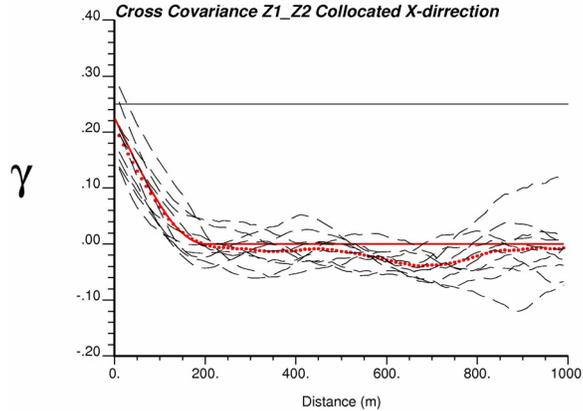
Figure 2: Cross-covariance reproduction of the simulated random variables Z_1 and Z_2, assuming both variables are collocated. The dots are the average taken over all realizations; individual realizations are in dashed lines; and the thin solid line corresponds to the analytical model. The analytical model is very close to the average over all realizations, which makes it difficult to differentiate the dots from the solid line.
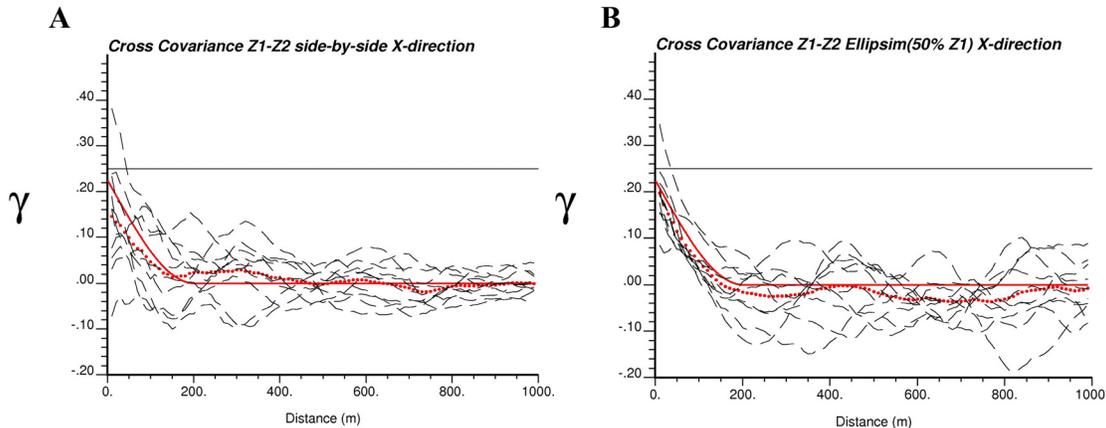
Figure 3: (A) Cross-covariance between Z_1 and Z_2 combined side by side. (B) Cross-covariance between Z_1 and Z_2 combined using `ellipsim` categorical model as a boundary model with a target proportion of Z_1 of 50%. The dots are the average taken over all realizations; individual realizations are in dashed lines; and the thin solid line corresponds to the analytical model.

For the second scheme, using a circular shape with radius of 150 meters and three target proportions of 25, 50 and 75%, the cross covariance between the experimental points derived from the average over all realizations compares well with the analytical model (Figure 3B) (Equation 1). The fluctuations at short lag distances are small. This confirms our expectation

that when more contact surfaces between domains are available and are more irregularly oriented; the determination of the nugget effect should have less uncertainty, compared to the case where a single contact surface exists between the two domains. A completely straight or planar boundary gives the least possible surface area to the boundary. This leads to the smallest possible transition zone between rock types and the fewest possible pairs for variogram calculation. This was also confirmed by a poorer reproduction at shorter lag distances, with lower covariances than the analytical model, when the target proportion of the domain Z_1 was lower than 10%. In addition, when the proportion of one domain decreases, the dispersion of the ergodic fluctuations increases (Figure 4).
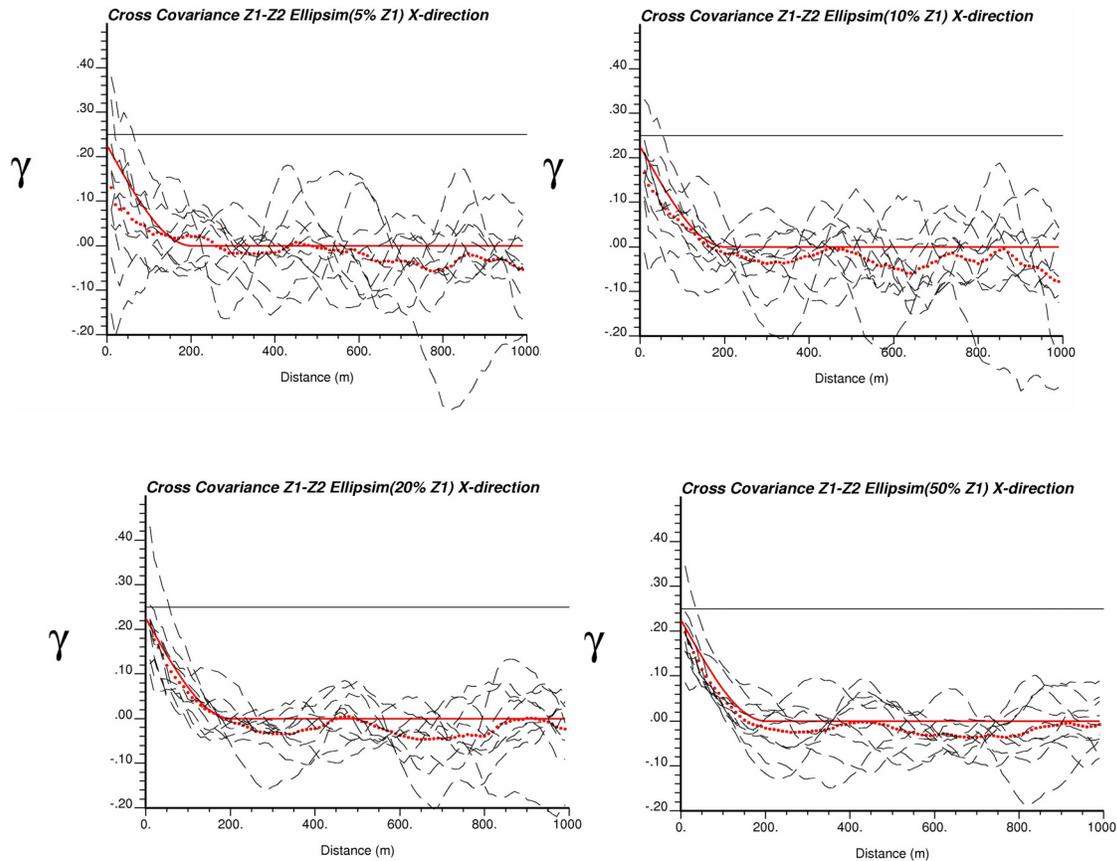
Figure 4: Cross-covariance between Z_1 and Z_2 combined using `ellipsim` categorical model as a boundary model, for target proportion of Z_1 of 5, 10, 20 and 50%. Note that as the target proportion of one of the domains decreases the experimental derived from the average over all showed a systematically lower covariance at shorter lag distances than the dashed analytical model. The dots are the average taken over all realizations; individual realizations are in dashed lines; and the thin solid line corresponds to the analytical model.

