A Short Note on the Comparison of Techniques for Recoverable Reserves Estimation

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Estimated recoverable reserves of mineral/resource is critical for making decisions and mine planning. These estimated reserves are sensitive to the method and parameters selected for estimation. In this paper we are going to discuss about the comparison of different estimation techniques - ordinary kriging, indicator kriging and simulation. The study is done with two sets of field data of the same area (exploration data and blast hole data). This study can be of use for implementation aspects and decision making, while estimating reserves.

Introduction

A reserve estimate is based on prediction of the physical characteristics of a mineral deposit through collection of data, analysis of data, and modeling the size, shape and grade of the deposit. These physical characteristics of the mineral deposit are inferred from sample data information. Reserve estimation requires analysis and synthesis of the sample data information to develop a model for the reserve. Selection of reserve estimation technique is critical and subjective to the available information, robustness of the chosen method. At the same time, CPU time for estimation and accessibility of algorithm/method of interest is also important factor.

There are several geostatistical (interpolation, simulations) techniques available for estimation of reserves. All these techniques have their applicability, advantages and limitations. The well established kriging estimator is a linear combination of known surrounding sample data:

$$z^*(u) = \sum_{i=1}^{n} \lambda_i . z(u_i) + \left[1 - \sum_{i=1}^{n} \lambda_i \right] . m$$

where, $z^*(u)$ is the estimate at location u, $z(u_i)$ is sample data at location u_i, n is the number of sample data, m is the global mean and λ_i is the weight assigned to i^{th} sample data. The kriging error variance is:

$$\sigma_E^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j C_{ij} - 2. \sum_{i=1}^{n} \lambda_i C_{i0} + \sigma^2$$

where, 0 refers to the unsampled location, C_{ij} is the covariance between data at i and j, C_{i0} is the covarianve between the data at i and the location to be estimated, n is the number of sample data and σ^2 is the variance of data. The weights are calculated by minimizing the kriging error variance.

The idea of ordinary kriging is to minimize the kriging error variance under the criteria of weights sum up to 1. The approach is to make lagrange formalism, take partial derivative and set it to zero. It gives the ordinary kriging system as follows:

$$\begin{cases} \sum_{j=1}^{n} \lambda_j C_{ij} + \mu = C_{i0} \\ \sum_{j=1}^{n} \lambda_j = 1 \end{cases} \quad , \quad i = 1, \ldots\ldots, n$$

The idea of indicator kriging for continuous variables is to estimate the distribution of uncertainty $F_z(u)$ at unsampled location u. The cumulative distribution function is estimated at a series of threshold values: $z_k, k = 1, \ldots\ldots, K$. The indicator formalism of the values can be done as follows:

$$i(u_i; z_k) = \Pr ob\{Z(u_i) \le z_k\}$$

$$= \begin{cases} 1, \text{if } z(u_i) \le z_k \\ 0, \text{otherwise} \end{cases}$$

The indicator kriging derived cumulative distribution function at an unsampled location at threshold z_k is calculated as:

$$F_{IK}(u; z_k) = \sum_{i=1}^{n} \lambda_i(z_k)[i(u_i; z_k) - F(z_k)] + F(z_k)$$

This indicator kriging procedure requires a variogram measure corresponding to each threshold $z_k, k = 1, \ldots\ldots, K$ so that the weights $\lambda_i(z_k), i = 1, \ldots\ldots, n; k = 1, \ldots\ldots., K$ can be determined. The thresholds are often chosen to be equally spaced quantiles, for example the nine deciles are often chosen.

Conditional simulation removes the smoothing effect generated by kriging. The smoothing effect of kriging makes the variance of kriged estimates too small. The variance of kriged estimate is:

$$Var\{y^*(u)\} = \sigma^2 - \sigma_E^2$$

where, $y^*(u)$ is the kriged estimate at location u, σ^2 is the variance of data and σ_E^2 is the kriging variance. In simulation the variance of the estimates is corrected by adding a random component in the simulated value, which removes the effect of missing variance.

$$y_S(u) = y^*(u) + R(u)$$

where, $y_S(u)$ is the simulated value at location u, and $R(u)$ is a random component with a mean of 0.0 and a variance of σ_E^2.

Comparative Case Study of Reserve Estimation Techniques

The idea here is to set a reference result with the blasthole data and compare different methods of estimation with the reference result using the exploration data. We do global comparison and panel wise comparison with in the area. Here we consider a panel size of 60m X 60m X 20m for panel comparison.

For setting a reference result we do ordinary kriging of blast hole data with a short radius of 15m and number of data used are 2 to get reproduction of the original data statistics. The statistics comparison of original blast hole data, declustered blast hole data and kriged blast hole data is shown in Table 3.

For comparison purpose, we take those panels from the reference model, which are more than 80% populated, where the SMU size for estimation is 5m × 5m × 10m. The area consist of 16650 number of panels, out of which 832 panel are more than 80% populated, which we select for comparison of different estimation methods (Figure 1) .

The data for the case study are of two types. One is exploration data, which has 943 numbers of drill holes sample information. Second is blast holes sample information which is exhaustive in nature. All the sample data are defined by easting, northing and elevation for their spatial location. The exploratory and blast hole data, both consist of gold quantity in the samples.

Considering spatial distribution of both exploratory and blast hole data, the domain of study defined is of 2220 m × 1080m × 500m size (19450E-20530E, 19500N- 21720N, -100 Elevation to 400 Elevation). We consider only those blast hole data, which are within the defined boundary.

