Flexible Change of Support Model Suitable for a Wide Range of Mineralization Styles

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A change of support model consists of a procedure to change a stationary histogram at a small data scale to represent a larger scale; typically a distribution of sample drillhole data is changed to represent a chosen selective mining unit (SMU) scale. Most grade variables average arithmetically. Thus, the mean stays the same for different scales and the variance changes according to well established theory using average variogram values. The longstanding challenge has been to predict how the shape of the histogram changes. The discrete Gaussian model is widely used because it appears reasonable and introduces few artifacts; there are no artificial minimum/maximum values and the target mean and variance are reproduced exactly. The resulting distribution shape, however, is strongly dependent on a multivariate Gaussian distribution. We generalize the approach by taking advantage of a property of the isofactorial model, which allows us to specify where the random function falls on the spectrum between a low-entropy mosaic model and a maximum-entropy diffusion model. A Measure of Dissemination (MD) is computed using a ratio of the madogram and the variogram. The change of support algorithm is developed that uses a Variable Measure of Dissemination (VMD) that is easily calculated and fit to data from the deposit. The resulting VMD change of support model is very flexible to represent different mineralization styles encountered in practice; global predictions of recoverable reserves are more accurate and decision-making is improved. A number of real examples are shown to demonstrate the approach.

Introduction

Predicting the grade and tonnage above a cutoff grade for practical mining volumes is an important problem. It is particularly critical early in resource evaluation when no actual mining has taken place. Practitioners rely on global change of support models, kriging with different search plans, and simulation to address this problem.

Simulating at point scale and then upscaling the realizations to the SMU's size provides a direct solution to the change of support problem, but it can be very demanding in computer resources and it is affected by the multivariate Gaussian methods adopted for simulation. If indicator based simulation is used, the block values are affected by an increased grade smoothing in the SMU scale due to the uncontrolled interclass transitions and randomness within classes that are characteristic of indicator simulation.

Global change of support models do not require a high amount of computer resources, are fairly simple to apply and can be very flexible in regard of the shape of distribution. A change of support model is used to estimate different block selectivity statistics for individual panels from the point-support samples. The principles that every change of support model must to fulfill are (Chilès and Delfiner, 1999):

- The marginal distribution of the block support grades $Z(v)$ depend on the complete spatial distribution of the point support grades $Z(x)$.
- The general approach is to establish a transformation from the distribution of $Z(x)$ to the less selective distribution of $Z(v)$.
- The distribution of block grades $Z(v)$ must have the same mean m as the distribution of the point support grades $Z(x)$. The variance distribution of block grades $Z(v)$ is the dispersion variance $D^2(v,V)$, which is defined by the well known volume-variance relation, which states:

$$D^2(v,V) = \bar{\gamma}(V,V) - \bar{\gamma}(v,v)$$

where $\overline{\gamma}(V,V)$ is the average variogram within the deposit, it approaches the sample scale variance as the deposit volume V increases in comparison to the variogram range; $\overline{\gamma}(v,v)$ is the average variogram within the block or SMU, if the samples volume is several orders of magnitude smaller than the block volume the average variogram within the block is practically equal to the samples dispersion variance:

$$D^2(\bullet,v)=\overline{\gamma}(v,v)$$

The most popular change of support models are affine, indirect log-normal and discrete Gaussian model. The affine change of support model (Journel and Huijbregts, 1978) is the simplest among all change of support models, it assumes no change in the shape of the data histogram, only reduction in the variance. It is known, however, that the distribution does change shape as the variable is averaged within larger blocks, thus affine correction is limited to minor changes in scale.

The indirect lognormal change of support model (Isaaks and Srivastava, 1989) assumes that the point and block distributions are lognormally distributed. The indirect log-normal correction is performed by maintaining the mean and reducing the variance (Chilès and Delfiner, 1999). The indirect lognormal change of support model is considered to be more realistic than affine, since it does not impose artificial maxima and minima and can assume arbitrary shape. However, again the assumption of the same shape for point and block distributions is very limiting, particularly when the point data histogram do not match satisfactorily the lognormal distribution. A methodology for transforming the point-scale lognormal distribution to a more symmetric block scale distribution has been proposed (Vargas-Guzmán, 2005).

The Discrete Gaussian model (Matheron, 1978) is a robust model for change of support. It does not make any assumptions on the point and block distributions. Discrete Gaussian correction is performed by transforming point data to the normal space and fitting the point distribution of the normal space by the Hermite polynomials. Subsequently, Hermite expansion of the block distribution is found in the normal space and the new distribution is back-transformed to the original units, as result, desired block distribution is found. One of the advantages of the discrete Gaussian model is that it allows the distributions of larger block volumes become more symmetric as the size of the block increases. Neither indirect lognormal or affine correction, obviously, allow such symmetrization.

The Hermitian model (the basis for the Discrete Gaussian change of support) is the most used model of the family of Isofactorial models. These models have proven flexible for fitting a wide range of univariate and bivariate distributions. Isofactorial models are based on the factorial decomposition of a transformed distribution into polynomials with an orthonomal base, such as the Hermite polynomials, Laguerre Polynomials, among several others.

An adequate transformation and the correspondent type of polynomial factors for a given grade distribution is chosen first. Then, the form of the covariance of these factors is defined according the type of spatial dispersion of the variable under study. Two models for extreme behaviour of spatial phenomena have been defined in Geostatistics (Matheron, 1989; Chilès and Delfiner 1999, Wackernagel 2003): the first is the diffusion isofactorial model, which corresponds to variables that exhibits a gradual spatial change, with high values separated of low values but with a continuous spatial transition between them (maximum entropy). Examples of diffusion model are the Sequential Gaussian Simulation realizations fulfilling the condition of multigaussianity (see Fig. 1). The second extreme model of spatial behaviour, the Mosaic model, corresponds to a high continuity (or even identity of values) within a region or partition followed by abrupt changes in the transition from one partition to another. Voronoi polygons have been frequently used as an example of Mosaic type spatial phenomena, as well as the "dead leafs model" (Fig. 2)

When sample support values of a Diffusion type phenomenon are upscaled to SMU size, the SMU scale distribution will become symmetric very quickly as the SMU size increases. Mosaic type variables will be more resistant to change of support, and its symmetrization will be clear only when the SMU size exceeds the size of the partitions.

