The Problem of Kriging when Estimating in a Finite Domain

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In this paper a commonly observed problem of Kriging when estimating a finite domain is discussed. Specifically, it is investigated from practical point of view why when estimating locations of interest in the finite domain using a string of data based on Ordinary Kriging (or sometimes even Simple Kriging), the boundary datum in the string on average receives more weight than any other data in that string. In order to explain this problem of Kriging, a comprehensive study of the influence of data in a string on the estimation of the finite domain based on Simple Kriging and Ordinary Kriging using different variogram models is conducted.

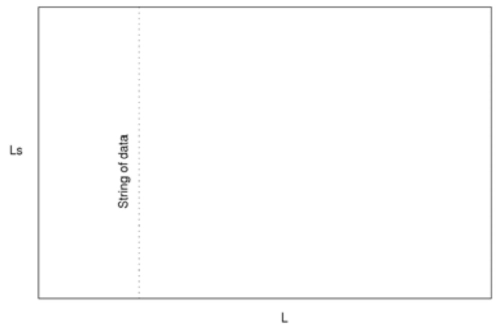
Introduction

When estimating locations of interest in the finite domain using a string of data based on Ordinary Kriging (or Simple Kriging), it is frequently observed that the boundary data in the string receives more weight than any other data in that string. The reason for this phenomenon is the following. When we adopt a random function model {Z(u), where u belongs to the study domain} and use kriging, we implicitly assume that this random function Z is stationary and ergodic. That is, we claim that our multivariate distribution is invariant under translation over the study domain and that the study domain is embedded within an infinite domain. A direct consequence of ergodicity assumption is less redundancy or stronger importance of the boundary data than any other data in the string of data in Kriging. While such weighting of the boundary samples is theoretically valid, we believe that it could lead to biased estimation of finite domain, especially if the data exhibits strange trends with boundary/border effects.

In this paper we conduct comprehensive study to investigate and explain the influence of data in a string on the estimation of the finite domain based on Simple Kriging and Ordinary Kriging. Moreover, we study the impact of different variogram models (including the impact of variogram type and range) on all locations in the finite domain and the impact of different variogram models on the average Simple Kriging and Ordinary Kriging weights for a string of data. The formula for calculation of the Simple Kriging and Ordinary Kriging average and accumulated weights is also introduced. The influence of the domain size on the average Simple Kriging and Ordinary Kriging weights is discussed.

Accumulated Simple and Ordinary Kriging Weights for a String of Data

Consider a string of data of length *Ls* located in a rectangular study domain of size *L* by *Ls*. Assume that the string of data is located parallel to two sides of the domain, see figure below.



Let consider estimation of the value of the variable of interest at arbitrary location *u** of the rectangular domain using Simple Kriging (SK) technique. It is known that the estimate in this case is a weighted average of the data in a string and a population mean. Now restrict our attention to calculation of the weights given to each data in a string.

The weights $\lambda_1, \ldots, \lambda_{Ls}$ given to the string of *Ls* data for Simple Kriging are obtained using the well-known system of normal equations

$$\begin{aligned} &C(u_1,u_1)\lambda_1(u^*) + C(u_1,u_2)\lambda_2(u^*) + \ldots + C(u_1,u_{Ls})\lambda_n(u^*) = C(u_1,u^*),\\ &C(u_2,u_1)\lambda_1(u^*) + C(u_2,u_2)\lambda_2(u^*) + \ldots + C(u_2,u_{Ls})\lambda_n(u^*) = C(u_2,u^*),\\ &\ldots\\ &C(u_{Ls},u_1)\lambda_1(u^*) + C(u_{Ls},u_2)\lambda_2(u^*) + \ldots + C(u_{Ls},u_n)\lambda_n(u^*) = C(u_{Ls},u^*), \end{aligned} \tag{1}$$

where $C(u_i, u_j)$ denotes the covariance between the data at locations u_i and u_j in the string and $C(u_i, u^*),\ i = 1, \ldots, Ls,$ denotes the covariance between the data in the string and the location where we are estimating value of the variable of interests, $i = 1, \ldots, Ls,\ j = 1, \ldots, Ls.$

In matrix form the system for weights (1) can be rewritten as

$$C_M \cdot \lambda_V(u^*) = C_V(u^*), \tag{2}$$

where C_M is a covariance matrix of size *Ls* by *Ls* with

$$C_M(i, j) = C(u_i, u_j),\ i = 1, \ldots, Ls,\ j = 1, \ldots, Ls; \tag{3}$$

$C_V(u^*)$ is a covariance vector of size *Ls* by 1 given by

$$C_V(u^*) = [C(u_1, u^*)\ C(u_2, u^*) \cdots C(u_{Ls}, u^*)]^T;$$

and $\lambda_V(u^*)$ is the vector of weights for the sting of data when estimating the value of the variable of interest at location *u**, that is,

$$\lambda_V(u^*) = [\lambda_1(u^*)\ \lambda_2(u^*) \cdots \lambda_{Ls}(u^*)]^T.$$

When estimating the value of the variable of interest at several different locations based on SK, the weights each time are calculated using the systems like (2). All these systems have the same left-hand side covariance matrix C_M, but different right-hand side covariance vectors $C_V(u^*)$ and, of course, different vectors of weights $\lambda_V(u^*)$ corresponding to the *Ls* data in a string. Specifically, the systems for the weights when estimating *m* different locations of the variable of interest are given by

$$\begin{aligned} &C_M \cdot \lambda_V(u^{*1}) = C_V(u^{*1}), \\ &C_M \cdot \lambda_V(u^{*2}) = C_V(u^{*2}), \\ &\dots \\ &C_M \cdot \lambda_V(u^{*m}) = C_V(u^{*m}), \end{aligned} \tag{4}$$

where C_M is specified by (3), $C_V(u^{*r}) = [C(u_1, u^{*r})\ C(u_2, u^{*r}) \cdots C(u_{Ls}, u^{*r})]^T$ and $\lambda_V(u^{*r}) = [\lambda_1(u^{*r})\ \lambda_2(u^{*r}) \cdots \lambda_{Ls}(u^{*r})]^T$, $r = 1, \dots, m.$

If we add up all matrix equations in (4) (note that we can add matrix equations because the dimensions of all systems are the same), we obtain

$$C_M \cdot [\lambda_V(u^{*1}) + \lambda_V(u^{*2}) + \cdots + \lambda_V(u^{*m})] = [C_V(u^{*1}) + C_V(u^{*2}) + \cdots + C_V(u^{*m})]. \tag{5}$$

Now let

$$[\lambda_V(u^{*1}) + \lambda_V(u^{*2}) + \cdots + \lambda_V(u^{*m})] = \tilde{\lambda}_V = m \cdot \bar{\lambda}_V, \tag{6}$$

where $\tilde{\lambda}_V$ and $\bar{\lambda}_V$ denote the vector of accumulated and averaged weights given to the string of *Ls* data. Moreover, note that

$$\frac{1}{m}[C_V(u^{*1}) + C_V(u^{*2}) + \cdots + C_V(u^{*m})] = \bar{C}_V(u^{*1}, u^{*2}, \dots, u^{*m}) = \bar{C}_V, \tag{7}$$

where $\bar{C}_V$ denotes an average covariance vector.

Thus, using notation (6), (7) we can rewrite system (5) as

$$C_M \cdot \bar{\lambda}_V = \bar{C}_V. \tag{8}$$

Then a vector of average SK weights for the string of data of length *Ls* can be found by solving equation (8), which involves the left-hand side covariance matrix accounting for redundancy of the data and the right-hand side covariance vector accounting for the average covariance between the locations of interest and the string of data. Note that accumulated SK weights for a string of length *Ls* data in a domain of Size $L \times Ls$ can be found by multiplying a vector of average weights by number of locations *m*, where we are estimating the value of the variable of interest.

In general, note that the result stated in (8) is not only restricted to the data with the 'string configuration', it applies to arbitrary data configuration. Moreover, equation (8) holds not only in 2D for rectangular domain, but also in *n*-variate space for *n*-dimensional rectangle.

If we estimate rectangular study domain using a sting of data based on Ordinary Kriging instead of Simple Kriging, then the average weights, $\bar{\lambda}_{V0}$, for a string data are given by (the derivation is absolutely analogous)

$$C_{MO} \cdot w = \overline{C}_{VO},$$

where

$$\overline{C}_{VO} = \frac{1}{m}[C_{VO}(u^{*1}) + C_{VO}(u^{*2}) + \cdots + C_{VO}(u^{*m})],$$

with $C_{VO}(u^*) = [C(u_1,u^*)\ C(u_2,u^*) \cdots C(u_{Ls},u^*)\ 1]^T$; $w = [\overline{\lambda}_{V0}{}^T\ \overline{\mu}]^T$, where $\overline{\mu}$ is an average Lagrange multiplier and C_{MO} is a matrix of the size *Ls* by *Ls* given by

$$C_M(i,j) = C(u_i,u_j),\ i = 1,\dots,Ls,\ j = 1,\dots,Ls;$$
$$C_M(Ls+1,i) = C_M(i,Ls+1) = 1,\ j = 1,\dots,Ls;$$
$$C_M(Ls+1,Ls+1) = 0.$$

Case Study

Let consider estimation of finite domain using a boundary string of data, see below.

Note that such configuration is considered only because of symmetry of the estimation problem. That is, it is known that locations of interest will receive the same estimate despite of the location of the string (string is right-hand side or left-hand side from the location of interest) provided they are on the same distance from all data in a string. Thus, there is no point of considering the full finite domain; we will consider only its right-hand side from the string of data portion.

Now let us investigate the influence of a particular data in the string on estimation of all locations in the finite domain based on Ordinary and Simple Kriging. As a measure of influence of the data in a string on the location of interest *u**, we can use the weight given to it by SK or OK. Note, however, that the Ordinary Kriging (OK) and Simple Kriging (SK) weights can take on different signs. That is, these weights can be positive, negative or zero. Thus, for better understanding and visualization of the influence of particular data in the string on all locations of the domain we propose to scale the weights for all locations of interest independently to be positive and sum to 100%. That is, as a measure of influence of the data in the string on the location of interest *u**, we propose to use the relative weight calculated based on the weights given to the string of data by SK or OK in the following way. If $\lambda_V(u^*) = [\lambda_1(u^*)\ \lambda_2(u^*) \cdots \lambda_{Ls}(u^*)]^T$ denotes the vector of SK or OK weights given to a string of data when estimating location *u**, then the influence of *j*-th data in a string on estimation of this location is

$$i_j(u^*) = \frac{|\lambda_j(u^*)|}{\sum_{k=1}^{Ls}|\lambda_k(u^*)|} \cdot 100\%.$$

This procedure can be repeated to define the influence for all locations in the domain.

