

A Short Note on Prediction of Uncertainty in Tonnage and Grade of Vein Type Deposits

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Geostatistical simulation is being used increasingly to quantify uncertainty and assess risk in mineral resources and ore reserves. The developments of geostatistical surface modeling, trend modeling and stochastic simulation permit a reliable scheme for the prediction of uncertainty in the tonnage and grade in vein type deposits. The Pangeos software provides a straightforward implementation of vein type modeling and uncertainty estimation. This short note documents some modeling workflow considerations and implementation details.

Introduction

Geostatistics provides a number of tools for simulation and risk analysis aimed at the mining industry. Vein type modeling is one particular niche area that is specifically targeted. The tools are available in a number of different software including the public domain GSLIB programs, some custom programs from CCG and commercial software including Pangeos; this document provides some details. In the context of this document, a vein type deposit is a tabular relatively thin deposit from a variety of geological settings including clastic sedimentation, weathering, and shear/contact zones.

Vein Uncertainty Estimation is broken down into four main steps:

- I. Data Assembly – preparing all of the necessary data and geological inputs
- II. Geometry Modeling – constructing models of surfaces and geometry
- III. Grade Modeling – modeling trends and grade realizations
- IV. Post Processing – processing the realizations to assess uncertainty

The following steps should be viewed as recommendations and suggestions rather than a recipe.

Data Assembly

There is more to Data Assembly than formatting the data in a spreadsheet or GSLIB-type data file: the overall geometry and a conceptual geological model must be established. There are two types of drillhole point data that must be loaded:

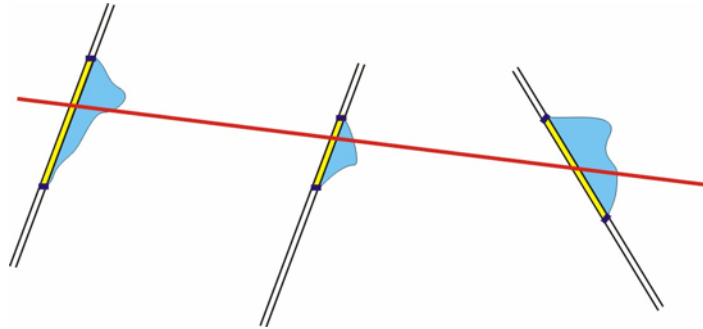
1. The drillhole markers for the hangingwall and footwall must be imported as a set of (X, Y, Z) points. These will be used primarily for geometric modeling. The vein thickness should also be loaded if it has been corrected to true thickness.

- The grades will come in as $(DH, X, Y, Z, Grade_1, \dots, Grade_N, Length)$ points. Logically, they should be between the markers; the user should check this by visualizing the results after loading the data.

The raw data of from-to grades and survey information is typically processed outside of geostatistical software to get to this format. A solid model should be converted to a regularly gridded indicator model. The indicator model will be used primarily for checking, but it could also be used for SIS with a locally varying mean.

Vein Coordinates

An estimate of the overall plane of the vein is required as an input. It is fairly straightforward to optimize the precise strike and dip: the *Fit Plane* function will provide best estimates of strike and dip given the assay data. The dip direction and dip angle are iteratively perturbed to minimize the variance of the Z values weighted by grade. The sketch below illustrates the concept on a cross section. The blue values represent grade values along the length of the drillholes. The red line is a good fit because it minimizes the deviation of the Z data from their mean.