Current implementations of the Discrete Gaussian Model consider by default that the variable used for change of support corresponds to the Diffusion Model. However, real data usually behaves as an intermediate model between the Mosaic and the Diffusion, and its behaviour can vary as the scale changes.

Intermediate models of change of support exist, such as the barycentric model (Chilès and Delfiner 1999; Wackernagel, 2003) and the Beta model (Hu, 1988), but only for the Beta model a methodology for its practical application has been proposed and implemented (Emery and Ortiz, 1984).

The main objective of the present work is to develop a new change of support model that is not based on the extreme of either the diffusive or the mosaic type model. After introduction of isofactorial models and the Discrete Gaussian Model procedure, an intermediate model between the Mosaic and the Diffusion, called Barycentric model is presented. Then, a Measure of Dissemination (MD) used to characterize the spatial dispersion is derived from the barycentric model as a function of variogram and madogram; and a representative value of this measure is calculated for a given SMU size. Afterwards, a variable Effective Measure of Dissemination (VEMD) change of support model is introduced and applied in small example and real data examples.

Isofactorial Models

Consider a pair of stationary random variables ($Y(\mathbf{x})$, $Y(\mathbf{x+h})$) representing the values of an attribute at locations **x** and **x+h**. This pair follows an isofactorial model if the bivariate distribution of the pair ($Y(\mathbf{x})$, $Y(\mathbf{x+h})$) can be factorized as the following linear combination (Matheron, 1984a; Chilès and Delfiner, 1999):

$$F(dy_x, dy_{x+h}) = F(dy_x)F(dy_{x+h})\sum_{k=0}^{\infty} T_k(h)\chi_k(y_x)\chi_k(y_{x+h}) \tag{1}$$

where $\chi_k, k \in N,$ is a set of orthonormal functions for the space $L^2(G)$, $\chi_k, k \in N,$ are referred to as the 'factors' of the isofactorial model; and the coefficients $T_k(h)$ are the covariances between the factors χ_k.

A variety of isofactorial models have been developed during past years. Known examples are bigaussian, bigamma, Hermitian, etc (Amstrong and Matheron, 1986 a, b; Chilès and Delfiner, 1999). A large number of these models can be grouped according to their applicability to two main types of patterns present in random functions - diffusion and mosaic patterns.

Diffusion type isofactorial models (Fig. 1) are characterized by almost continuous realizations and used for phenomena that exhibit a gradual spatial change, with almost continuous realizations. Characteristic feature of diffusion type models is disconnectedness of the extreme values: the occurrence of extreme-high or low values are purely random, so there is no clustering in space. In diffusion type models the k factor correlation functions are given by

$$T_k(h) = (\rho(h))^k. \tag{2}$$

A diffusion type phenomena is defined by the following variogram vs. madogram ratio,

$$\frac{\gamma_1(\mathrm{h})}{\sqrt{\gamma(\mathrm{h})}} = \frac{C_1}{\sqrt{C}} \qquad \forall \mathrm{h}\ , \tag{3}$$

where $\gamma_1(h)$ and $\gamma(h)$ are the theoretical madogram and variogram models with sills C_1 and C, respectively.

The Mosaic type models (Fig. 2) present partitions or cells within which the phenomenon stays constant while it shows sudden change from one cell to another. This corresponds to the idea of abrupt change when stepping from one compartment to another. Voronoi polygons, polyhedra based on a Poisson process, among others, are classical examples of mosaic models. In Mosaic type model the factor covariances are the same for al k in Eq. (1):

$$T_k(h) = \rho(h). \tag{4}$$

A mosaic type phenomena is defined by the following variogram vs. madogram ratio,

$$\frac{\gamma_1(\mathrm{h})}{\gamma(\mathrm{h})} = \frac{C_1}{C} \qquad \forall \mathrm{h}\,, \tag{5}$$

where, as before, $\gamma_1(h)$ and $\gamma(h)$ are the theoretical madogram and variogram models with sills C_1 and C, respectively. The variogram sill is the variance, while the madogram sill is equal to the *dispersion indicator*, whose analytical expression is given by:

$$S = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |z' - z| F(dz)F(dz') \tag{6}$$

Or, it can be calculated numerically as:

$$S = \frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} |z_i - z_j| \tag{7}$$

Isofactorial Change of Support: Discrete Gaussian Model

The discrete Gaussian model (Chilès and Delfiner, 1999) is the change of support model which relies on the following assumptions and hypotheses:

- $Z(x)$ is a known stationary random function at point support; $Z(v)$ is an unknown stationary random function at block support.
- The domain of interest is divided into k same size blocks v_k. Each datum is randomly and uniformly located within the block. If there are several samples within the block, their locations are independent from each other.
- $Z(x) = \phi(Y(x))$, where ϕ is the known anmorphosis function and $Y(x) \sim N(0,1)$. $Z(x) = \phi_v(Y_v)$, where ϕ_v is the unknown anmorphosis function and $Y_v \sim N(0,1)$.
- For any block v_k and point x, the pair $(Y(x), Y_v)$ is bigaussian, with the same correlation coefficient r.
- Cartier's Relation (Matheron, 1984b): If the location of x is randomly and uniformly located within the block v_k, the value of x is equal to the block value, that is,

$$E(Z(x) \mid Z_v) = Z_v\,. \tag{8}$$

The Discrete Gaussian model for the change of support involves the following 5 steps.

