Short Note: Sequential Indicator Simulation with Local Probabilities Derived from a Deterministic Rock Type Model

Steven Lyster and Clayton V. Deutsch

Department of Civil & Environmental Engineering
University of Alberta

Abstract

A deterministic rock type model should always be constructed if possible. 3-D solid or wireframe models could be used or 2-D interpretations on sections and maps can be used. Even though the rock type model is deterministic, there is uncertainty and it is desirable to construct alternative geostatistical realizations of rock type for further processing. Sequential indicator simulation (SIS) and truncated Gaussian simulation (TGS) are common geostatistical methods. These methods, however, require local control from the deterministic model to generate reasonable results; otherwise, rock types are located in unreasonable locations. This note describes how a deterministic rock type model is used in SIS/TGS to control deterministic features.

Introduction

Sequential indicator simulation (SIS) is used to generate realizations of regionalized variables coded as indicators (Journel, 1978). Indicator kriging (IK) is performed to determine the conditional probabilities of each rock code (indicator) at all unknown points. A single rock code is then drawn from these probabilities for each point. A variogram for each rock code must be inferred before IK can be performed.

For simple IK, the estimate of the probability of indicator k at location $\mathbf{u}$ is given by

$$\begin{aligned}\left[i(\mathbf{u};k)\right]^*_{SK} &= \left[Prob\left\{k(\mathbf{u})=k \mid n(\mathbf{u})\right\}\right]^*_{SK} \\ &= \sum_{\alpha=1}^{n} \lambda_\alpha^{SK}(\mathbf{u};k)\cdot i(\mathbf{u}_\alpha;k) + \left[1-\sum_{\alpha=1}^{n}\lambda_\alpha^{SK}(\mathbf{u};k)\right]\cdot p(k)\end{aligned} \tag{1}$$

where i^* is the estimate of the probability, n is the number of previous data in the estimation neighbourhood, *p(k)* is the probability distribution function (pdf) value for rock code k, and the λ values are the simple IK weights found by the simple kriging system of equations

$$\sum_{\beta=1}^{n} \lambda_\beta^{SK}(\mathbf{u};k)\cdot C_I(\mathbf{u}_\beta-\mathbf{u}_\alpha;k) = C_I(\mathbf{u}-\mathbf{u}_\alpha;k) \quad \alpha=1,\ldots,n \tag{2}$$

The global pdf of all rock codes must be known to use simple IK. In cases where the pdf is unknown or varies locally, ordinary IK can be used. Ordinary kriging involves constraining the sum of the kriging weights to equal 1; in the case of IK this causes equation (1) to become

$$\left[i(\mathbf{u};k)\right]^*_{OK} = \sum_{\alpha=1}^{n} \lambda_\alpha^{OK}(\mathbf{u};k)\cdot i(\mathbf{u}_\alpha;k) \tag{3}$$

with the system of equations (2) now having the added restriction

$$\sum_{\alpha=1}^{n} \lambda_{\alpha}^{OK} = 1 \qquad (4)$$

Ordinary IK is more robust than simple IK, but it has added complexity and requires more data to give unbiased results. If sampled locations are sparse, ordinary IK can give results that are too deterministic because data points for many rock codes are not found inside the variogram range. In this case ordinary IK may not reproduce the variability of the data.

Local Probabilities

To improve SIS using simple IK, a local pdf can be used for each simulated point rather than the global pdf. This is advantageous for situations where sampling data is sparse. Using a local pdf with simple IK gives the same primary advantage as ordinary IK – the flexibility and robustness to account of a locally varying pdf. When a local pdf is used equation (1) accounts for the difference in proportions of the indicators at each point. A global pdf may lead to situations where a rock type appears in an area that is unreasonable. For example, if there is no data to suggest a rock type occurs in a given area but its presence in the global pdf allows it to be drawn during simulation.

