Short Note:
The Maximum Allowable Variance to Meet Probabilistic Resources Classification Criteria

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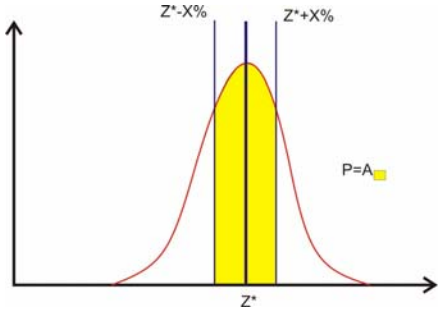
Probabilistic criteria could be applied directly to assess whether or not an area meets the classification limits; however, there is a problem. Classification criteria are for volumes that are relevant for technical and economic decision making: typically a month. Orebody models are not represented very precisely at the scale of a nominal month. There is a need to assess the criteria at a smaller scale. This short note describes a simple procedure.

Probabilistic Criteria

Some engineers and geologists would advocate the use of probabilistic criteria for resource/reserve classification. One possible probabilistic criteria for a measured resource is that *there should be greater than 90% probability that the grade of a monthly production volume be within 15% of the estimated grade*. There are three parameters: the volume (*monthly*), the confidence probability (*90%*) and the precision (*15%*). These values could be changed to meet site specific considerations.

Regardless of the distribution of grades, the distribution of uncertainty in a large production volume (say a month) will be nearly Gaussian. This is based on the central limit theorem that states that the sum of a large number of independent equally distributed random variables tends to a Gaussian distribution. The grades within a monthly production volume do not meet all these criteria, but the tendency toward a Gaussian distribution is commonly observed. We could then visualize the probabilistic criteria as follows:



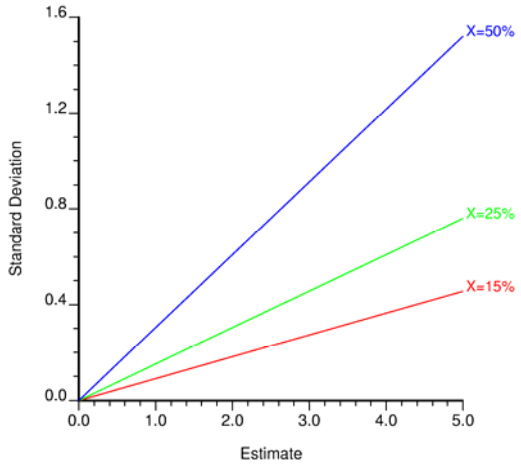
The maximum allowable standard deviation of the monthly grades can be calculated based on the confidence probability P (set to 90% above) and the precision X (set to 15% above). This calculation is based on the standard Gaussian distribution $G(y)$ and the specified parameters.

$$\sigma_{\text{maximum}}\left(Z^*, X, P\right) = \frac{X}{G^{-1}\left(\frac{1+P}{2}\right)} \bullet Z^*$$

This is universal and does not depend on the grade. The units of the standard deviation $\sigma_{maximum}(Z^*,X,P)$ are the same as the grade Z^*. Any volume, with estimate Z*, that has a standard deviation greater than $\sigma_{maximum}(Z^*,X,P)$ will have a lower than $P\%$ probability to be within $X\%$ of the estimate. The slope can be tabulated for different X and P values.

Slope: $\frac{X}{G^{-1}\left(\frac{1+P}{2}\right)}$		*X* Precision Value		
		50%	25%	15%
P Confidence Value	50%	0.741	0.371	0.222
	75%	0.435	0.217	0.130
	90%	0.304	0.152	0.0912

A cross plot of the local standard deviation and the local mean could be used to assess the confidence in the estimates, see below. Points with standard deviations below the line would have probability values in excess of the specified P value. The following plot shows the lines for P=90% and the three X values in the table. The range of the estimates is arbitrary.