1. Original data distribution is transformed to the Gaussian using normal score transformation. Anamorphosis function $\phi = F^{-1} \circ G$ describes this transformation. That is,

$$z_i = \phi(y_i)\,, \tag{9}$$

   where z_i and y_i, $i = 1, \ldots, n$, are original and normal score transformed data. Can check anamorphosis by plotting $\phi(y_i)$ versus original data distribution.

2. A variogram model for the original data is fitted and the variance for the block support, $\bar{\gamma}(v,v)$ is calculated.
3. The distribution of the normal score transformed data is fitted with Hermite polynomials. For a stationary Gaussian random function $Y(\mathbf{x})$ the Hermite polynomials are orthonormal functions for

the space L²(G). That is, they have a mean of zero, a variance of one, and are uncorrelated. Hermite polynomials are a polynomial family defined by the Rodrigues' Formula:

$$H_p(y) = \frac{1}{\sqrt{p!} \cdot g(y)} \cdot \frac{d^p g(y)}{dy^p}. \tag{10}$$

The first two Hermite polynomials are

$$H_0(y) = 1 \text{ and } H_1(y) = -y. \tag{11}$$

For higher orders, the following recurrence formula is used

$$H_{p+1}(y) = -\frac{1}{\sqrt{p+1}} yH_p(y) - \sqrt{\frac{p}{p+1}} H_{p-1}(y). \tag{12}$$

The distribution of the normal score transformed data expanded using Hermite polynomials is given below

$$\phi(y) = \sum_{p=0}^{\infty} \phi_p H_p(y), \tag{13}$$

where $\phi_p = E(\phi(Y(x))H_p(Y(x)))$.

4. A new distribution for the block support is found in terms of Hermite polynomials.

$$\phi_v(Y_v) = E(\phi(Y(x)) \mid Y_v) = \sum_{p=0}^{\infty} \phi_p E(H_p(Y(x) \mid Y_v)). \tag{14}$$

Note that the first equality follows directly from Cartier's Relation. Since the pair $(Y_v, Y(x))$ is bivariate standard normal (pure diffusion model),

$$E(H_p(Y(x) \mid Y_v)) = r^p H_p(Y_v), \quad p \geq 0, \tag{15}$$

the distribution for the block support in terms of Hermite polynomials is given by

$$\phi_v(Y_v) = \sum_{p=0}^{\infty} \phi_p r^p H_p(Y_v)),. \tag{16}$$

where constant *r*, referred to as the change of support coefficient is found from

$$\sigma_v^2 = \sigma^2 - \bar{\gamma}(v,v) = \sum_{p=1}^{P} \phi_p^2 r^{2p}, \tag{17}$$

where σ^2 and σ_v^2 are the point and block variances, respectively. Due to uniqueness of the Hermite polynomial decomposition, the coefficients of the block anamorphosis are $\phi_p r^p$, $p \geq 0$.

5. The Hermite-expanded distribution for the block support is back transformed to the original units.

Barycentric Model

An intermediate isofactorial model between diffusion and mosaic models can be obtained as a linear combination of both models, that is, an intermediate isofactorial model

$$F(dy_x, dy_{x+h}) = F(dy_x)F(dy_{x+h})\sum_{k=0}^{\infty} T_k(h)\chi_k(y_x)\chi_k(y_{x+h}) \tag{18}$$

with:

$$T_k(h) = \beta(\rho(h))^k + (1-\beta)\rho(h), \tag{19}$$

where $\beta \in [0,1],$ is called *barycentric model* (Chilès and Delfiner, 1999) . The variogram vs. madogram relation which defines this model is:

$$\frac{\gamma_1(\mathrm{h})}{C_1} = \beta\sqrt{\frac{\gamma(\mathrm{h})}{C}} + (1-\beta)\ \frac{\gamma(\mathrm{h})}{C} \tag{20}$$

The parameter β of the barycentric model is referred as a measure of dissemination (MD) and is calculated from Eq. (20) to be

$$\beta(\mathrm{h}) = \frac{\frac{\gamma_1(\mathrm{h})}{C_1} - \frac{\gamma(\mathrm{h})}{C}}{\sqrt{\frac{\gamma(\mathrm{h})}{C}} - \frac{\gamma(\mathrm{h})}{C}} \tag{21}$$

The measure of dissemination, β, becomes 0 when the spatial distribution of grades correspond to a mosaic type model (Eq. (2)), and it is equal to one, when it correspond to a pure diffusion model (Eq. (4)).

The measure of dissemination (MD) is not necessarily constant and changes according the variogram direction and lag distance, h, in response to the corresponding changes in the variogram and madogram. An Effective Measure of Dissemination (EMD), or β_{eff} , gives an 'expected' value for parameter β and for a given SMU size V, and sample support volume v, can be calculated by the expression:

$$\beta_{eff} = \frac{1}{|V|^2}\iint_v \beta(v, v')dvdv' \tag{22}$$

A Variable Effective Measure of Dissemination (VEMD) change of support model

A Variable Effective Measure of Dissemination (VEMD) change of support model based on the above development of the effective MD can be proposed as follows:

- $Z(x)$ is a known stationary random function at point support; $Z(v)$ is an unknown stationary random function at block support.
- The domain of interest is divided into k same size blocks v_k. Each datum is randomly and uniformly located within the block. If there are several samples within the block, their locations are independent from each other.
- $Z(x) = \phi(Y(x)),$ where ϕ is the known anamorphosis function and $Y(x) \sim N(0,1).$ $Z(x) = \phi_v(Y_v),$ where ϕ_v is the unknown anamorphosis function and $Y_v \sim N(0,1).$
- The bivariate distribution of the pair (Y(**x**), Y(**x+h**)) is given by

  $$F(dy_x, dy_{x+h}) = F(dy_x)F(dy_{x+h})\sum_{k=0}^{\infty} T_k(h)\chi_k(y_x)\chi_k(y_{x+h})$$

  where $T_k(h) = \beta(h)\cdot r^k + (1-\beta(h))r$.