Using the same synthetic examples, the impact of different drill hole data spacing was examined. Overall, the reproduction of the cross covariance analytical model is as good as when all simulated values were used, although a wider range of fluctuation between realizations is observed. If the data spacing is larger than the range of the cross-covariance, the calculation of a cross-covariance will be meaningless.

2D Example

A real categorical geological model (Figure 5) was used to forecast the results for geometries of a real deposit. The grades, assumed to be percentage of copper, within the five rock types, Z_1 to Z_5 were constructed as a linear combination of four underlying non-standard normal random variables,

$$Y_1 \sim N(0.01, 0.5) \qquad \text{with } \gamma_{Y_1}(\mathbf{h}) = 0.05 + 0.45 \cdot Sph_{\left(\substack{h\max=100 \\ h\min=100}\right)}(\mathbf{h})$$

$$Y_2 \sim N(2.0, 1.0) \qquad \text{with } \gamma_{Y_2}(\mathbf{h}) = 0.1 + 0.9 \cdot Sph_{\left(\substack{h\max=400 \\ h\min=50}\right)}(\mathbf{h})$$

$$Y_3 \sim N(0.2, 0.75) \qquad \text{with } \gamma_{Y_3}(\mathbf{h}) = 0.05 + 0.70 \cdot Exp_{\left(\substack{h\max=50 \\ h\min=300}\right)}(\mathbf{h})$$

$$Y_4 \sim N(0.75, 1.5) \qquad \text{with } \gamma_{Y_4}(\mathbf{h}) = 0.3 + 1.2 \cdot Exp_{\left(\substack{h\max=400 \\ h\min=250}\right)}(\mathbf{h})$$

where Y_2 and Y_4 have a 55° anisotropy. The coefficients that multiplied the underlying variables in the summation that originates Z_1 to Z_5 are:

	Z_1	Z_2	Z_3	Z_4	Z_5
Y_1	$\sqrt{0.5}$	0.0	0.0	$\sqrt{0.2}$	$\sqrt{0.7}$
Y_2	0.0	$\sqrt{0.7}$	$\sqrt{0.2}$	0.0	0.0
Y_3	$\sqrt{0.5}$	$\sqrt{0.3}$	0.0	0.0	$\sqrt{0.25}$
Y_4	0.0	0.0	$\sqrt{0.8}$	$\sqrt{0.8}$	$\sqrt{0.05}$