Figures 1-3 show the influence (and the sign of the influence) of the first, second and middle data in a string of 11 and 7 data values on all locations of the finite domain calculated based on Ordinary and Simple Kriging using Spherical variogram model

$$\gamma(h) = \begin{cases} 1.5\frac{h}{a} - 0.5\frac{h^3}{a^3}, & h < a, \\ 1, & h \geq a, \end{cases}$$

where a denotes the range of correlation. When comparing the influence contours in Figures 1-3, one can easily note that they have the same pattern. That is, the influence of particular data in a string (first, second and third) by Ordinary Kriging is the strongest on the closest locations to that data and reduces with distance. Moreover, note that this influence reduces with the distance the least in the direction perpendicular to the string of data (and, of course, it reduces the most in the direction parallel to the string of data). In Ordinary Kriging most of the time the weights, given to the string of data when estimating finite domain, are positive, only the locations on which the data has small effect have negative weight by Ordinary Kriging. From the influence maps obtained by Simple Kriging based on Spherical variogram, then we can see that the influence of particular data in a string (first, second and middle) is the strongest on the closest data locations and reduces with distance. However, when the distance from the data in a string to the location, where we are estimating, is close to the range of correlation, we observe that the influence of that data in a string increases again provided location of interest is located on the line perpendicular to the string of data. Also note that, at distances larger than the range of correlation, the influence of a string of data on finite domain estimation is zero. And, as before, only the locations on which the data has small effect receive negative weight by Simple Kriging.

Figures 4-6 show the influence (and the sign of the influence) of the first, second and middle data in a string of 11 and 7 data values on all locations of the domain calculated based on Ordinary and Simple Kriging using the Exponential variogram model

$$\gamma(h) = 1 - e^{-\frac{3h}{a}},$$

where a denotes the range of correlation. When comparing the influence contours in Figures 1-6, one can easily note that the pattern of the influence by OK is very similar in this case to the one obtained by Ordinary Kriging based on Spherical variogram model. Moreover note, that the pattern of influence of the particular data in string on all locations in the finite domain obtained by Simple Kriging is virtually the same as by Ordinary Kriging, with the only difference that the influence contours obtained based on Simple Kriging are much wider, that is by Simple Kriging larger number of data is more influenced by particular data in a string than in the case of Ordinary Kriging. Note also that when estimating finite domain using Simple Kriging or Ordinary Kriging based on Exponential variogram model, the influence of any data in string on the locations in this domain is positive, that is, the weights given by Kriging to data in string by finite domain locations in the domain are always positive.

Figures 7-9 display the results for the influence of the first, second and middle data in a string of 11 and 7 data values on all locations of the domain based on Ordinary and Simple Kriging using Gaussian variogram model

$$\gamma(h) = 1 - e^{-\left(\frac{3h}{a}\right)^2},$$

where a denotes the range of correlation. The influence maps obtained by Ordinary Kriging are quite similar to the ones obtained by Ordinary Kriging using two other variogram models. However, one can easily note that the same is not true for the influence maps obtained by Simple Kriging based on Gaussian variogram model for any data in string. The influence of a particular data in string on the locations in the domain is more uniform. That is, all locations which lay on the same line as data in a string influence of which we are assessing receive the most influence if the line is perpendicular to the string of data. Then the influence reduces uniformly in the direction perpendicular to that line. As before, negative influence is observed only for locations on estimation of which the data does not have strong impact.

The other question of interest to the finite domain study is a behavior of the average weights for the string of data obtained based on Simple and Ordinary Kriging using different variogram models. Figure 10 shows several examples for average weights given by OK and SK to the strings of data of different length. From this figure, we can clearly see that Gaussian model is the most unstable; the fluctuation in the average weights for different data in string is rather large. Moreover, the average weights can be negative. For the other two variogram models, we can observe that in Ordinary Kriging the boundary samples receive more weight on average, however, the same can not be said about the average SK weights.

To understand better how the average SK and OK weights and their patterns develop with change in the range of correlation for different variogram models, several sensitivity studies were conducted. Results of these studies are shown in Figure 11 for Spherical variogram model, in Figure 12 for Exponential variogram model and in Figure 13 for Gaussian variogram model. In general, it was observed the following. When applying Ordinary Kriging to estimate the finite domain using either Spherical or Exponential variogram model, the boundary data will have on average several times more influence on the estimation than any other data in a string, see Figures 10-12. Moreover, this influence will increase with increase in the range of correlation. The average weights obtained using Simple Kriging based on Spherical and Exponential variogram models behave differently from the ones obtained based on Ordinary Kriging. That is, when the range of correlation is small, boundary data values in the string receives less weight than any other data values in a string by SK. However, with increase in the range of correlation, the boundary data values starts to receive larger weight. Finally, when range of correlation is large, the patterns for the average weights produced by Simple Kriging are very similar to the ones produced by Ordinary Kriging. Note also that the weights calculated based on Exponential variogram change more smoothly with respect to location of the data in the string than produced based on Spherical variogram model. For Gaussian variogram model, we can note that (as was already mentioned above) the weights are very unstable. As the range of correlation increases, the fluctuation in the magnitude of the weights increases dramatically. Moreover, the pattern of the average weights behavior also changes with the range of correlation. Specifically, when the range of correlation is small, the middle data or boundary data receive maximal average weights; however, when the range of correlation increases only middle data in the string receive maximum average weight (the boundary data receives the smallest weight on average). Note also that the average weights produced by Simple Kriging based on Gaussian variogram are positive when the range of correlation is small, but they can be either positive or negative when the range of correlation is large. The average weights produced by Ordinary Kriging fluctuate between positive and negative values for any range of correlation.

Note also that with increase in the finite domain size, the average SK and OK weights given to the string of data reduces, this can be observed directly from Figures 1-9.

To further understand the patterns of the average OK and SK weights, the maps of maximal influence of the data in the string were created. Results are shown in Figures 14, 15 for Spherical variogram model, Figures 16, 17 for Exponential variogram model and Figures 18, 19 for Gaussian variogram model. Note that Figure 15 for Spherical variogram model shows that some portion of the finite domain receive maximal weight equal to zero. This simply means that all SK estimates for these locations are independent of the data values in the string (SK weights for these locations are zero, thus, maximal influence is also zero).

Now let us analyze maps of Figures 14-19 more precisely. From Figure 15 a),b) one can clearly note that the data in a string which is located on the same line as the location of interest perpendicular to the string of data obtains by Simple Kriging based on Spherical variogram model with small range of correlation (2 and 5, respectively) the largest weight. However, with increase in the range of correlation, the structure of the maximal influence becomes different. That is, the data points in the string which are the closest to the location where we are estimating using SK do not receive the largest weight, the largest weight are received by the first or second boundary data. The impact of this behavior can also be observed from Figure 11. Specifically, we observe from this figure that when the range of correlation in Spherical variogram is small, boundary data points receive on average the SK weight (thus, influence) similar to all other data in the string. On the other hand, when the range of correlation in Spherical variogram is large, the boundary data values receive on average the weights (thus, influence) several times larger than all other data in the string. To similar conclusions we can come from Figure 17 for Simple Kriging with Exponential variogram. With respect to Ordinary Kriging with either Exponential or Spherical variogram we can say that the estimation in finite domain is always biased toward boundary data (see also Figures 11 and 12), since all locations in the finite domain except the closest to the string of data give in Ordinary Kriging the largest weight to the boundary data values in the sting. The only model which virtually preserves the correct influence structure in Simple Kriging is the Gaussian variogram model. However, this can not be concluded about the Gaussian variogram model in Ordinary Kriging. On the contrary to two other variogram models, when estimating finite domain using Ordinary Kriging based on Gaussian variogram model, most of the locations in finite domain give the largest weight to the middle data in the string.

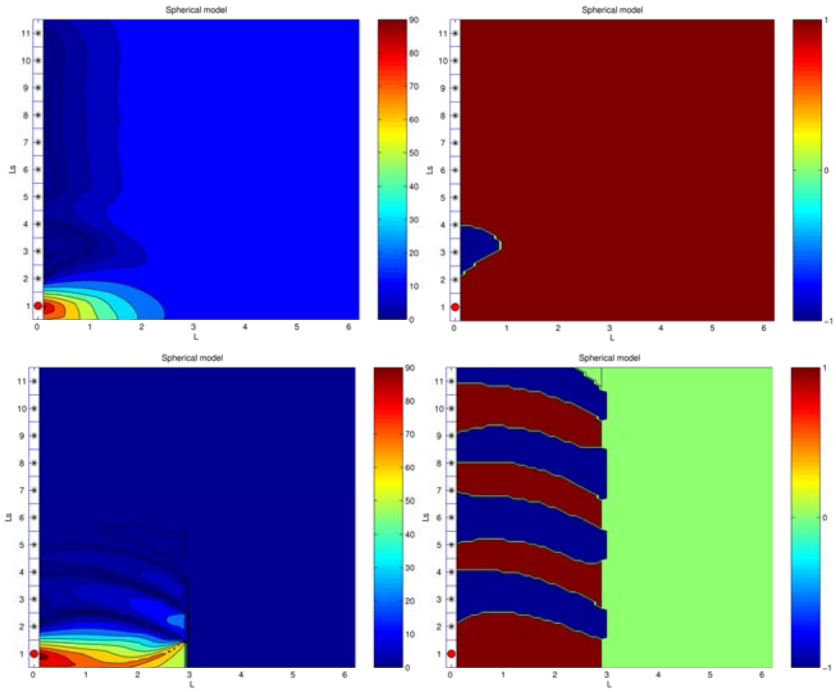
In order to explain (especially in the case of Ordinary Kriging) biased structure of the maximal influence of the data in the string, the estimation variances for Simple Kriging and Ordinary Kriging were obtained for each location in the finite domain, see Figures 20-25. Note that estimation variances for the locations which are the closest to the string of data are the smallest. With increase in the distance from the data to the locations, the estimation variance increases. However, the structure of the estimation variances is different from the patterns of maximal influence maps.

Conclusions

In this paper a problem of estimation of finite domain using a sting of data based on Simple and Ordinary Kriging was considered. In order to assess and analyze the influence of the data in the string on the estimation of the finite domain, a comprehensive study was conducted. Specifically, the following questions were investigated:

1. How strong is the influence of each data in the string of data values on estimation of finite domain based on SK and OK;
2. Which data in the string has the strongest influence on estimation of the locations in finite domain;

Moreover, in this paper, formulae for average Simple Kriging and Ordinary Kriging weights for a string of data were derived. The change in the average weights with change in the variogram parameters was investigated. The influence of the domain size on the average Simple Kriging and Ordinary Kriging weights was also discussed.