- Cartier's Relation: If the location of *x* is randomly and uniformly located within the block v_k, the value of x is equal to the block value, that is,

$$E(Z(x)\,|\,Z_v) = Z_v\,.$$

The Variable Effective Measure of Dissemination (VEMD) change of support model involves the following 6 steps.

1. The original data distribution is transformed to the Gaussian using normal score transformation. Anamorphosis function $\phi = F^{-1} \circ G$ describes this transformation.
2. The effective measure of dissemination, β_{eff}, is calculated from Eq.(23).
3. A variogram model for the original data is fitted and the variance for the block support, $\bar{\gamma}(v,v)$ is calculated.
4. The distribution of the normal score transformed data is fitted with Hermite polynomials.
5. A new distribution for the block support is found in terms of Hermite polynomials.

$$\phi_v(Y_v) = E(\phi(Y(x))\,|\,Y_v) = \sum_{p=0}^{\infty} \phi_p E(H_p(Y(x)\,|\,Y_v)). \tag{23}$$

where:

$$E(H_0(Y(x)\,|\,Y_v)) = 1,$$

$$E(H_p(Y(x)\,|\,Y_v)) = \frac{1}{|V|^2}\iint_V T_p(u,u')\,du\,du', \quad p > 0. \tag{24}$$

Because

$$T_p(h) = \beta(h)\cdot r^p + (1-\beta(h))r, \tag{25}$$

then,

$$\begin{aligned} E(H_p(Y(x)\,|\,Y_v)) &= \frac{1}{|V|^2}\iint_V T_p(v,v')\,dv\,dv' = \frac{1}{|V|^2}\iint_V [\beta(v,v')r^p + (1-\beta(v,v'))r]\,dv\,dv' \\ &= r^p \frac{1}{|V|^2}\iint_V \beta(v,v')\,dv\,dv' + r\frac{1}{|V|^2}\iint_V (1-\beta(v,v'))\,dv\,dv' \\ &= \beta_{eff} r^p + (1-\beta_{eff})r, \quad p > 0. \end{aligned}$$

And the distribution for the block support in terms of Hermite polynomials is given by

$$\phi_v(Y_v) = \phi_0 + \sum_{p=1}^{\infty} \phi_p [\beta_{eff} r^p + (1-\beta_{eff})r] H_p(Y_v)), \tag{26}$$

where constant *r*, referred to as the change of support coefficient is found from

$$\sigma_v^2 = \sigma^2 - \bar{\gamma}(v,v) = \sum_{p=1}^{P} \phi_p^2 [\beta_{eff} r^p + (1-\beta_{eff})r], \tag{27}$$

where σ^2 and σ_v^2 are the point and block variances, respectively and β_{eff} is the effective measure of dissemination.

6. Finally, the Hermite-expanded distribution for the block support is back transformed to the original units.

Software Implementation

Several existing programs of the extended GSLIB group were modified in order to implement the flexible change of support and the required previous steps. First, the `GAMV` program for calculating experimental variograms was updated for calculating the dispersion indicator, and thus, allowing the standardized semimadogram calculation. The parameter file for the updated program remains unchanged.

The second program modified was `GAMMABAR`, this new version allows to calculate the Beta effective for a given SMU volume and a user defined block discretization. The new parameter file for `GAMMABAR` is presented below. As it can be observed, the specification for the madogram model is required if the EMD has to be calculated.

```
                  Parameters for GAMMABAR
                  ***********************

START OF PARAMETERS:
6.0  6.0   6.0          -X,Y,Z size of block
 6     6     1            -X,Y,Z discretization
1                            - calculate measure of dissemination? 0-no, 1-yes
2   0.03                     - VARIOGRAM. nst, nugget effect
1 0.2    0.0     0.0   0.0   -  it,cc,ang1,ang2,ang3
      9.5  9.5  9.5          -  a_hmax, a_hmin, a_vert
1 0.77    0.0     0.0   0.0  -  it,cc,ang1,ang2,ang3
     88.0  88.0  88.0        -  a_hmax, a_hmin, a_vert
2   0.1                      - MADOGRAM. nst, nugget effect
1 0.39    0.0     0.0   0.0  -  it,cc,ang1,ang2,ang3
      9.2  9.2  9.2          -  a_hmax, a_hmin, a_vert
1 0.51    0.0     0.0   0.0  -  it,cc,ang1,ang2,ang3
     99.0  99.0  99.0        -  a_hmax, a_hmin, a_vert
```

The third program modified was `DGM`. The new version of this program allows calculating the change of support coefficient *r* and block distribution according to the effective measure of dissemination, β_{eff}. The new parameter file for `dgm_md` is given below.

```
                  Parameters for DGM
                  ******************

START OF PARAMETERS:
ns_nw_data.trn       -file with input transformation table
0                    - effective measure of dissemination (beta effective)
0.5                  -variance at block support (calculated with gammabar)
0.001                -acceptable error for the block variance calculated using Hermite polynomials
(e.g. e=0.0001)
100                  -number of Hermite polynomials to use (e.g. np=100)
dgm_point.out        -file for point scale output to check the anamorphosis
dgm.dbg              -file with Hermite Coefficients, fi(p)
dgm.out              -file for output with Z values at the block support
```

A Small Synthetic Example

Fig. 3 shows a distribution of 101 grade values. The mean and standard deviation of the simulated data are 1.25 and 1.96, respectively, and the coefficient of variation is 1.56. The distribution of the data is highly skewed. Assume that the target average variogram is 1.9 (approximately half the data variance). Let us now consider how the block distribution would change for these data when considering different levels of the effective measure of dissemination.

