Calculation of Full Permeability Tensor in an Unstructured Grid Block

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Heterogeneity in a hydrocarbon reservoir is captured by the rock permeability. This heterogeneity exists in reservoir both in fine scale and large scale. The fine scale variability is commonly modeled with geostatistical methods. These models can be as large as a hundred million cells; it is inefficient to feed them into a flow simulator due to the computational cost and time. Upscaling techniques are applied to average the fine scale permeability up into the final flow simulation grids. In cases where unstructured grids are used, full permeability tensors arise instead of a diagonal tensor. The focus of this work is on development of a method to characterize the full permeability tensor for an unstructured grid block using fine scale heterogeneity information. A prototype program called `ptensor` *is developed, based on the* `flowsim` *program, to calculate a full tensor permeability on a polygonal grid block.*

Introduction

Geostatistical realizations provide models of reservoir properties for millions of grid blocks to better capture heterogeneity based on multiscale information. Feeding these fine scale models to flow simulation is impractical due to the computational inefficiency. Upscaling techniques are often considered to average the fine scale models to coarser scale models. A simple averaging is sufficiently and reasonable for variables that average linearly; however, in the case of permeability which does not average linearly, a simple arithmetic averaging is inadequate.

Commonly unstructured grids are used in order to better capture the flow response near complex reservoir features such as faults and wells. Irregularly shaped grids do not conform to the underlying fine scale model and this irregularity changes the assumption of simulators which consider that the pressure equation has a diagonal permeability tensor. Flow Simulation on unstructured grids requires directional permeability or full tensor permeability to be specified. Recent modifications to flow simulators permit the solution of the flow equations using a full tensor.

This paper presents a new method to calculate the full permeability tensor in an unstructured grid based on a numerical finite difference solution of the steady state flow equations in presence of a fine scale heterogeneous model of permeability. The method presented here is for a 2-D polygonal shaped grid but can be easily generalized for any grid shape in 3-D.

Background

Permeability upscaling refers to a procedure in which the underlying fine scale permeability is averaged up to return the effective permeability of a larger domain. There are several upscaling techniques available. In an ideal case, the equivalent permeability for a group of fine grid blocks serially arranged has been analytically proven to be equal to their harmonic average. If the blocks are arranged in parallel to the flow direction, the equivalent permeability is equal to their arithmetic average (see Figure 1) (Deutsch, 1987; Kelkar and Perez, 2002). Here the assumption is that the permeability tensor is a scalar (k_{eff}).

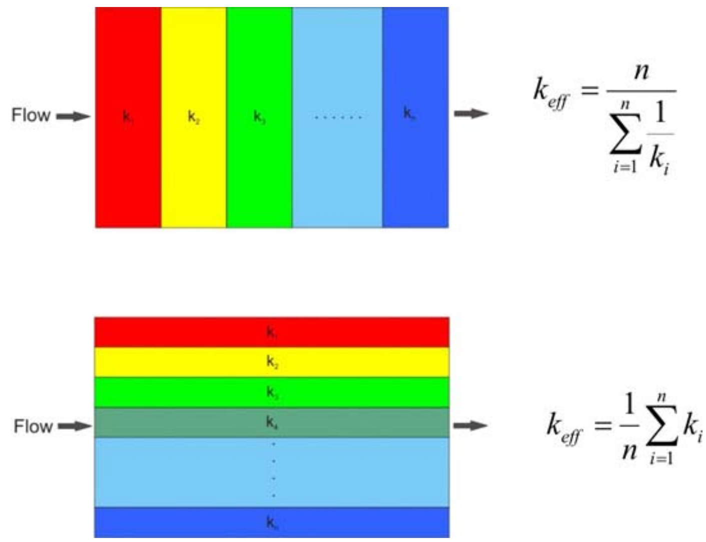

Figure 1. Upscaled effective permeability for simple cases of series (top) and parallel (bottom) layers. Redrawn from Kelkar and Perez (2002).

Gomez-Hernandez and Wen (1994) showed that a simple arithmetic averaging is valid as long as the spatial variability of permeability does not display strong anisotropy.

