Local Recoverable Reserves Prediction with Block LU Simulation

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Prediction of local recoverable reserves is an important problem in ore reserve evaluation. The relatively wide spacing of exploration data leads to unavoidable uncertainty in the grades at unsampled locations. Deterministic methods such as kriging do not explicitly account for this lack of perfect information and the anticipated selectivity of the mining operation. We present a methodology and software for assessing recoverable reserves at selective mining unit (SMU) resolution over larger production panels. These reserves and their uncertainty are calculated by performing matrix (LU) simulation at a fine resolution over panels, and then scaling these simulated models to SMU size to calculate the expected tonnage and average grade above several cutoffs. Measures of uncertainty in the SMU grade distribution in the panels are also presented and future enhancements are discussed.

Introduction

This paper presents a technique to predict local recoverable reserves in a single pass without storing and processing multiple simulated realizations. The approach consists on considering panels of a relatively large size, where the distribution of selective mining units (SMUs) is estimated by locally simulating the grade values via the matrix decomposition (LU) method (Davis, 1987).

We first review the existing methods for estimating recoverable reserves. Then, the proposed methodology is described along with the program that has been implemented. The method of local LU has been around for a while (see Glacken, 1996); this paper presents the methodology and a GSLIB-like program. The paper concludes with a case study where we show the use of the program and discuss possible enhancements and conclusions.

Review of Existing Methods

There are many techniques for recoverable reserves estimation. They may be grouped into (1) global recoverable reserves estimation, (2) deterministic mapping, (3) probabilistic mapping, and (4) simulation.

Global recoverable reserves estimation without any local precision is possible by correcting a representative data-scale histogram (Isaaks and Srivastava, 1989). The distribution of sample values, usually composited to a constant volume, are used to build a declustered histogram, that is, one where the relative frequencies assigned to each sample are corrected to account for the spatial clustering due to preferential sampling commonly found in exploration campaigns. The representative histogram can be obtained with many techniques, such as polygonal weighting, cell declustering, and declustering based on the accumulation of ordinary kriging weights. This representative histogram is then corrected to account for the volumes of the anticipated selective mining units. The results provide no local precision, but could be used to calibrate deterministic mapping. Volume-variance relations based on average variogram values are used to calculate the variance reduction from the exploration data to the SMU scale. Recalling the expression for calculating the dispersion variance of the regionalized variable measured over a volume v within a larger domain of volume V:

$$D^2(v|V) = \bar{\gamma}(V,V) - \bar{\gamma}(v,v)$$

where $\bar{\gamma}(v,v) = \frac{1}{v^2}\int_v \int_v \gamma(y-y')dy\,dy'$ represents the average variogram value with the head and tail of a separation vector spanning the corresponding volume v (Journel and Huijbregts, 1978). In the case of global recoverable reserves, the volumes of interest are v = volume of a SMU and V =volume of the entire

domain or deposit. The dispersion variance reflects the expected variability around the mean of grades measured over volumes the size of a SMU. Once the variance of SMUs over the domain is known, the variance reduction factor can be calculated as:

$$f = \frac{D^2(\bullet \mid A) - D^2(v \mid A)}{D^2(\bullet \mid A)} = 1 - \frac{\bar{\gamma}(A,A) - \bar{\gamma}(v,v)}{\bar{\gamma}(A,A) - \bar{\gamma}(\bullet,\bullet)} = \frac{\bar{\gamma}(v,v)}{\sigma^2}$$

The variance reduction factor is the standardized difference between the dispersion variance of points within the domain and blocks within the domain. Each one can be replaced by the corresponding average variograms and these can be simplified, since $\bar{\gamma}(A,A) = \sigma^2$ and $\bar{\gamma}(\bullet,\bullet) = 0$. σ^2 is the representative sample variance.

A change of shape model must be applied to adjust the sample histogram, which is considered at point support, to the volumetric support of the SMU (v). Several models are available: affine correction, indirect lognormal correction and the discrete Gaussian model are the more popular. The tonnage above cutoff and the grade above cutoff can be calculated from the corrected histograms (Figure 1).

