A Short Note on:
The Uncertainty Matrix for Assessment of Risk and Uncertainty

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Assessing and presenting uncertainty from multiple realizations is a challenge in modern mining geostatistics. Most practitioners have developed their own procedures and tricks to present uncertainty. The aim of this short note is to collect some of that practice together for newer practitioners. One important goal of risk assessment is to understand where the uncertainty is coming from: the geologic rock types or the grades within the rock types. An uncertainty matrix is used to assess the sources of uncertainty.

Starting Point

Selective mining units (SMUs) are used to represent a mineral deposit. There is no expectation that the mine will extract each SMU; however, the size of the SMU captures the anticipated selectivity of the mining equipment, the number and quality of grade control sampling, the efficacy of the grade control program and the procedures for dig limit determination and implementation. Paper number 27 in the CCG 2003 report discussed the determination of the SMU size by calibration to mining practice. This was expanded and published at the MININ conference in 2004.

SMU-scale realizations are required for risk and uncertainty assessment. We do not simulate the rock types or the grades at the SMU-scale directly; Gaussian techniques cannot be reliably used with multiscale data. Direct simulation techniques have not been fully developed nor tested in real deposits; however, nearly 1/5 papers in the geostatistical theory section of this report are aimed at direct simulation. At present, rock types are simulated at a small scale, then grades are simulated at the same small scale using rock type control. Sensitivity studies over the past 30 years (from before the time of Mining Geostatistics) have shown that we should simulate at least 9 small scale values within an SMU, that is, the discretization should be 3x3x1 where we often simulate at the bench height. The resolution in the vertical direction is increased when there is a possibility of split benching or changing the bench toe elevation locally. Most practitioners use a discretization to 4x4 or even 5x5. For example, a SMU size of 20m by 20m would be simulated by points at a 5m by 5m grid spacing.

The point-scale realizations are often very big. It is common to have more than 10 million points direcretizing one particular orebody within a deposit. These values are upscaled to the SMU scale. There is no need to keep the point-scale realizations; however, it is good practice to save the compressed small scale realizations for checking and future calculations. The categorical rock type variable upscale to proportions at the SMU scale. We should keep those proportions, but they may add needless complexity to subsequent calculations. An alternative is to keep the most common rock type in the SMU. In almost all cases, the continuous grade variables upscale by arithmetic averaging. Certain peculiar variables such as color, permeability and other geometallurgical or geomechanical variables may not average in a simple arithmetic fashion.

Multiple realizations are required to assess uncertainty. It is desirable to have $L=100$ realizations; however, some orebodies are so large that we are fortunate to have $L=20$. Each realization is processed one at a time to get an SMU scale realization. The sketch on Figure 1 illustrates one realization of one SMU. The point scale realization (4 by 4 discretization in this case) is scaled to an SMU. There are multiple grades and derived variables that must be saved.

The SMU parameters (tonnes, rock types, grades,…) should all be indexed by $i=1,\dots N$ SMU locations within the deposit and by $l=1,\dots,L$ realizations. The tonnes per block may be constant, may be modeled as a specific gravity random variable or may be derived on the basis of the grades. Some of the grades contribute to profit and some are contaminants. The potential revenue is the value of the SMU (normally expressed in $/t), which is often calculated from the grade variables. The revenue may be represented by a number of destinations (mill, heap leach, stockpile, waste dump,…) and by different potential revenues for each destination. In this note we will use this revenue simply as a variable to establish ore and waste for reporting and uncertainty assessment. The storage required to represent the SMU realizations is often manageable because the number of SMUs and large storage available on our computers. The simpler representation amounts to take the most common rock type for the particular block in the particular realization. This is often adequate; preserving the details of the proportions within the SMU may not be required.

The creation of the fine scale realizations (the point values to the left in the figure above) is very important and there are many implementation decisions. These are addressed by numerous CCG papers and other references. Our concern here is to summarize the uncertainty and sources of uncertainty in the SMU realizations.

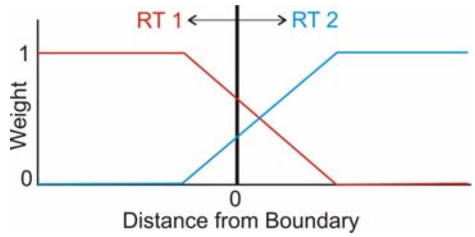
Realizations

The emphasis of this paper is the post-processing steps to summarize and present uncertainty in rock types and grades. A common approach to construct realizations is to (1) build multiple rock type realizations using some categorical-variable simulation technique (SIS, OBM, MPS,…), (2) build grade realizations for each rock type, and (3) merge the grade realizations within each rock type realization. Some software permits constructing the grade realizations within the rock type realizations; it seems inefficient to construct an entire grade realization for each rock type and then merge by some cookie-cutter approach.

The schematic illustration on Figure 2 shows how realizations are often merged. Two binary rock type realizations are shown at the left. Two grade realizations for each rock type are shown at the top. The four realizations on the lower right (inside the red box) are different resulting models after merging. It is common to merge the first grade realization with the first rock type model and so on; these realizations (on the diagonal) are circled.

The merging of realizations must often be done at the point-scale to preserve local details in the rock type models; however, the computer storage and CPU requirements become significant. The SMU-scale is often used for computer speed. This is a reasonable compromise when the irregularity of the rock type boundaries is at a relatively large scale.

