Calculating Recoverable Reserves with Uniform Conditioning

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The theory of uniform conditioning is presented. This includes the discrete Gaussian model for change of support, block kriging to estimate the panel grades, and the calculation of the quantity of metal and proportion of the panel above the cutoff grade. The place of uniform conditioning is discussed. This paper is a short note that summarizes the Guidebook.

Introduction

The calculation of recoverable reserves must consider the application of a cutoff grade and a selective mining unit (SMU) size. The cutoff grade is calculated using economic and management parameters and is considered known. The SMU size is based on the deposit type and the mining equipment. The change of support from the data to the SMU scale is accomplished under a Gaussian model. Uniform conditioning (UC) is one Gaussian model. We present uniform conditioning in a complete and unified manner.

The earliest reference to UC was Matheron (1974). Remacre and Guibal published a paper on recoverable reserves that included UC (Guibal and Remacre, 1984). An Appendix in Guibal's 1987 paper (Guibal, 1987) describes UC in a Gaussian context. Remacre presents a summary of UC in the same collection (Remacre, 1987). Remacre compares UC and indicator kriging (Remacre, 1989). Rivoirard's book derives the theory for UC (Rivoirard, 1994). An interesting paper on the history of non-linear geostatistics was presented by Vann and Guibal where they discussed the place of indicator kriging and UC (Vann, Guibal, and Harley, 1998). Chilès and Delfiner mention UC in their book (Chilès and Delfiner, 1999). Assibey-Bonsu and Krige presented a paper comparing several different methods for estimating recoverable resources (Assibey-Bonsu and Krige, 1999). Chris Roth and Jacques Deraisme (2000) proposed a model to incorporate the information effect in UC.

Uniform Conditioning

The workflow for UC can be summarized in 5 steps (adapted from Remacre, 1987):

1. Estimate the panel grades.
2. Fit the Discrete Gaussian Model (DGM) to the data.
3. Determine the change of support coefficients for the SMU and panel sized blocks.
4. Transform the Z panel estimates to Y using the panel anamorphosis function and the Z cutoff grades to Y using the SMU anamorphosis function.
5. Calculate the proportion and quantity of metal above each cutoff.

Each step will be discussed in detail below.

Step 1: Estimation of the panel grade

UC relies on a robust estimate of the panel grade. Consider the typical data available during the exploration phase of a mining project. The data are widely spaced conform to a coarse grid. Estimating very small blocks with relation to the data spacing does not produce reliable results. Block kriging a larger mining panel will give more reliable results. Recall the kriging estimator:

$$Z^*(\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u}) z(\mathbf{u}_\alpha) \tag{1}$$

and the ordinary block kriging system of equations (Journel, 1978):

$$\begin{aligned} \sum_{\beta}^{n} \lambda_\beta C\left(\mathbf{u}_\beta, \mathbf{u}_\alpha\right) + \mu(\mathbf{u}) &= \overline{C}\left(V(\mathbf{u}), \mathbf{u}_\alpha\right), \quad \alpha = 1, \ldots, n \\ &\approx \frac{1}{N} \sum_{j=1}^{N} C\left(\mathbf{u}'_j, \mathbf{u}_\alpha\right) \\ \sum_{\beta}^{n} \lambda_\beta &= 1 \end{aligned} \tag{2}$$

Step 2: Fitting the Distribution

Data are small scale. These small samples are not representative for large scales. The sample distribution is fit using a Hermite polynomial expansion. Once the polynomials have been fitt, the function maps the point variable, Z, to the Gaussian variable, *Y*:

$$\begin{aligned} Z(\mathbf{u}) &= \Phi(Y(u)) \\ &\approx \sum_{n=0}^{np} \phi_n H_n\left[Y(\mathbf{u})\right] \end{aligned} \tag{3}$$

where *np* is the highest order term in the polynomial expansion, ϕ_n is a fitted coefficient for each term, and $H_n[Y(\mathbf{u})]$ is the hermite polynomial value defined by the term of the expansion and the y value. Eq. (3) is referred to as the Gaussian anamorphosis. The ϕ coefficients must be calculated for the anamorphosis function. The first coefficient is:

$$\phi_0 = E\left\{Z(\mathbf{u})\right\} \tag{4}$$

or the expected value of $Z(\mathbf{u})$. Higher order coefficients can be calculated using:

$$\begin{aligned} \phi_p &= E\left\{Z(\mathbf{u}) \cdot H_p\left(Y(\mathbf{u})\right)\right\} \\ &= \int \Phi\left(y(\mathbf{u})\right) \cdot H_p\left(y(\mathbf{u})\right) \cdot g\left(y(\mathbf{u})\right) \cdot dy(\mathbf{u}) \end{aligned} \tag{5}$$

Eq. (5) can be approximated with the data at hand, as a finite summation:

$$\phi_p = \sum_{\alpha=2}^{N} \left(z(\mathbf{u}_\alpha - 1) - z(\mathbf{u}_\alpha)\right) \cdot \frac{1}{\sqrt{p}} H_{p-1}\left(y(\mathbf{u}_\alpha)\right) \cdot g\left(y(\mathbf{u}_\alpha)\right) \tag{6}$$

The fitted coefficients must satisfy the following equality:

$$Var\{Z(\mathbf{u})\} = \sum_{p=1}^{np} \phi_p^2 \tag{7}$$

where $Var\{Z(\mathbf{u})\}$ is the variance of Z at the point support. If the summation is significantly different, the anamorphosis modelling should be checked. See Journel (1978) and Chilès and Delfiner (1999) for additional information.