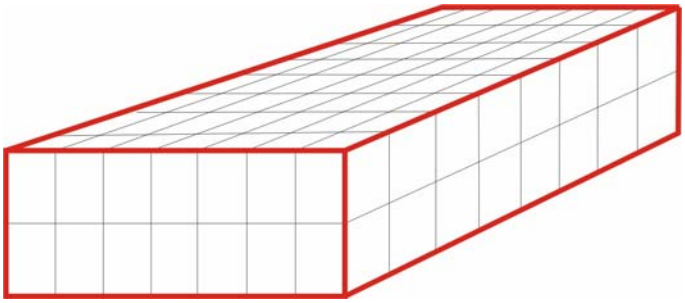
The scale (monthly or otherwise) does not have to be specified; we assume that the estimate and standard deviation are calculated at the right scale. Different X values could be used for measured, indicated, and inferred. The following workflow could be applied to use this methodology/plot for classification:

1. Simulate realizations of the grade at a reasonably small scale.
2. Block average the realizations to nominal monthly production volumes.
3. Calculate the local mean (E-type) and the local standard deviation.
4. Check the standard deviation relative to the mean using the relations given above and assign a resource/reserve classification.

In many cases, it is unreasonable to average the simulated realizations to the large scale considered relevant for classification (monthly). It is desirable to classify based on SMUs or the original scale of simulation. The following section addresses this possibility.

Block Scale Classification

In general, hundreds of SMU blocks contribute to a volume considered relevant for classification (a monthly scale). We would prefer to classify each SMU rather than classify arbitrarily large volumes. Any particular monthly production volume will be irregular in shape and we would prefer boundaries between the different categories that have more local precision than large nominal volumes.



Assume the grades average arithmetically (with equal weight in this example):

$$Z_{monthlyvolume} = \frac{1}{N}\sum_{i=1}^{N} Z_i$$

Considering the small scale very small compared with the monthly scale, the relationship between the variances of small and large volumes can be approximated by:

$$\sigma^2_{monthlyvolume} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} Cov\{Z_i, Z_j\}$$

$$= \frac{\sigma^2}{N} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} Cov\{Z_i, Z_j\}$$

This expression can be calculated using the programs gammabar or gammabar-irr, which gives the average variogram all over the monthly production volume of regular or irregular shape, respectively. Thus, if a standardized variogram model is used, the standardized average covariance can be obtained by the expression:

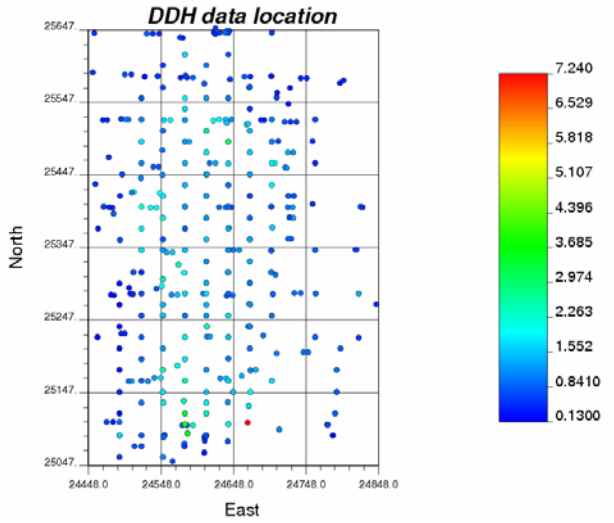
$$\bar{C}(V,V) = 1 - \bar{\gamma}(V,V)$$

And the relation becomes:

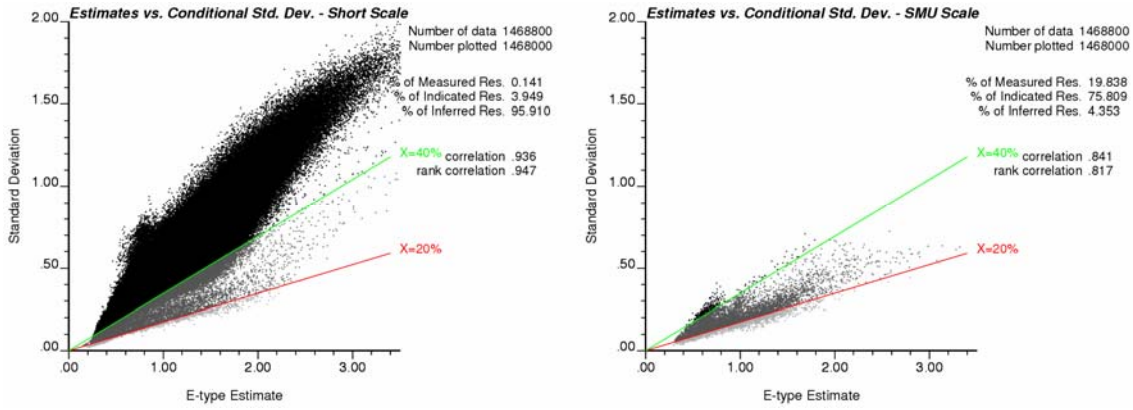
$$\sigma^2_{monthlyvolume} = \sigma^2_{smallscale}\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} Cov\{Z_i, Z_j\} = \sigma^2_{smallscale}\bar{C}(V,V)$$