Trends near geologic boundaries present problems. The cookie-cutter approach may not a good one. Realizations could be *blended* near boundaries with some kind of weighting function:



This weighting function is asymmetric. This captures some features of soft boundaries. The techniques of Paula Larrondo could be applied. Data can be used within some distance of boundaries. This subject is not dealt with further in this short note.

Reporting Zones

The rock types and grades must be reported with different groupings or zones. These groupings may be production time periods (daily, monthly, yearly,..), benches, rock types, pit shells or any other type of grouping. These zones could be as small as individual blocks or as large as the entire block model. The sketch on Figure 3 shows a grade model with seven arbitrary zones. Software implementations would require some specification of zones through a block model or an algorithm. In the context of simulation, there is uncertainty in the tonnes, rock types and grades within each zone.

We are interested in the expected value and the uncertainty in each variable within each reporting zone. There are many ways to report the uncertainty. The expected results for each reporting zone could be presented as shown at the top of Figure 4. The blue cells show tonnes, the yellow show grades and the orange show revenue. The darker color cells show the total over all rock types. Additional variables could be tabulated such as ore destination, stockpile, metal content, and geometallurgical rock types. Uncertainty in each variable is commonly shown by P10/P50/P90 values; see the bottom of Figure 4.

We could show different quantiles such as the P05/P50/P95 or the P25/P50/P95. We could show the expected value in place of the P50. The variance or standard deviation could be shown with the expected values to represent uncertainty. Provided enough realizations are available, it is probably better to show quantiles, since we need not make any parametric assumption to calculate probability intervals.

The numbers in the tables of Figure 4 are calculated from the SMU-scale realizations and the full or simple representation mentioned above in the Starting Point Section. There are four indices:

Number of blocks in the reporting zone:	$i=1,\ldots,N_b$	(1 to nxyz)
Number of realizations:	$l=1,\ldots,L$	(1 to 200)
Number of rock types:	$k=1,\ldots,N_{RT}$	(1 to 20
Number of grades:	$j=1,\ldots,N_G$	(1 to 10)

The range given to the right is a typical allowable range. There could be more than 200 realizations, 20 rock types and 10 grade variables, but not often. The L realizations are typically merged rock type and grade realizations (as discussed above) and are assembled as the first rock type realization combined with the first grade realization and so on; the diagonal realizations shown on the schematic of Figure 2. The variables for every block, realization, rock type, and variable could be summarized as:

Tonnage:	$T_{i,l}$
Revenue:	$Rev_{i,l}$
Rock type proportions:	$P_{i,l,k}$
Grades:	$P_{i,l,k,j}$

The entries required in the tables of Figure 4 are easily calculated; the bookkeeping is simple. There are some implementation/interpretation challenges. Calculating quantiles such as the P10/P50/P90 values requires storage of the value for each realization, sorting the values, and extracting the specified quantile. The tonnage within particular rock types is calculated assuming the probability of a rock type can be interchanged with the proportion of that rock type. For example, a P_k=0.2 value for a block means that there is a 20% that the block is rock type *k*. If we had 100 blocks with P_k=0.2, then we would expect 20 of them to actually be rock type k. A value of P_k=0.2 does *not* mean that 20% of the block is rock type *k* and 80% other rock types. Nevertheless, for reporting and accounting purposes, we make the interpretation that a probability is a proportion of the block. The global accounting will be unbiased. Inconsistencies may arise if the reporting zones are very small.

Tables of expected results and uncertainty contain the summary information we need to convey; a graphical display is often convenient to simplify presentations. An example is shown in Figure 5. The expected value is shown as the red dot. The upper and lower limits are the specified probability intervals (P10/P90 values). Time is a natural ordering for the reporting zones. The zones could also be ordered by location, bench elevation or some other natural coordinate.

The tabulated results or the figures mentioned above do not permit assessment of joint uncertainty between multiple zones/time periods. We would have to go back to the original realizations to extract the joint realizations; five joint realizations are shown schematically on Figure 6 – the realizations are connected by solid lines. The tabulated uncertainty was calculated directly from the joint realizations, so the results are consistent, that is, the 80% probability intervals will contain 80% of the realizations. The expected results (red dots illustrated above) are not, of course, consistent with the results calculated from kriging. The smoothing of kriging makes the results inconsistent with expected values from resource calculations.

Uncertainty Matrix

Uncertainty can be measured by the variance or the difference between the specified symmetric percentiles, for example, the P10 and P90 values. We discussed how to tabulate and present the results in the preceding section. It is often interesting to know what fraction of the uncertainty comes from the rock types and what fraction comes from the grades. We could combine the realizations in different ways to see how the results vary. In many cases, we have a deterministic rock type model coming from interpretation (or the most common rock type from a set of realizations) and a deterministic grade model coming from kriging or conventional interpolation algorithm.

The number of possible models that could be combined is shown graphically on Figure 7. *D* represents the deterministic model. *L* represents the number of realizations. There are $(L+1)^2$ possible models that could be assembled. Of course, the number of rock type and grade realizations need not be the same. The *L* realizations identified by the red circles are the ones specified above for reporting. The realizations do not have to be matched in order; however, we would not be sampling the uncertainty correctly if we correlated the choice of a rock type realization with the choice of a grade realization. Moreover, we would underestimate uncertainty if we used the same grade realization for multiple rock type realizations or vice versa. The uncertainty sampled by the *L* realizations (the diagonal ones identified by red circles) is fair, but we do not know how much of the uncertainty is due to the grade models and how much is due to the rock type models.