The variables Z_1 up to Z_5 were merged together using the categorical rock type model (Figure 6)
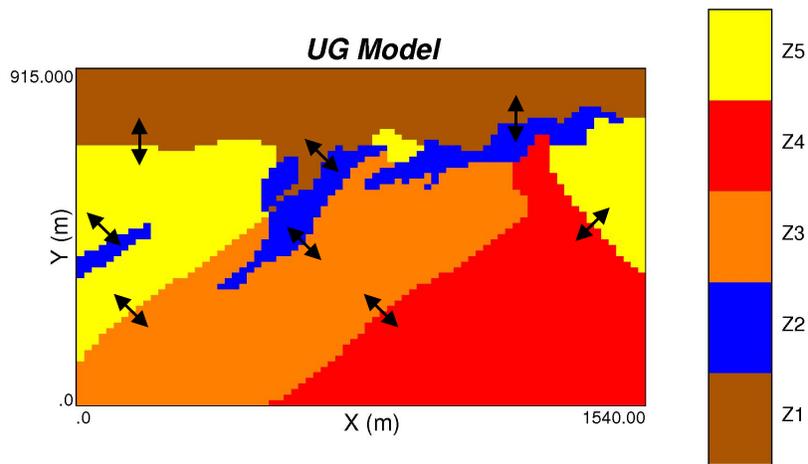
Figure 5: Categorical rock type model. The arrows indicate the directions in which the cross-covariance between domains was calculated.
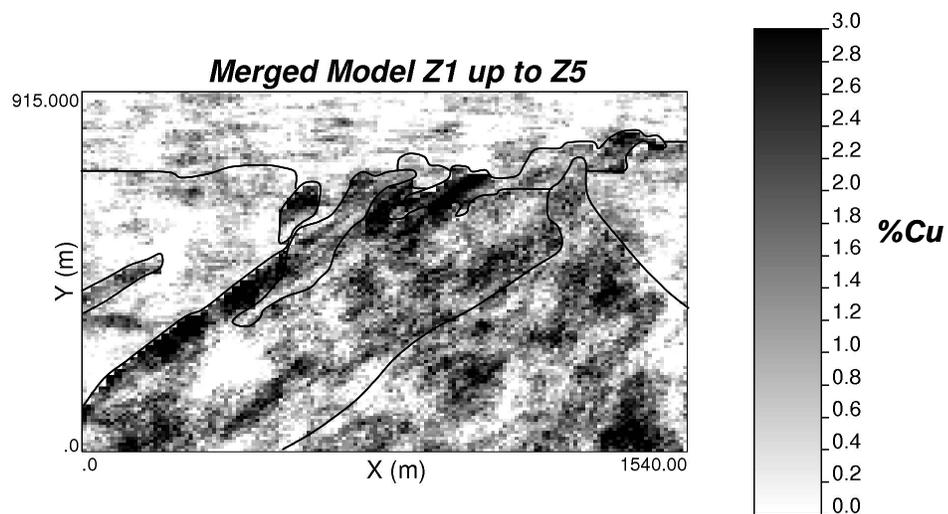
Figure 6: A realization of the merged grades Z_1 up to Z_5 using the categorical rock type model of Figure 5.

The cross covariance was calculated for each pair of Z_i and Z_j, $i \neq j$, in the directions sketch in Figure 5 and compared with the corresponding analytical model:

$$Cov_{Z_1,Z_2}(\mathbf{h}) = 0.29 - 0.019 - 0.271 \cdot Exp_{\left(\substack{h\max=50 \\ h\min=300}\right)}(\mathbf{h})$$

$$Cov_{Z_1,Z_5}(\mathbf{h}) = 0.561 - 0.047 - 0.266 \cdot Sph_{\left(\substack{h\max=100 \\ h\min=100}\right)}(\mathbf{h}) - 0.248 \cdot Exp_{\left(\substack{h\max=50 \\ h\min=300}\right)}(\mathbf{h})$$

$$Cov_{Z_2,Z_3}(\mathbf{h}) = 0.374 - 0.037 - 0.337 \cdot Sph_{\left(\substack{h\max=400 \\ h\min=50}\right)}(\mathbf{h})$$

$$Cov_{Z_2,Z_5}(\mathbf{h}) = 0.206 - 0.014 - 0.192 \cdot Exp_{\left(\substack{h\max=50 \\ h\min=300}\right)}(\mathbf{h})$$

$$Cov_{Z_3,Z_4}(\mathbf{h}) = 1.2 - 0.24 - 0.96 \cdot Exp_{\left(\substack{h\max=400 \\ h\min=250}\right)}(\mathbf{h})$$

$$Cov_{Z_3,Z_5}(\mathbf{h}) = 0.3 - 0.06 - 0.24 \cdot Exp_{\left(\substack{h\max=400 \\ h\min=250}\right)}(\mathbf{h})$$

$$Cov_{Z_4,Z_5}(\mathbf{h}) = 0.487 - 0.079 - 0.168 \cdot Sph_{\left(\substack{h\max=100 \\ h\min=100}\right)}(\mathbf{h}) - 0.24 \cdot Exp_{\left(\substack{h\max=400 \\ h\min=250}\right)}(\mathbf{h})$$