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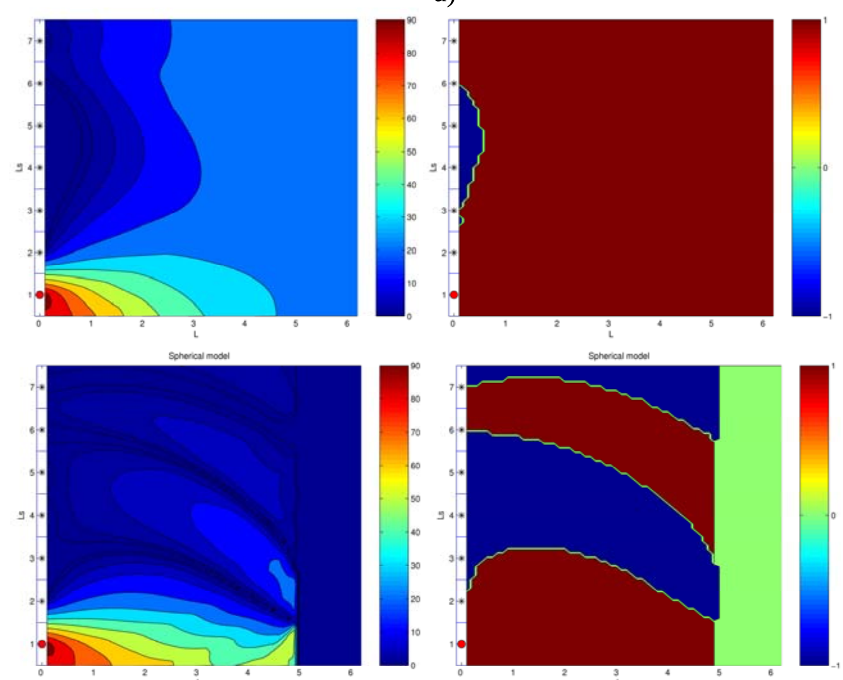
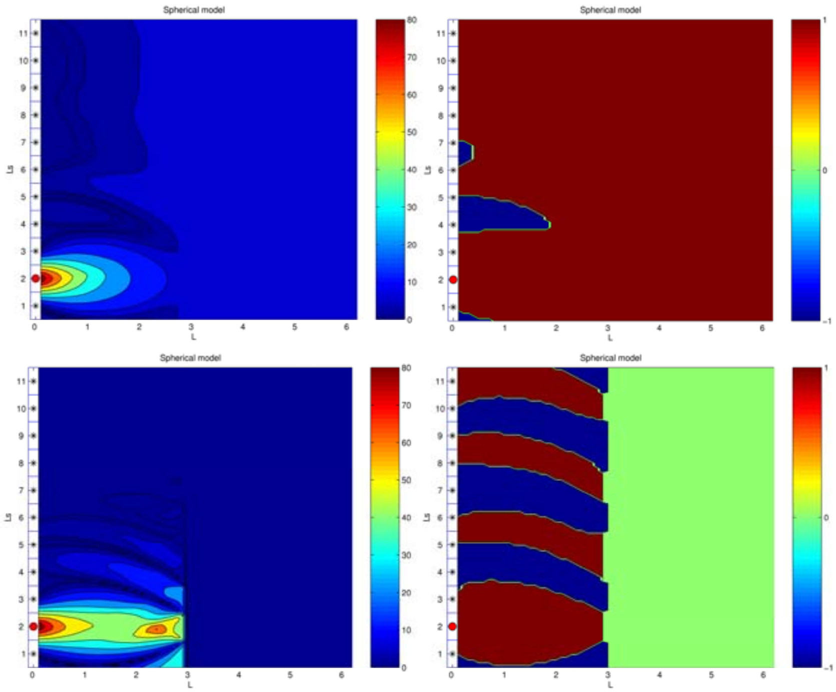


b)

Figure 1: Influence (first column) and sign of influence (second column) of the first data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Spherical variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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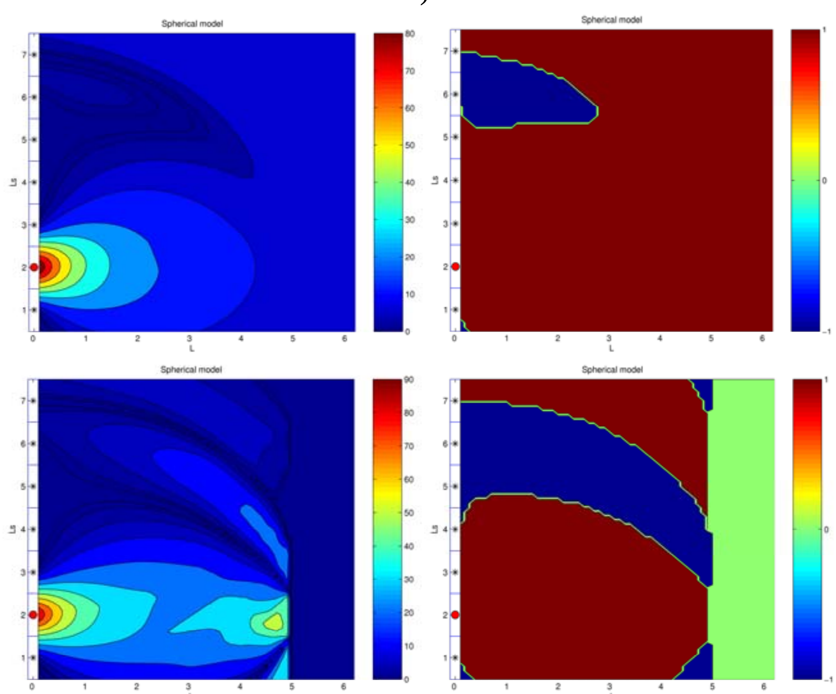

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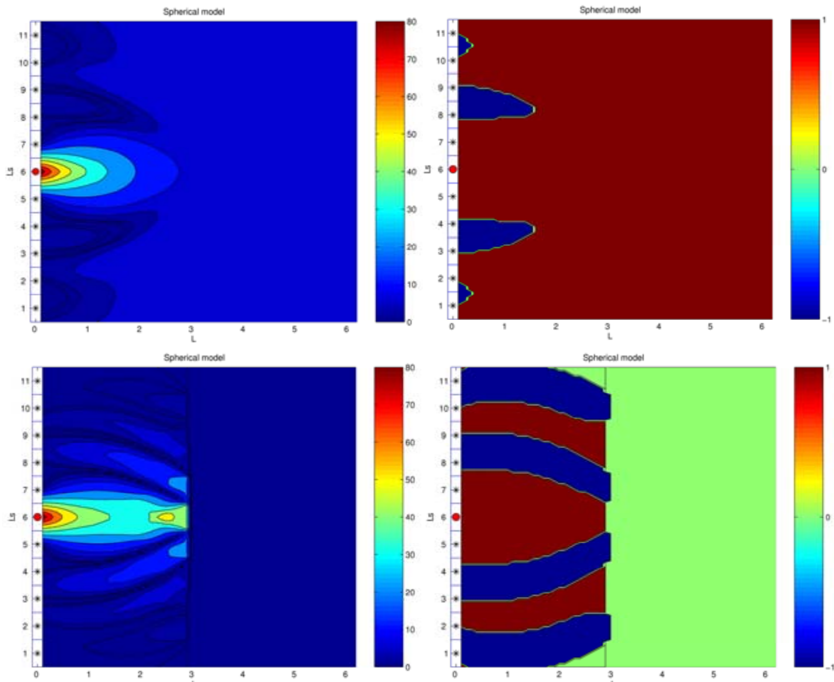
Figure 2: Influence (first column) and sign of influence (second column) of the second data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Spherical variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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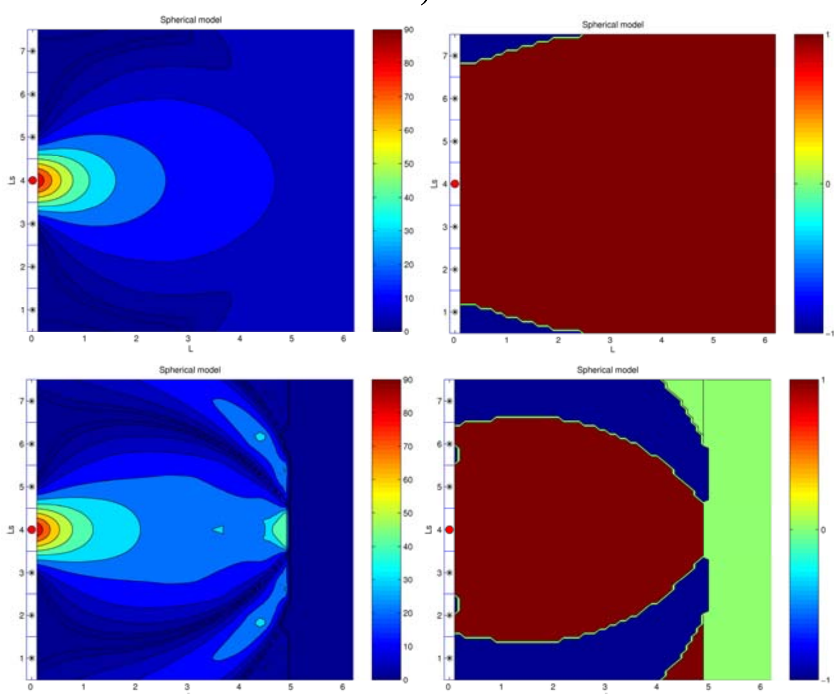

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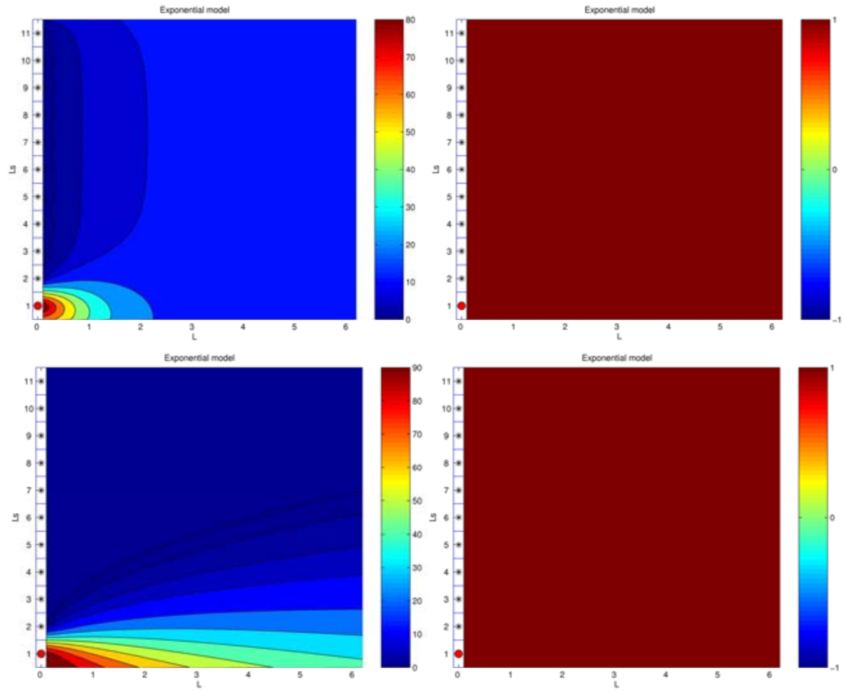
Figure 3: Influence (first column) and sign of influence (second column) of the middle data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Spherical variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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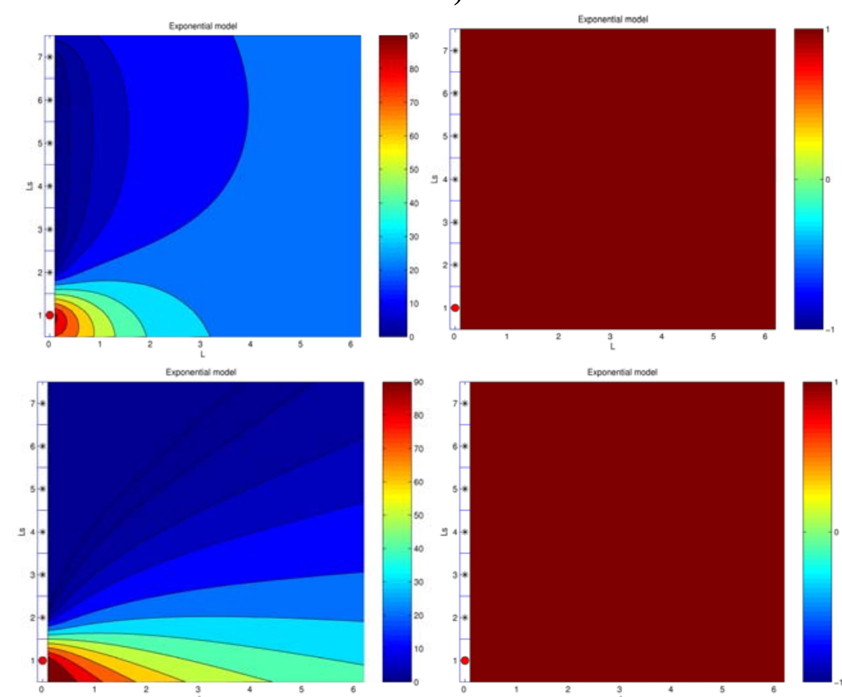
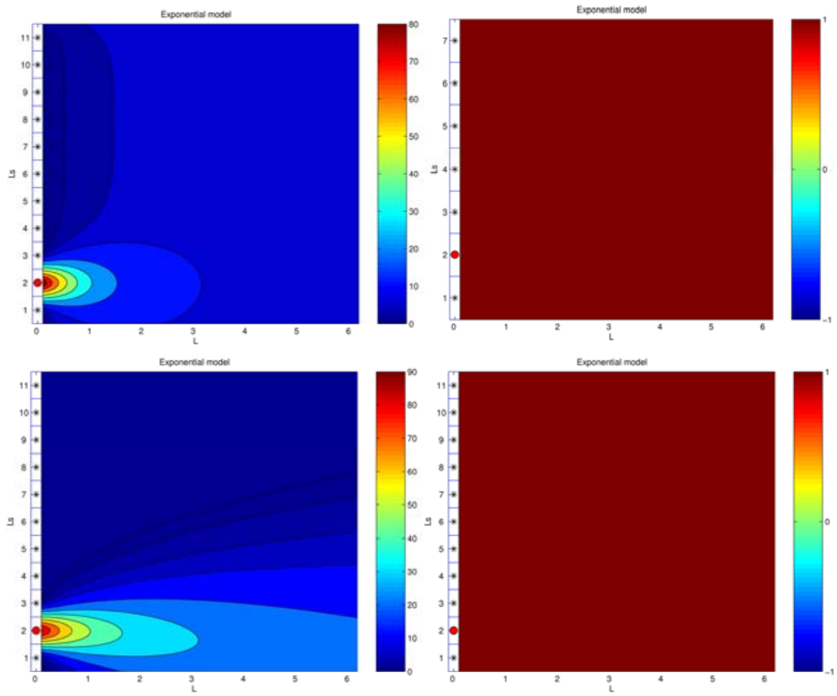