To determine the pdf at each location a previously generated deterministic model may be used. The deterministic model is not perfect, but the proportions of different indicators are approximately correct in each neighbourhood; at the very least the proportions for each region will be better than an overall global pdf.

Extracting the local pdf for each point is quite simple. A spherical search window is used to read all nearby points in the deterministic model; these points give the local pdf. Using this method a local pdf can be easily determined for every point. Any shape of search window could be used, to account for anisotropy in the geologic bodies.

Example

An example is shown in Figure 1. The top left image is a deterministic model with five rock codes represented by the five different colours. The other five pictures chart the likelihood of finding the rock codes within a search radius; red is 100% probability and blue is 0%. Each image represents one rock code. All of the images are 100 units wide by 80 units high; the search radius is 20 units. The values represented in the five likelihood images may be used as the local pdf values for simple IK.

Figure 2 shows 20 regularly-spaced samples taken from the deterministic image as conditioning data. Below that are four SIS realizations: the top two are generated using a global rock code pdf, the lower two using the local proportions shown in Figure 1. Visual inspection of these realizations, along with others, shows that the realizations using a global pdf have more noise and less continuity than those utilizing a local distribution. The global pdf realizations also have rock types in locations that are unreasonable based on the sample data. The same variograms and parameters were used for both approached to SIS.

Probability maps for each of the five facies over 100 SIS realizations using a global pdf are shown in Figure 3. It is notable that there are significantly fewer points with zero probabilities than in Figure 1; the use of a global pdf results in at least a small chance of every facies at every point. This causes small pockets of rock types in areas where the sampling does not suggest or it is known that rock type does not occur.

Figure 4 shows the probability maps of each of the five facies, taken over 100 SIS realizations using the local pdf shown in Figure 1. These maps show the facies are significantly more confined to areas near where they are known to occur. The grid nodes with at least one occurrence for each rock type in Figures 1 and 4 are very similar; however, Figure 4 illustrates the variability inherent to simulation. The nodes with zero probability in Figure 1 for a given facies also have zero probability for that facies in Figure 4. To extend the areas where a rock type is permitted, a larger search radius can be used to scan the deterministic image.

Histograms are shown in Figure 5 for all of the probability maps shown in Figures 1, 3, and 4. These histograms show the distribution of probabilities of finding each of the five rock types at each point in the simulation fields. It is notable that compared to the reference local pdfs, the global SIS realizations do not show enough points with probabilities of 0 or 1 for any facies. This is due to the possibility of finding any of the facies at any point, which in inherent to simple IK with a global pdf.

Conclusions

Local probabilities for SIS can return realizations with better continuity; this is useful when geologic bodies are generally unbroken. The deterministic model used to derive the local pdfs may not be perfect, but the local distributions are almost certainly better than a global mean imposed uniformly. The local pdf approach can be used to soften the assumption of stationarity.