The spatial distribution of sample locations of exploratory data is shown in Figure 2. The data show highly clustered behavior (between19880E, 19990N, 280 Elevation and 20200E, 21400N, -150 Elevation) in a big area and some area is sparsely sampled. The data shows a highly skewed distribution of gold grades (Figure 3) in the domain, which tends to lognormal distribution. The gold data distribution has a mean of 0.4456 and variance of 1.6656.

The declustered data (Figure 4) of gold grade distribution has a mean of 0.3033, variance of 0.9823 shows reduction in both mean and variance of data distribution as a result of cell declustering. The spatial distribution of blast hole data is shown in Figure 2. The data shows highly skewed distribution of gold Figure 3. The gold data distribution has a mean of 0.7247, variance of 2.1981. The majority of data lie between range of 0 and 5.

The correlation between gold and exploration data for different search radius to select pairs are shown in Table 1 and Table 2, where in the Table 1, we select the pair of exploration data from blast hole data which is the closest with in the search radius, similarly Table 2 shows correlation taking all pairs, within the given search radius. These table shows that at very short radius of 1m data shows some reasonable correlation, and as we increase the search radius, the correlation goes down.

The variogram measure for the exploration data in real space and normal space are defined in the major direction of -10° azimuth, minor direction of 80° azimuth and 90° dip, in vertical direction for both blast hole and exploratory data. The fitted variogram models for real space gold exploration data:

$$\gamma(h) = 0.25 + 1.12\exp_{\substack{h\max=85 \\ h\min=60 \\ hvert=17}}(h) + 0.30sph_{\substack{h\max=440 \\ h\min=120 \\ hvert=250}}(h)$$

The fitted variogram models for normal score gold exploratory data:

$$\gamma(h) = 0.15 + 0.52\exp_{\substack{h\max=60\\ h\min=45\\ hvert=80}}(h) + 0.33sph_{\substack{h\max=1400\\ h\min=320\\ hvert=150}}(h)$$

For indicator kriging of exploratory data we define nine thresholds at each decile for gold data. The thresholds for gold data → 0.025, 0.035, 0.05, 0.07, 0.105, 0.16, 0.255, 0.46, 1.015. The variograms fitted for each threshold are as follows:

Threshold 1 $\quad \gamma(h) = 0.19 + 0.43sph_{\substack{h\max=48\\ h\min=48\\ hvert=8}}(h) + 0.38sph_{\substack{h\max=1050\\ h\min=350\\ hvert=80}}(h)$

Threshold 2 $\quad \gamma(h) = 0.20 + 0.40sph_{\substack{h\max=48\\ h\min=48\\ hvert=8}}(h) + 0.40sph_{\substack{h\max=1050\\ h\min=350\\ hvert=150}}(h)$

Threshold 3 $\quad \gamma(h) = 0.45 + 0.20sph_{\substack{h\max=110\\ h\min=150\\ hvert=35}}(h) + 0.35sph_{\substack{h\max=1050\\ h\min=300\\ hvert=200}}(h)$

Threshold 4 $\quad \gamma(h) = 0.20 + 0.38sph_{\substack{h\max=25\\ h\min=15\\ hvert=15}}(h) + 0.42sph_{\substack{h\max=900\\ h\min=270\\ hvert=180}}(h)$

Threshold 5 $\quad \gamma(h) = 0.20 + 0.40sph_{\substack{h\max=25\\ h\min=15\\ hvert=15}}(h) + 0.40sph_{\substack{h\max=650\\ h\min=245\\ hvert=180}}(h)$

Threshold 6 $\quad \gamma(h) = 0.25 + 0.42sph_{\substack{h\max=45\\ h\min=20\\ hvert=25}}(h) + 0.33sph_{\substack{h\max=550\\ h\min=200\\ hvert=280}}(h)$

Threshold 7 $\quad \gamma(h) = 0.28 + 0.39sph_{\substack{h\max=20\\ h\min=25\\ hvert=28}}(h) + 0.33sph_{\substack{h\max=270\\ h\min=90\\ hvert=288}}(h)$

Threshold 8 $\quad \gamma(h) = 0.45 + 0.20sph_{\substack{h\max=5\\ h\min=20\\ hvert=45}}(h) + 0.35sph_{\substack{h\max=120\\ h\min=45\\ hvert=270}}(h)$

Threshold 9 $\quad \gamma(h) = 0.45 + 0.42sph_{\substack{h\max=25\\ h\min=20\\ hvert=150}}(h) + 0.13sph_{\substack{h\max=200\\ h\min=45\\ hvert=200}}(h)$

The fitted variogram models for real space gold blast hole data:

$$\gamma(h) = 0.01 + 0.53sph_{\substack{h\max=5\\ h\min=5\\ hvert=21}}(h) + 0.24sph_{\substack{h\max=27\\ h\min=12\\ hvert=22}}(h) + 0.22sph_{\substack{h\max=81\\ h\min=28\\ hvert=22}}(h)$$

Results and Comparison

The global comparison of ordinary kriging, indicator kriging and simulations are shown in Figure 5. Figure 5(a) shows restrictive kriging with less number of data (minimum 2 to maximum 4). The restrictive ordinary kriging grade-tonnage curve shows closeness to the reference grade-tonnage kriging. Indicator kriging shows a deviation from the reference grade-tonnage curve at higher cutoff. The simulation results are very close to the reference results, where the grade-tonnage curve of simulations is the average of 10 grade-tonnage curve of their 10 corresponding realizations.

Figure 5(b) shows a smooth ordinary kriging, and smooth indicator kriging results, where kriging is done using more number of data (minimum 2 to maximum 32). Both ordinary kriging and indicator kriging shows lower average grade and more tonnage fraction as compare to reference results.