The experimental cross covariance obtained from the average overall realizations compares very well with the analytical models (Figure 7), except for the pairs Z_3/Z_4 and Z_4/Z_5, that have a side by side arrangement that shows lower covariances at shorter lag distances.
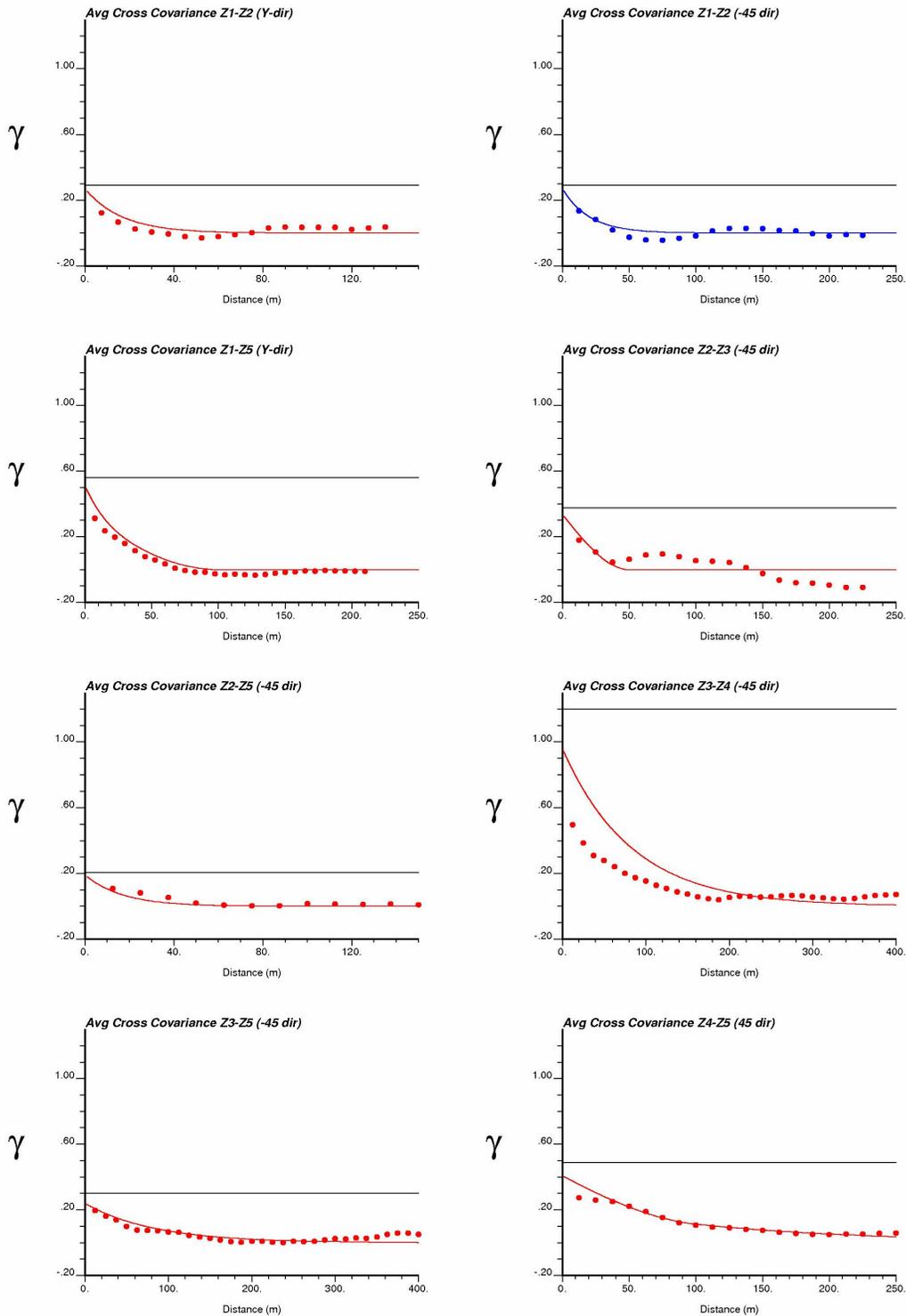
Figure 7: Cross-covariance reproduction of the simulated pairs Z_i and Z_j for $i \neq j$, combined by the categorical rock type model. The experimental points correspond to the average over ten realizations, and the thin solid line corresponds to the analytical model.

Application

A synthetic example was created in order to use a full LMC cosimulation and compare it with the results obtained from simulating two adjacent rock types independently. The LMC model was obtained by calculating the cross variograms between values of the different domains and the direct variograms within each rock type.

Using a similar methodology, two random variables, Z_1 and Z_2, generated as the linear combination of three underlying standard normal random variables were used to populate a synthetic geological model; this will be considered as the 'true' image (Figure 8) for comparison. The 2D reference image (2000 by 1000 meters, with a 10 meters grid spacing in both directions) was sampled at a spacing of 70 meters in the X-direction and 10 meters in the Y-direction yielding a total of 2800 samples.
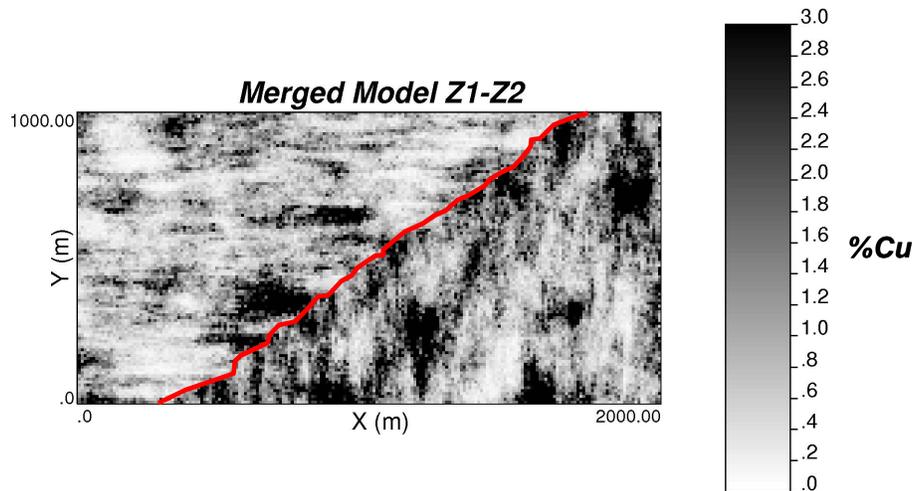
Figure 8: Reference or 'true' image. Z_1 is represented by **RT₁** (left), Z_2 by **RT₂** (right).

