An Update on Kriging Near Boundaries

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Gradational or soft boundaries are common in several types of geological settings due to the transitional nature of geological mineralisation processes. Contacts were grades change transitionally across the boundary are usually characterized by a non-stationary behavior of the variable of interest, that is, the mean, variance or covariance are no longer constant within a zone of influence of one rock type into the other, and their values depends on the location relative to the boundary. The M.Sc. thesis of Larrondo developed the framework for non-stationary (co)kriging in the presence of a non-stationary soft boundary. This approach reduces the misclassification of ore and waste within the transition zone. This paper describes advances in the methodology since that publication, debugged software and application notes.

Background

Copper mineralisation in porphyry-type deposits often occurs as disseminated and stockwork mineralisation. The grade usually changes gradually across geological boundaries. These types of deposit are usually embedded in a large-scale fault system that gives a structural signature to contacts between domains. Faults are usually active throughout deposit formation; therefore, some mineralisation occurs along extension fractures around the major faults, leading to a non-stationary behavior of grades towards boundaries. Post mineralisation process such as leaching occurs preferentially along faults, which also results in non-stationary behavior in the vicinity of contacts.

The references (Larrondo, 2004; Larrondo and Deutsch, 2004a/b) provide details of the approach to estimate in presence of a non-stationary soft boundary. The developed methodology will be referred to as Soft Boundary Kriging (SBK). SBK consists of (1) identification of the boundaries between rock types, (2) selection of a distance of non-stationary influence into each rock type and the precedence/timing of rock types to break ties when three or more rock types influence a particular area, (3) optimization of non-stationary mean and variance functions within each boundary zone, and finally (4) non-stationary cokriging to calculate estimates. This workflow is robust and reasonable from a geological perspective. The non-stationary model is based on the addition of stationary rock-type specific random variables and non-stationary boundary zone random variables. The result is a licit random variable that is very practical.

Reproduction of the trends and non-stationary features of soft boundaries has a great impact on mine planning, expected dilution and ore reserves. Boundary areas are often associated with greater uncertainty; classification may also be affected. Two synthetic examples are shown to highlight the differences between alternate methodologies to estimate in presence of a soft boundary. Areas of current and future work are identified. The latest SBK kriging program is very clean and we are extending the approach to simulation.

Small Example

Figure 1 shows the setting of a small synthetic example. Data on a 70m by 70m grid spacing were sampled to approximate an exploration setting. These data were used to construct three kriged models: (1) kriging assuming the boundary is hard, that is, no influence of data across the boundary, (2) kriging assuming the boundary is soft, that is, data from across the boundary can be used although the variogram is rock type specific, and (3) soft boundary kriging (SBK). A reference model was constructed by kriging a set of blasthole data sampled at a 20m by 20m spacing. Figure 2 shows plots of the three estimates and the reference blasthole kriging results.

The three different estimates can be compared to the reference within the boundary zone. SBK shows a 20% improvement in the percentage of blocks misclassified with respect to a hard boundary modelling approach and a 10% improvement compared with soft boundary estimation. Table 1 shows (mis)classification blocks for the three methodologies within the boundary zone.

The boundary zone is separated into a high grade boundary zone (HGBZ) corresponding to the area of influence of the contact inside RT1 and a low grade boundary zone (LGBZ) corresponding to the area of influence between RT1 and RT2 inside RT2, see Figure 3. The misclassification in the HGBZ and LGBZ is shown on Tables 2 and 3. In the LGBZ, the hard boundary modelling scheme underestimates ore, while a soft boundary approach overestimates ore compared to waste (Table 2). In the case of the HGBZ, the hard boundary scheme over estimates ore, while a soft boundary appears unbiased.

Table 4 shows a comparison of the tonnage, metal content and average grade above cutoff. The SBK kriging methodology performs better in all cases.

Big Example

A 3-D example was built using a geological model of a porphyry copper deposit from Northern Chile. Selected sections through the rock type and grade model are shown on Figures 3 and 4. Non-stationary cokriging requires a rock type model with the boundary zone and distance to boundary assigned to each block. Considering the geology of the deposit, the matrix of maximum distance of influence (in meters) between rock types required to perform non-stationary kriging was chosen (see right).