b)

Figure 4: Influence (first column) and sign of influence (second column) of the first data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Exponential variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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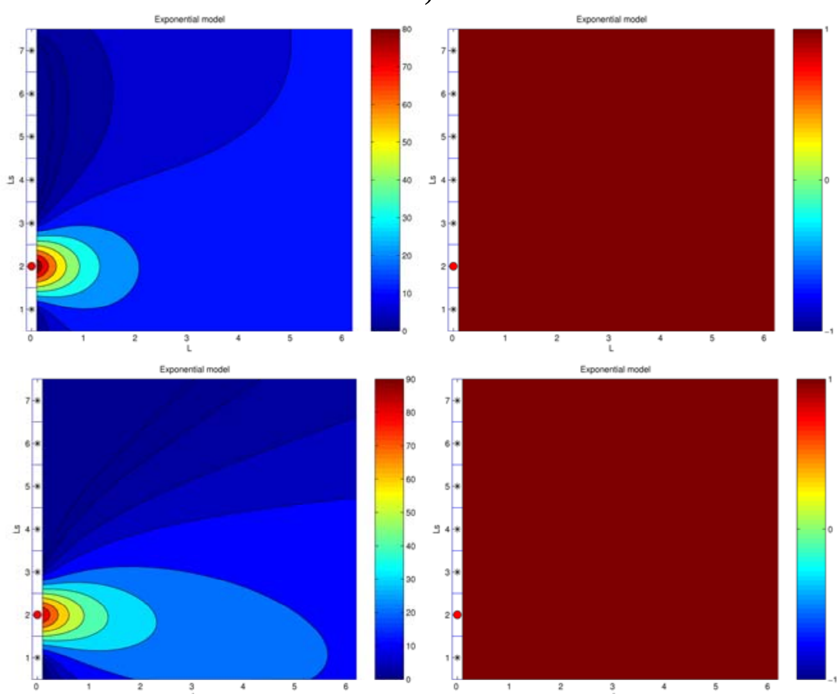

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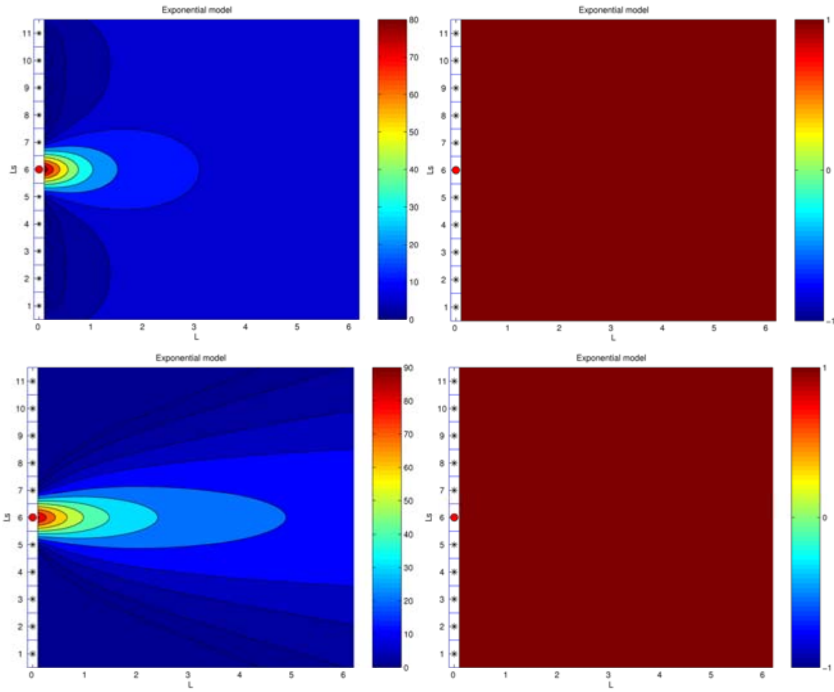
Figure 5: Influence (first column) and sign of influence (second column) of the second data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Exponential variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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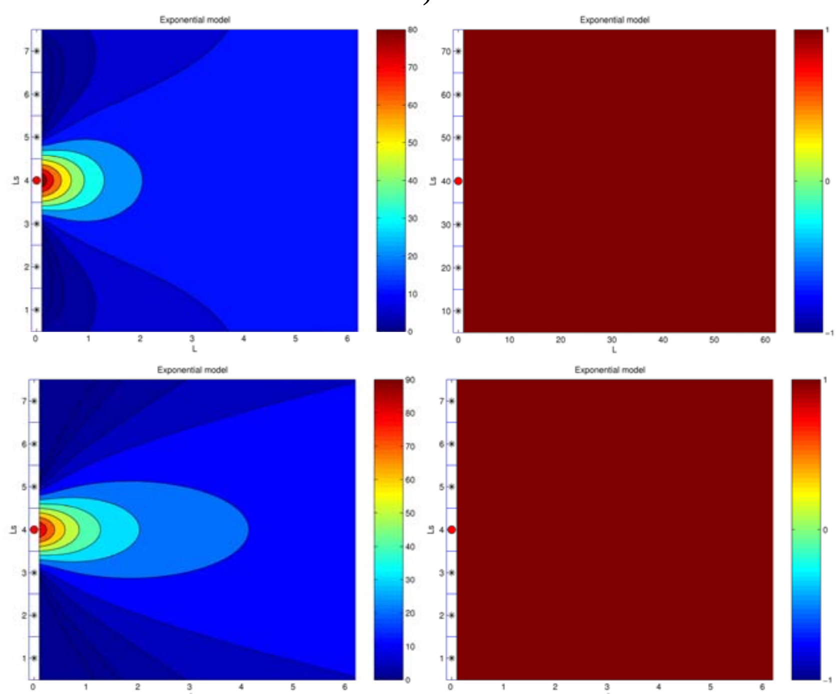

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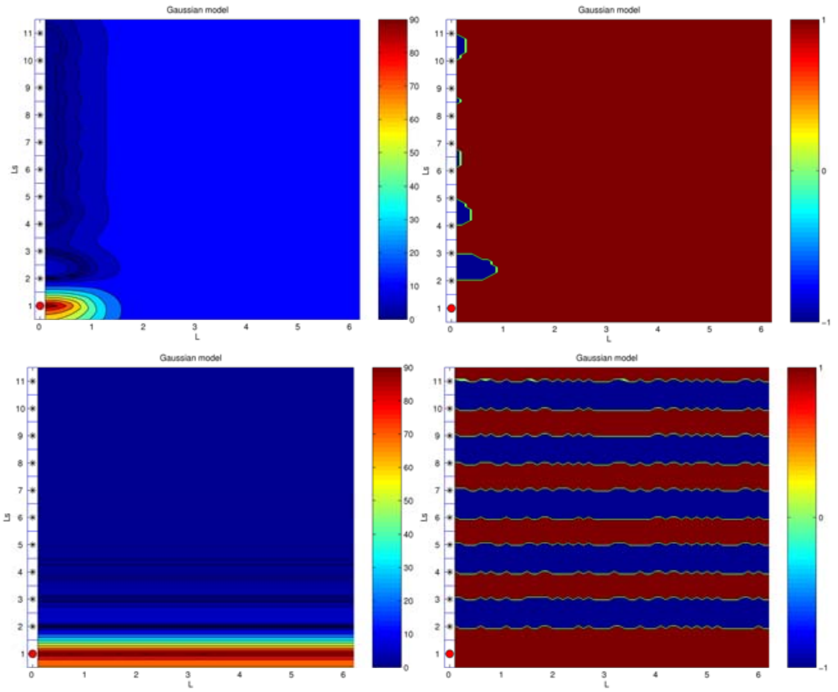
Figure 6: Influence (first column) and sign of influence (second column) of the middle data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Exponential variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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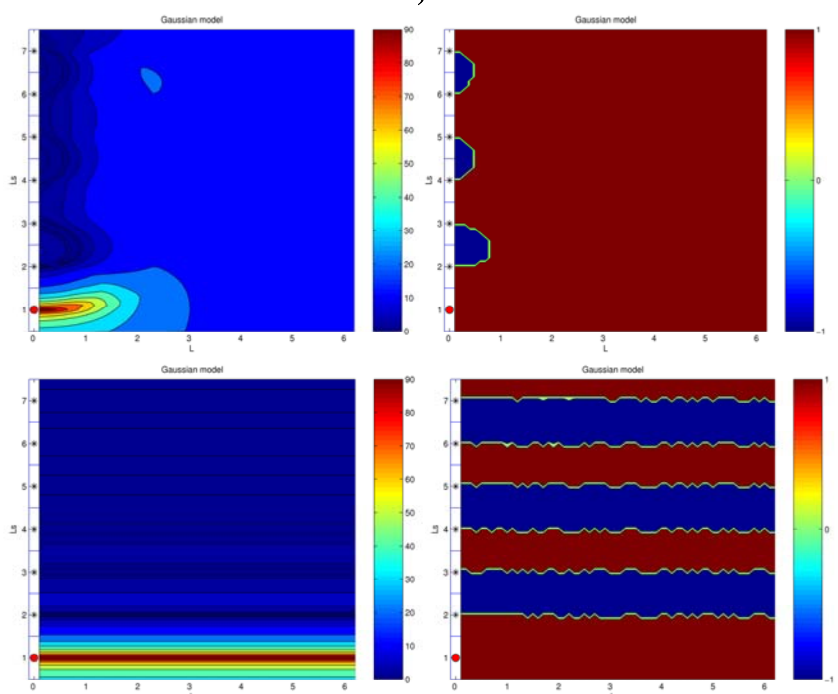