Variograms were calculated from the normal scores transform values from each rock type, **RT₁** and **RT₂**. Cross variograms can not be calculated if the variables are not collocated, which is the case here since we are trying to characterize the spatial variability across the boundary between **RT₁** and **RT₂**. An alternative (Wawruch *et. al.* 2003) is to (1) calculate the cross covariance between the variables, (2) extrapolate the experimental points at lags near to zero to obtain the structured cross covariance ($B_{Z1\text{-}Z2}$) (Figure 9), (3) determine the relative nugget effects for Z_1 and Z_2, and (4) calculate the sill of the cross variogram between Z_1 and Z_2 as:

$$Cov_{Z1\text{-}Z2}(0) = \frac{B_{Z1\text{-}Z2}}{1 - \left[\frac{1}{2}\left(\frac{Cov_{z1}^{0}}{\sigma_{z1}^{2}} + \frac{Cov_{z2}^{0}}{\sigma_{z2}^{2}}\right)\right]}$$

In this example, the relative nugget effects obtained from the direct variograms of each rock type were both 0.1, the structured cross covariance was chosen at 0.4, so the sill of the cross variogram is 0.44. With this value the experimental points from the cross covariance can be inverted to obtain the cross variogram between Z_1 and Z_2.
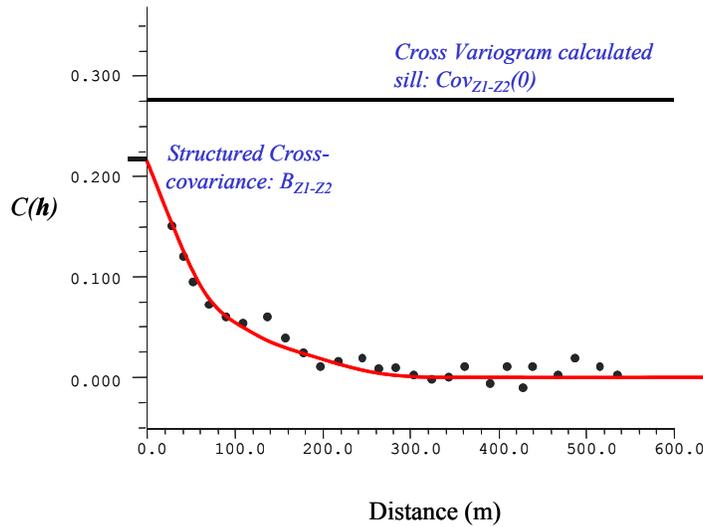
Figure 9: Sketch with the structured cross covariance and calculated sill of a cross variogram given an experimental cross covariance between two non-collocated variables.

As we discussed in the previous section, there will be some surface contacts that will present a higher uncertainty in the determination of the structured cross covariance. In this case, however, the nugget effect between the grade at each side of the boundary is not needed in any calculations because there are no collocated data nor do we estimate collocated grid blocks; most cokriging and cosimulation programs require the LMC to be defined with variogram models, which requires the nugget effect and the sill of the cross variogram.

The direct and cross variograms of Z_1 and Z_2 were model using a linear model of coregionalization obtained by a semi-automatic variogram fitting program (Larrondo *et. al.*, 2003). Since independent simulations of Z_1 and Z_2 were also performed, the direct variograms of each variable were modeled independently.

The cosimulation was performed using the full LMC cokriging option of the `ultimate sgsim` program (Deutsch and Zanon, 2002); in this case each rock type was simulated using the samples of the other rock type, as a secondary variable. For the comparative case, sequential Gaussian simulation with the same parameters was used to simulate each rock type independently as the contact between **RT₁** and **RT₂** was a hard boundary.