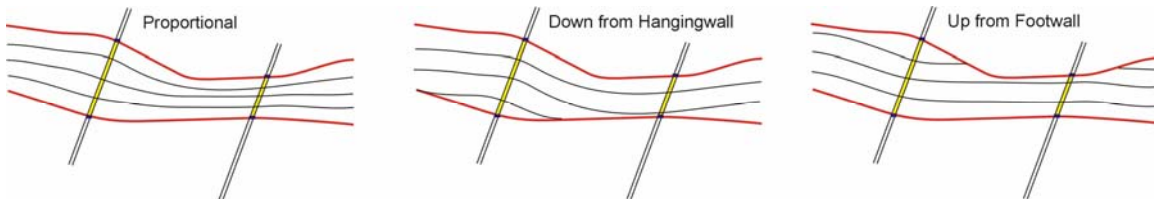


The criteria for fitting the vein dip and dip direction is to minimize the variability weighted by the grades:

$$\min \left(\sum_{i=1}^{N_{data}} grade_i \left[Z_{plane,i} - Z_{Vi} \right]^2 \right)$$

Where $Z_{plane}(X_{v,l}, Y_{v,i})$ is the fitted plane values at the data locations. High grades may have to be cut so that they do not have an inordinate influence on the fitted plane.

Rotation by the dip direction and dip leads to orthogonal vein coordinates: X_v , Y_v , and Z_v . Geometry and grade modeling will be done in the X_v/Y_v coordinate system; however, the perpendicular direction must usually be corrected to account for the continuity of the grades within the vein structure. The vertical coordinate for modeling the grades will be set to proportional (PROP), down from the hangingwall (HW-D), or up from the footwall (FW-U). The sketch below illustrates the three common options.



The corrected local vertical coordinate is referred to as Z_{VL} and is calculated as a function of the Z_V coordinate. For the three styles above:

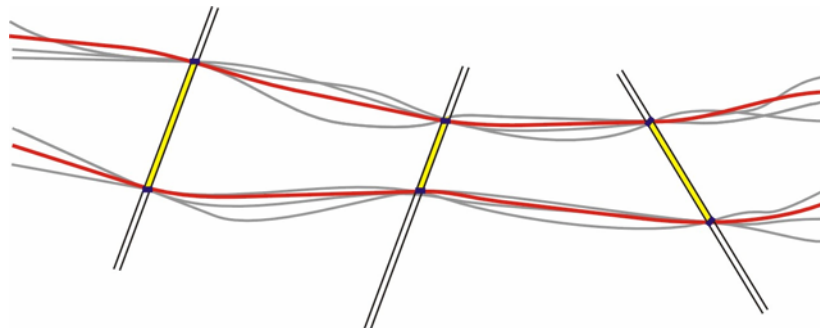
$$Z_{VL}^{PROP} = \frac{Z - Z_{FW}(X_V, Y_V)}{Z_{HW} - Z_{FW}(X_V, Y_V)} \cdot T \quad Z_{VL}^{HW-D} = (Z - Z_{HW}(X_V, Y_V)) \quad Z_{VL}^{FW-U} = (Z - Z_{FW}(X_V, Y_V))$$

where T is the average thickness of the vein structure. The proportional style does distort the volume support of the data and care should be taken to avoid bias. The coordinate in the HW-D system is always negative and the coordinate in the FW-U system is always positive. Grades will be modeled in the local Z_{VL} coordinate.

Geometry Modeling

Local variability and uncertainty in geometry can be very important in vein type deposits. It is important to reasonably represent the deterministic features of geometry while accurately capturing and modeling the stochastic features.

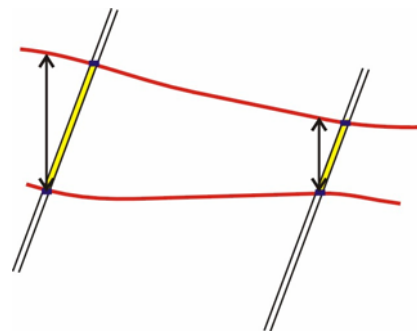
The geometry is specified by a hangingwall surface and footwall surface. The hangingwall surface is typically modeled first by estimation. Then, simulation is used to create multiple realizations. The sketch below shows three drillhole intersections (in yellow with blue markers), an estimated hangingwall contact (the upper red line), an estimated footwall contact (the lower red line), and three realizations of both the hangingwall and footwall.