Deterministic mapping such as kriging or inverse distance does not account for uncertainty due to widely spaced data. In areas where the samples are sparse, the estimated grades appear smooth and all approach the local or global mean. This gives an unrealistic sense of low variability. These techniques could be calibrated to give similar global results to the corrected global estimation mentioned above. This is done at a cost of a higher conditional bias of the estimated grades as compared to the actual grades that are eventually mined out. Ordinary kriging with a restricted search is very common in the mining industry (Figure 2).

Probabilistic mapping techniques are based on kriging and provide a local probability distribution. The methods include indicator kriging (IK), multivariate Gaussian kriging (MG), disjunctive kriging (DK), uniform conditioning (UC) and other variants.

- Indicator kriging (Journel, 1983; Journel and Alabert, 1989) estimates the probability of not exceeding several cutoffs, by performing simple or ordinary kriging over indicator coded sample values. This procedure is repeated at different thresholds and requires the estimation and modeling of an indicator variogram for each cutoff. The estimated probabilities are used to construct a discretized conditional distribution, which allows inferring the local uncertainty at point support. This distribution can be corrected to account for change of support, providing a model for the local uncertainty of block grades. Incorporating secondary variables and managing trends is possible, but cumbersome.

- MutliGaussian kriging (Verly, 1983) approaches the problem by considering that the normally transformed samples are distributed as a multiGaussian variable. Under this assumption, the conditional distributions can be calculated with ease, since the local mean and variance of the conditional distributions, the only parameters required to fully define the full local distribution, are provided by the simple kriging mean and kriging variance. The quantiles of these distributions can be backtransformed to give any summary statistic required for characterizing the uncertainty (Ortiz and Deutsch, 2004). Trends and change of support are problematic with this method, mainly, because of theoretical reasons: ordinary kriging should not be used for estimating the parameters of the local distributions, and values do not average linearly in transformed units, hence, change of support should not be done by block kriging the normally transformed values. Considering secondary variables is not straightforward.

- Disjunctive kriging is rarely used because of its theoretical complexity (Rendu, 1980; Rivoirard, 1994). The method estimates any function of the samples by decomposing this function in an infinite series of polynomials, whose coefficients must be estimated. The lack of flexible commercial software for using DK has provoked its selfish use in industrial applications.

- Uniform conditioning (Remacre, 1989) aims at determining the distribution of blocks within a panel, by proceeding in two steps: firstly, an ordinary kriging estimation of the grades of large panels is done to define the local conditioning value for the second step, which is to compute the

distribution of block (SMU) grades within the panel, conditional to its estimated value, by means of a change of support model, based on a bivariate Gaussian assumption of the distribution of blocks and panels. This method allows accounting for local fluctuations in the average grade, but cannot straightforwardly account for secondary information.

These techniques are not that common because they depend heavily on stationarity and are sensitive to many modeling decisions. These techniques are awkward to implement in presence of multiple variables and anticipated grade control practices. They are proven successful in cases where they can be calibrated to actual production data.

The ***simulation*** alternative consists of simulating a large part of the deposit at a high resolution, scaling the realizations to an SMU scale and accounting for the results (Journel, 1974; Gómez-Hernández and Journel, 1992). This is the "modern" approach to calculate the recoverable reserves. Among the advantages of this approach, one can mention the consistency of the fine resolution model when considering change of support to several block sizes and the possibility to account for secondary variables and trends. Also, the simulation approach provides realizations that can be used to emulate the performance of the large volumes in relation to grade uncertainty, metallurgical recovery, grade control, etc. The simulated realizations allow accounting for joint uncertainty, that is, the uncertainty over a function that considers several points at the same time. Advanced applications consist of simulating the grade control procedures such as the spacing of grade control samples, sample error, and estimation such as blasthole kriging.

One of the main limitations of approaching the problem of estimating recoverable reserves by simulation is the size of the models. This complicates the handling and storage of the models and also requires a significantly large CPU time to compute the large number of realizations.

We propose an approach that keeps the advantage of a full simulation study, but allows a much faster and direct computation of the summary statistics of interest.