The reproduction of the direct variograms for both the cosimulation and independent simulation was fairly good (Figure 10). Although the reproduction of the cross variogram was poor compared with the analytical model, the first 100 meters (total range) in the X-direction showed a similar amount of correlation (Figure 11). The case where the contact between **RT₁** and **RT₂** was assumed to be a hard boundary, resulted in almost no correlation for lags less than the range of the cross variogram, and is significantly lower than the correlation of the conditional data across the boundary. This correlation is a remnant correlation from data, not from modeling. While for a soft boundary assumption, the correlation of the average over all realizations is closer to the correlation shown by the 'truth' reference.
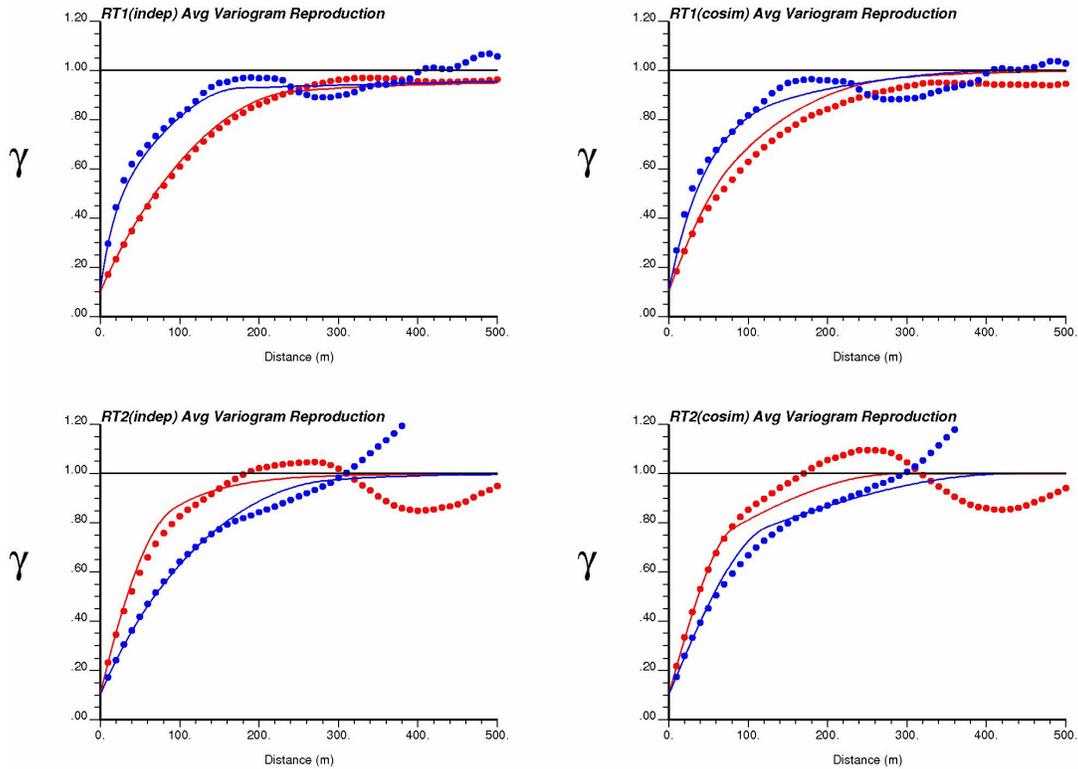
Figure 10: Direct variograms reproduction for Z_1 and Z_2, cosimulated (right) and independently simulated (left). The dots represent the average of simulated values over ten realizations, the dashed line corresponds to the cross covariance calculated for the training image, and the solid line is the analytical model derived from the theoretical expression.
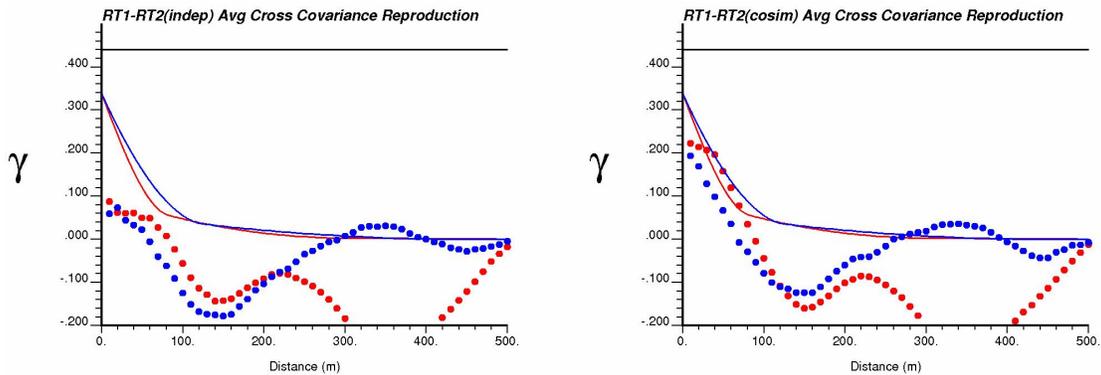
Figure 11: Cross covariance reproduction for Z_1 and Z_2, cosimulated (right) and independently simulated (left). In a soft boundary scheme (right) the correlation between the simulated values is very close to the 'truth' reference. In the hard boundary assumption, the correlation at short lag distances is significantly lower. The dots represent the average of simulated values over ten realizations, the dash line correspond to the cross covariance calculated for the training image, and the solid line is the analytical model derived from the theoretical expression.

Cross validation of the model obtained by independently simulating Z_1 and Z_2 showed that the model is accurate and precise (Deutsch, 2002). The cosimulated model is also accurate, and equally precise for **RT$_1$**, while for **RT$_2$** is slightly less precise than the model obtained from independent simulations (Figure 12). This is not surprising since the fitted LMC model for this rock type did not fit the data as well as for **RT$_1$**. This is a common disadvantage when using a linear model of coregionalization. The cosimulated model did, however, show less smoothing (Figure 13), which translates to less conditional bias in the estimation.