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 30 & 30 & 30 \\ 0 & 0 & -1 & 120 & 60 \\ 0 & 0 & 90 & -1 & 90 \\ 0 & 0 & 30 & 30 & -1 \end{bmatrix}$$

The boundary zones defined by the contact between the primary mineralization units correspond to a wide fault zone, therefore, they were assumed to be more extensive than the ones defined by the secondary mineralization. Also, the secondary mineralization grades influence the primary mineralization, but the reverse is not allowed (the zero distances in the matrix). A set of precedence rules is also required that captures the timing of the mineralization; the influence of rock type 2 is the youngest (see right).

$$\left.\begin{matrix} 4\text{-}5 \\ 3\text{-}4 \\ 4\text{-}3 \end{matrix}\right\} \textit{primary mineralization}$$

$$\left.\begin{matrix} 5\text{-}2 \\ 4\text{-}2 \\ 3\text{-}2 \end{matrix}\right\} \textit{secondary mineralization}$$

The optimum mean, variance and covariance models for the non-stationary boundaries required for the kriging program `kt3d_bound` where calculated as in Larrondo and Deutsch (2004b). Kriging with a non-stationary boundary was performed. A minor bug in the kriging program was fixed; all kriging systems are solvable, kriging variances are positive and the resulting estimates appear reasonable.

The best conventional technique was considered to be ordinary kriging with soft boundaries. Cross validation was performed with this approach and the recommended SBK approach. Figure 6 shows the cross validation results. SBK works significantly better than ordinary kriging. The non-stationary behavior of the mean is very well reproduced by the proposed non-stationary cokriging as shown in Figure 7. Although the variance of the estimates in the boundary zone is lower than the reference, the increasing trend toward the boundary is well reproduced (Figure 8).

Conclusions and Future Work

Setting up a multipart random variable with stationary and non-stationary factors in the presence of geological boundaries provides a theoretically robust methodology to handle non-stationary soft boundaries. The soft boundary kriging (SBK) approach has been applied to a number of different datasets.

The large 3D geological model of a porphyry copper deposit illustrates the utility of this technique for a practical application. The SBK kriging estimates reproduce the non-stationary behavior of the conditioning data at the geological contacts, and it also reproduces the stationary means of each rock type in the model. A decrease in the global variance is due to the smoothing effect of kriging. Cross validation results show that the result works much better than assuming a hard boundary.

The required optimization programs were included in last year's report. The latest code for kriging is released as part of this CCG report. These programs will be extended to simulation software in the near future.

References

- C.V. Deutsch and A.G. Journel. *GSLIB: Geostatistical Software Library: and User's Guide*. Oxford University Press, New York, 2nd Edition, 1998.
- P. F. Larrondo. *Accounting for Geological Boundaries in Geostatistical Modeling of Multiple Rock Types.* M.Sc. thesis, University of Alberta, Edmonton, AB, 2004
- P. F. Larrondo and C. V. Deutsch. Methodology for Geostatistical Model of Gradational Geological Boundaries: Local Non-stationary LMC. In *Centre For Computational Geostatistics*, Volume 6, Edmonton, AB, 2004.
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	Hard Boundaries		Soft Boundaries		Non-stationary Soft Boundaries	
Reference \ Estimate	Ore	Waste	Ore	Waste	Ore	Waste
Ore	47.6	9.3	50.9	6.0	50.5	6.3
Waste	5.1	38.1	7.2	35.9	5.7	37.4
Misclassified:	14.4%		13.2%		12.0%	

Table 1: Percentage of blocks correctly classified as ore and waste and misclassified compared to the reference for the three methodologies within the boundary zone.

	Hard Boundaries		Soft Boundaries		Non-stationary Soft Boundaries	
Reference \ Estimate	Ore	Waste	Ore	Waste	Ore	Waste
Ore	15.3	14.2	23.8	5.7	22.3	7.2
Waste	3.0	67.5	7.6	62.9	5.0	65.5
Misclassified:	17.2%		13.3%		12.2%	

Table 2: Percentage of blocks correctly classified as ore and waste and misclassified compared to the reference for the three methodologies within the low grade side of the boundary zone (LGBZ).