Local LU Simulation

Rather than simulating the full domain of interest, which can account for several millions of nodes in a high resolution model, we propose a local simulation approach, where panels are considered independent from each other and the computation of recoverable reserves is done on a SMU scale within the panels and considering the local information. This idea of moving neighborhood LU simulation has been around for some time. The thesis of Ian Glacken (1996) presents a comprehensive review of the method (Glacken, 1997).

The method works as illustrated in Figure 3, by considering a panel and the SMUs within it. A single search is done to find the nearest samples in the neighborhood. These samples will condition the simulation of a high resolution local simulation, which is performed with the matrix decomposition algorithm considering the discretization nodes of the panel.

The LU simulation algorithm provides an appropriate tool for the fast simulation of several hundreds joint realizations of the discretization points within the panel.

The method requires:

- Transforming the data to a standard Gaussian distribution.
- Calculate and model the variogram of these normally transformed data.
- Compute the covariance matrix for the locations of the available data in the search neighborhood and the locations of the discretizing nodes within the panel.
- Decompose the covariance matrix via the Cholesky decomposition into a lower and an upper triangular matrix. This decomposition also implies that the upper triangular matriz is equal to the transpose of the lower triangular matrix.
- Compute simulated values for the nodes within the panel, by multiplying the lower triangular matrix with a vector of random normal deviates. This operation can be quickly repeated several hundreds (or thousands) times.

- Back-transform the simulated values to grade units.

These simulated values are then internally stored and manipulated to calculate and display relevant statistics. The resulting models from the LU simulation within a panel can be easily handled to calculate the mean grade of the panel, the distribution of SMU grades within the panel –which can be summarized as the variance of SMU grades with respect to the average panel grade for a specific simulation, or with respect to the global average value of the panel–, the expected proportion of SMUs and their grade, above a specified cutoff, etc.

Since the software works like a kriging program, the size of the model is not a significant limitation. The only limit is derived from the number of nodes with which the panel is discretized. Furthermore, the manipulation of multiple realizations operates behind the scenes, easing the work of the practitioner. The program can be modified to provide any relevant summary statistics.

BLUSIM Program

The BLUSIM program was assembled from the KT3D and LUSIM codes in GSLIB with significant modifications to post process the local distributions. The program runs as a GSLIB program (Deutsch and Journel, 1998).

Input parameters

A parameter file is created when first run (Figure 4).

The parameter file requires the following:

- Information about the sample data (lines 5 to 7): Name of the datafile, columns for the coordinates X, Y, Z and the variable and declustering weight, trimming limits to discard missing values coded with a particular value.
- Transformation options (lines 8 to 15): The program offers the possibility of using the original sample data or an already transformed database, name of the transformation table (for checking), switch to consider a reference distribution. If turned off, the transformation is done considering the weighted sample distribution as the reference, name of the file with the reference distribution, columns for variable and declustering weight on the reference distribution file, and lower and upper tail options.
- Debugging options (lines 16 and 17): Debugging level. If set to three or higher, large amounts of information about the calculations will be done, debugging file. It stores all the information. This file can be considerably large if a high debugging level is used, and as an additional feature, the program writes out the last full realization at point and SMU support, for checking purposes. These are automatically output in files: dbgsim.out and dbgsmu.out.
- Simulation grid and discretization parameters (lines 19 to 24): Number of realizations for panel and SMU statistics calculations, grid definition for panel support, number of SMUs per panel, panel total discretization.
- Cutoff reporting parameters (lines 25 to 26): The number of cutoffs for reporting the proportion and grade of SMUs within the panel, and the cutoffs considered.
- Seed for random number generator (line 27)
- Search parameters for nearby samples (lines 28 to 31): Minimum and maximum number of samples in the neighborhood for simulation, Maximum samples per octant, if octant search is on, and search radii and angles for the search ellipsoid.
- Variogram model parameters (lines 32 to 34): Nugget effect and number of nested structures for the three-dimensional variogram model, variogram type, sill contribution, angles and ranges for each variogram structure considered.