References

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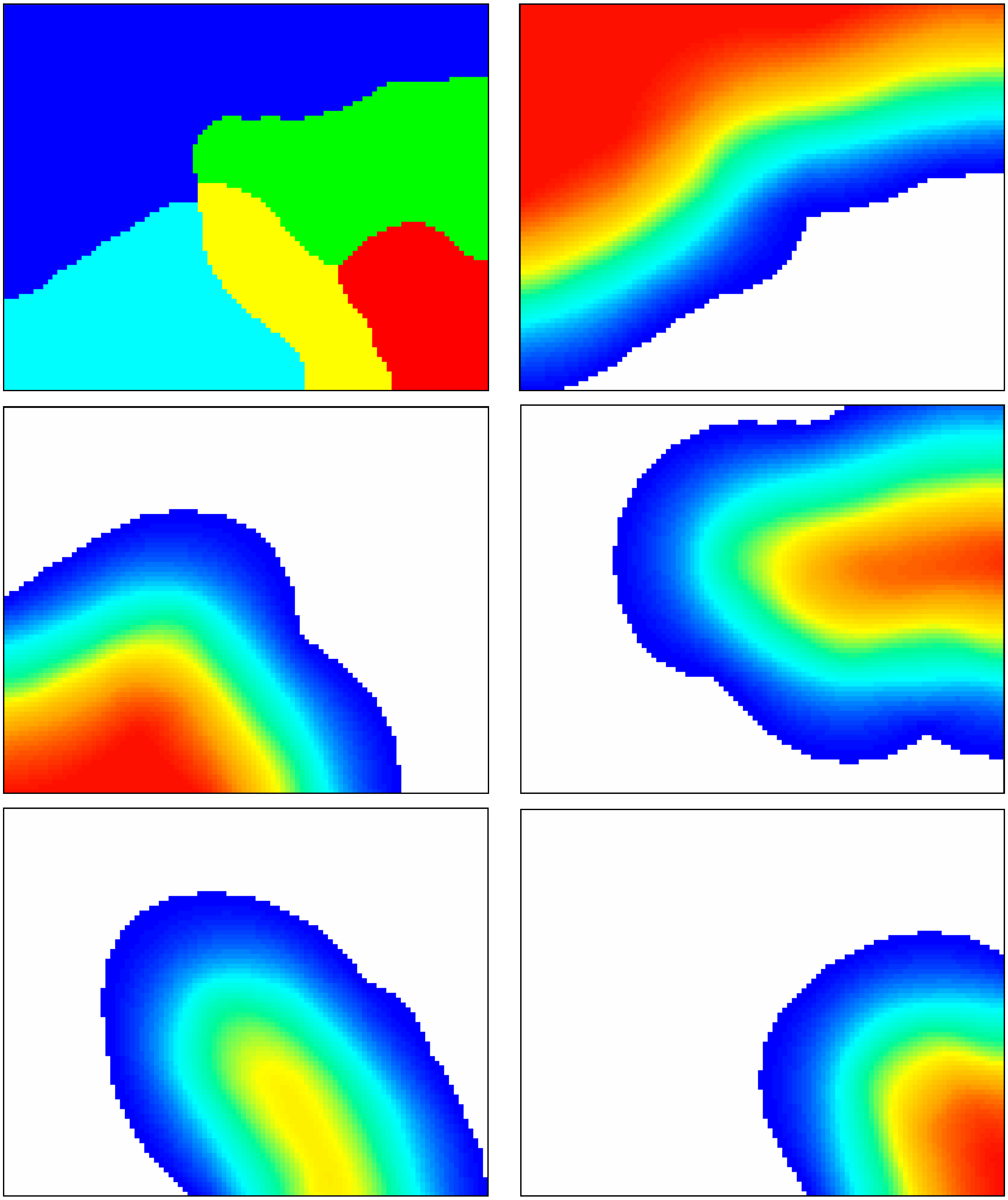

Figure 1: Top left: A deterministic model with five different rock codes. Rest: The local probabilities of finding each of the five rock codes at every point, one rock code per picture. The colours in the deterministic model image only serve to differentiate the different indicators. The colours in the five probability charts indicate the pdf value for the rock code represented in that graph; white is zero probability, blue is low probability and red is one.