For panel comparison, we compare ordinary kriging, indicator kriging, and simulation with the reference results. We compare both average grade and proportion of ore at cutoff 0.5, cutoff 1 and cutoff 1.5. The results are shown in Figure 6, Figure 7, Figure 8, and Figure 9. If we look at the trend line of individual method, simulations are showing some biased results. It might happen due to choosing restrictive parameters for getting the global result nearer to the reference results at the same time here the simulation results are outcome of only 10 realizations.

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Table 1: Scatter plot statistics for blast hole and exploration data, where for each exploration data, the closest pair of blast hole data (with different search radius) has been taken.

Pair Radius (m)	Number of data	Correlation	Rank correlation	Mean (exploration)	Mean (blast hole)	Std. dev. (exploration)	Std. dev. (blast hole)
1	74	0.597	0.622	1.102	0.869	3.014	1.435
2	614	0.265	0.434	0.928	0.978	1.907	1.817
3	1875	0.197	0.437	0.872	0.933	1.719	1.927
5	4759	0.188	0.418	0.844	0.875	1.708	1.723
7	6105	0.197	0.426	0.831	0.857	1.718	1.680
10	7152	0.193	0.434	0.8	0.832	1.703	1.616
15	8654	0.194	0.443	0.751	0.810	1.610	1.562
20	10007	0.187	0.443	0.725	0.797	1.583	1.516

Table 2: Scatter plot statistics for blast hole and exploration data, where for each exploration data, all the blast hole data as pair (with different search radius) has been taken.

Pair Radius (m)	Number of data	Correlation	Rank correlation	Mean (exploration)	Mean (blast hole)	Std. dev. (exploration)	Std. dev. (blast hole)
1	75	0.590	0.612	1.131	0.861	3.004	1.427
2	624	0.264	0.430	0.935	0.982	1.9	1.823
3	2067	0.208	0.434	0.890	0.948	1.711	1.912
5	9595	0.170	0.418	0.873	0.898	1.735	1.732
7	26355	0.183	0.418	0.88	0.89	1.735	1.645
10	76539	0.189	0.418	0.885	0.882	1.783	1.592
15	255602	0.197	0.407	0.884	0.881	1.79	1.544
20	598638	0.164	0.385	0.888	0.882	1.815	1.693

Table 3: Blast hole and reference data (kriged data) statistics

Statistics	Original blast hole data	Declus 10mX10mX10m of Size	Kriged data (15m search radius) 2 nos of data
Number of Data	165867	165867	338237
mean	0.7247	0.6960	0.6469
std. dev.	1.4826	1.4727	1.0602
coef. of var	2.0457	2.1161	1.6389
Maximum	227.8	227.8	115.7721
Upper quartile	0.8000	0.76	0.7538
Median	0.3400	0.32	0.3439
lower quartile	0.1400	0.14	0.1531
Minimum	0.0000	0.0000	0.0000