	Hard Boundaries		Soft Boundaries		Non-stationary Soft Boundaries	
Reference \ Estimate	Ore	Waste	Ore	Waste	Ore	Waste
Ore	79.8	4.4	77.9	6.3	78.7	5.5
Waste	7.1	8.7	6.8	8.9	6.4	9.4
Misclassified:	11.5%		13.1%		11.8%	

Table 3: Percentage of block correctly classified as ore and waste and misclassified compared to the reference for the three methodologies within the HGBZ.

	Reference			**Hard Boundaries**					
	Metal	Tonnes	Avg Grade	Metal	%	Tonnes	%	Avg Grade	%
HGBZ	58,466	1,782,763	3.28	54,642	93	1,840,865	103	2.97	91
LGBZ	15,964	624,205	2.56	9,029	57	386,398	62	2.34	91

	Reference			**Soft Boundaries**					
	Metal	Tonnes	Avg Grade	Metal	%	Tonnes	%	Avg Grade	
HGBZ	58,466	1,782,763	3.28	52,456	90	1,794,510	101	2.92	89
LGBZ	15,964	624,205	2.56	16,931	106	665,480	107	2.54	99

	Reference			**Non-stationary Soft Boundaries**					
	Metal	Tonnes	Avg Grade	Metal	%	Tonnes	%	Avg Grade	
HGBZ	58,466	1,782,763	3.28	56,304	96	1,801,178	101	3.13	95
LGBZ	15,964	624,205	2.56	14,165	89	579,120	93	2.45	96

Table 4: Tonnage, metal content and average grade above cutoff comparison between a hard boundaries approach, the soft boundaries approach and the non-stationary soft boundary kriging.

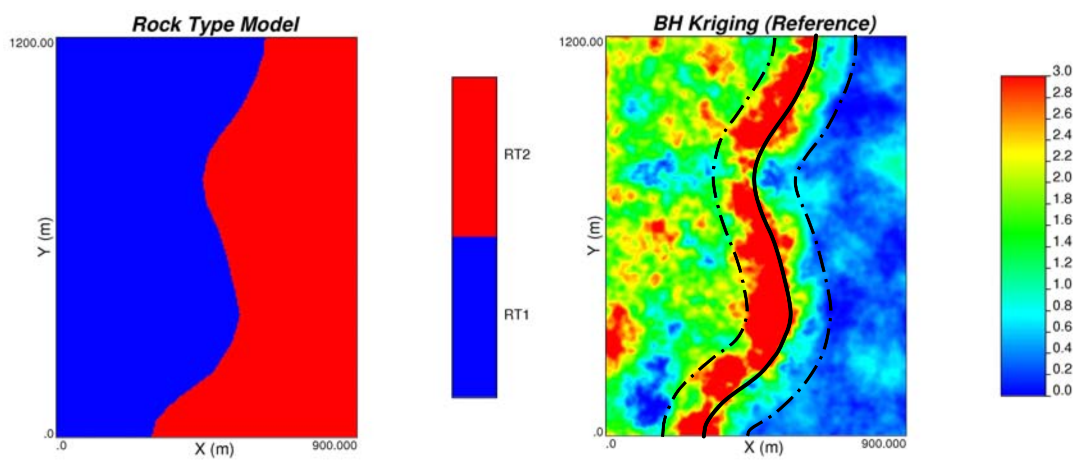

Figure 1: Synthetic example of two rock types with a soft boundary. The reference was sample at 20x20 meters mimicking a blast hole grid spacing and then estimated using kriging.