There are different measures of uncertainty that could be used when we assess the sources of uncertainty. The width of the P90-P10 could be used. The variance or the standard deviation could be used. The standard deviation would be comparable to a selected probability interval since most large-scale distributions of uncertainty are nearly Gaussian. These measures could be standardized by the P50 or the expected value for a relative measure of uncertainty. The standard deviation will be used to assess the relative uncertainty. One advantage of the standard deviation is that it can be calculated without having all of the values from all of the realizations; we just accumulate the sum and sum of squares.

$$\sigma_T = \sqrt{\frac{1}{L}\sum_{l=1}^{L} T^2 - \left(\frac{1}{L}\sum_{l=1}^{L} T\right)^2}$$

The second advantage is that the standard deviation is linearly related probability intervals, which are being used increasingly as a measure of uncertainty.

The uncertainty is different for each output parameter. There are many parameters (recall the tables on Figure 4). The amount of uncertainty coming from the rock type realizations and the amount coming from the grade realizations are referred to as the *source of uncertainty*. We could calculate the source of uncertainty for every parameter, but that becomes difficult to present and understand. It may be reasonable to pick just one parameter. Perhaps the most relevant parameter is revenue; however, revenue is not an intrinsic geologic parameter. The grade above a cutoff is notoriously variable and often increases when the tonnes decrease. For this reason, it may be better to use ore tonnes or the quantity of metal (product of the tonnes and grade). The focus in this short note is to develop the methodology. We use ore tonnes denoted *T* as the parameter of interest.

First Approximation – Source of Uncertainty

A relatively quick approach to calculate the amount of uncertainty coming from the rock types and grades is to process the 2·*L* realizations colored blue in Figure 7, that is, merge the deterministic rock type model with each grade realization and the deterministic grade model with each rock type realization. We assemble the 2·*L* realizations and calculate the resource tables for each zone and each realization. The relative uncertainty due to the rock type model and grade model are calculated as:

$$\%RT_z = \frac{\sigma_{T,z,R-\text{variable}}}{\sigma_{T,z,R-\text{variable}} + \sigma_{T,z,G-\text{variable}}} \cdot 100\% \qquad (1)$$

$$\%G_z = 100 - \%RT_z$$

Where $\sigma_{T,z,R\text{-variable}}$ is the standard deviation of the ore tonnes (*T*) over the *L* realizations assembled with the deterministic grade model and the *L* different rock type realizations and $\sigma_{T,z,G\text{-variable}}$ is the standard deviation of the ore tonnes (*T*) over the *L* realizations assembled with the deterministic rock type model and the *L* different grade realizations. The source of uncertainty is zone dependent; hence the subscript *z*.

Knowing the source of uncertainty could be useful. Attention could be focused on the rock type interpretation or the grades. There may be implications for blending and stockpiling. The schedule could be adapted to mitigate uncertainty coming from different areas. Infill drilling and other data collection programs could be targeted. Resource and reserve classification strategies could be affected. Spatial variations in grade versus rock type uncertainty could affect grade control and the mining strategy. An area with a large component of rock type uncertainty may be dealt with differently.

The uncertainty from the merged realizations is of primary importance. The source of uncertainty (for example 70% uncertainty due to rock types and 30% due to grades) is interesting to report. This percentage changes in different zones. It could be presented graphically, see Figure 8. Uncertainty can be shown in absolute or relative terms.

Better Approximation – Source of Uncertainty

The deterministic grade and rock type models, which lead to the 2·L realizations colored blue in Figure 7, may not represent the variability of the stochastic realizations. Merging the stochastic models together may lead to better results. We could calculate the row wise and column wise standard deviations and calculate the source of uncertainty due to the stochastic rock types and the stochastic grades. Each of the standard deviations will now be calculated as an average:

$$\sigma_{T,z,R-\text{variable}} = \frac{1}{L}\sum_{l=1}^{L}\sigma_{T,l,z,R-\text{variable}}$$

In the case of rock type uncertainty – each of the $l=1,\ldots,L$ grade realizations is held constant and merged into each rock type realization in turn (columns of the matrix on Figure 7). This ensures that a realistic representation of grade variability is considered. In the case of grade uncertainty – each of the $l=1,\ldots,L$ rock type realizations is held constant and merged with each grade realization in turn (rows of the matrix on Figure 7).

The source of uncertainty is calculated just like in Equation 1 – the standard deviations take longer to calculate. The same graphical presentation could be used. Few comparative studies are available. The results of the *first approximation* and the *better approximation* seem to be quite similar.