Although both model are similarly accurate and precise the overall uncertainty, defined as the average kriging variance of all samples, is significantly lower for the cosimulated model (0.3 for both Z_1 and Z_2) than for the independently simulated model (0.62 for Z_1 and 0.91 for Z_2).
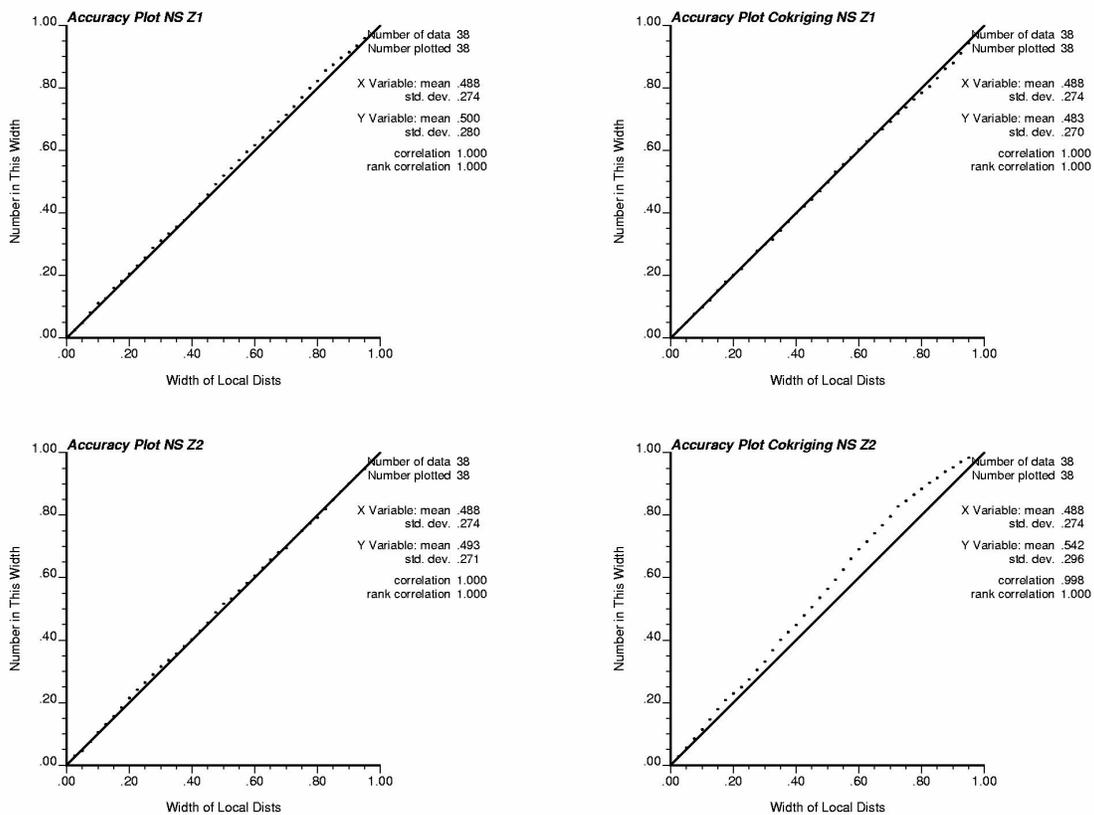
Figure 12: Accuracy plot for Z_1 and Z_2, estimate independently (left) versus cosimulated (right). Cross validation show that the models from independent simulation or cosimulation of Z_1 are accurate and precise. For Z_2 the parameters used for cosimulation results a slightly less precise model than in the case of independent simulation.

The distribution of errors (true-estimated) should be symmetric and centered at zero, as occurs for both schemes, but the standard deviation of the errors for cokriged estimates is significantly lower than independently kriged values; 0.56 against 0.75 for Z_1 and 0.45 against 0.95 for Z_2.

The cumulative distribution of back transformed simulated values shows a very good reproduction of the data histograms, for both schemes. The target mean and variance are well reproduced for both cosimulation and independent simulations.
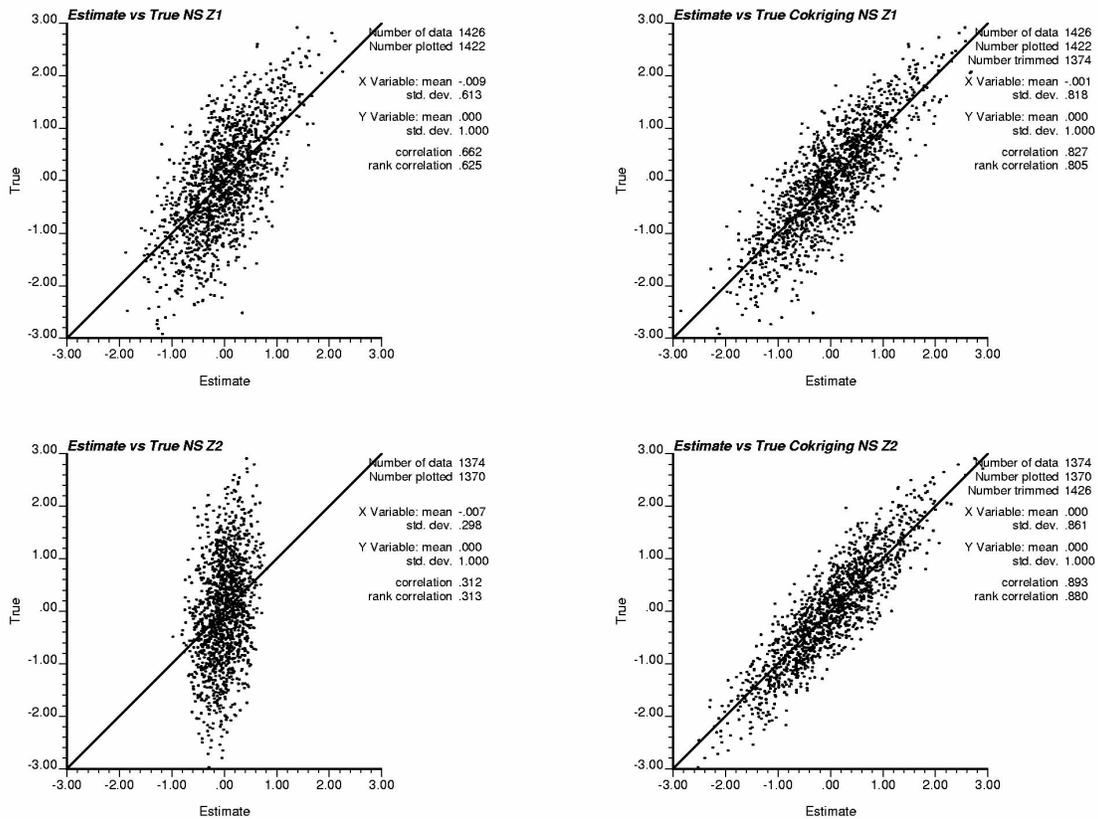
Figure 13: Cross validation of data values in RT_1 and RT_2, estimate independently (left) versus cosimulated (right). The cokriging cross validation shows far less conditional bias and a much higher correlation than the estimation of each rock type independently, especially for RT_2.