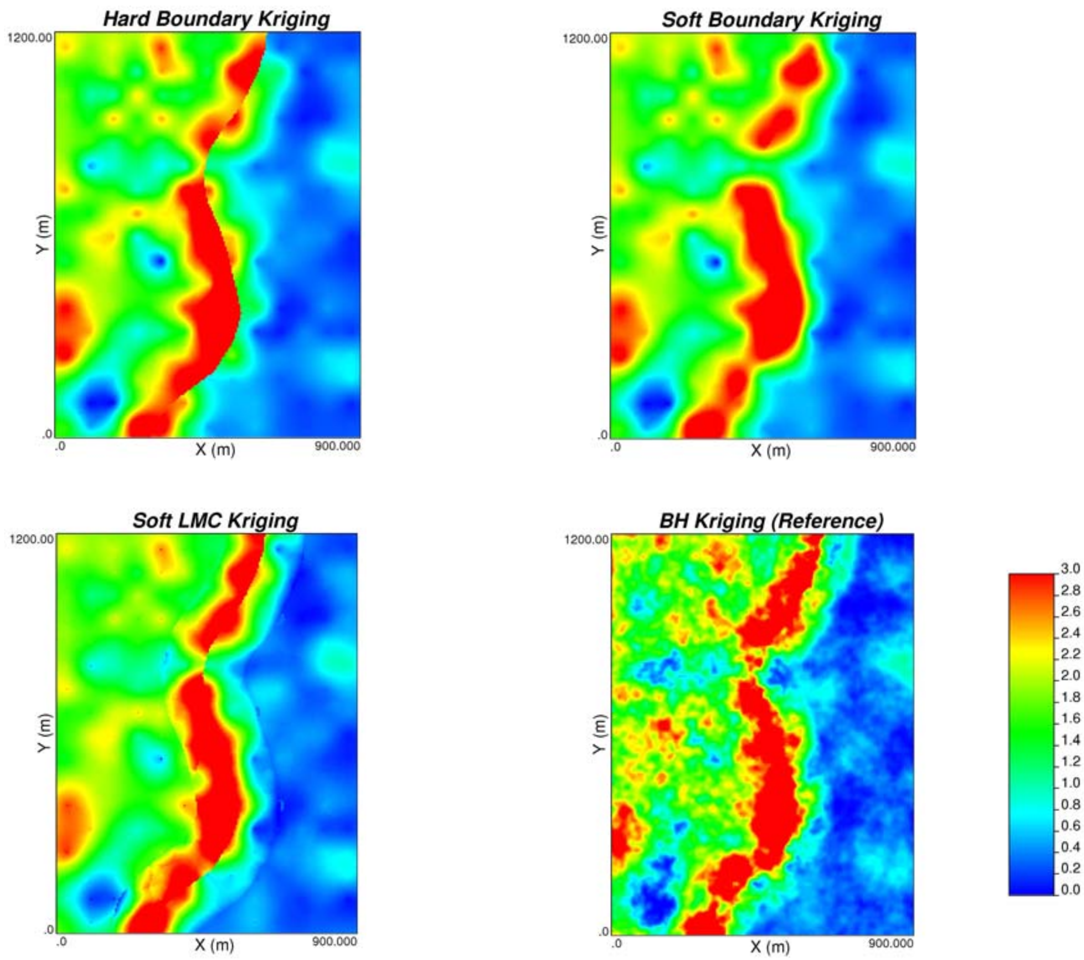

Figure 2: Results from the alternative kriging method applied to this synthetic example.

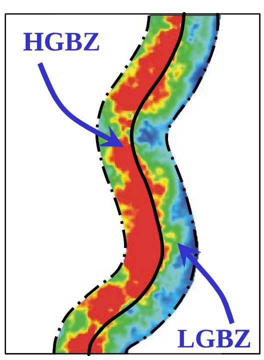

Figure 3: Boundary zone for the synthetic example separated into high grade and low zones for checking the estimation techniques