Program

The `calcres` program was written to perform the basic resource/reserve calculations within zones. This program is very simple, but useful anyway. The `calcres` program follows standard GSLIB conventions. The parameters for the program:

```
START OF PARAMETERS:
50 100 1                 -grid size: nx, ny, nz
1                        -number of realizations
zones.dat                -file with zones
1                        -   column for zone ID
-1.0e21  1.0e21          -   trimming limits
sisim.out                -file with rock types
1                        -   column for rock type
2  1 2                   -   number of rock types; rock types
tonnes.out               -file with tonnes per block
1                        -   column for tonnes
1                        -number of grades
sgsim01.out              -   file with  grade one RT one
```

```
13      1                          -   column for grade
14      sgsim02.out                -   file with  grade one RT two
15      1                          -   column for grade
16      rev.out                    -file with revenue/grade
17      1                          -   column for revenue
18      1.0                        -   cutoff (> considered ore)
19      calcres-exp.out            -file for expected output
20      calcres-unc.out            -file for uncertainty output
```

Lines 1 and **2** specify the size of the grid and the number of realizations. The zones are defined in a gridded model defined on **lines 3-5**. The number of zones and zone names are identified in the input gridded file. The zones must be integer codes. The zones do not need to be contiguous. If there is no zone file, then the entire model is treated as one zone. **Lines 6-8** specify the input rock type realizations and the rock types. If there is no rock type file, then the model will be considered as one rock type. Each rock type will be reported separately and together. **Lines 9-10** specifiy a gridded tonnes-per-block model. The tonnes per block will be default to 1.0 if this file does not exist. The number of grades is specified on **line 11**. The program requires a grade model for each rock type – the model input cycles fastest over rock type and then over multiple grades. The example given here is for one grade and two rock types (**lines 12 – 15**). A gridded revenue model is specified in **lines 16** and **17**. If the revenue file does not exist, then the first grade is considered for the cutoff. The cutoff is specifed in **line 18**. The expected output (corresponding to the top table in Figure 4) is written to the output file specified on **line 19**. The probabilistic output is written to the output file specified on **line 20**.

There are two other programs. The `calcsources` program merges the $nsim^2$ realizations and writes the results to a file for calculating the standard deviations. The parameter file for the `calcsources` program is exactly the same as for the `calcres` program. The programs could be merged; however, the `calcsources` program is really quite slow. It is often desirable to get the results of model merging right away and let the sources of uncertainty program run in the background. The `calcrtu` program calculates the standard deviations and the uncertainty due to the rock type model. Of course, the uncertainty due to the grade model is the rest.

Example

A synthetic example was constructed to illustrate the methodology. Figure 9 shows the setup of the problem with a total of 5000 grid blocks, 190 data and 10 zones. Figure 10 shows the first four realizations out of 100. The RT models were constructed by SIS and the grade models were constructed by SGS.

Figure 11 and 12 show the expected results and results with uncertainty. The P50 values are very close to the expected results because the distributions of uncertainty are all quite symmetric. The uncertainty for the total ore tonnes (RT 1 and 2) is shown on Figure 13. Many plots could be made of different tonnes, grade and revenue uncertainty; we will only show one. The realizations that went into the total ore uncertainty are all shown on Figure 14. The realizations on Figure 14 could be passed through economic calculations to assess economic uncertainty. The summaries on Figure 13 would be adequate for most planning purposes.

The sources of uncertainty for each zone were calculated for the tonnes of ore and the ore grade. The results are summarized below. We see that the contribution of uncertainty to ore tonnes and grade are very similar. The uncertainty is different within each zone, but overall, we see about 40% of the uncertainty due to the rock types and 60% due to the grades.

	Ore Tonnes		Ore Grade	
Zone	RT	Grade	RT	Grade
1	20.8%	79.2%	30.4%	69.6%
2	29.4%	70.6%	39.4%	60.6%
3	49.9%	50.1%	44.2%	55.8%
4	43.5%	56.5%	42.2%	57.8%
5	38.6%	61.4%	41.6%	58.4%
6	48.8%	51.2%	38.7%	61.3%
7	45.7%	54.4%	35.4%	64.6%
8	33.5%	66.6%	31.8%	68.3%
9	33.6%	66.4%	37.4%	62.6%
10	30.9%	69.1%	31.9%	68.1%
Average	37.5%	62.5%	37.3%	62.7%

Conclusions

This short note discusses some detailed steps of post processing realizations in a mining context. Reporting resources/reserves within specific areas/zones is important. We are interested in the expected value as well as the uncertainty. Rock type uncertainty and grade uncertainty are important – the uncertainty we report is a combination of both. Special steps must be taken to understand how much of the uncertainty is coming from rock types and how much is coming from grades. The procedures and steps for this assessment were presented. A similar scheme could likely be adapted to assess the uncertainty due to multiple grades in the context of profit/revenue being calculated as a function of multiple variables. The main limitation is a computer time.

References

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- Deutsch, C.V. and Journel, A.G., 1998. *GSLIB: Geostatistical Software Library: and User's Guide*. Oxford University Press, New York, 2nd Ed.
- Journel, A.G., Huijbregts, C.J., 1978: *Mining Geostatistics*. Academic Press, New York.
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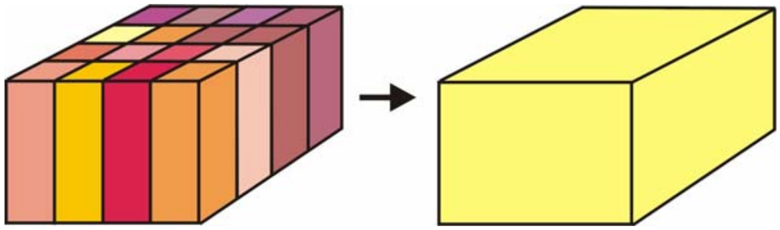

Full Representation:	T	tonnes
	$P_k, k=1,\ldots,N_{RT}$	proportion of rock types
	$Z_{1,1} \ldots Z_{Ng,1}$	grades in RT 1
	.	
	$Z_{1,NR} \ldots Z_{Ng,NR}$	grades in RT N_{RT}
	Rev	potential revenue
Simple Representation:	T	tonnes
	RT	most common rock type
	$Z_1 \ldots Z_{Ng}$	grades
	Rev	potential revenue

Figure 1: Representation of uncertainty at the SMU scale. Upscaling is illustrated schematically at the top and the parameters retained are shown below.