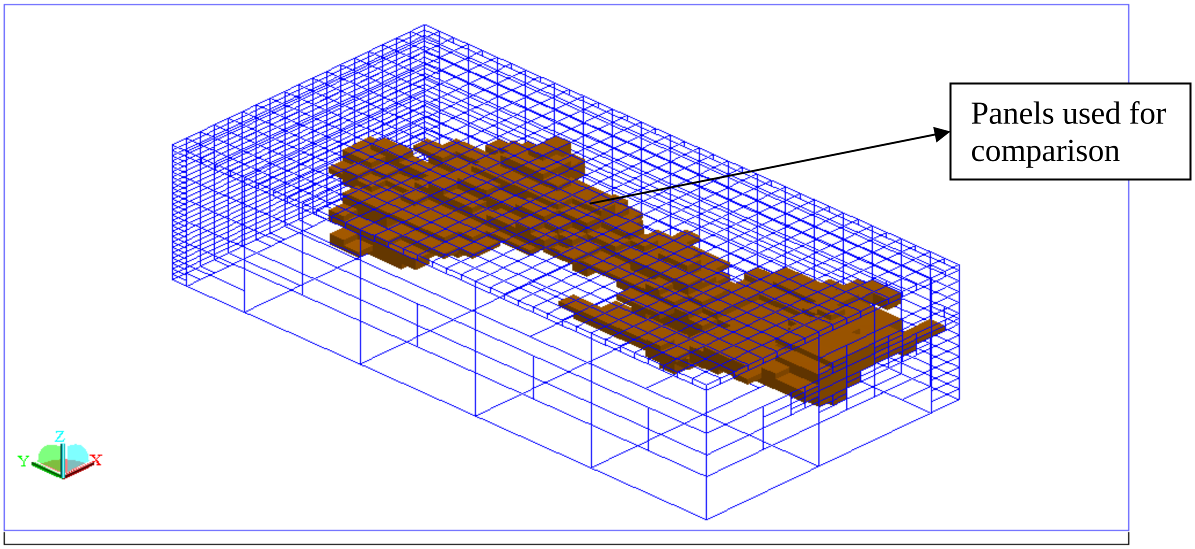
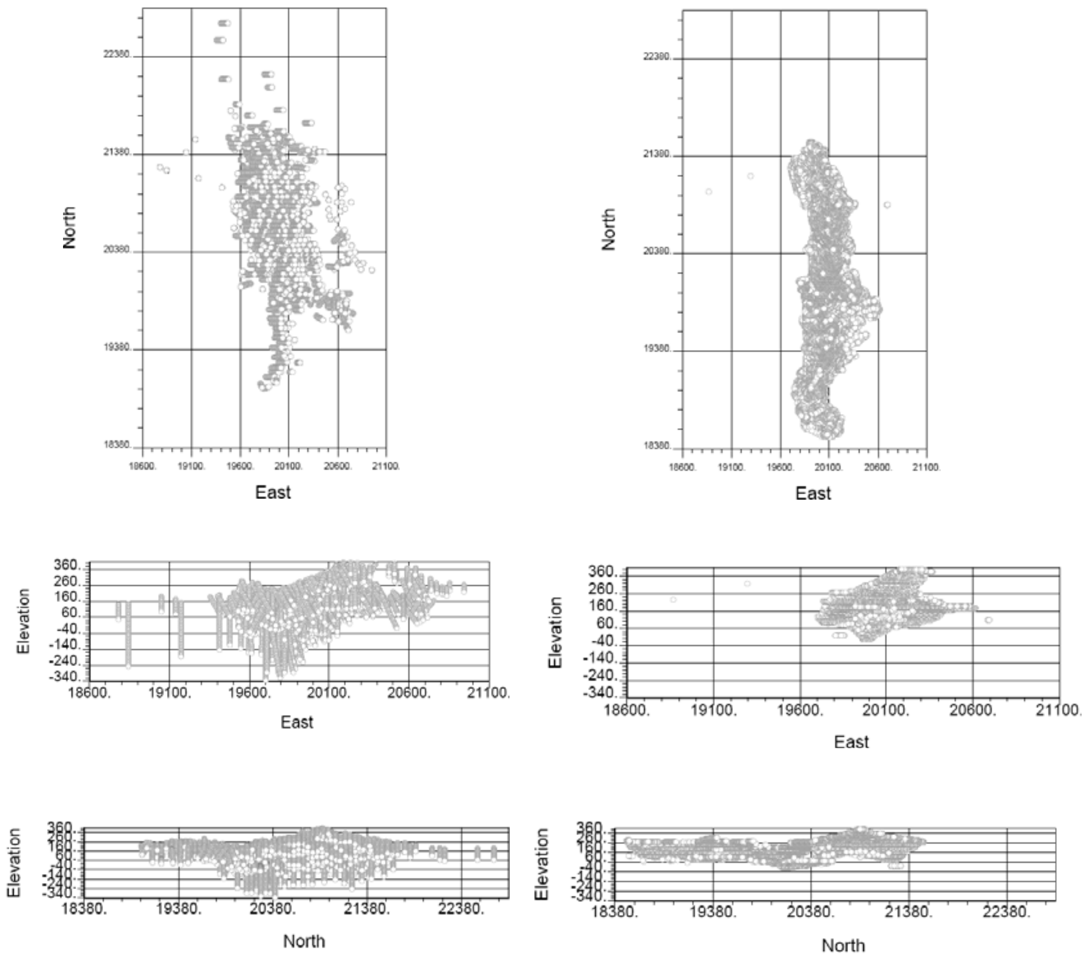
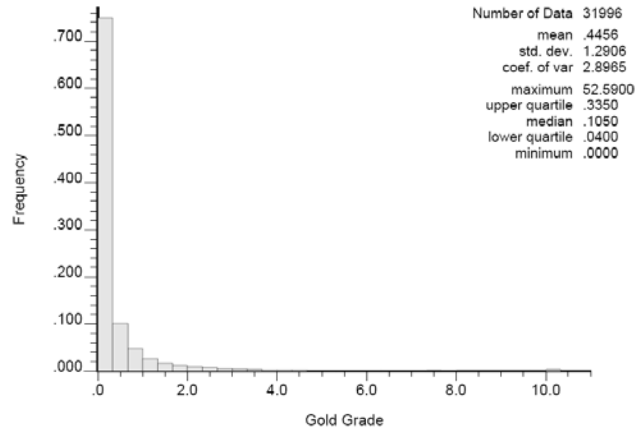