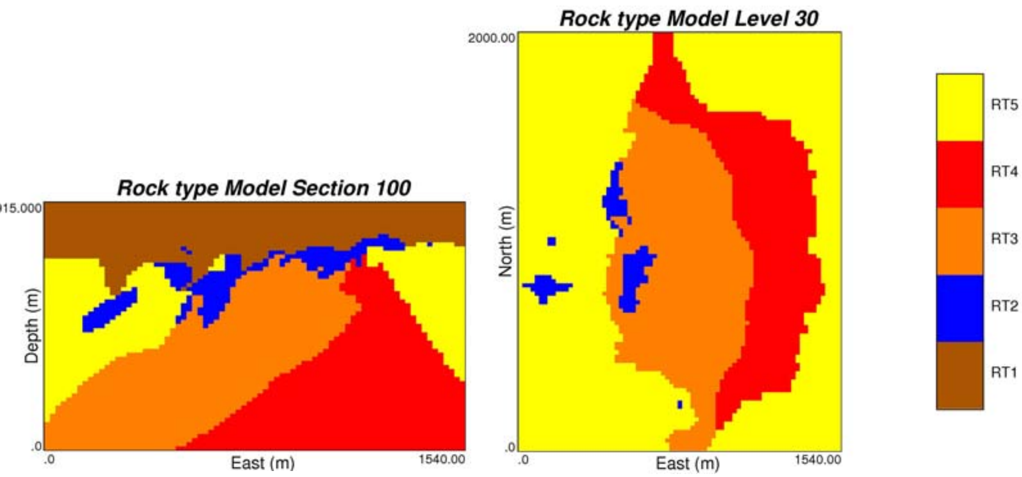

Figure 4: Categorical rock type model of a porphyry copper deposit in Northern Chile.

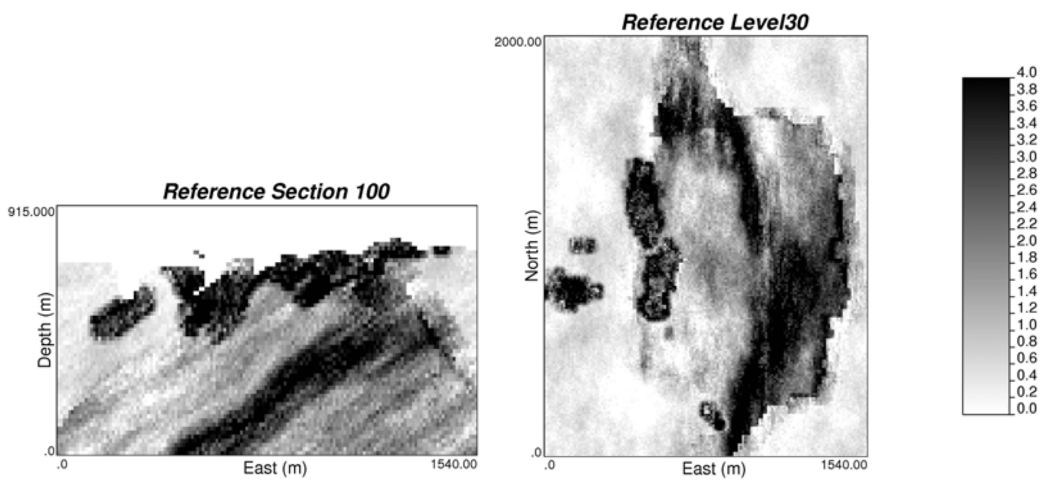

Figure 5: Section and Level maps of the reference distribution. Values from rock type 1 were assigned a default value of –9 since this unit is of no economic interest.

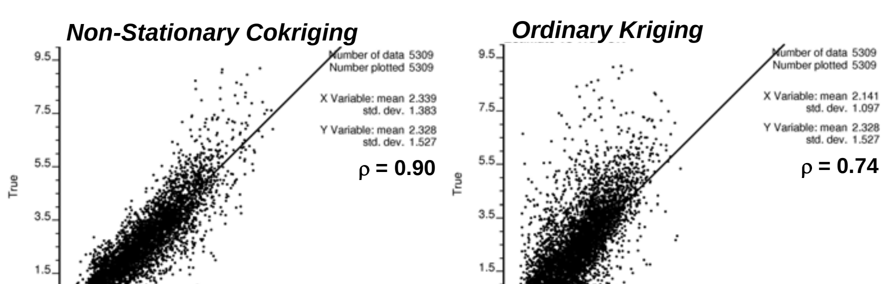

Figure 6: Cross validation comparison between the proposed methodology, non-stationary cokriging, and ordinary kriging with soft boundaries.