Output results

The program outputs a file that contains, for every panel, the following summary statistics:

- Panel average grade
- Average variance of the SMU grades within the panel with respect to the average grade of the panel for each particular realization
- Average variance of all the SMU simulated grades (for all SMU within the panel and for all realizations) within the panel with respect to the average grade of the panel
- Mean grade of SMUs above cutoffs
- Proportion of SMUs within the panel above cutoff

As mentioned earlier, to check the results, the program also outputs one full point and SMU realization.

Examples

BLUSIM can be used to provide point support statistics, by not discretizing the panel, that is, by considering a panel with only one SMU, with a single discretizing point at its center. A result equivalent to applying multiGaussian kriging at a point support is then obtained. To check the program, a dataset of copper grades is used. BLUSIM and KT3D are compared, when working in Gaussian units, that is, using the dataset already transformed to a standard normal distribution as input. Figure 5 shows the comparison of these results:

- First, the comparison of mean and variance is done for the results in Gaussian units of BLUSIM and the result of performing NSCORE transformation of the sample values and then running KT3D for the estimation of the ordinary kriging mean and variance of the points. Slight differences are due to precision in BLUSIM, since only 1000 realizations of the point values are calculated. KT3D provides the exact calculation of the variance.
- Second, a comparison of block simulation in Gaussian units is done. A bias in the estimation of the variance can be seen for high valued blocks, which can be explained by the difficulty in obtaining by simulation very high values to reproduce correctly the true variance obtained by multiGaussian kriging.
- Third, a point simulation of grades is done, to check if the back transformed values are correctly computed. These values are compared with the post-processed multiGaussian kriging results with POSTMG. The result is quite good and the differences can be assigned to the precision of BLUSIM due to the number of realizations (1000) and to the post-processing of the multiGaussian kriging results to estimate the variance in original units.
- The last test shows the comparison of the statistics of block simulated values computed with BLUSIM and those calculated by multiGaussian kriging. Notice that in multiGaussian kriging the back transformation is done from the Gaussian average of simulated nodes within each block and considering the transformation table at point support. This is not theoretically correct and it explains the bias in mean and variance.

A second example is prepared to show the assessment of uncertainty that provides BLUSIM. The same copper data are used to compute panel grades and the dispersion of SMU grades within the panels. Results are illustrated in Figure 6.

Figure 7 shows the computation of mean grades and proportions above increasing cutoffs. This allows the calculation of recoverable reserves above the cutoff grade and could also be used to forecast the dilution expected during mining, since higher SMU variability within panels implies more complicated selection in grade control.

Figure 8 shows a realization of the point values and the SMU values. The obvious discontinuity seen between panels is explained by the fact that each panel is simulated independently hence no correlation

across the boundary of a panel is used to simulate the adjacent panels. However, it is clear that inside each panel, point grade simulated show spatial correlation.

Discussion and Proposed Future Work

Although this a fairly straightforward implementation of the idea of locally simulating the grades in a fine grid and then getting block averaged statistics for uncertainty quantification, there are several possible extensions that could be of interest:

- We could enhance the method to simulate mining (add sampling errors, simulate grade control and dilution, simulate the scaling to trucks, classify based on the estimated grades, then report the true results).
- Other measures of heterogeneity for the values within the block could be reported. One is to calculate the kriged surface (in Gaussian units) and to calculate a gradient of that surface. A second idea is to quantify the dispersion of the grades/SMUs within the mining volume.
- Something is going on whenever the "proportion/tonnage of ore" differs from 0 or the full tonnage of the mining volume (T). The two principal reasons are (1) uncertainty due to sparse data, and (2) the natural variability of the grades in the volume. We could devise a simple measure of how much the Δ from 0/T is due to sparse data and how much is due to local geological variability.
- We should report measures of uncertainty such as scaled up probability to be within a probabilistic tolerance – this could be used to support a geometric classification of resources.
- Consider reading in a high resolution rock type model and simulate only within a specified rock type. The program could consider only one rock type at a time. Perhaps we simulate the entire block as the rock type under consideration and report a fraction of the block that the results apply to.
- It may be important to consider multiple correlated variables under a full LMC or a Markov model. This may be important for practical problems. It would be easy to convert all secondary elements/assays to the primary one under an equivalent grade model. More complex calculations that include grade tonnage curves are probably beyond the scope of our analysis.
- An important modern consideration is the consideration of multiple ore dispositions (e.g., stockpile, leach, or plant). We should at least discuss this and comment on how the approach could be tailored to calculate the probability that the SMUs should report to each disposition – associated to tonnage.