Estimation of the Hangingwall/Footwall

This is done by ordinary kriging. It is a good idea to use a variogram with a zero nugget effect so that the drillhole data are closely matched. The variogram range could be set arbitrarily between $\frac{1}{4}$ and $\frac{1}{2}$ of the domain size to provide smooth variations. It is also a good idea to use a global search (all of the drillhole data to estimate each location), which avoids circular/elliptical artifacts due to the search.

The hangingwall surface in the Z_V units is kriged first. Then, the footwall surface in Z_V units can be estimated in similar manner or thicknesses can be used. Estimating with thickness is slightly more problematic. One of two approaches must be considered: (1) true corrected thickness values at the drillholes are treated as a variable, which is directly estimated, or (2) thickness values are calculated from the footwall elevations and the collocated (in X_V/Y_V space) estimated hangingwall elevation; see



the arrows on the sketch to the right. The estimated thicknesses are subtracted from the estimated hangingwall surface to provide an estimated footwall surface.

Simulation of the Hangingwall/Footwall

SGS is often used with the normal scores variogram of the hangingwall markers in Z_{VL} coordinates. A fixed number of realizations (normally, 5, 20 or 100) are created. The same number of hangingwall and footwall realizations are constructed. They are treated as pairs throughout all subsequent calculations.

Special Considerations

There are times when we want to simulate the deviation from a user input hangingwall surface, footwall surface, or thickness surface. The challenge is coming up with reasonable Δ distributions. There are also times we want to control pinchouts at the margins of the vein or, perhaps, internal to the vein. It would be reasonable to permit negative thicknesses and then remove the negative thickness from visualization and all post processing – perhaps just set the footwall to the hangingwall if the footwall elevation exceeds the hangingwall.

Grade Modeling

There are many reasons for a reliable grade model including (1) the grade is essential for reliable economic forecasts, (2) increasingly, there is a desire to selectively extract part of the vein structure due to economic constraints, and (3) there is a need to understand the impact of dilution on average grade.

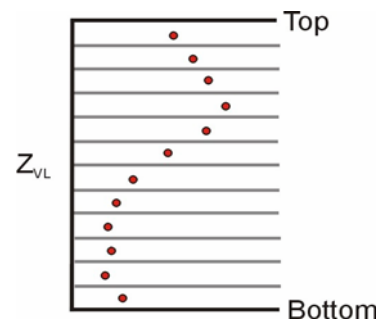
All grade modeling is done using the X_V/Y_V coordinates in the areal plane and the appropriate Z_{VL} coordinate in the vertical direction. Of course, in a steeply dipping vein, the Z_{VL} coordinate is far from vertical, but we refer to it as vertical in any case. In the case of the proportional correlation style, the average thickness T is discretized into a fixed number of Z_{VL} slices of varying thickness. In the case of the hangingwall-down or footwall-up correlation style, a maximum thickness is chosen and constant thickness Z_{VL} slices are chosen.

A kriged model is a good starting point. A variogram of the grades (no transformation) should be calculated and modeled. Ordinary kriging is considered robust and can be used to create this model. Conventional wisdom in surface mining is to restrict the number of data in the kriging to arrive at reasonable estimates of recoverable reserves, that is, try to correct for the information effect with a restricted search. This is not good practice in underground mining. Conditional bias, that is, overestimation of high grades and underestimation of low grades is a common result of a restricted search. A large search is recommended. This may smooth the grades, but the resulting model will not systematically overestimate the high grade stopes.

Simulation is required to realistically appraise the importance of spatial variability and to transfer that variability through to grade uncertainty and financial risk. Simulation requires trend modeling before simulation.

Trend Modeling

A vertical trend is almost always required. It is often calculated in slices of the Z_{VL} coordinate. The number of slices will depend on the number of drillholes; more data will permit more slices. It is important that the vertical trend be smooth. The



schematic to the right is an example. There are minor fluctuations in the bottom three slices, but the overall behavior is smooth. If there are more variations than shown here, a larger smoothing window should be considered. It may be necessary to cut high grades before constructing the vertical trend. A schematic trend is shown at the right. The vertical trend is represented by $m_Z(Z_{VL})$ for all Z_{VL} slices in the model.