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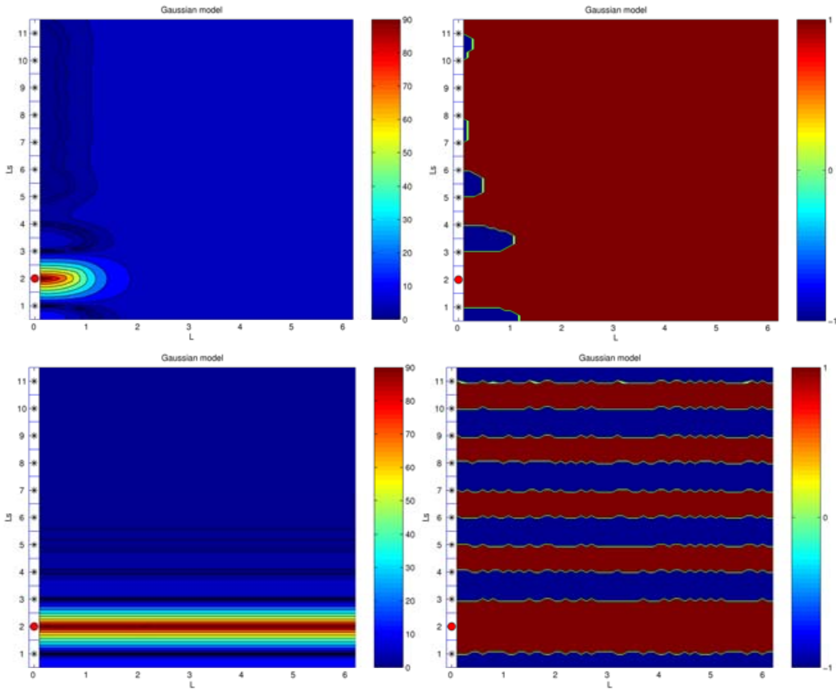
Figure 7: Influence (first column) and sign of influence (second column) of the first data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Gaussian variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



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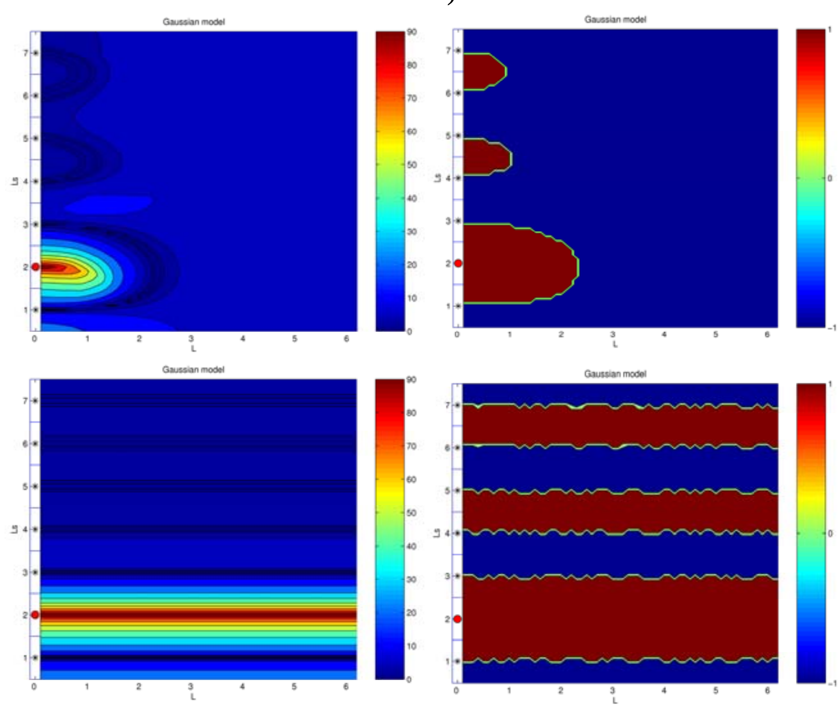
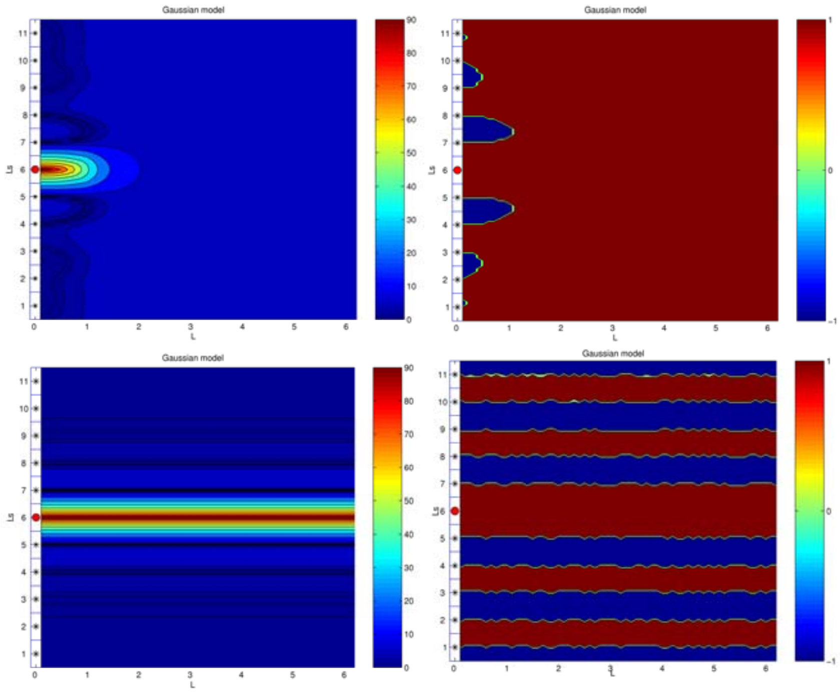
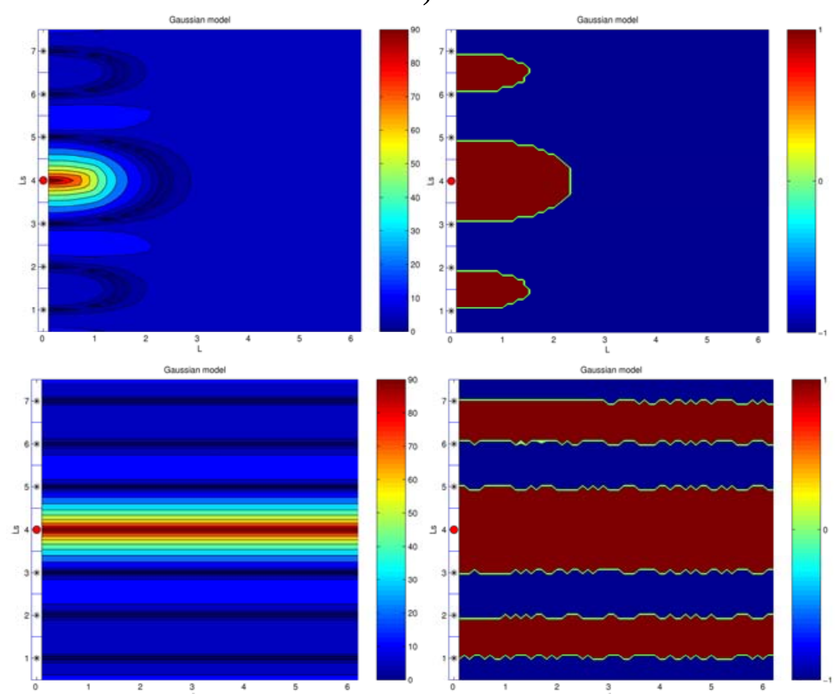


b)

Figure 8: Influence (first column) and sign of influence (second column) of the second data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Gaussian variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).

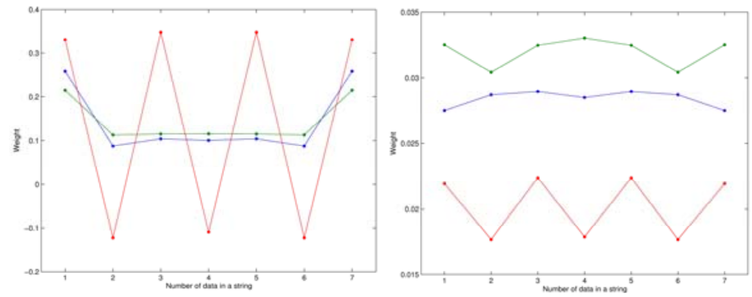


a)

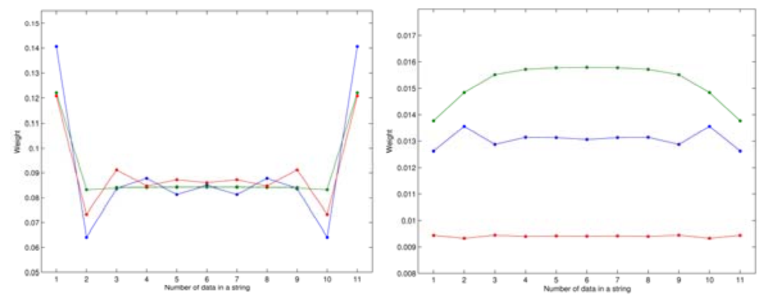


b)

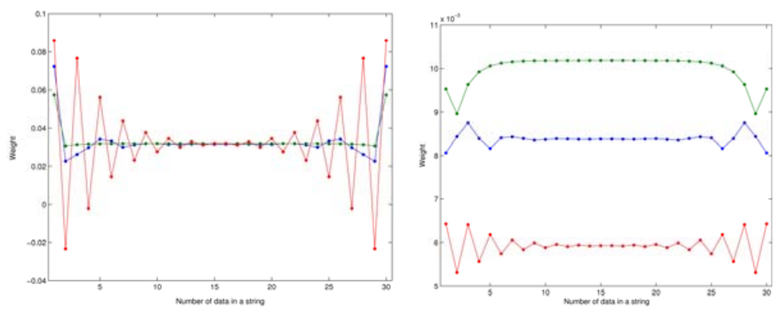
Figure 9: Influence (first column) and sign of influence (second column) of the middle data in a string on all locations of finite domain of size a) L = 6, Ls = 11 and b) L = 6, Ls = 7 obtained using OK (top figures) and SK (bottom figures) based on Gaussian variogram model (with range of correlation equal to 3 in a) and with range of correlation 5 in b)).