Fig. 4 shows the change in the shape of the block support distribution for $\beta_{eff} = 0, 0.25, 0.5, 0.75$ and 1. Note that value $\beta_{eff} = 1$ corresponds to conventional discrete Gaussian model for the change of support. For convenience, Fig. 4 also shows the original data distributions. When comparing graphs in Fig. 4, we note that as β_{eff} increases the shape of the block support distribution becomes more and more similar to the shape obtained using conventional discrete Gaussian model for the change of support. Moreover, with an increase in β_{eff}, we can also note that the number of small values in the block support distribution increases. The mosaic model ($\beta_{eff} = 0$) is characterized by the smallest number of low data values. The mosaic model is characterized by dramatic changes between large and small values. High and low grades quickly average with adjacent grades that are potentially very different.

Results for the block distribution obtained based on affine correction and the indirect lognormal correction are shown in Fig. 5. Note that the resulting affine block distribution is very similar to the one obtained by the pure mosaic model; while the indirect lognormal block distribution is very similar to the pure diffusive model or intermediate model with high EMD (β_{eff}).

Fig. 6 shows grade-tonnage curves obtained for the same levels of β_{eff} as considered above. The highest value for the grade above cut-off is obtained for the mosaic model (β_{eff} = 0), the lowest value for the grade above cut off is obtained for the diffusion model (β_{eff} = 1). In general, with an increase in the value of β_{eff}, we observe a decrease in the grade above cut-off.

Practical Applications

Now let us present three examples of the application of the newly developed flexible change of support model: two epithermal disseminated gold deposits, and a skarn zinc deposit. For all these examples the omnidirectional experimental variograms and madograms were generated and their models fitted, the measure of dissemination was then calculated from these models at several lag distances. In practice, a more detailed analysis could be undertaken by calculating the measure of dissemination from the directional variogram models in the major directions of continuity.

The data for first example come from the drilling campaign of an epithermal gold deposit located in the central Andes. The declustered grade distribution is skewed and has a mean of 0.42g/t, a standard deviation of 0.87 , a high coefficient of variation of 2.16, and a high skewness value of 6.3. The Measure of dissemination fluctuates around a relatively low value at short lag distances and then grows up approaching the pure diffusive behaviour at long lag distances (see Fig. 7a). With a SMU size of 5x5x5m the EMD is 0.51. In this case the difference between the intermediate and pure diffusion model are clear only for high cut-off's in the corresponding grade-tonnage curves presented in Fig. 7b.

For the second example the data set used consists of 2m drillhole composites from a Sn-Zn skarn deposit. For Zn grades the mean is 2.99%, the standard deviation 2.84% and the coefficient of variation 0.95. Here the MD takes moderate values at short lag distances and then decreases towards zero for lag distances that approach the variogram and madogram ranges which are short, around 16m (see Fig. 8a). For a SMU size of 4x4x4m the variance of the SMU scale values within the deposits becomes 3.86 and the EMD is 0.71. The curves of average grade above cutoff for the pure diffusion, mosaic and intermediate models (Fig. 8b) are different but close to each other. This is due to the relatively high EMD, a moderate coefficient of variation, and a low skewness.

The third example consists of a very skewed drillhole sampling data from a disseminated gold deposit. The three models, pure diffusion, intermediate, and pure mosaic, yield very different grade-tonnage curves (see Fig. 9b). The average gold grade of this data set is 1.13Au g/t, the standard deviation 4.25, the coefficient of variation is very high, 3.76, and the coefficient of skewness is also very high: 12.8. The fitted variogram and madogram models are very close (see Fig. 9a), this translates to a low and practically constant measure of dissemination at distances shorter than the variogram range. At a block size of 4m x 4m x 4m, the calculated EMD is 0.22.

From these examples, we conclude that the use of the intermediate model is relevant in certain cases and particularly when the data set is highly variable.

Limitations

Under the assumption of stationarity, the EMD is considered constant over the study area, which may be adequate for global estimation purposes. Local variations in the spatial distribution are not accounted for. The concept of a fixed-size SMU is somewhat artificial. It may be reasonable for feasibility studies or long term planning. Specific grade control practices must be considered in practice. It is also important to note that the results presented are valid only under the assumption of perfect ore/waste selectivity. A lack of perfect selection could be accounted for by using a slightly larger block size, but this is difficult to calibrate. Like other change of support models, we assume that the change of support coefficient r is a constant and, in general, that correlation functions in the isofactorial model are of the simple form

$$T_k(h) = \beta(h) \cdot r^k + (1 - \beta(h))r \ .$$

To make the Variable Effective Measure of Dissemination (VEMD) change of support model even more realistic we could consider r variable and

$$T_k(h) = \beta(h) \cdot (\rho(h))^k + (1 - \beta(h))(\rho(h)).$$

However, in this case we could encounter problems of non-positiveness of the resulting distributions (Chiles, 1999).

Conclusions

Predicting a grade-tonnage curve representative of practical SMU support before any mining has taken place is an important challenge for mining geostatisticians. The variable measure of dissemination (VMD) model presented here should prove useful when the real data do not fit the well-defined Gaussian pattern of spatial variation. The application of the intermediate model becomes relevant when the data set is highly variable and the measure of dissemination does not match the Gaussian model. A correct calibration of the measure of dissemination will yield more accurate predictions of recoverable reserves.