For more complex cases with increasing heterogeneity, flow-based upscaling techniques yield more accurate results. In this type of upscaling the flow equation is solved for pressure and the results are used to obtain the block permeability. Warren and Price (1961) applied this technique for regular coarse grids to obtain the diagonal tensor. Usually cases that involve the use of irregular grid or heterogeneous permeability field at fine scale require calculating the full permeability tensor. White and Horne (1987) were the first to propose a technique to determine full non-diagonal block permeability tensors. The resulting block permeability tensors are not always symmetric or positive-definite. Choosing appropriate boundary conditions is very important in flow-based upscaling. Durlofsky's (1991) idea of periodic boundary returns symmetric and positive definite full permeability tensor in medium with periodic condition (repetitive geological structures).

Almost all of the above mentioned techniques are applied on regular grids. Tran (1995) proposed a method in which the pressure is calculated in the smallest rectangle that includes the irregular block. However, the calculated permeability is diagonal. He (2000) applied Durlofsky's periodic boundary condition and solved the flow equations with a finite element method for general quadrilateral grids.

Methodology

Flow based upscaling technique is used to calculate effective permeability of coarse block. Consider single rectangular block imposed on a fine scale model (Figure 2). The idea here is to calculate the pressure at fine scale with specific boundary conditions applied at the boundary of the coarse block and then use the solution to calculate the full permeability tensor for that coarse block. In order to calculate pressure at fine scale in this 2D case, the 2-D single phase steady state flow equation with the assumption of incompressible fluid and rock is considered:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial P}{\partial y}\right)=0$$

where P is the pressure, and k_x and k_y are the fine scale permeability in x and y direction respectively.

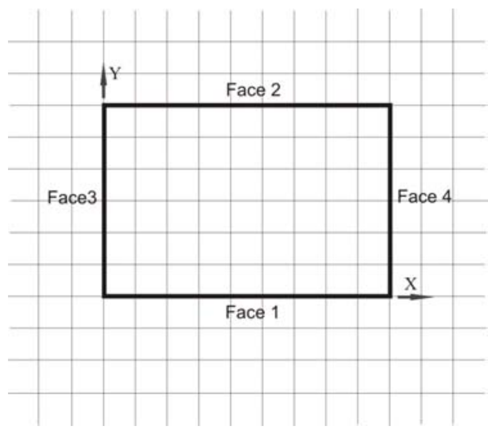

Figure 2. A coarse block consisting of fine scale permeability.

In order to calculate all components of the permeability tensor, the flow equation should be solved twice with two different boundary conditions. A constant pressure and no flow boundary conditions are applied at the perpendicular boundaries of the coarse block. In the first case, a no flow boundary is assigned to the Y-direction while constant pressure is considered in X-direction.

The second case considers that the flow direction is perpendicular to the first case, that is, the flow direction is along Y-direction and the no-flow boundaries are imposed in the X-direction (see Figure 3).

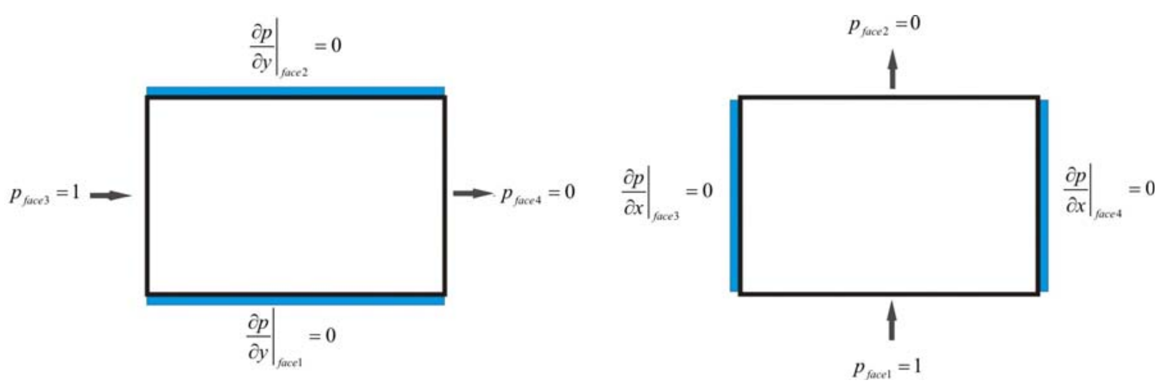

Figure 3. Boundary conditions applied on coarse grid when flow is in x direction (left) and y direction (right). Thick blue lines indicate no flow boundaries on those edges.