Comparison at the boundary

In order to compare the performance of the two methods, we need to focus on the results near the boundary where we can expect to have greater differences.

One comparison was done using the expected value (E-type value) in original units at each location compared to the 'true' value in the reference map. The expected value is the average of the simulated realizations at each location. The block values obtained from cosimulation show systematically higher correlation coefficients with the true values. As expected, the difference between the two methods becomes smaller beyond the range of correlation of the cross variogram (Figure 14).
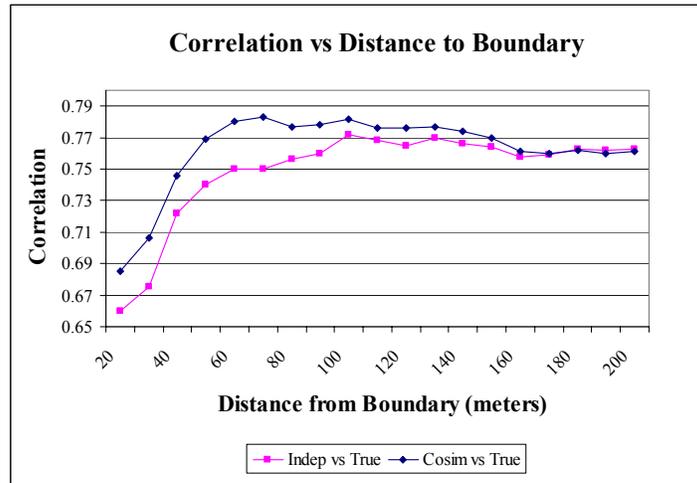
Figure 14: Correlation coefficient between E-type estimates of cosimulated and independently simulated models, and the "true" values considering blocks within a given distance from the boundary between Z_1 and Z_2. The higher correlation coefficient with the true values shown by the blocks estimated by cosimulation indicate this model better represents the underlying correlation that exists between Z_1 and Z_2.

The variance of the blocks from each realization within a given distance from the boundary was also compared. As expected, the average of the variance over all the realizations showed lower variance for the block values obtained using cosimulation (Figure 15). This variance is also closer to the average variance calculated from the same group of blocks in the 'true' reference map.

Conclusions

The geological mechanisms involved in the formation of a deposit are in most cases transitional in nature, which yields contacts between domains that are diffuse or gradational. These soft boundaries are widespread in different types of deposits and their correct reproduction by geostatistical methods has a great impact on mine plan design, expected dilution and final mineral resources. The areas close to contacts are usually areas of higher uncertainty.

The estimation of a domain with a soft boundary implies that samples from either side of the boundary should be used in the estimation. A common practice is to include samples or previously estimated nodes from outside the domain within a certain distance. Whether kriging or simulation is used, the assumption that the samples or nodes outside the domain follow the same distribution and spatial model as the samples inside is often incorrect and lead to the corruption of the statistical parameters near the boundary.
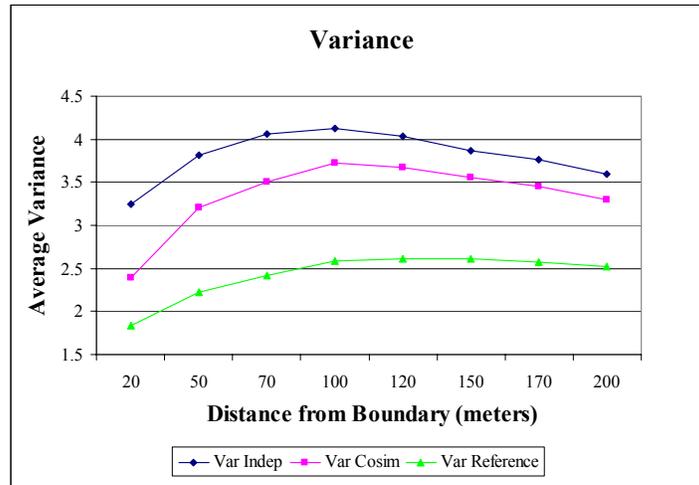
Figure 15: Average variance calculated from blocks within a given distance from the boundary between Z_1 and Z_2. The average variance obtained from cosimulation is lower than the variance obtained from independent simulations. The difference between the two methods decreases as we consider blocks further away from the boundary, where the influence of the data from the adjacent rock type in the cosimulation decreases. The average variance from cosimulated blocks is closer to the variance of the same blocks in the 'true' reference map, than the average variance obtained from independent simulations.

A linear model of coregionalization (LMC) can be used to capture the spatial correlation of the variable across a boundary between domains. This model allows the correlation of the grades across the boundaries to be captured through a legitimate spatial model of coregionalization, which can then be used to cokrige or cosimulate grades using data from adjacent rock types. This approach guarantees the correct reproduction of representative statistics of the individual geological units used for resource estimation.

This approach, assumes that the variable is stationary in each domain, and therefore can be used to model a **global** correlation across a boundary. However, nature provides us with several examples where the behavior of our variable of interest is no longer stationary as it gets closer to a boundary. Modeling of **local** non-stationary soft boundaries is addressed in Larrondo and Deutsch (2004).

The proposed methodology in this contribution has the advantage of improved resource estimation by reducing the uncertainty in transitional zones near boundaries and reproducing the correlation of the conditioning data across a soft boundary. It also shows a decrease of smoothing in the estimates if kriging is the tool to obtain the resources.

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