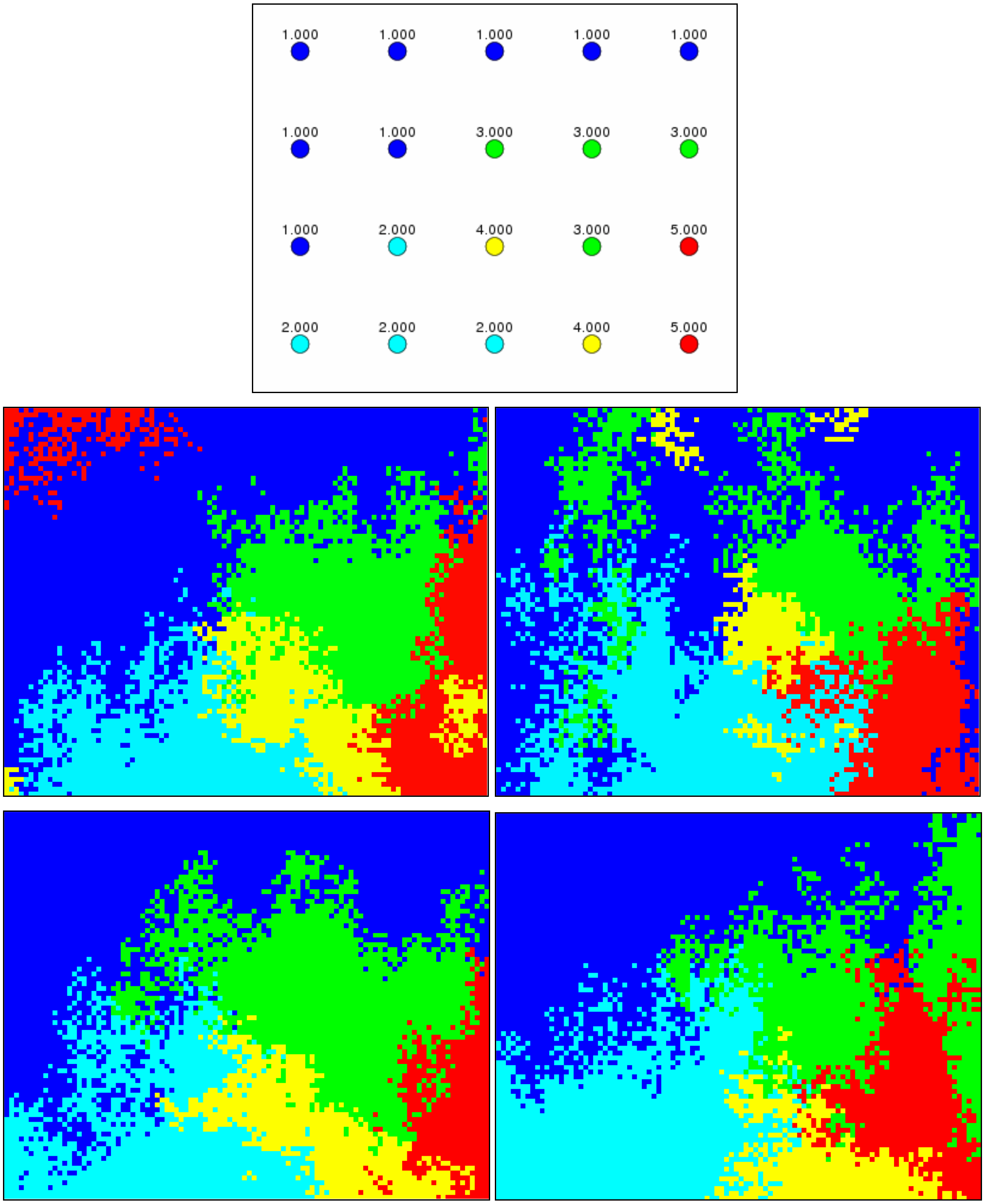

Figure 2: Top: 20 regularly-spaced samples taken from the deterministic image in Figure 1. These samples are used as conditioning data for SIS. Middle: Two SIS realizations using a global pdf. Bottom: Two SIS realizations generated by using the local indicator probabilities shown in Figure 1.