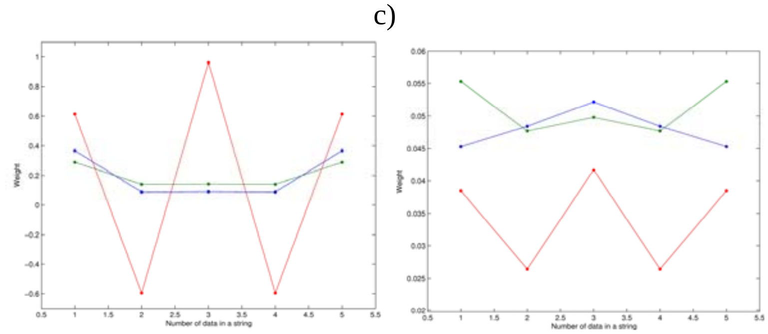
a)



b)

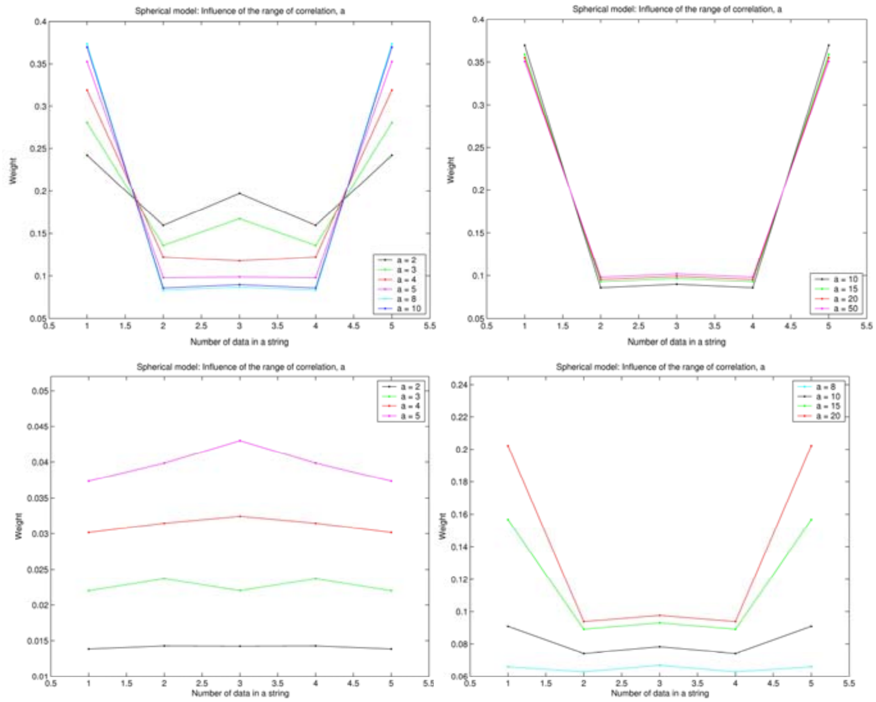


c)

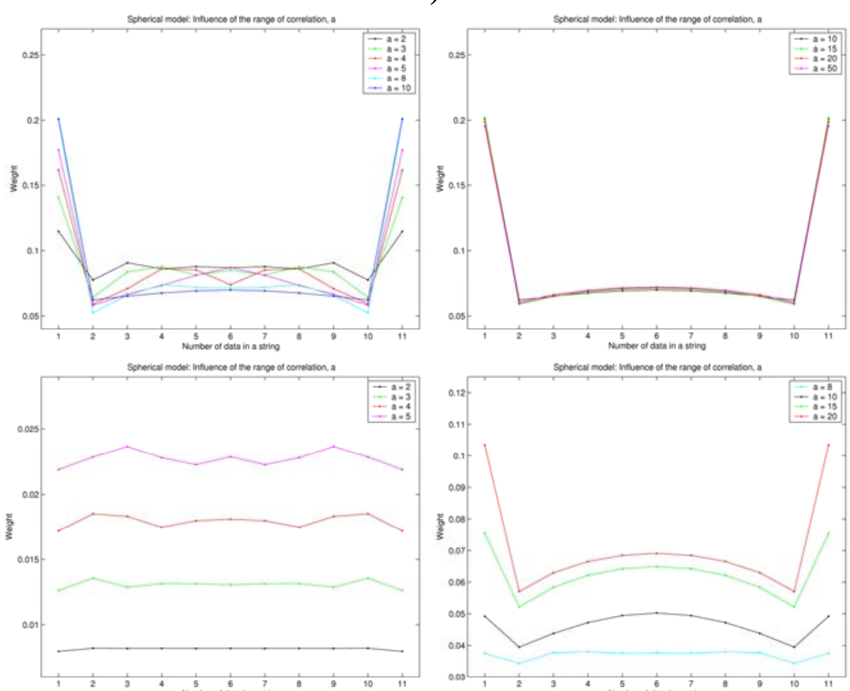


d)

Figure 10: Result for the average OK (first column) and SK (second column) weights calculated based on Spherical, Exponential and Gaussian variogram models in domain of size: a) L = 10, Ls = 7 (range of correlation is 5); b) L = 8, Ls = 11 (range of correlation is 3); c) L = 8, Ls = 30 (range of correlation is 5); d) L = 10, Ls = 5 (range of correlation is 6). Blue, green and red lines denote results for the average weights obtained based on Spherical, Exponential and Gaussian variogram models, respectively.

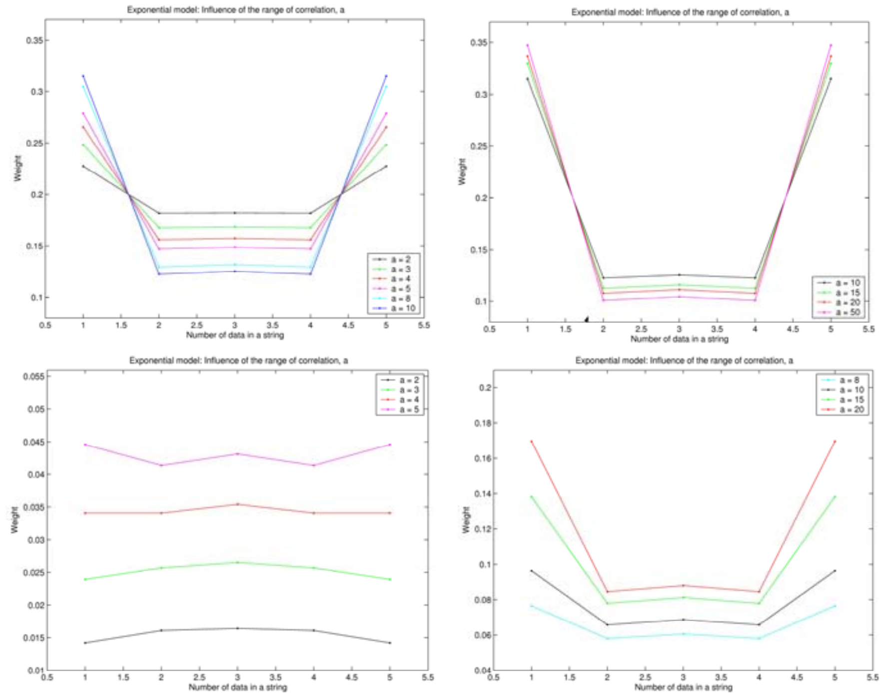


a)

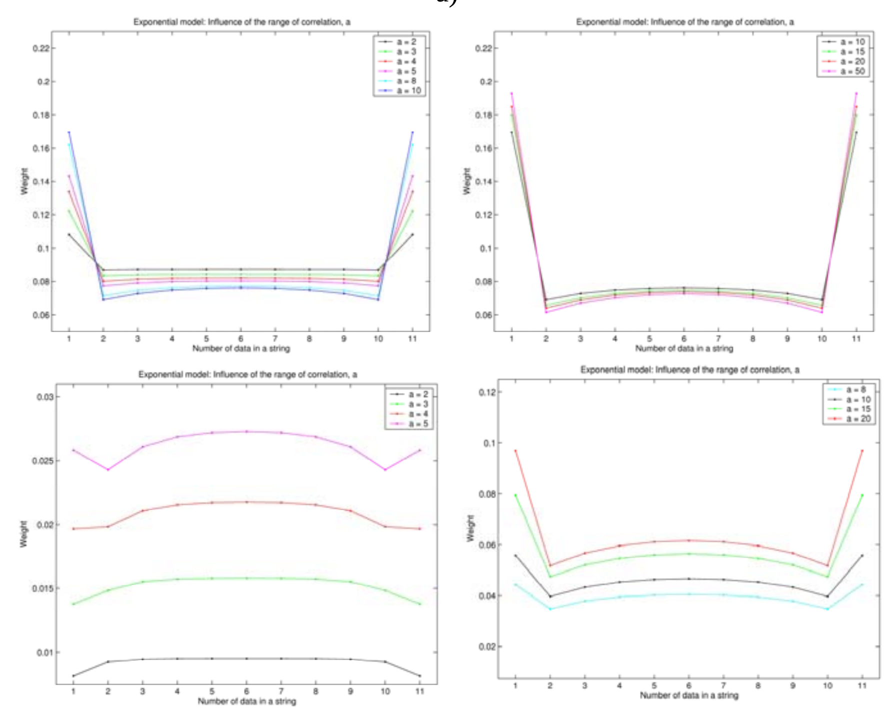


b)

Figure 11: Impact of the range of correlation on the average OK (top figures) and SK (bottom figures) weights calculated in domain of size: a) L = 10, Ls = 5 and b) L = 8, Ls = 11 based on Spherical variogram model.

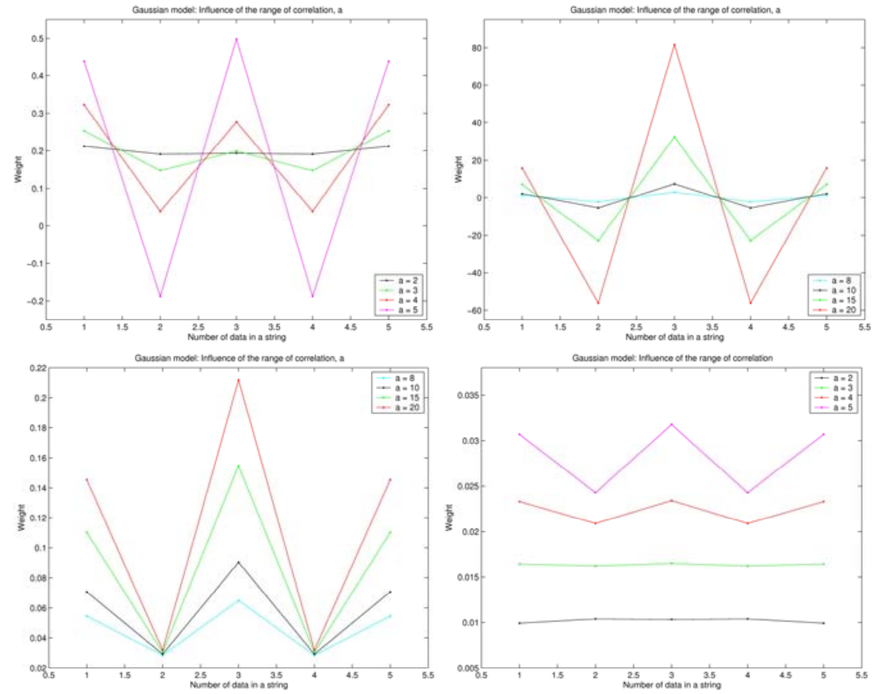


a)