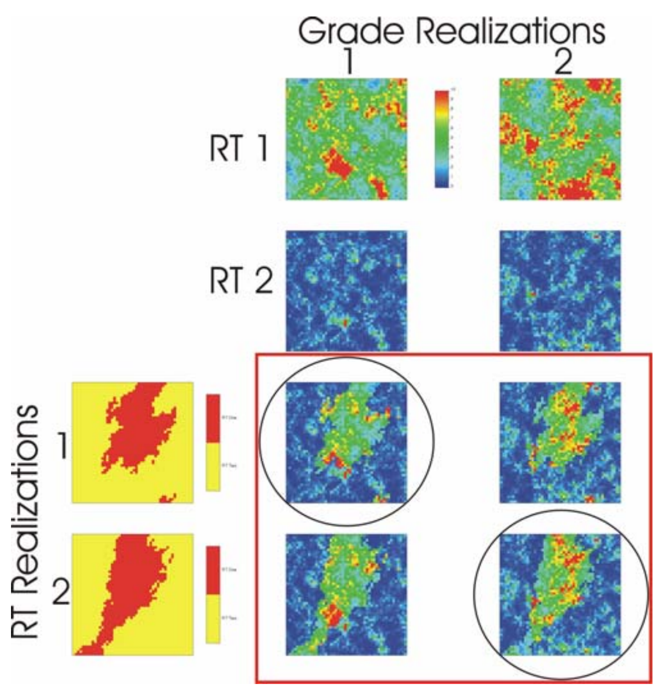

Figure 2: Merging of two rock type realizations and two grade realizations. There are four possible realizations. The circled ones are normally retained for further analysis.

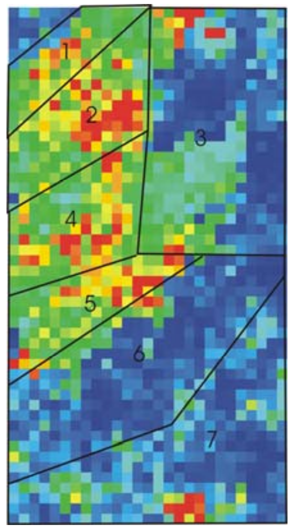

Figure 3: Example bench map with seven different zones. These zones were arbitrarily sketched; often, they correspond to time periods or mining areas.

	Waste (t)	Ore (t)	Grade 1	...	Grade Ng	Revenue
RT 1						
RT 2						
...						
RT Nrt						
Total						

	Waste (t)			Ore (t)			Grade 1			...			Grade Ng			Revenue		
	P10	P50	P90	P10	P50	P90	P10	P50	P90	P10	P50	P90	P10	P50	P90	P10	P50	P90
RT 1																		
RT 2																		
...																		
RT Nrt																		
Total																		

Figure 4: Schematic tables that could be used to represent the results within a particular reporting zone. The upper table is for the expected results and the lower table is for the results with uncertainty.

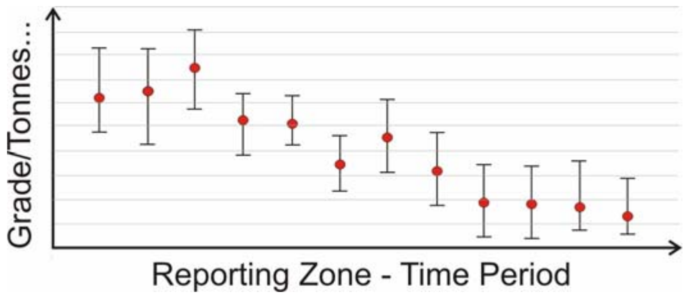

Figure 5: Uncertainty versus reporting zone (time period). The red dots illustrate the expected or P50 result and the black lines indicate a specified probability interval.

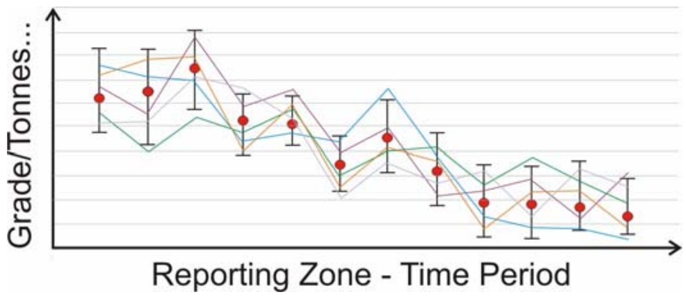

Figure 6: Joint uncertainty versus reporting zone (time period) represented by five realizations (the different color lines).