Figure 1: Panels used for comparison purpose, in the domain



(a) **(b)**

Figure 2: Location map (a) exploratory data (b) blast hole data



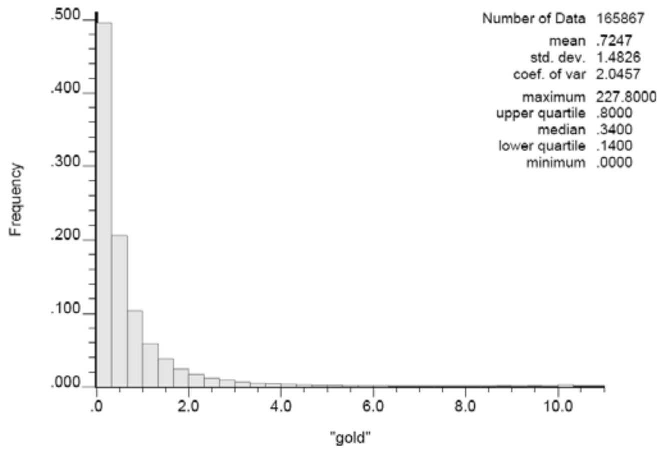


Figure 3: Histogram plot of (a) exploration data (b) blast hole data

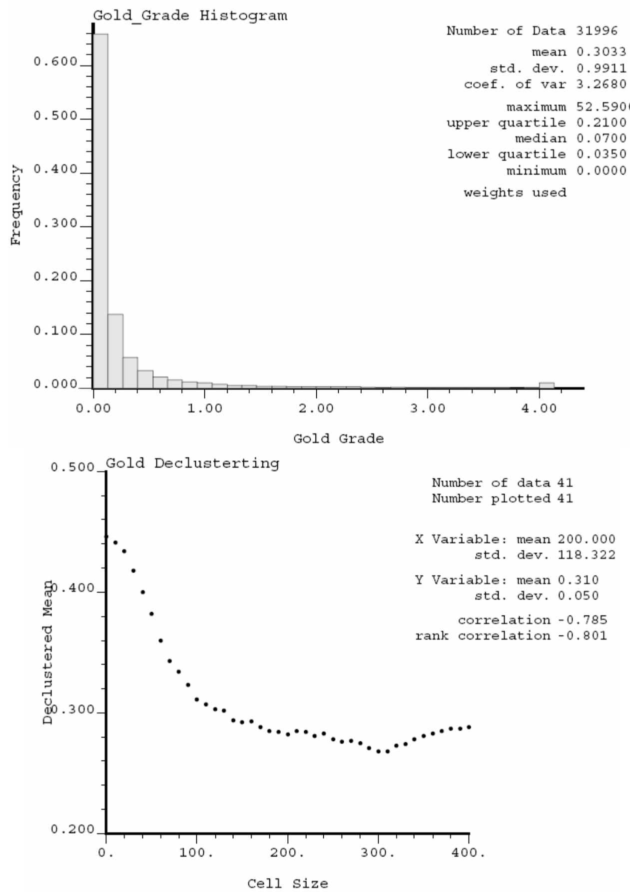


Figure 4: Cell declustering of exploration data

(a)

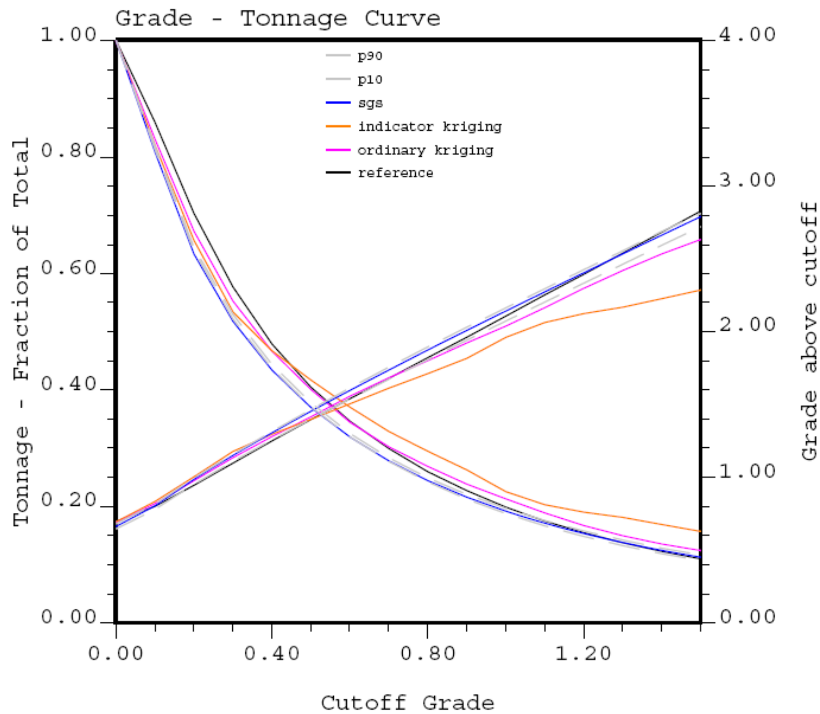


Figure 5: Cell declustering of exploration data

(b)