By using results from these two solutions and applying Darcy's law, volume averaged velocities and pressure differences are calculated over the entire rectangular grid. Now applying generalized Darcy's law, four tensor components are calculated by solving the following system of equations.

$$(u_x)_x = -\left[k_{xx}\left(\frac{\partial p}{\partial x}\right)_x + k_{xy}\left(\frac{\partial p}{\partial y}\right)_x \right]$$

$$(u_y)_x = -\left[k_{yx}\left(\frac{\partial p}{\partial x}\right)_x + k_{yy}\left(\frac{\partial p}{\partial y}\right)_x \right]$$

$$(u_x)_y = -\left[k_{xx}\left(\frac{\partial p}{\partial x}\right)_y + k_{xy}\left(\frac{\partial p}{\partial y}\right)_y \right]$$

$$(u_y)_y = -\left[k_{yx}\left(\frac{\partial p}{\partial x}\right)_y + k_{yy}\left(\frac{\partial p}{\partial y}\right)_y \right]$$

where $(u_x)_x$ and $(u_y)_x$ are velocity components when flow is in *x*-direction, $(u_x)_y$ and $(u_y)_y$ are velocity components when flow is in *y*-direction, $\left(\frac{\partial p}{\partial x}\right)_x$ and $\left(\frac{\partial p}{\partial y}\right)_x$ are pressure gradient in *x* and *y* direction, respectively when flow is in the *x*-direction, $\left(\frac{\partial p}{\partial x}\right)_y$ and $\left(\frac{\partial p}{\partial y}\right)_y$ are pressure gradients in *x* and *y* direction respectively when flow is in the *y*-direction.

Using this boundary condition does not ensure that the resulting permeability tensor will be symmetric. A least square method can be applied to ensure symmetric tensor (Durlofsky, 2005). Adding a $\left.\frac{\partial p}{\partial y}\right|_{face2} = 0$ constraint on cross terms ($k_{xy} = k_{yx}$) can be another option. In general, one expectation is that the calculated tensor will be positive definite, that is:

$$k_{xx} > 0,\ k_{yy} > 0 \text{ and } k_{xx}.k_{yy} > k_{xy}.k_{yx}$$

Dealing with irregular grids

A similar approach can be used for irregular grids. We can discretize the irregularly shaped coarse scale block using the underlying fine scale model. This simply requires determining whether a fine scale block lies within a coarse scale grid; this can be determined by evaluating whether or not the centre point of the fine scale block falls inside the irregular coarse block (Figure 4). A more accurate approximation to the irregular coarse scale block can be obtained if the underlying heterogeneity model is at a sufficiently fine scale. The irregular grid is surrounded by the smallest rectangle around it (dark black lined rectangle in Figure 4). Boundary conditions are applied at the boundary of this rectangular domain and pressure is calculated for the fine grid within this domain. The area between the rectangular domain and irregular grid is called the buffer zone.