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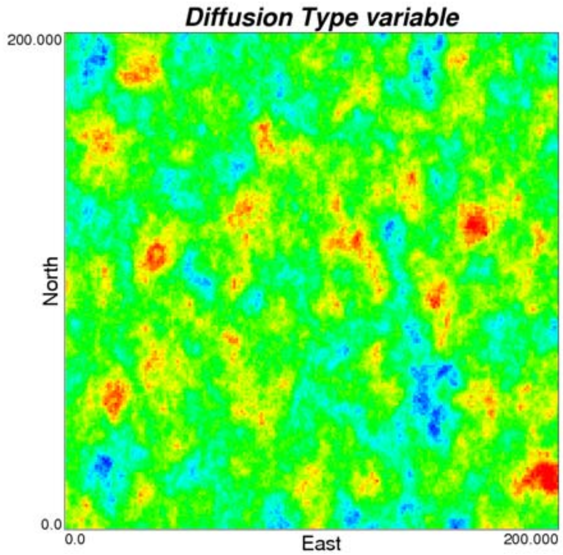


Figure 1: Example Diffusion Model (Sequential Gaussian simulation realization).

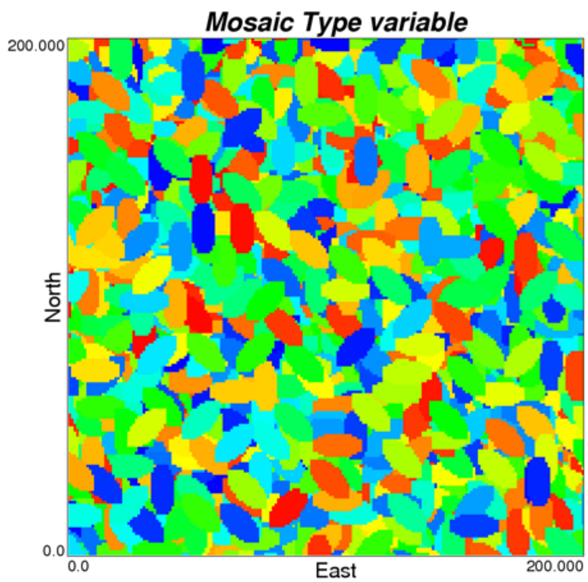


Figure 2: Example Mosaic Model. (Dead leafs model)

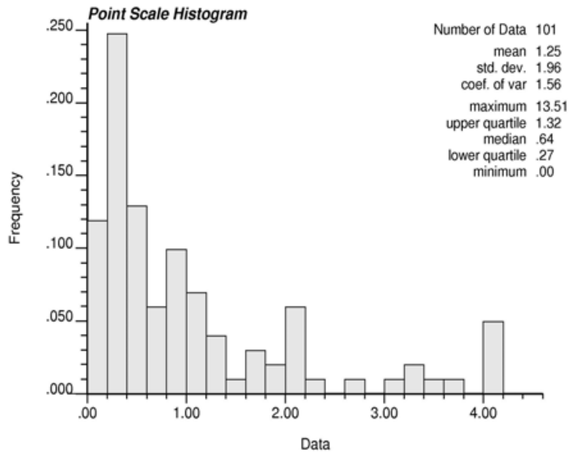


Figure 3: Distribution of the 101 simulated grade values.