Conclusions

Predicting local recoverable reserves with exploration data is a critical problem. The program presented here combines the best features of simulation and estimation to lead to direct predictions of local reserves. It allows inferring the variability of SMUs within panels and the expected proportion and grade above cutoff to determine the recoverable reserves.

The methodology permits forecasting the recoverable reserves for long term planning, from drillhole sample data. It can incorporate other factors affecting the recovery, such as sampling errors, dilution, and could be adapted to better understand the uncertainty due to data sparcity and due to geological heterogeneity.

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References

Davis, M. W., *Production of conditional simulations via the LU triangular decomposition of the covariance matrix*. Mathematical Geology, vol. 19, no. 2, 1987, p. 99-107.

Deutsch, C.V. and Journel, A.G., 1998, GSLIB: Geostatistical Software Library: and User's Guide. Oxford University Press, New York, 2nd Ed.

Glacken, I. M., Change of Support and Use of Economic Parameters for Block Selection, in Geostatistics Wollongong '96, E. Y. Baafi and N. A. Schofield (eds.), volume 2, pages 811-821, Kluwer, 1997.

Gómez-Hernández, J. J., and Journel, A. G. *Joint Sequential Simulation of Multigaussian Fields*. In Geostat Troia 1992, ed. Soares, Kluwer Publ., 1992.

Isaaks E. and Srivastava R., 1989, *An Introduction to applied geostatistics*, Oxford University Press, New York

Journel, A. G. *Geostatistics for conditional simulation of orebodies*, Economic Geology, Vol. 69, p. 673-687, 1974.

Journel, A. G. *Nonparametric estimation of spatial distribution*. Mathematical Geology, 15(3):445–468, 1983.

Journel, A. G., and Alabert, F. *Non-Gaussian data expansion in the Earth Sciences*. Terra Nova, 1:123–134, 1989.

Journel, A. G., and Huijbregts, C. J., *Mining geostatistics*, London: Academic Press, 1978, 600 p.

Ortiz, J. M., and Deutsch, C. V., A Methodology to Assess Block Grade Uncertainty, CIM Conference and Exhibition, Edmonton 2004, May 9 to 12, 2004.

Remacre, A. Z. Uniform conditioning versus indicator kriging, in Geostatistics, M. Armstrong (ed.), Volume 2, pages 947-960, Kluwer, 1989.

Rendu, J.-M., *Disjunctive Kriging: Comparison of Theory with Actual Results*, Mathematical Geology, 12(4): 305-320, 1980

Rivoirard, J. *Introduction to disjunctive kriging and nonlinear geostatistics*. 1994, Oxford, Oxford University Press.

Verly, G. *The multiGaussian approach and its applications to the estimation of local reserves*. Mathematical Geology, 15(2):259–286, 1983.

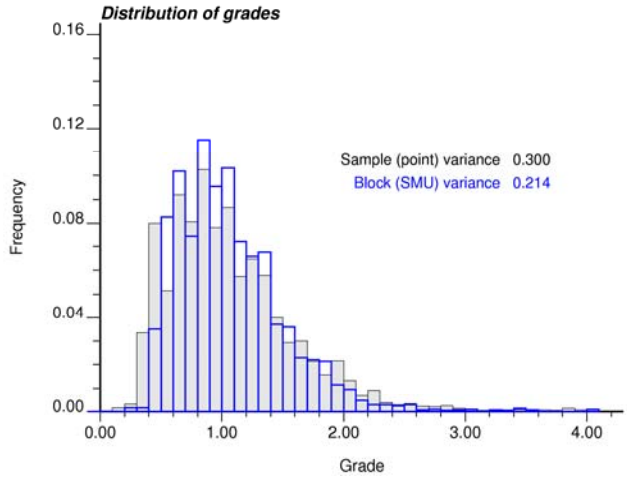

Figure 1: Point and block support distributions of copper grades. The block support distribution was calculated by affine correction from the point support declustered histogram.

