Summary Statistics for Geologic Continuity, Grid Geometry and Permeability Tensor Characterization

R.M.Hassanpour and C.V.Deutsch

Centre for Computational Geostatistics
Department of Civil and Environmental Engineering
University of Alberta

This short note summarizes the results of permeability tensor characterization and geologic and geometric parameters. The methodology for calculation and characterizations are presented in separate papers in this report. An important direction of research is the study of the multivariate distribution of geologic variables summarizing depositional setting and architecture, geometric variables summarizing the grid system used to discretize the domain of interest, and effective flow properties such as permeability tensors. This multivariate distribution is complex and relevant to performance forecasting with uncertainty. A number of papers in this report (201, 202, 203 and 204) by John Manchuk and the authors of this note are focused on this problem.

Permeability Tensor

Geological environments are always anisotropic. Some geological variables are scalars at a small scale, but turn to tensors at larger scale. This is particularly true of rate constants such as permeability. Tensors are commonly used to show the value of an anisotropic property with respect to all coordinate axes. Tensors are widely used in mathematics and physics. In Mechanics, tensors are used to describe the strain and stresses and moment of inertia. In petroleum engineering, the directional rock permeability is characterized as a tensor. The permeability tensor is required to solve the flow equation more realistically.

$$\bar{\bar{K}} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

The second order permeability tensor has four components in 2D and nine components in 3D. Diagonal terms in permeability tensor represent the permeability in x, y and z direction when the pressure gradients are in the same direction. The off-diagonal terms, for example k_{xy}, represents the component of permeability in *x* direction when the pressure difference is applied in *y* direction.

Shape of permeability tensor is still under discussion of researcher. Some researcher believes that the permeability tensor is symmetric (Bear 1972, Gelhar and Axness 1983), other believes that it is not necessarily symmetric and can be asymmetric (Ababou 1988; King 1993). There is evidence that the tensor could be asymmetric with non-linear boundary conditions.

Methods of full and symmetric permeability tensor calculation are presented in paper 201 in this report. In the symmetric cases the permeability tensor can be characterized by six values (three diagonal and three off-diagonal) in 3D and three values in 2D (two diagonal and one off-diagonal).

Geological Continuity Direction

Geological interpretation reveals the geological continuity in subsurface lithofacies. This continuity is direction dependent and depends on the direction of deposition and subsequent diagenetic alteration (Deutsch 2002). The Geometric anisotropy can be detected in such depositional cases. In case of geometric anisotropy the range of variogram changes with direction. Figure 1 shows three variograms with different ranges calculated at different directions.

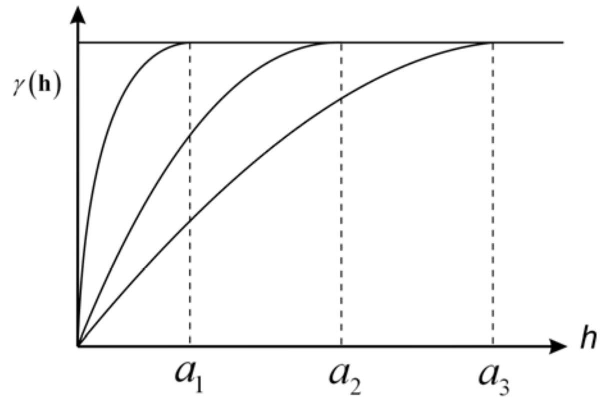

Figure 1: Three variograms calculated ate different directions show the geometric anisotropy in depositional setting.

Generally, directions of continuity in variables are determined prior to geostatistical modeling. If the variograms are calculated for large number of directions and distances, then for each direction, we can find the distance at which the variogram reaches its sill. If the distances are plotted on a rose diagram, the shape would be an ellipsoid in 3D or an ellipse in 2D (Isaaks and Srivastava 1989). This elliptical shape can be fully characterized by two orthogonal vectors in 2D and three orthogonal vectors in 3D. Method of calculating the direction of continuity using the moment of inertia is presented in paper 112 in this report. In 2D cases, two azimuth angles of major and minor diameter of ellipse and anisotropy ratio of two diameters are determined.

Grid Block Orientation

The orientation of irregular grid block is important for calculation of permeability tensor (See paper 201 in this report). This orientation can be extracted from the moment of inertia tensor (paper 112). Two angles for 2D grids and three unit vectors for 3D grids are determined. These directions show the directions of major and minor diameters of an ellipsoid that surrounds the grid blocks.

An Interesting Research Avenue

The multivariate space of the geological continuity, grid geometry, and permeability characteristics is high dimensional. For example, there may be up to 20 variables describing geological parameters (facies, porosity, directions and magnitude of continuity), up to 20 variables describing each grid element (size and orientation), up to nine variables describing the permeability tensor. There are other variables including boundary conditions, the spatial distribution of grid blocks etcetera. One could imagine a multivariate distribution with more than 100 dimensions. Data points to inform this multivariate distribution could be created by processing high resolution training images (form, for example, the CCG training image library) through flow simulation. The high dimensional distribution could be modeled with a variety of techniques. This will be the subject of future CCG research.

References

Ababou, R., 1988, Three-dimensional Flow in Random Porous Media, Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, USA.

Bear, J., 1972, Dynamics of Fluids in Porous Media, American Elsevier Pub. Co., New York, 764 pages.

Deutsch, C.V., 2002, Geostatistical Reservoir Modeling, New York, Oxford University Press, 376 pages.

Gelhar, L.W., and Axness, C.L., 1983, Three-dimensional Stochastic Analysis of Macrodispersion in Aquifers, Water resources Research, 19, 161-180.

Isaaks, E.H., and Srivastava, R.M., 1989, An Introduction to Applied Geostatistics, New York, Oxford University Press, 561 pages.

King, M.J., 1993, Application and Analysis of Tensor Permeability to Crossbedded Reservoirs, SPE paper 26118, Western Regional Meeting, Anchorage, Alaska, USA.