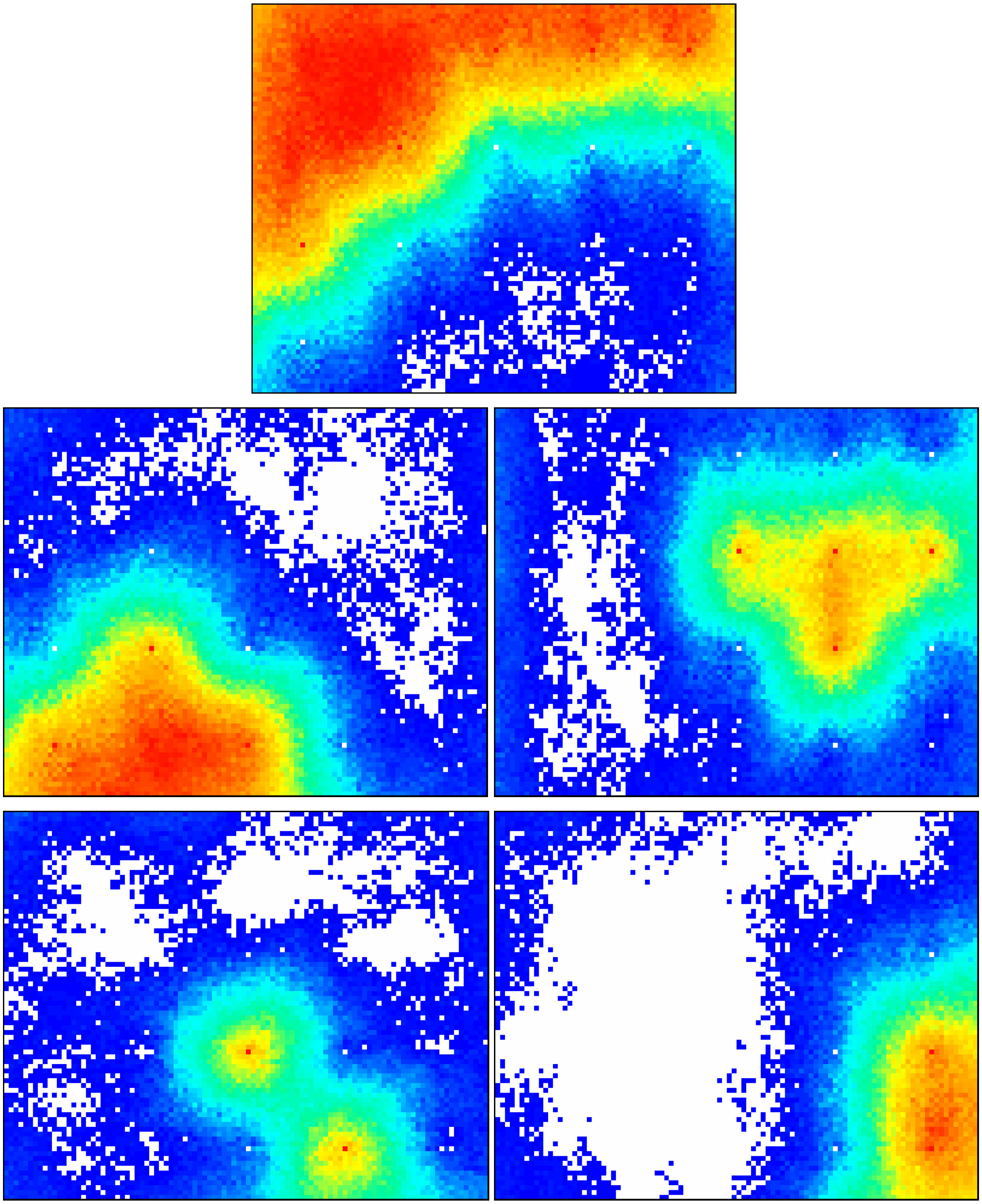

Figure 3: Probability maps of each of the five facies, calculated from 100 SIS realizations using a global pdf.