Step 3: Change of Support Coefficient Calculation

The discrete Gaussian model is used for calculating the change of support. It controls the shape and variability of the distribution at the larger scale. The anamorphosis function in Eq. (3) can be modified to account for the change of support from point data to block data by the addition of a change of support coefficient r:

$$\begin{aligned} Z(v) &= \Phi(Y(v)) \\ &\approx \sum_{n=0}^{np} r^n \phi_n H_n\left[Y(v)\right] \end{aligned} \tag{8}$$

The distribution of grades for large volumes can be determined by calculating r, which requires the variance of the larger support volumes. Typically, there is not enough data available to do this explicitly. The dispersion variance of the larger blocks can be estimated using the modeled variogram of the point data:

$$\sigma_v^2 = \sigma_{\mathbf{u}}^2 - \bar{\gamma}_{v,v} \tag{9}$$

where v is the smu support volume, σ_v^2 is the variance of the smu sized blocks, $\sigma_{\mathbf{u}}^2$ is the variance of the point data, and $\bar{\gamma}_{v,v}$ is the average variogram value for the smu. This equality is true for the point support and the block support:

$$\begin{aligned} Var\{Z_v\} &= \sigma_v^2 \\ &= \sigma_{\mathbf{u}}^2 - \bar{\gamma}_{v,v} \\ &= \sum_{n=1}^{np} r^{2n} \phi_n^2 \end{aligned} \tag{10}$$

where $Var\{Z_V\}$ is the variance of Z at the smu support. The only unknown parameter is r. A bisector search method can be applied to find the value of r that satisfies the equality.

This procedure has to be applied again to find the change of support parameter, r', for the panels. The panel anamorphosis function is defined as:

$$\begin{aligned} Z(V) &= \Phi(Y(V)) \\ &\approx \sum_{n=0}^{np} \left(r'\right)^n \phi_n H_n\left[Y(V)\right] \end{aligned} \tag{11}$$

The panel variance should be estimated from the variance of the kriged panels. A more robust estimate for the panel change of support can be obtained by incorporating the information effect, see Roth and Deraisme (2000). The panel change of support coefficient can be estimated by solving the following equation:

$$\begin{aligned} Var\{Z_V\} &= \sigma_V^2 \\ &= \sum_{n=1}^{np} \left(r'\right)^{2n} \phi_n^2 \end{aligned} \tag{12}$$

Step 4: Transformation of the Panel Estimates and Cutoff Grades to Gaussian Units

If the panel estimation was done in original grade units, each estimate will need to be transformed to Gaussian units using the panel anamorphosis function from Eq. (11). Each cutoff grade also needs to be transformed to Gaussian units. The cutoffs grade should be transformed using the SMU anamorphosis from Eq. (8).

Step 5: Calculation of the Proportion and Quantity of Metal above Cutoff

Given that the panel grade is known, the distribution of the SMU's within that panel can be calculated. By definition, the average of the SMU's within the panel is the panel grade, and the variance is based on the change of support model. The recoverable reserves are defined by the proportion and quantity of metal above the cutoff grade.

The proportion above the cutoff grade is calculated as follows:

$$\begin{aligned} P(z_c) &= P\left[Z(v) \geq z_c \mid Z(V)\right] \\ &= P\left[Y_v \geq y_c \mid Y_V\right] \\ &= 1 - G\left(\frac{y_c - \left(\frac{r'}{r}\right) Y_V}{\sqrt{1 - \left(\frac{r'}{r}\right)^2}} \right) \end{aligned} \tag{13}$$

The quantity of metal can be calculated in one of two ways. The first is an integration of the conditional distribution above the cutoff grade (Remacre, 1984):

$$Q(z_c) = \int_{y_c}^{\infty} \Phi_r(Y_v) g(Y_v \mid Y_V) d(y_v) \tag{14}$$

The second is by using the fitted hermite polynomials (Rivoirard, 1994):

$$
\begin{aligned}
Q(z_c) &= E\left[Z(v) I_{Z(v)\geq z_c} \mid Z(V) \right] \\
&= E\left[\Phi_v(Y_v) I_{Y_v \geq y_c} \mid Y_V \right] \\
&= \sum_{n=0}^{np} q_n \left(\frac{r'}{r} \right)^n H_n(Y_V) \\
&= \sum_{n=0}^{np} \sum_{p=0}^{np} \phi_p r^p U_p^n(y_c) \left(\frac{r'}{r} \right)^n H_n(Y_V)
\end{aligned} \tag{15}
$$

where

$$
\begin{aligned}
&U_0^0(y_c) = 1 - G(y_c) \\
&U_k^0(y_c) = U_0^k(y_c) = \frac{-1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) \\
&U_p^n(y_c) = \frac{-1}{\sqrt{n}} H_p(y_c) H_{n-1}(y_c) g(y_c) + \sqrt{\frac{p}{n}} U_{p-1}^{n-1}
\end{aligned}
$$

The grade above cutoff is calculated from the quantity of metal and proportion:

$$
M(z_c) = \frac{Q(z_c)}{P(z_c)} \tag{16}
$$

Discussion

Recoverable reserves are driven by the change of support model and its parameters. At early stages of exploration, the change of support is controlled by the variogram. Short range variogram structure has a large impact. Sensitivity analysis should be undertaken. More data will be available as the mine moves into production. The change of support model can be validated by production. UC is best suited for diffusion style deposits. Multiple indicator kriging may perform better in cases where the connectivity of the extreme values is important. Vann and Guibal (1998) present some easy statistical tests to determine the suitability of UC.

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