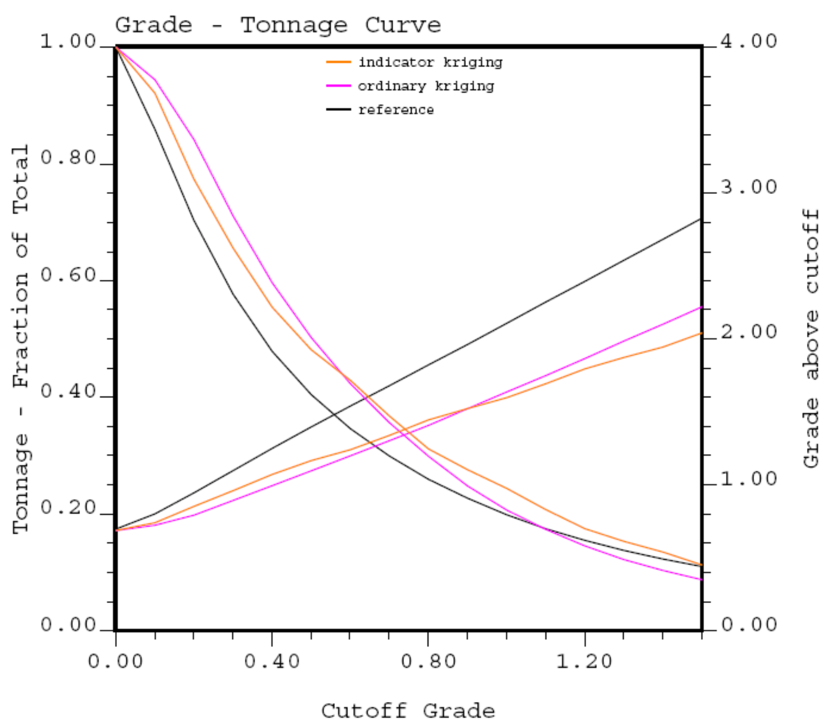
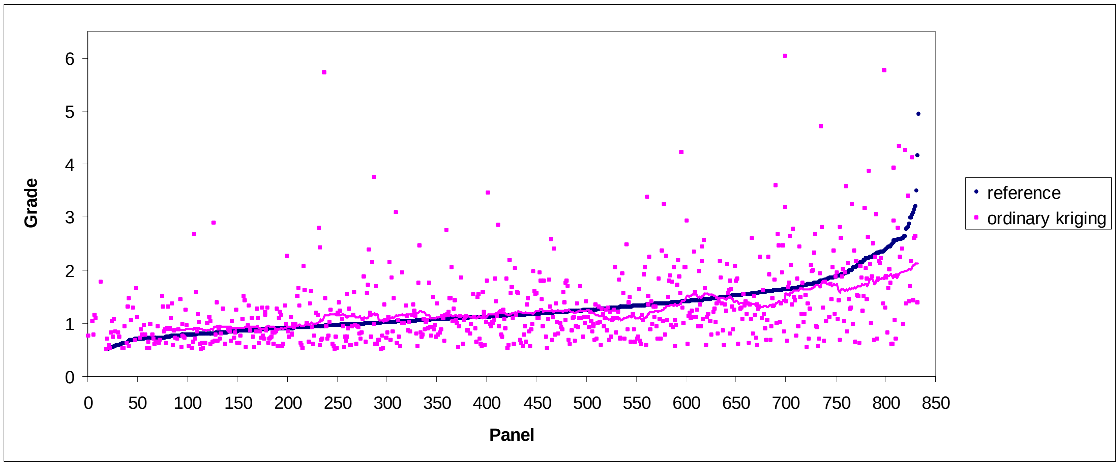
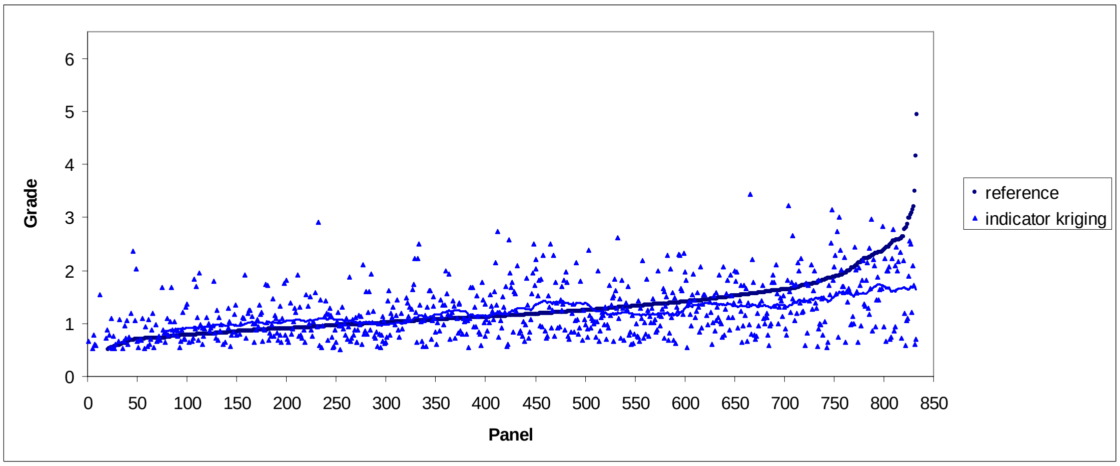


Figure 6: Grade-Tonnage curve (a) restrictive kriging (b) smooth kriging





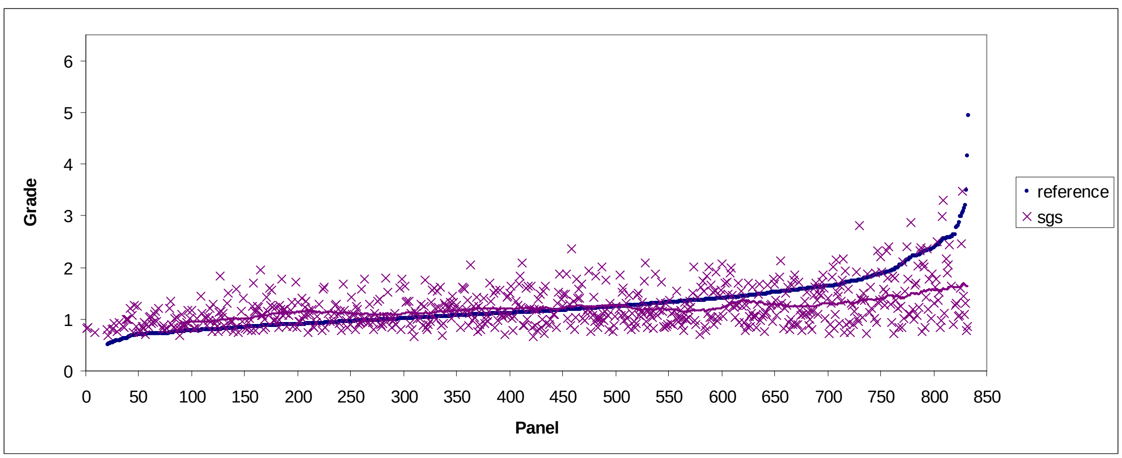


Figure 7: Panel grade comparison at cutoff 0.5.