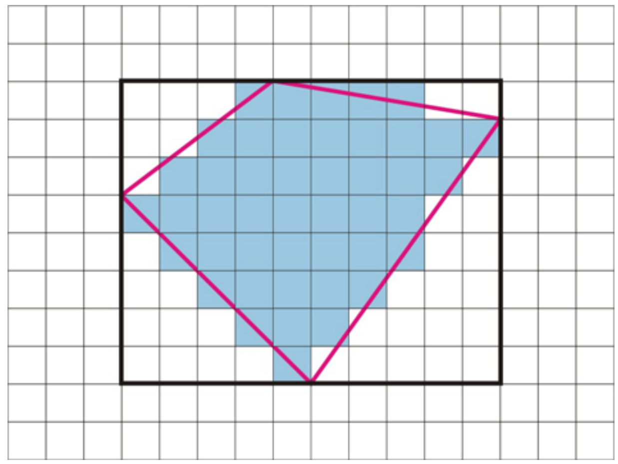

Figure 4. A diamond shape coarse grid and the underlying fine scale model. Shaded fine scale blocks lie inside the coarse irregular grid and are considered for the calculation of effective permeability. The rectangular region (dark black line) is the smallest rectangle that will encompass the irregular grid.

Program

The `ptensor` program is developed to calculate a full permeability tensor for a 2D unstructured grid block. It is based on the `flowsim` (Finite difference flow simulator) program which is a finite difference flow simulator that assumes flow in one direction and no-flow boundaries in the perpendicular directions. The `ptensor` program permits different options related to the shape of coarse grid, size of the buffer zone, the permeability value inside this zone, and the requirement for symmetry in the resulting tensor. The parameters required for this program are:

```
                              Parameters for PTENSOR
                              ********************
Line   START OF PARAMETERS:
 1     perm.dat                   -Input datafile with permeabilities
 2     1  2  0  0  0              - columns for kx,ky,kz, ky/kx, kz/kx
 3     perm.out                   -output file for permeability tensor
 4     100  60  1                 -input : nx, ny, nz
 5     1.0  1.0  1.0              -input : dx, dy, dz
 6     4                          - Number of U.S. grid vertices
 7      10     20                 - Vertex 1; X,Y
 8      30     40                 - Vertex 2; X,Y
 9      50     60                 - Vertex 3; X,Y
10      70     80                 - Vertex 4; X,Y
11     1 1 1 1                    -Buffer Zone Size: Left,Right,Top,Bottom
12     1                          -Buffer Zone: Homogenous(1),Heterogeneous(0)
13     20                         -   if (1),Constant Permeability Value
14     0                          - Symmetric Tensor? Yes(1),No(0)
```

The information about the fine scale permeability data file is input in Lines **1** and **2**. In Line **3,** the name of output file is specified. The number and size of grid cells in the input file should be specified in Lines **4** and **5**. A general irregular shape grid can be considered as coarse grid for which permeability tensor is desired. The number of unstructured grid's vertices is specified in line **6** and the corresponding vertices coordinates are specified in the following lines. Vertices should be provided starting from the top one and numbered counter clockwise. Four integer numbers in line **10** control the size of buffer zone in the left, right, top and bottom of the rectangular domain respectively. Each integer value shows the number of fine grid which should be added to the smallest rectangular region. For example “1 2 2 1” means that the rectangle should be extended 1 cell from left, 2 cells from right, 2 cells from top and 1 cell from bottom (Figure 5). There is an option for the permeability value of buffer zone. Lines **12** and **13** enable the user to choose if the buffer zone is homogeneous or heterogeneous and what is the homogenous permeability value is (if put 1 in line 8). In line **14** there is an option to let the output tensor be symmetric.

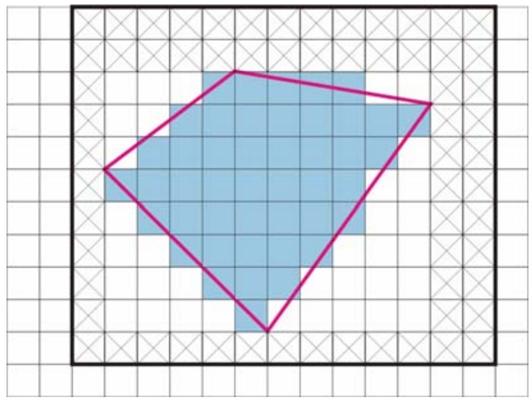

Figure 5. An unstructured grid with the buffer zone around it. Size of buffer zone is controlled by four integer values which indicates how many cells should the smallest rectangle be expanded on each side (2 from top, 1 from left, 2 from right and 1 from bottom).