Example

With the objective to illustrate the resources classification methodology explained above, a real data set was selected. This is a subset of 1081 drillhole samples from an exploration campaign of a Chilean copper deposit. The samples are located in a volume of 200m x 304m x 24m, which was discretized in a 2m x 2m x 2m short scale block model for sequential Gaussian simulation. 100 hundred realizations were generated and averaged in a 12m x 12m x 12m SMU scale. Both, the short scale model and the SMU scale model were post processed to obtain the mean and variance of the simulated values for each block.

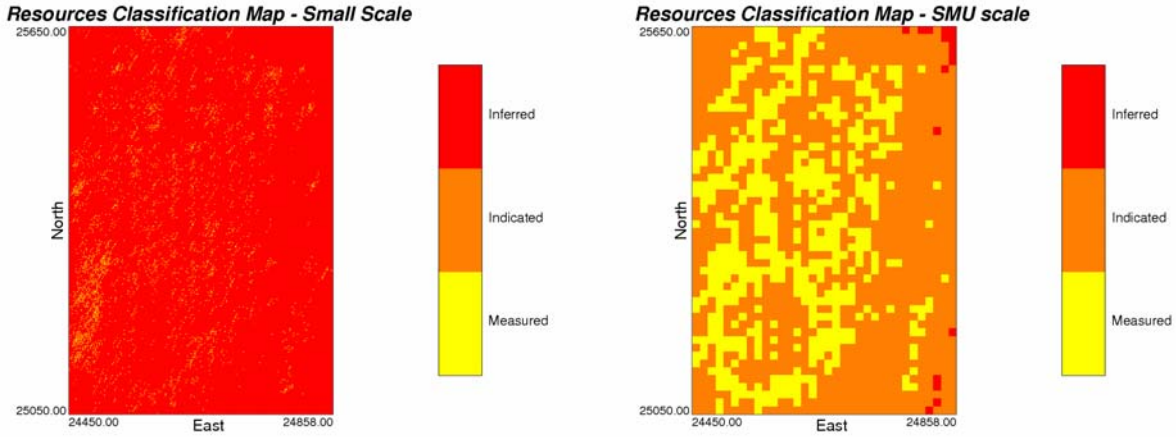


The slope of the limit between measured and indicated resources was calculated using a Confidence Value P = 75% and a Precision value X = 20%. For the limit between Indicated and Inferred resources the same Confidence Value was used, but the Precision Value was chosen as 40%. These limits were applied to post processed results of both the small scale block model and the SMU size averages. With the aim to illustrate the impact of scale in the probabilistic criteria of classification the scatterplots of E-type estimates vs. standard deviations for both models is plotted next.



In the plots above it can be observed that the variability in the short scale model is much bigger than in the SMU size model. While in the short scale model only a 0.14% can be classified as

measured resources and a 3.95% as indicated, in the SMU scale model the measured resources increases to 19.84%, the indicated to 75.81%, and the inferred resources drop from 95.91% to 4.35%.



The maps above also show the impact of scale in the probabilistic resources classification, while in the small scale map the resources classified as inferred are dominant, they are not in the SMU scale map. Resources classified as measured and indicated prevails in the SMU scale classification map.

Limitations

There is a strong assumption that the monthly (or the large scale) grades follow a Gaussian distribution. There is also an assumption that the standard deviations are calculated from simulated realizations. The kriging variance (simple or ordinary kriging) will ***not*** work because it is independent of the data values.

Conclusions

There are many schemes for classification. This short note does not help matters. Yet another basis for probabilistic classification is presented. The main contribution of this short note is for people to gain a better understanding of scale-dependent uncertainty and commonly proposed measures of relative uncertainty.