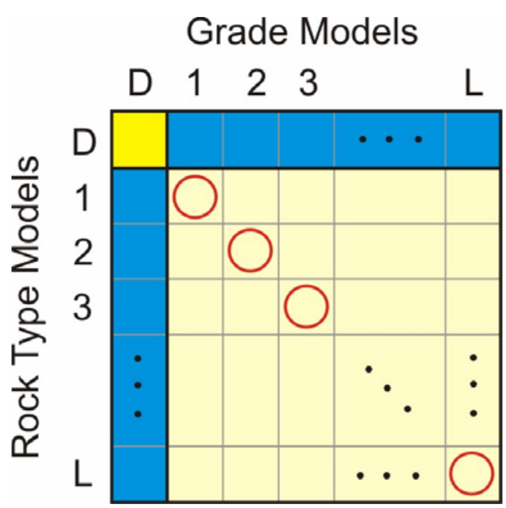

Figure 7: Matrix of possible realizations to construct. D represents a deterministic model, and L represents the number of realizations.

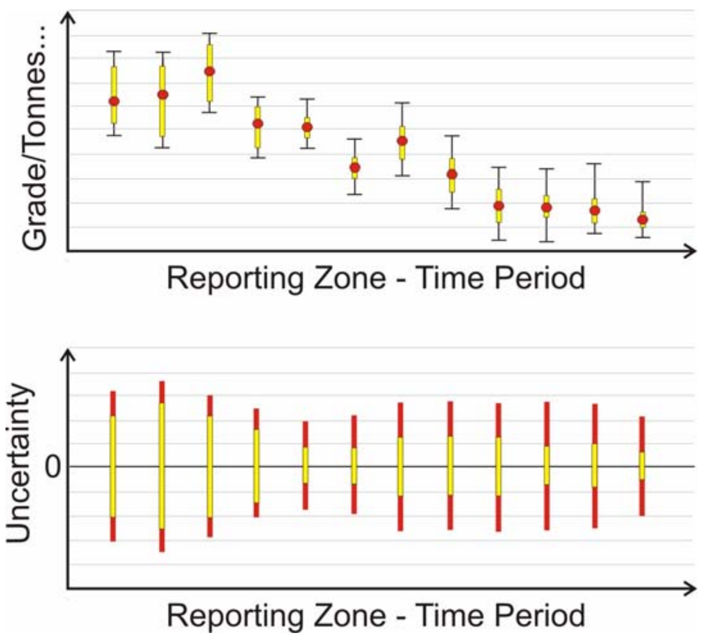

Figure 8: Two ways to present uncertainty. The yellow bars represent geologic rock type uncertainty and the extent of the black or red bars represents a total of rock type and grade uncertainty.

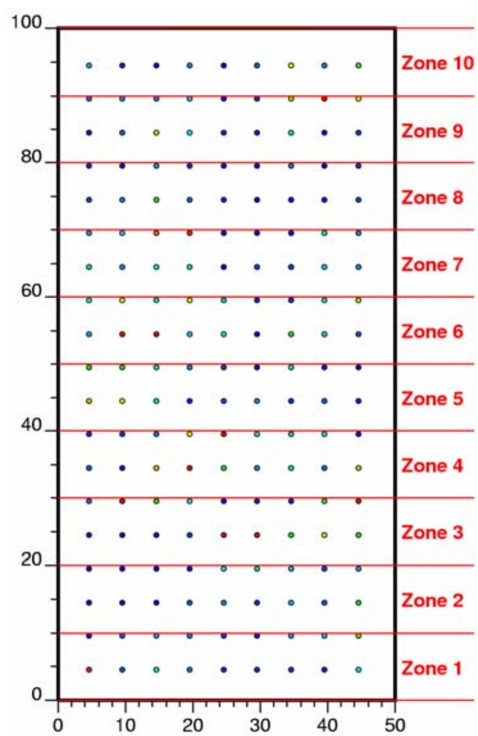

Figure 9: Setup for example problem. The coordinates are in grid block numbers. 190 drillholes are used for conditioning. The resources will be reported in 10 zones of 500 grid blocks.