An areal trend in the X_V/Y_V coordinates is often constructed by kriging. The first step is to vertically average the drillhole data to permit 2-D interpolation. The parameters for kriging are quite different than in the surface modeling discussed above. The grid spacing in X_V/Y_V is the same as the ultimate close spacing; however, block kriging is used to avoid reproducing the drillholes exactly. There should be a 20-40% nugget effect, a global search, and a large range in the ordinary kriging. A schematic trend is shown at the right. The areal trend is represented by $m_{XY}(X_V, Y_V)$ for all areal grid cells in the model. Sometimes the areal trend should be created using the thickness as an external trend model.

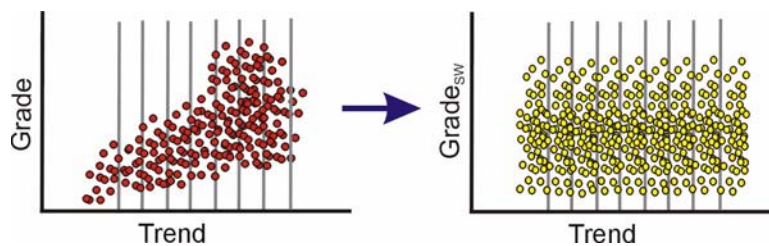
The vertical and areal trends must be combined by some formula. The most common approach is to assume conditional independence between the two trends:

$$m_{XYZ}(X_V, Y_V, Z_{VL}) = m_{global} \square \frac{m_Z(Z_{VL})}{avg(m_Z(Z_{VL}))} \square \frac{m_{XY}(X_V, Y_V)}{avg(m_{XY}(X_V, Y_V))}$$

The global mean is often very close to the average of the vertical trend $avg(m_Z(Z_{VL}))$ and the average of the areal trend $avg(m_{XY}(X_V, Y_V))$; however, declustering and/or debiasing may be necessary to establish the representative mean m_{global} .

Transformation

The trend values could be used as a locally varying mean, as an external drift, or as a local mean to calculate residuals. A robust approach to calculate residuals is the stepwise transformation. This is considered in **Pangeos**. The grades are transformed conditional to the trend. The following picture shows a schematic. The original grades will be highly correlated with the modeled trend. A number of classes are chosen (the vertical gray lines). The grades within each class are transformed to a standard normal distribution.



We need to have the transformation outside of SGS to permit variogram calculation, but it should also be embedded within SGS to facilitate back transformation and direct simulation of grade models.

Variograms of the resulting transform are used in SGS. The variogram may not show an easily interpreted structure, particularly if there is a lot of variability in the trend. In some cases, the short scale structure from the grade variogram (normal scores) should be used. The long scale trend features where the variogram systematically goes above the sill should be ignored; this variability is captured in the grade modeling.

The normal scores coming out of SGS should be checked for normality (shape, mean of zero, and variance of one) and the variogram reproduction. The back transformed values should be checked for the correct mean and trend reproduction.

Post Processing

Multiple realizations of the geometry and the grades provides the starting point for interesting post processing calculations. The uncertainty at the scale of the simulated model can be presented including the average thickness, average grade, P_{10}/P_{90} thickness and grade and so on.

A stope plan is required to assess joint uncertainty. If available, the stope outlines should be imported to Pangeos. If none are available, then stope polygons should be constructed on the basis of a nominal rectangular stope size.

Final Comments

This short note is the *start* of a detailed workflow for vein type modeling. The Pangeos software implements common geostatistical calculations and the specific workflow and computational ideas that come out of CCG. Of course, virtually all of the calculations could be undertaken with non-commercial software coming from the Software Catalogue.