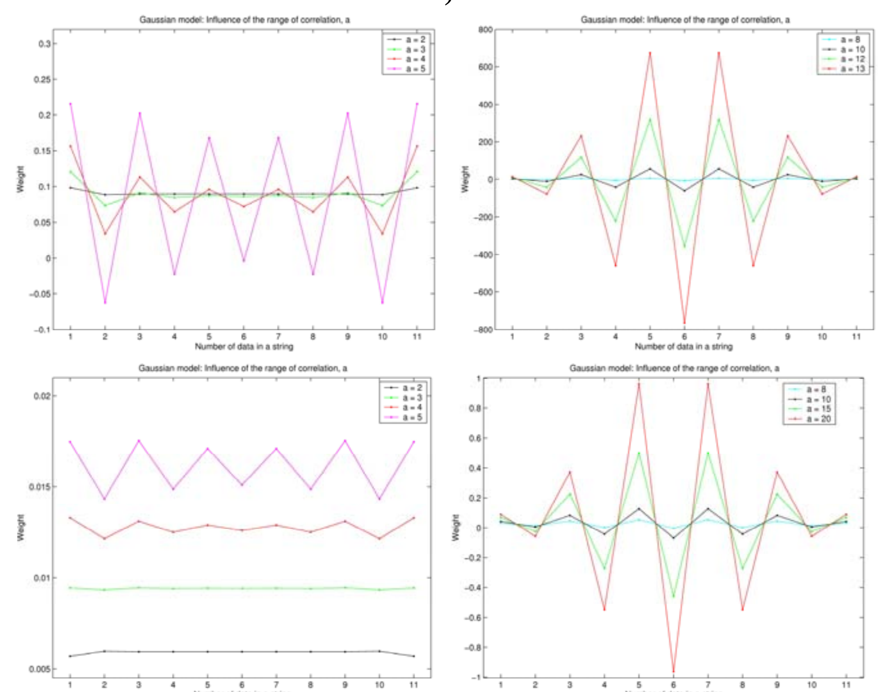


b)

Figure 12: Impact of the range of correlation on the average OK (top figures) and SK (bottom figures) weights calculated in domain of size: a) L = 10, Ls = 5 and b) L = 8, Ls = 11 based on Exponential variogram model.



a)



b)

Figure 13: Impact of the range of correlation on the average OK (top figures) and SK (bottom figures) weights calculated in domain of size: a) L = 10, Ls = 5 and b) L = 8, Ls = 11 based on Gaussian variogram model.

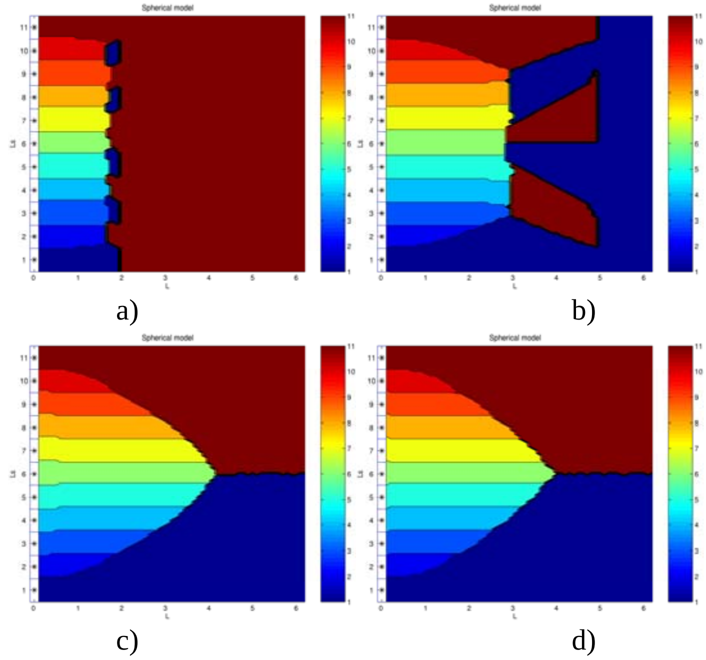


Figure 14: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by OK based on Spherical variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

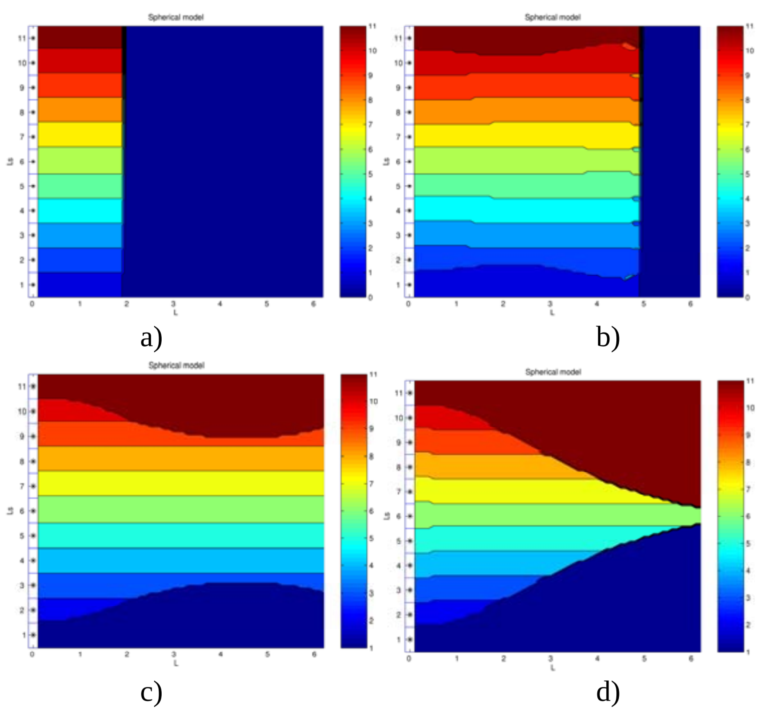


Figure 15: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by SK based on Spherical variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

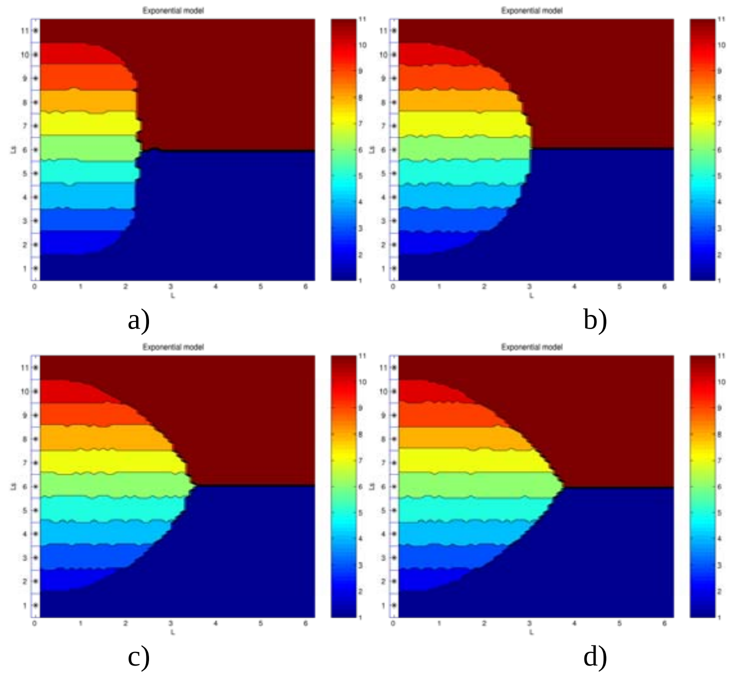


Figure 16: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by OK based on Exponential variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

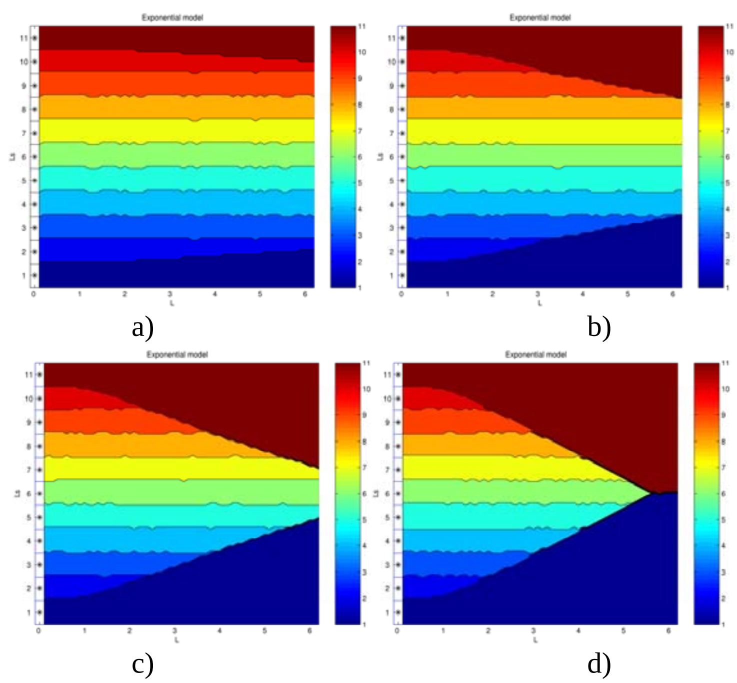


Figure 17: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by SK based on Exponential variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

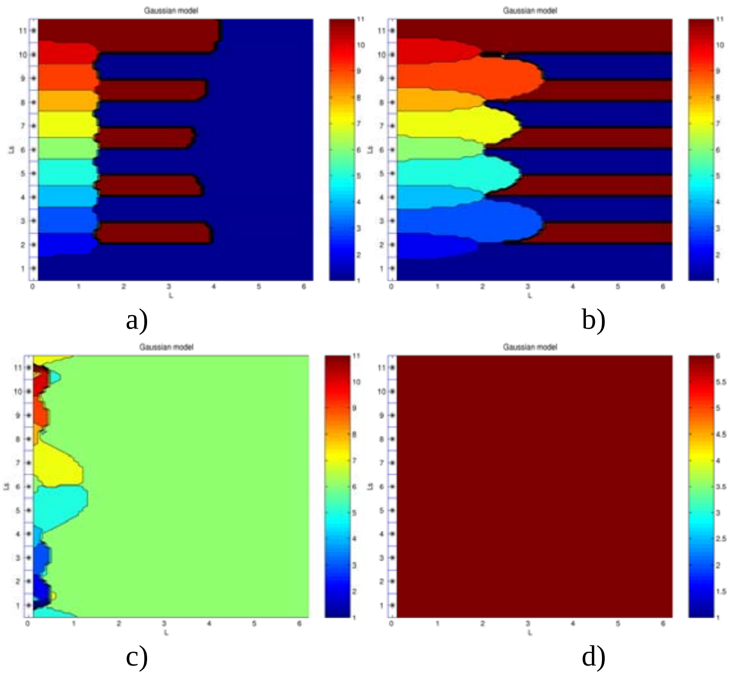


Figure 18: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by OK based on Gaussian variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

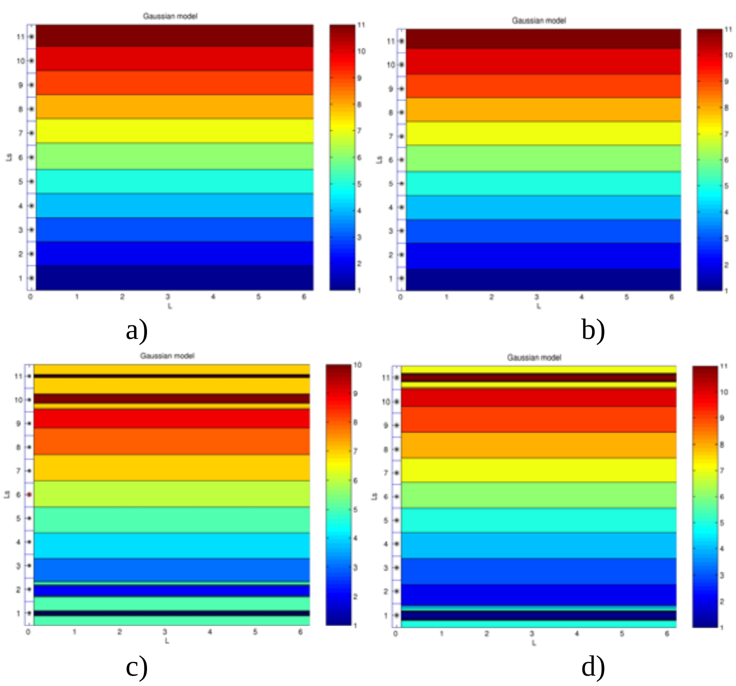


Figure 19: Map of maximal influence of the data in a string. Maximal influence of the data in a string on the particular location was defined as maximum relative weight obtained by SK based on Gaussian variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20).