Example 1

A simple diamond shape grid is imposed on a fines scale permeability model. The heterogeneous model is composed of five different facies. Constant permeability values are assigned to each facies (Figure 66 and Table 1).

Categories	Kx (mD)	Ky (mD)
Sand in Seq. 1	25	5
Sand in Seq. 2	20	4
Lime in Seq. 2	100	20
Sand in Seq. 3	11	2.2
Shale in Seq. 3	2	0.4

Table 1. Five different facies and their permeability values.

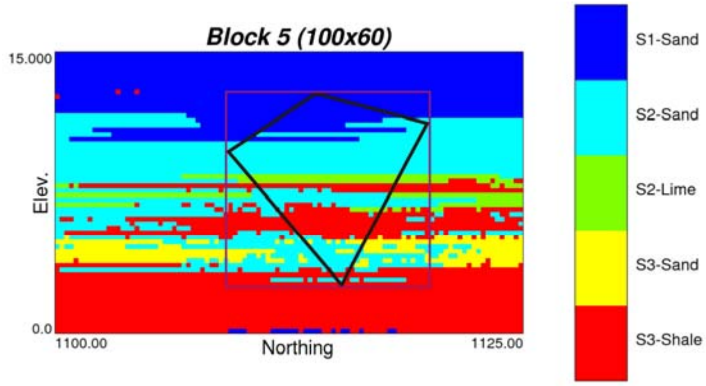

Figure 6. An irregular grid is imposed on a heterogeneous medium (see Paper 206 for construction of this facies model).

Initially, no buffer zone is considered and the option for symmetric tensor is turned on. The resulting tensor is as follows:

$$\bar{\bar{k}} = \begin{bmatrix} 41.315 & 0.5874 \\ 0.5874 & 3.7087 \end{bmatrix}$$

Different buffer zone conditions are applied to the irregular grid to check the sensitivity of the resulting tensor to the surrounding conditions. For this case, the option for symmetric tensor is turned off. **Error! Reference source not found.** shows the result of eight different buffer conditions. Here the size of buffer is shown in the same notation as the code. In the case of a homogeneous buffer zone, a constant permeability value of 20 mD is assigned to each cell in the surrounding buffer zone. The arithmetic, harmonic and geometric averages of permeability values in rectangle region are also shown in the table.

In all cases the calculated permeability tensor was positive definite. In heterogeneous buffer zone case, using different buffer zone size seems to give consistent tensor values. It has been observed by a number of authors that improved accuracy in *k* can be achieved if a larger local problem is solved (Gomez-Hernandez and Journel (1994); Holden and Lia (1992)). In the homogeneous buffer zone case, results are different from heterogeneous case but they are consistent as the size of buffer changes.

Table 3 shows the result for three cases which different homogeneous permeability values are assigned to the buffer zone. This table shows that as the value assigned to the homogeneous permeability of buffer zone increases it seems that the permeability tensor diverges quickly.

Example 2

An irregular polygonal grid with seven sides is imposed on a fine scale permeability model (100x60 cells). The medium is completely heterogeneous and the permeability values assigned to each facies are the same as Example 1 (Figure 7). A large heterogeneous buffer zone is considered and pressure is calculated on fine scale. The result permeability tensor is as follows:

$$\bar{\bar{k}} = \begin{bmatrix} 41.245 & 0.7204 \\ -0.8474 & 8.4145 \end{bmatrix}$$