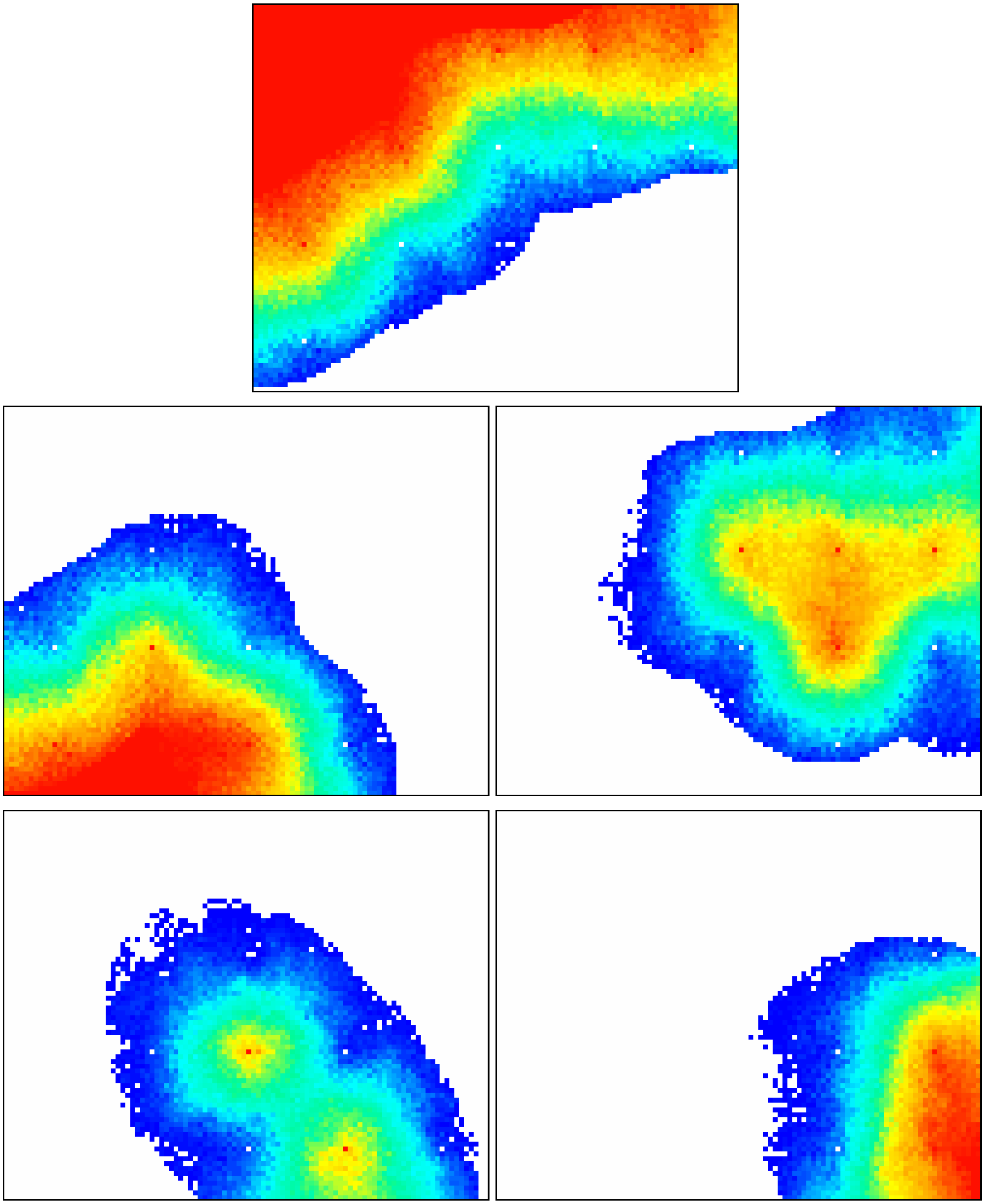

Figure 4: Probability maps of each of the five facies, calculated from 100 SIS realizations using a localized pdf.