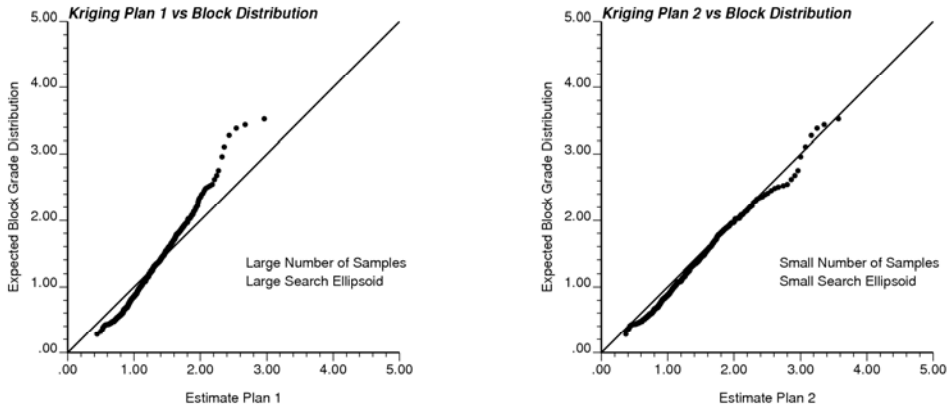

Figure 2: Quantile-quantile plots showing the comparison between the distribution of block estimated obtained by ordinary kriging and the expected block grade distribution. As the search neighborhood is constrained, the distribution of estimated block grades increases its variance, appearing closer to the distribution of actual block grades.

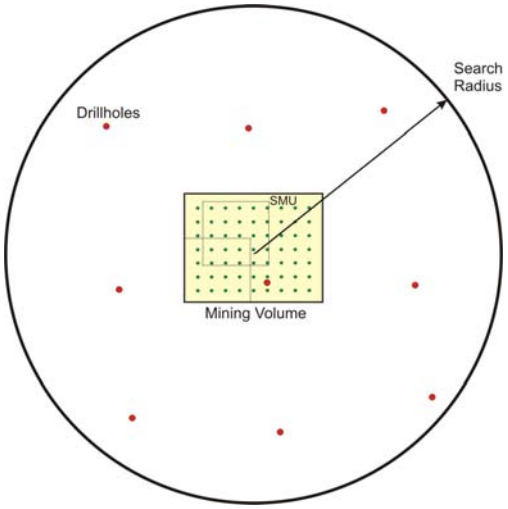

Figure 3: schematic 2-D illustration of the problem setting including the local neighborhood, exploration data, a mining volume discretized by some number of points and SMUs within the mining volume.

```
                  Parameters for BLUSIM
                  *********************

START OF PARAMETERS:
cluster.dat                       -file with data
1  2  3  4  5                     -   columns for X,Y,Z,var,wt
-1.0e21   1.0e21                  -   trimming limits
1                                 -transform the data (0=no, 1=yes)
blusim.trn                        -  file for output trans table
0                                 -  consider ref. dist (0=no, 1=yes)
histsmth.out                      -  file with ref. dist distribution
1  2                              -  columns for vr and wt
0.0    15.0                       -  zmin,zmax(tail extrapolation)
1       0.0                       -  lower tail option, parameter
1      15.0                       -  upper tail option, parameter
3                                 -debugging level: 0,1,2,3
blusim.dbg                        -file for debugging output
blusim.out                        -file for kriged output
100                               -number of realizations to generate
50   0.5    100.0                 -nx,xmn,xsiz - Pannel Size
50   0.5    100.0                 -ny,ymn,ysiz
1    0.5    1.0                   -nz,zmn,zsiz
5   5   5                         -nbx,nby,nbz- SMUs per Pannel
3    3   3                        -x,y and z block discretization
5                                 -number of cutoffs for reporting
0.2 0.5 0.8 1.0 2.0               -cutoffs
9784585                           -random number seed
4    8                            -min, max data for kriging
0                                 -max per octant (0-> not used)
20.0  20.0  20.0                  -maximum search radii
 0.0   0.0   0.0                  -angles for search ellipsoid
1    0.2                          -nst, nugget effect
1    0.8  0.0   0.0   0.0         -it,cc,ang1,ang2,ang3
          10.0  10.0  10.0        -a_hmax, a_hmin, a_vert
```