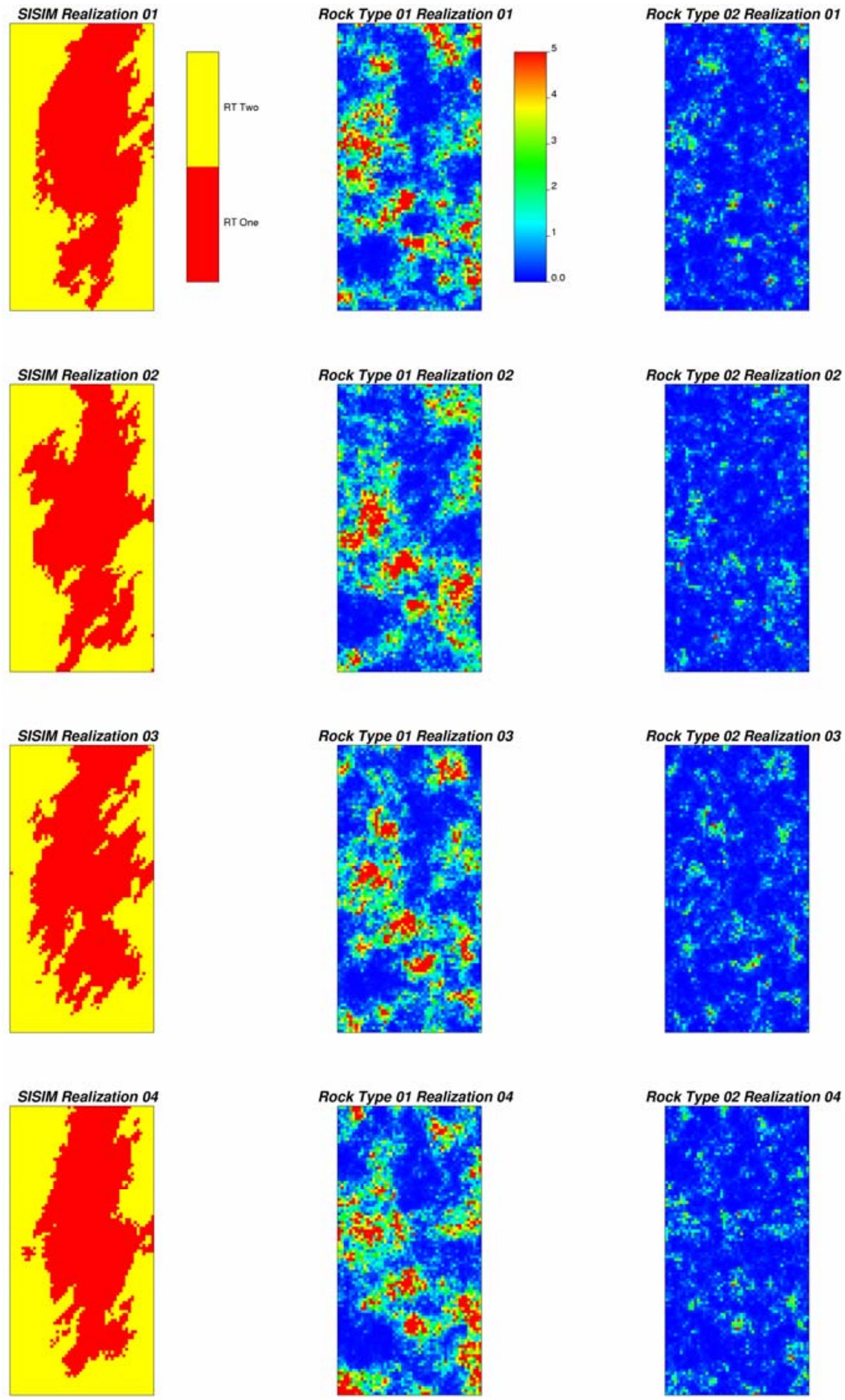

Figure 10: Four realizations of rock type and the grades within each rock type.

Zone	RT	Waste (t)	Ore (t)	Grade 1
1	1	41.5	12.3	1.880
1	2	421.6	24.6	2.068
1	0	463.1	36.9	2.005
2	1	108.3	52.3	2.233
2	2	328.3	11.1	1.808
2	0	436.6	63.4	2.159
3	1	85.9	138.6	4.263
3	2	255.5	19.9	2.161
3	0	341.5	158.6	3.999
4	1	115.9	171.6	4.291
4	2	192.8	19.7	2.190
4	0	308.7	191.3	4.075
5	1	202.4	124.2	3.025
5	2	158.2	15.2	1.998
5	0	360.6	139.4	2.913
6	1	186.9	168.6	3.634
6	2	132.1	12.4	2.056
6	0	319	181	3.526
7	1	192.5	121.3	3.229
7	2	171.7	14.5	2.021
7	0	364.1	135.9	3.100
8	1	225.5	58.1	3.003
8	2	207.8	8.7	1.921
8	0	433.3	66.7	2.863
9	1	163.4	64.3	2.883
9	2	253.9	18.4	2.027
9	0	417.3	82.7	2.692
10	1	123.3	84.3	3.491
10	2	274.8	17.6	2.028
10	0	398.1	101.9	3.238

Figure 11: Expected results for rock types and zones. A cutoff grade of 1.0 was used.

Zone	RT		Waste (t)			Ore (t)			Grade 1	
		P10	P50	P90	P10	P50	P90	P10	P50	P90
1	1	4	32	84	0	9	28	0.000	1.857	2.904
1	2	359	427	464	13	23	40	1.611	1.961	2.599
1	0	445	466	480	20	34	54	1.637	2.001	2.766
2	1	68	104	152	28	52	76	1.767	2.156	2.756
2	2	281	329	383	4	10	18	1.338	1.705	2.337
2	0	412	437	460	40	63	88	1.773	2.117	2.590
3	1	49	85	123	105	140	166	3.466	4.162	5.085
3	2	208	255	306	11	19	29	1.729	2.042	2.837
3	0	315	340	371	125	160	184	3.264	3.943	4.748
4	1	83	115	148	132	170	204	3.447	4.280	5.001
4	2	137	190	251	9	18	32	1.578	2.036	2.902
4	0	273	308	343	154	191	226	3.335	4.054	4.702
5	1	164	203	241	90	124	157	2.392	2.984	3.691
5	2	111	158	205	6	16	23	1.494	1.869	2.631
5	0	328	361	396	104	139	172	2.298	2.847	3.594
6	1	157	186	217	134	167	203	2.711	3.606	4.450
6	2	87	133	173	2	12	23	1.455	1.898	2.632
6	0	283	318	350	149	182	215	2.693	3.440	4.400
7	1	167	188	222	85	119	152	2.543	3.091	3.982
7	2	128	174	214	5	12	26	1.420	1.864	2.644
7	0	333	366	395	104	133	166	2.478	3.001	3.740
8	1	192	225	258	37	58	77	2.242	2.882	4.045
8	2	173	209	243	3	6	17	1.380	1.759	2.592
8	0	414	434	453	47	66	86	2.236	2.764	3.739
9	1	130	162	195	40	61	87	2.174	2.806	3.624
9	2	215	251	295	8	17	30	1.562	1.860	2.743
9	0	393	418	442	57	81	107	2.122	2.642	3.424
10	1	84	117	164	43	81	125	2.490	3.379	4.579
10	2	225	272	329	7	16	28	1.547	1.859	2.660
10	0	358	401	432	68	99	138	2.444	3.152	4.037

Figure 12: Uncertain results.