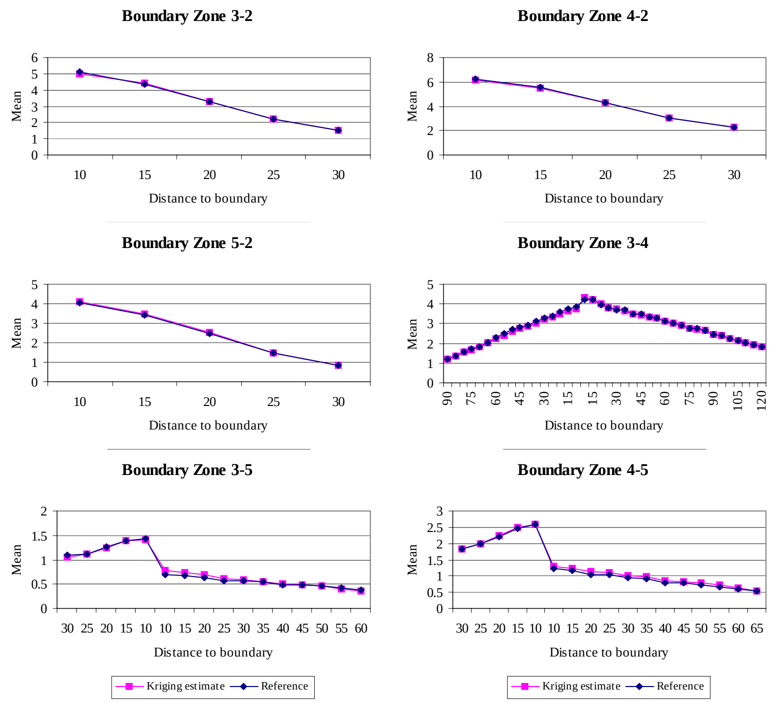

Figure 7: Mean at the non-stationary boundary zone. A 5 meters interval of the distance to the boundary was chosen to calculate the mean of the estimate value of all grid nodes

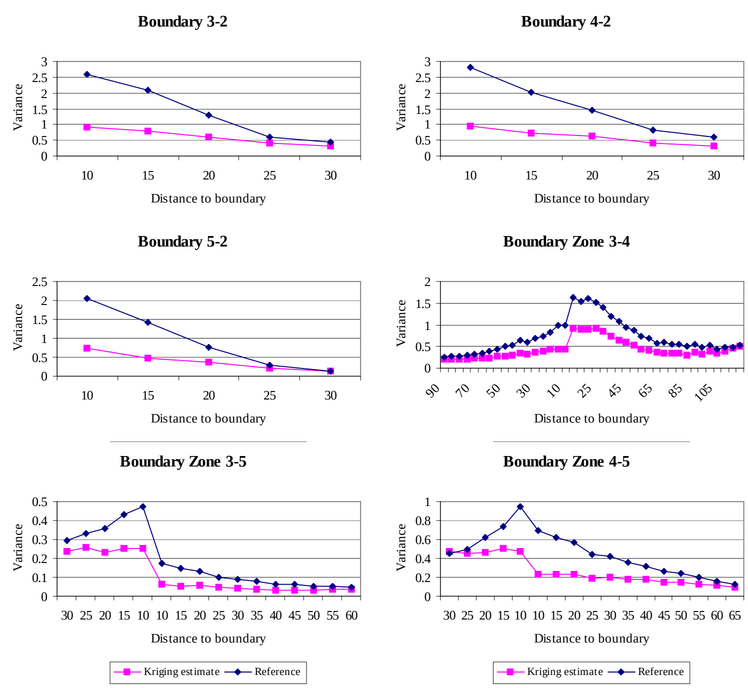

Figure 8: Variance at the non-stationary boundary zone. A 5 meters interval of the distance to the boundary was chosen to calculate the variance of the estimate value of all grid nodes