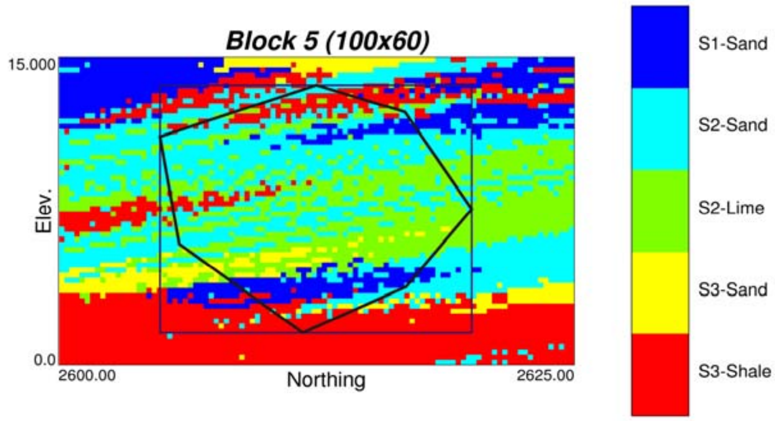

Figure 7. A polygonal irregular grid imposed on completely heterogeneous medium.

Conclusions and Future Work

A flow based upscaling techniques is examined using finite difference to calculate local pressures. The resulting tensor is positive definite and it can be symmetric in the case that a symmetric tensor is specified. For example, a symmetric tensor may be expected for a regular symmetric grid with homogeneous permeability field. Results are quite sensitive to the size and permeability of the cells within the buffer zone. The `ptensor` code is as fast as conventional `flowsim` program and the computational time is related to the calculation of pressure on fine scale.

Generalizing the method to 3-D is straightforward and will be undertaken. This will require a 3-D fine scale permeability model and solution of the 3-D steady state flow equation. The important issue here is how to deal with shape of 3-D irregular grid. The shape can be controlled by the coordinates of the vertices, which will define planar surfaces of the unstructured grid. A slight modification to the `ptensor` program is foreseen to handle these requirements.

Validation of the results from this approach is required. This will involve more detailed sensitivities to be undertaken, as well as comparisons against other tensor calculation approaches. For instance, if we consider a regular 3D block, we could compare the results of this simulation approach to that proposed by (Aasum *et. al.*, 1993). Of course, the results obtained via the proposed method can be checked by running flow simulation on both fine scale and the upscaled permeability model, and comparing simulation results such as recovery factor.

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Table 2. Eight examples with different buffer zone conditions.

Buffer Zone		k_{xx}	k_{xy}	k_{yx}	k_{yy}	KA	KG	KH	$Q_{x\text{-}in}$	$Q_{x\text{-}out}$	$Q_{y\text{-}in}$	$Q_{y\text{-}out}$
Heterogeneous	1 1 1 1	41.4914	-0.1766	0.6144	3.7864	33.2727	15.8638	6.5008	11.1682	11.1770	2.0884	0.7638
	2 2 2 2	41.4720	-0.1561	0.6050	3.8455	32.2665	15.1962	6.2455	10.9921	10.9967	2.0860	0.6994
	5 5 5 5	41.5900	-0.1198	0.6210	4.0132	30.2901	13.7855	5.6969	10.6044	10.6044	2.1683	0.5773
	5 3 1 2	41.5751	-0.1315	0.5621	3.8340	32.4685	14.9698	6.1051	9.9689	9.9729	2.2183	0.7616
Homogeneous	1 1 1 1	36.7364	-1.2821	0.2402	5.1884	19.6001	3.8746	16.4031	9.6888	9.6042	6.8161	6.2423
	2 2 2 2	36.5943	-1.2195	0.2746	5.1795	17.9535	3.4579	17.9075	9.5731	9.4802	7.1727	6.5885
	5 5 5 5	36.3405	-1.1096	0.3266	5.1533	14.0954	2.6486	22.8091	9.3906	9.2880	8.0142	7.4001
	5 3 1 2	36.3217	-1.1661	0.3356	5.1759	17.1332	3.2673	18.7641	8.5519	8.4742	8.3410	7.7308

Buffer zone permeability (md)	k_{xx}	k_{xy}	k_{yx}	k_{yy}	KA	KG	KH
1	31.3164	-0.0851	0.0363	3.9745	17.9535	3.4579	17.9075
20	36.5943	-1.2195	0.2746	5.1795	17.9535	3.4579	17.9075
100	38.2964	-2.1390	0.7123	5.7806	17.9535	3.4579	17.9075

Table 3. Three examples with different homogeneous buffer zone permeability.