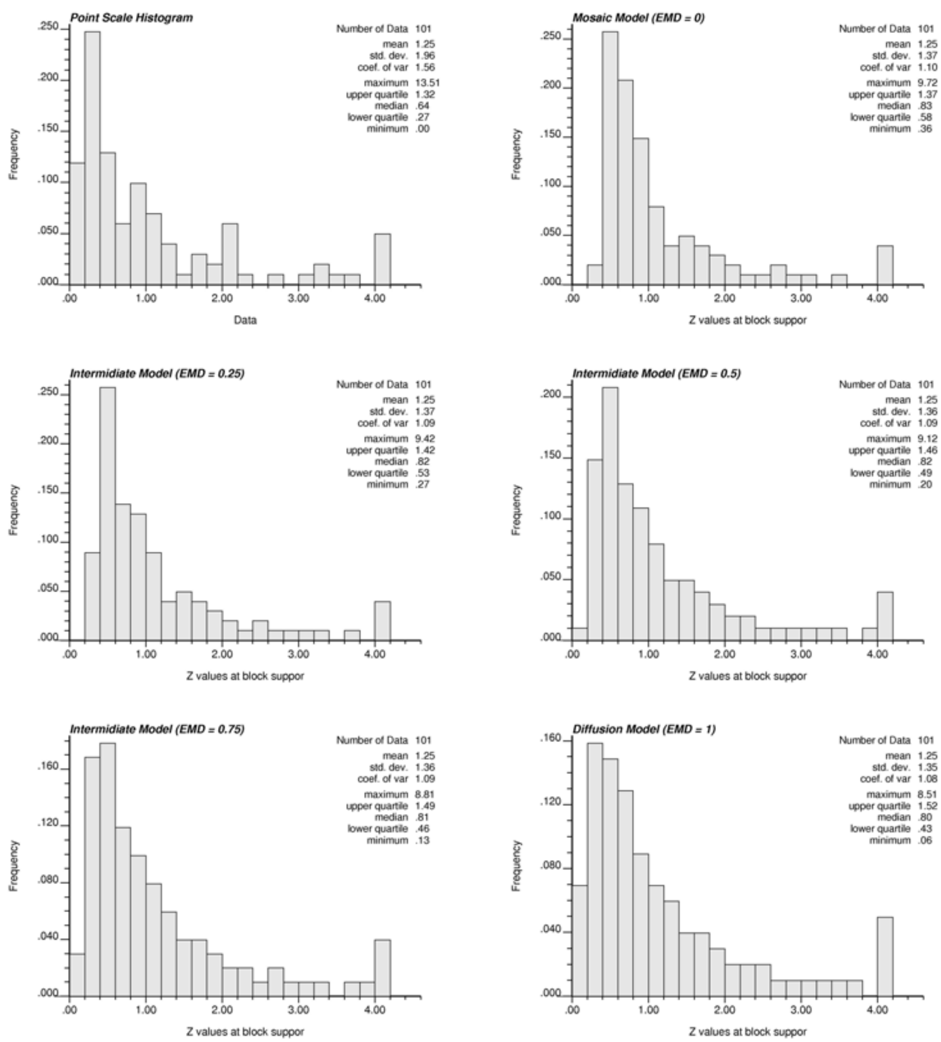


Figure 4: Original data distribution for 101 simulated values (top left), block support distribution obtained based on β_{eff} equal to 0 (top right), 0.25 (middle left), 0.50 (middle right), 0.75 (bottom left) and 1 (bottom right).

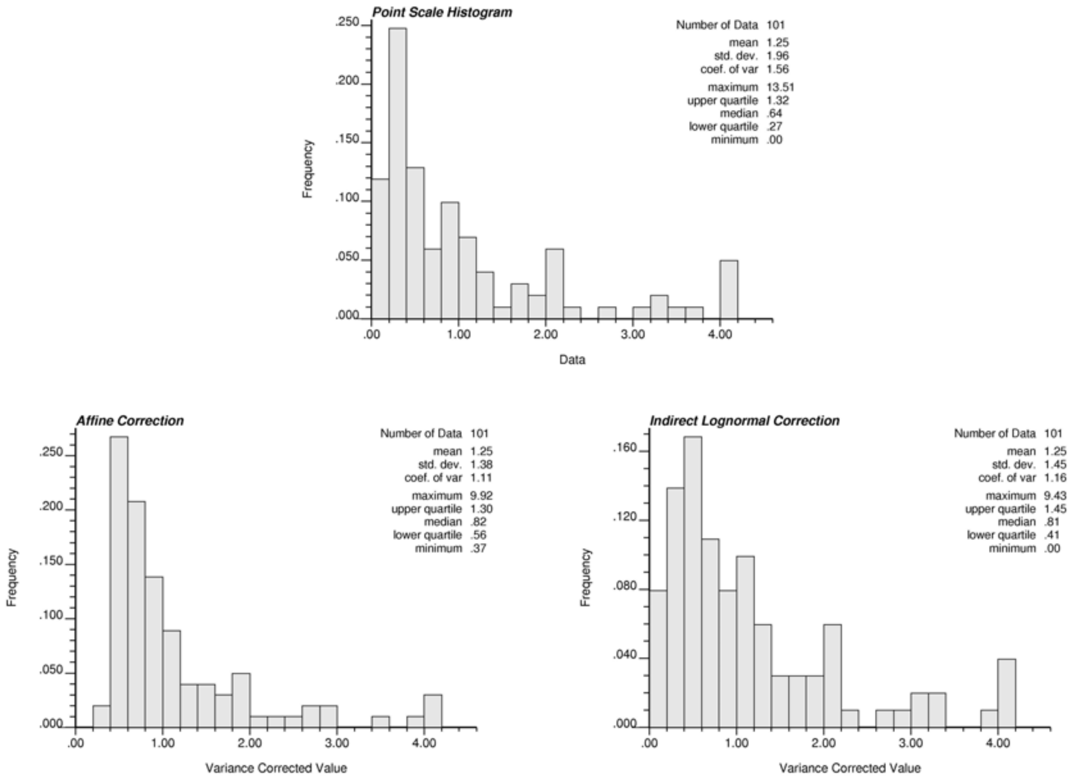


Figure 5: Original data distribution for 101 simulated values (top), block support distribution obtained based on affine correction (bottom left) and block support distribution obtained based on affine correction (bottom right).