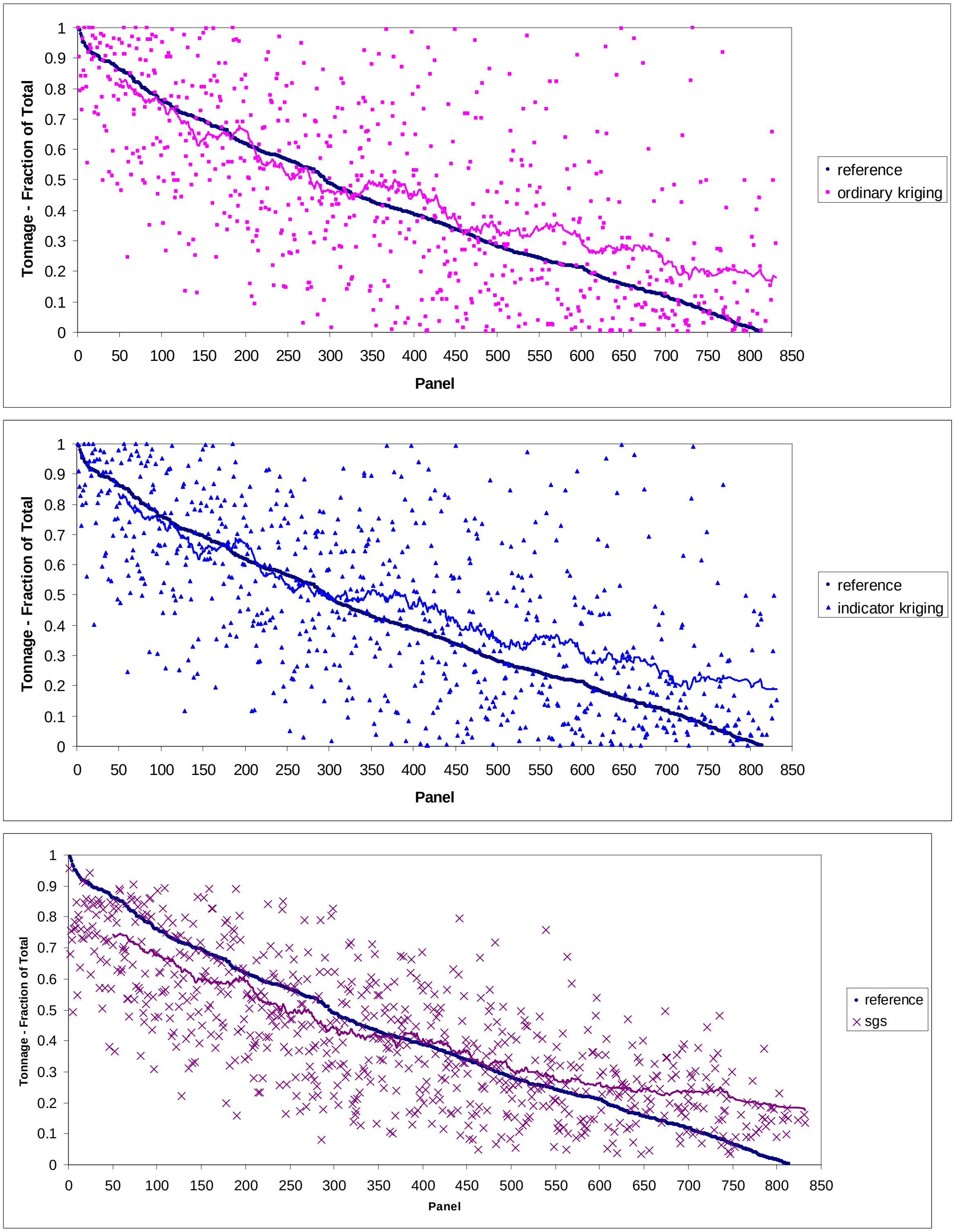


Figure 8: Panel tonnage proportion comparison at cutoff 0.5.