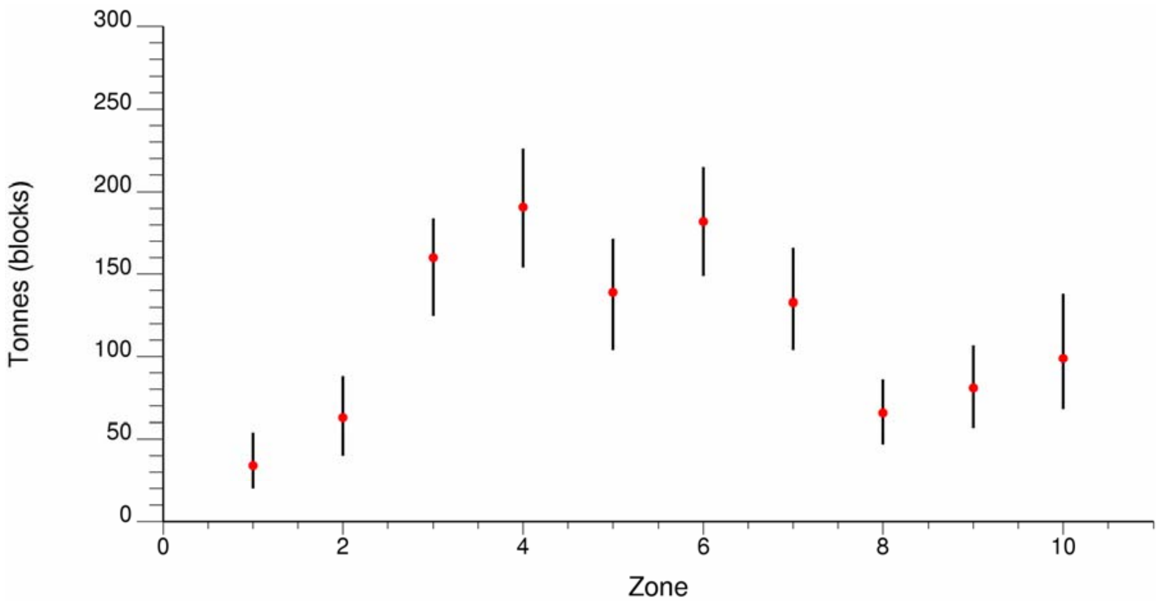

Figure 13: Total ore tonnes in each zone. The red dots are the P50 results and the solid line starts and ends at the P10 and P90, respectively.

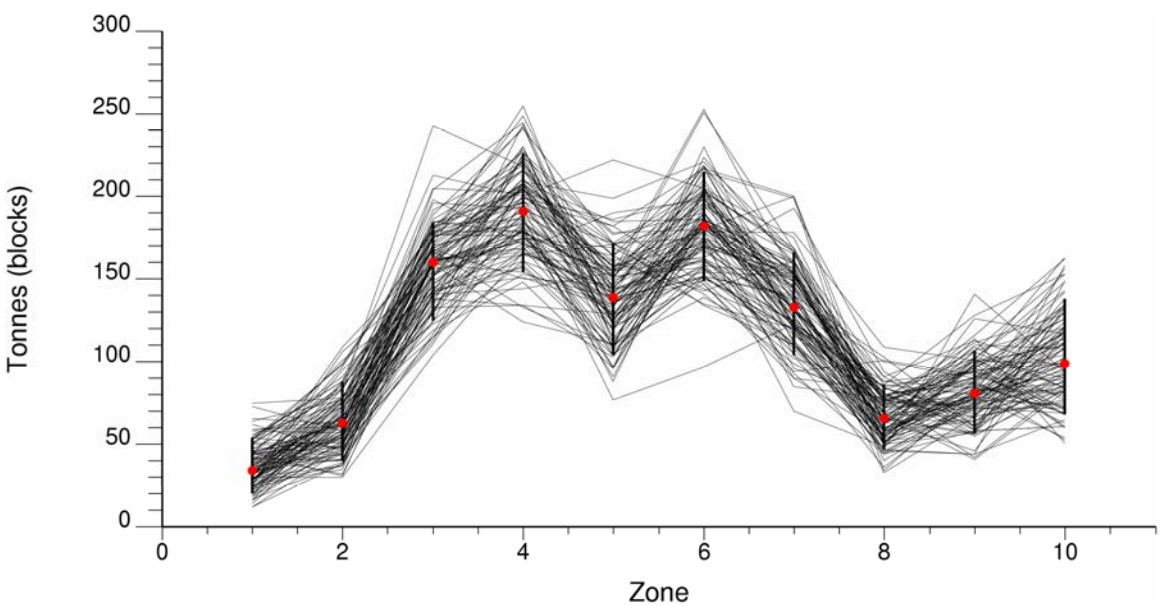

Figure 14: Total ore tonnes in each zone. The summaries of uncertainty are shown and the 100 realizations are shown by the solid connected lines.