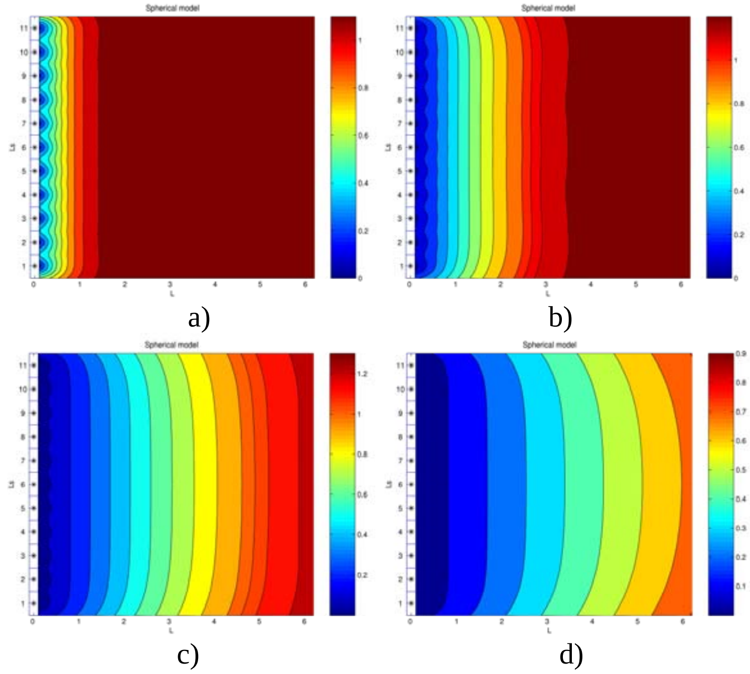


Figure 20: Map of the OK variances for estimated locations in the finite domain using a string of data. In estimation Spherical variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.

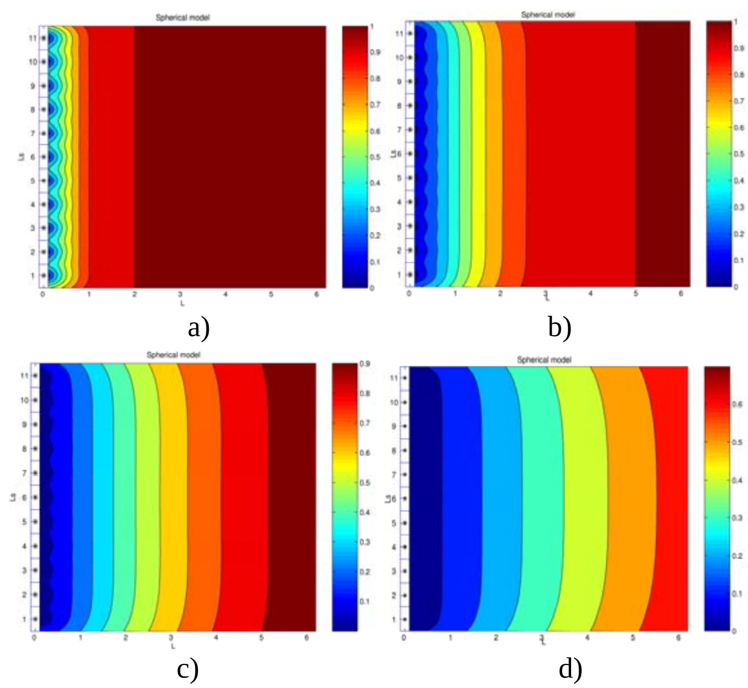
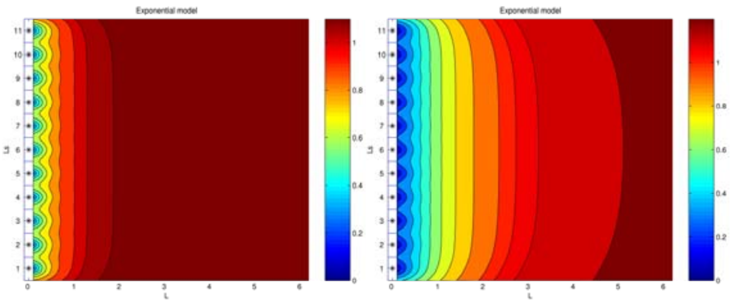
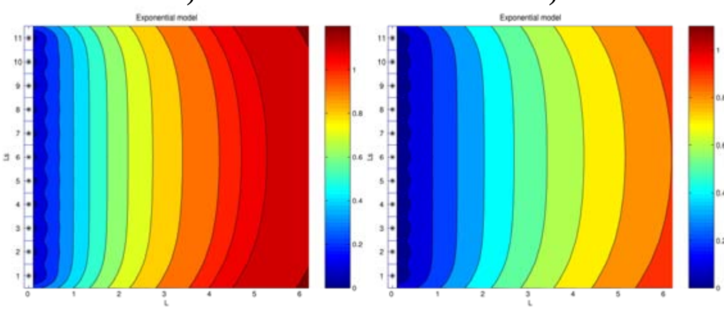


Figure 21: Map of the SK variances for estimated locations in the finite domain using a string of data. In estimation Spherical variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.

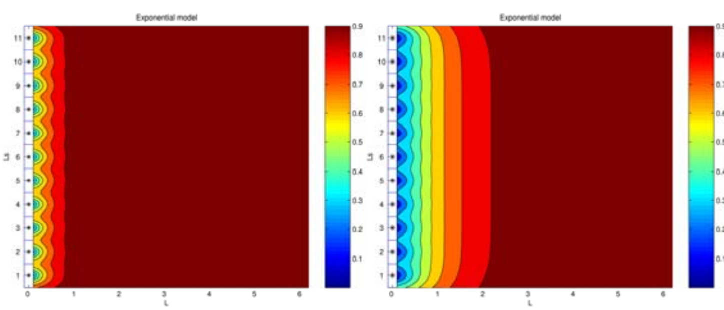


a) b)

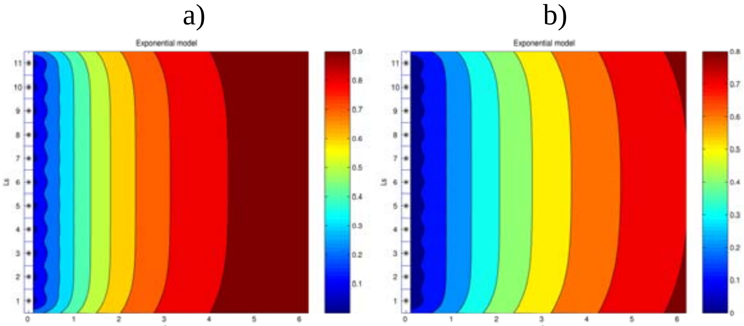


c) d)

Figure 22: Map of the OK variances for estimated locations in the finite domain using a string of data. In estimation Exponential variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.

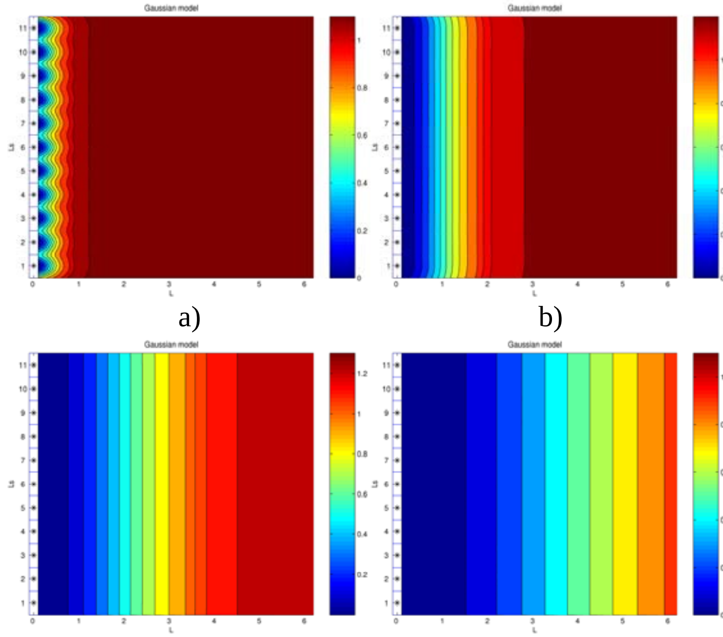


a) b)



c) d)

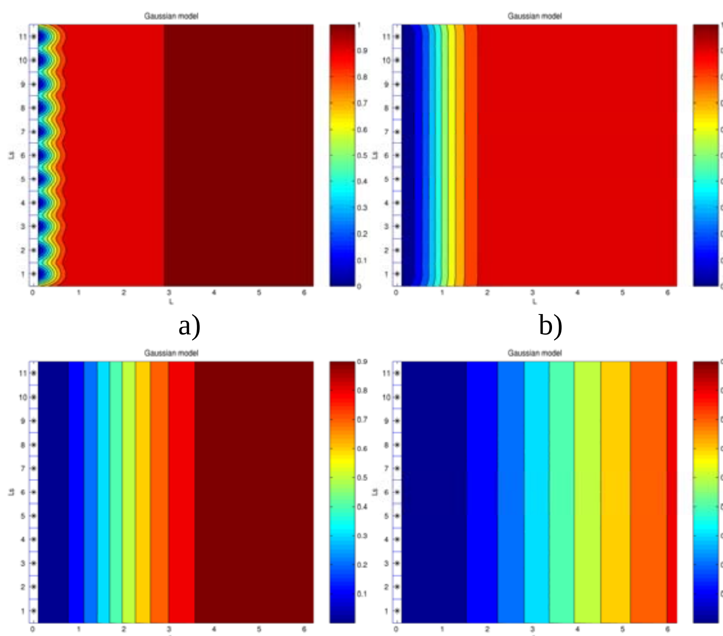
Figure 23: Map of the SK variances for estimated locations in the finite domain using a string of data. In estimation Exponential variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.



a) b)

c) d)

Figure 24: Map of the OK variances for estimated locations in the finite domain using a string of data. In estimation Gaussian variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.



a) b)

c) d)

Figure 25: Map of the OK variances for estimated locations in the finite domain using a string of data. In estimation Gaussian variogram model with range of correlation: a) 2; b) 5; c) 10; d) 20) was used.