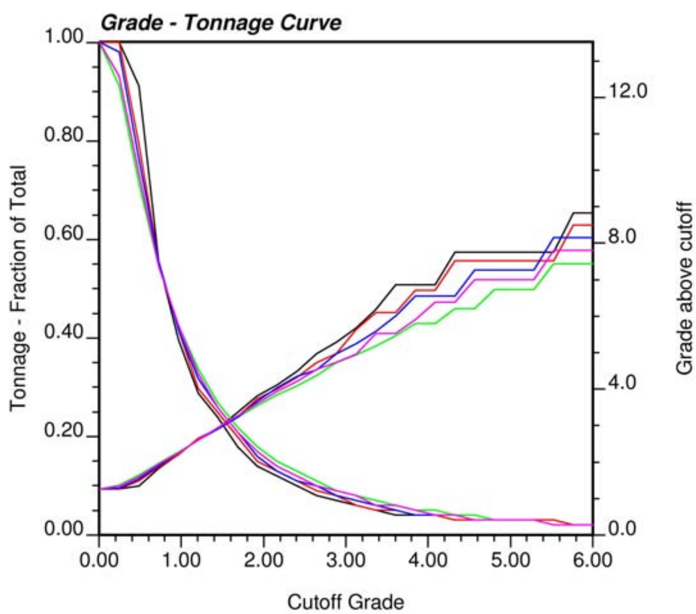
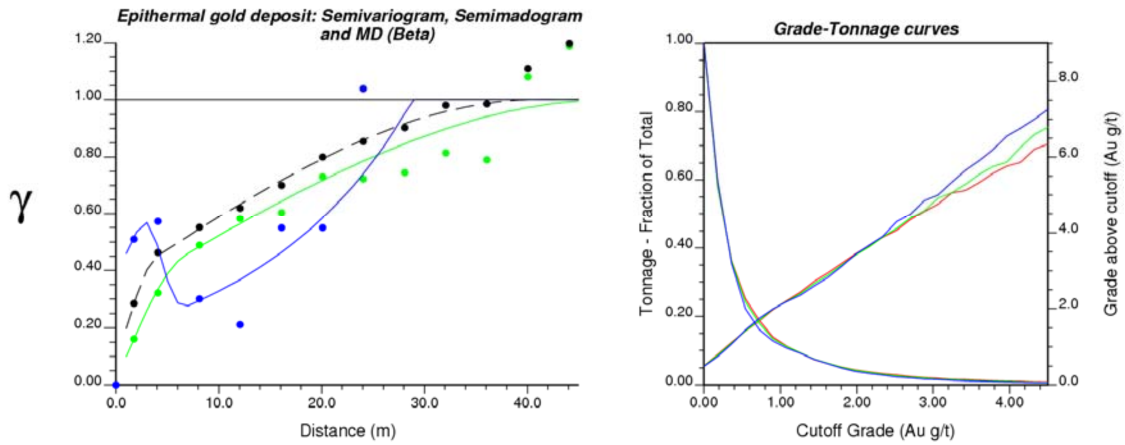
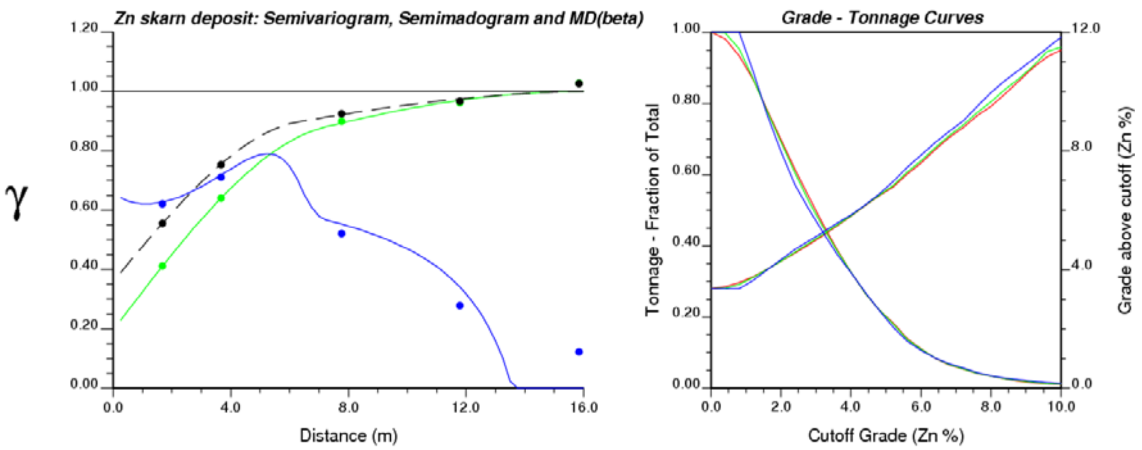


Figure 6: Grade-tonnage curves for β_{eff} equal to 0 (black curves), 0.25 (red curves), 0.50 (blue curves), 0.75 (purple curves) and 1 (green curves).



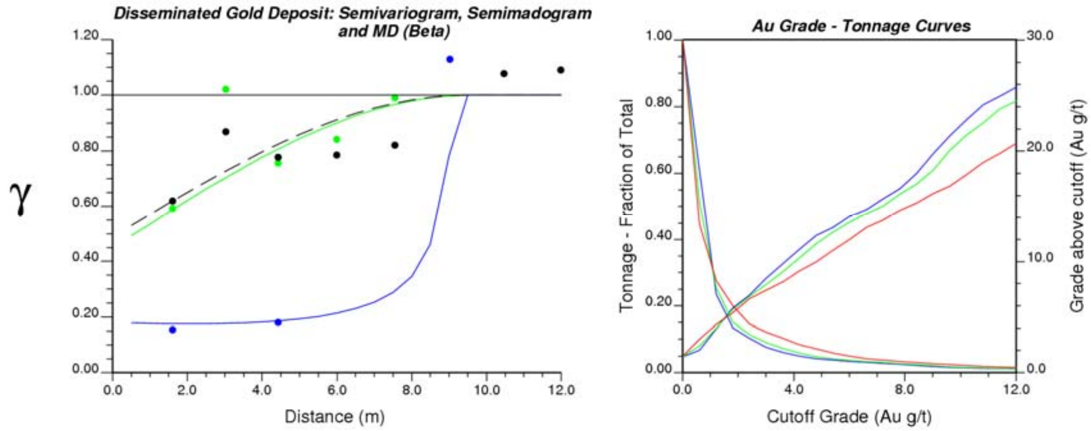
a) **b)**

Figure 7: Epithermal gold deposit: **(a)** Omnidirectional Variogram (green curve), Madogram (dashed black curve) models and the Measure of dissemination (continuous Blue curve). **(b)** Grade/tonnage curves for β_{eff} equal to 1 (diffusive model, red lines), equal to 0 (Mosaic model, blue lines) and 0.51 (intermediate model, green lines).



a) **b)**

Figure 8: Zn skarn deposit: **(a)** Omnidirectional Variogram (green curve), Madogram (dashed black curve) models and the Measure of dissemination (continuous Blue curve). **(b)** Grade/tonnage curves for β_{eff} equal to 1 (diffusive model, red lines), equal to 0 (Mosaic model, blue lines) and 0.71 (intermediate model, green lines).



a) **b)**

Figure 9: Disseminated Gold deposit**: (a)** Omnidirectional Variogram (green curve), Madogram (dashed black curve) models and the Measure of dissemination (continuous Blue curve). **(b)** Grade/tonnage curves for β_{eff} equal to 1 (diffusive model, red lines), equal to 0 (Mosaic model, blue lines) and 0.25 (intermediate model, green lines).