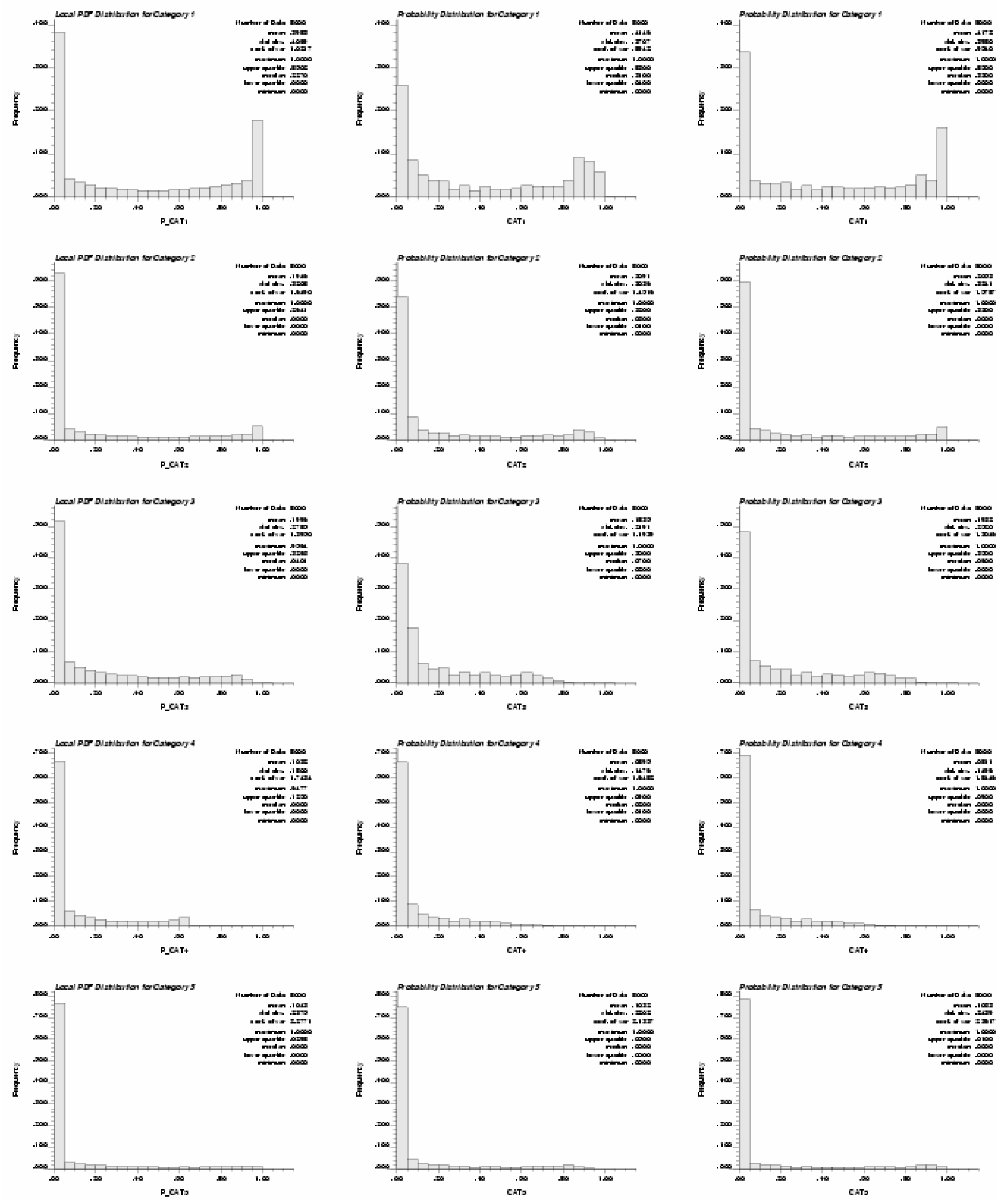

Figure 5: Histograms for all of the probability plots shown in Figures 1, 3, and 4. The categories start as 1 in the top row and increase to 5 moving to the bottom row. Left: Probabilities of each rock type at every point in the local pdf model from Figure 1. Centre: The probabilities from 100 SIS realizations using a global pdf, shown in Figure 3. Right: The probabilities taken from 100 SIS realizations using a localized pdf, shown in Figure 4.