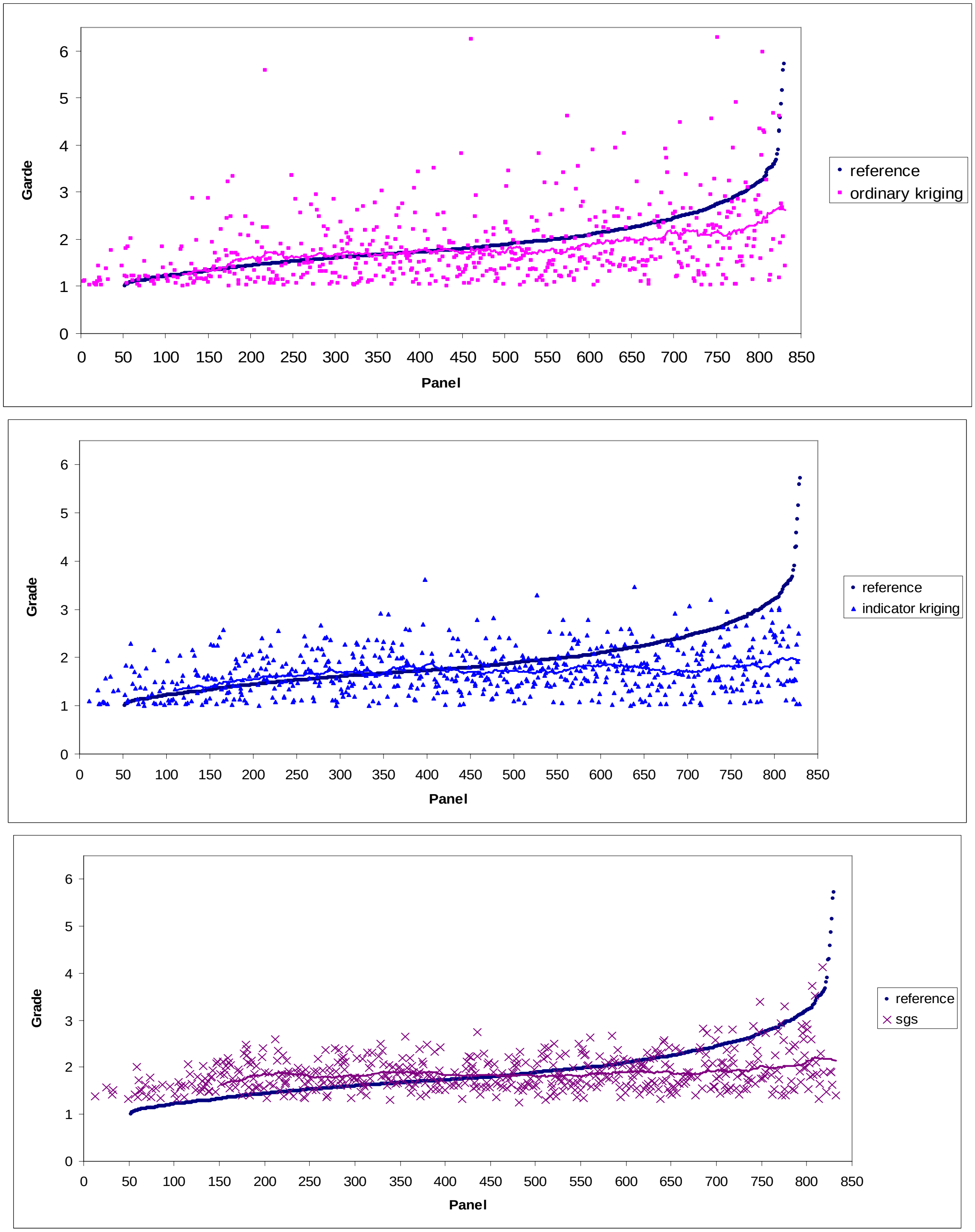


Figure 9: Panel grade comparison at cutoff 1

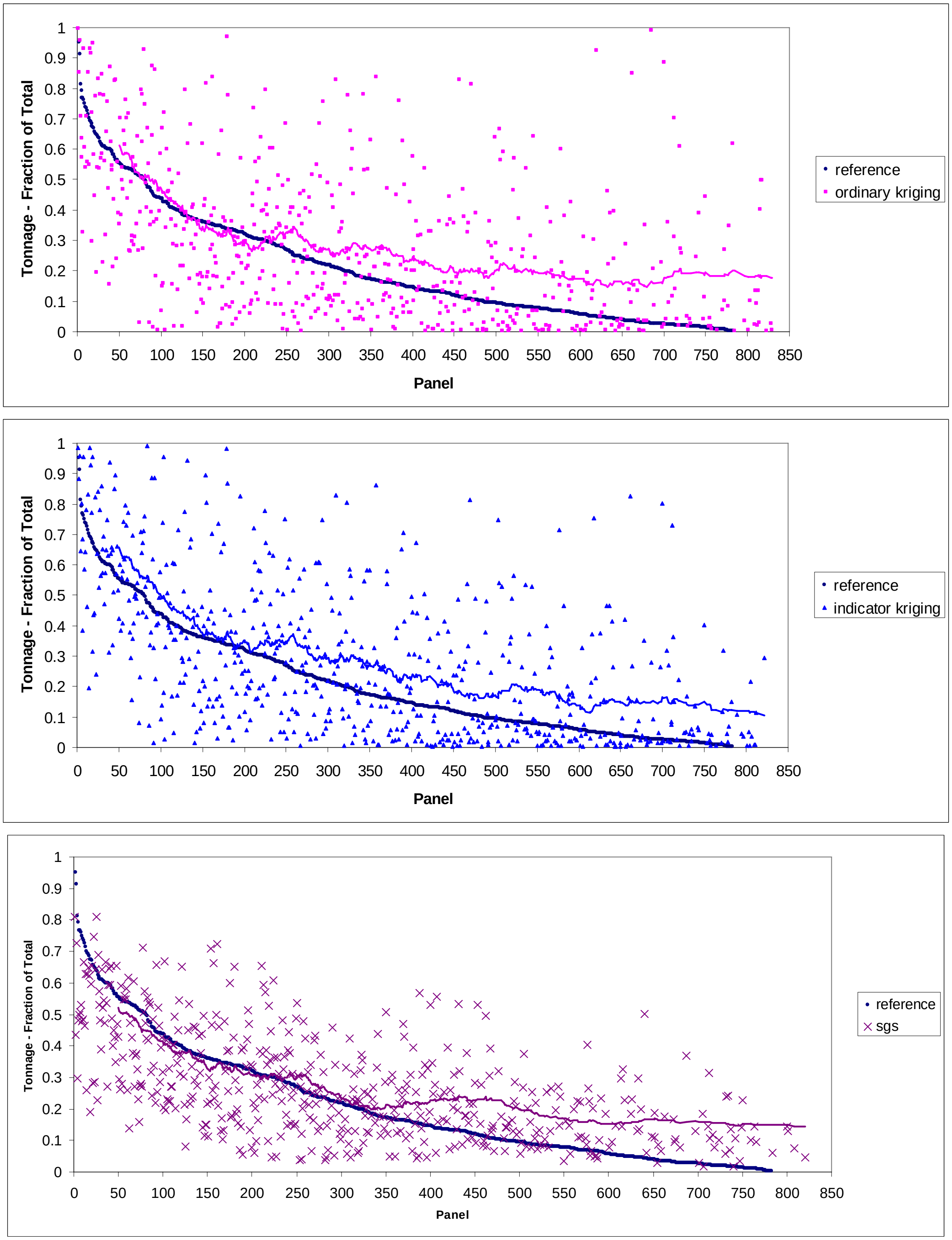


Figure 10: Panel tonnage proportion comparison at cutoff 1.