Figure 4: parameter file for BLUSIM.

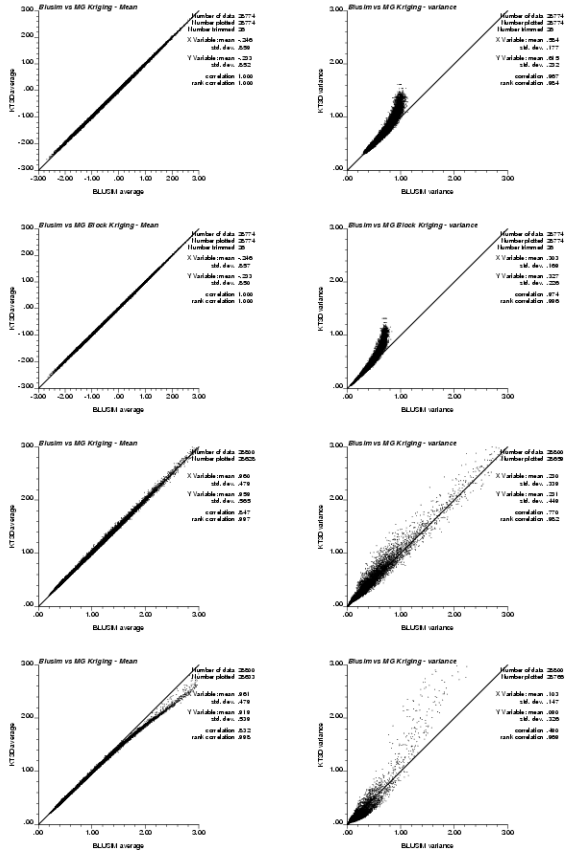

Figure 5: Comparison of BLUSIM with multiGaussian kriging using KT3D of the normal scores and POSTMG for back-transformation.

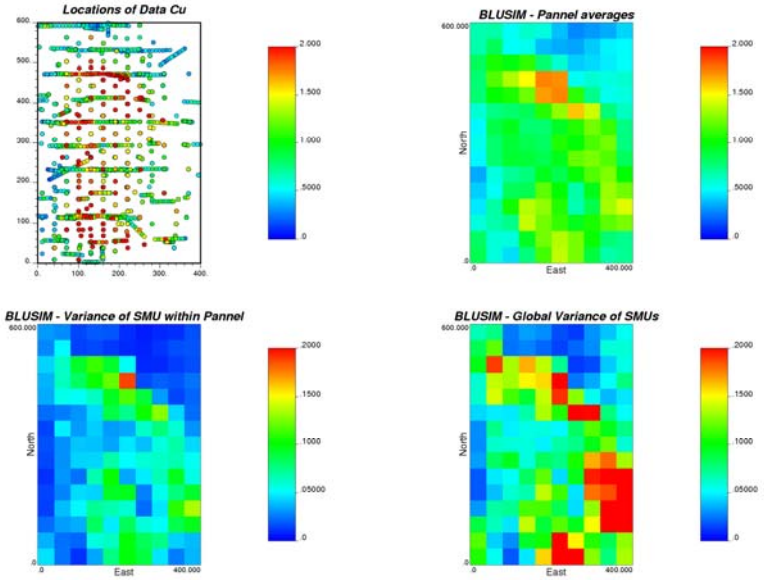

Figure 6: Location map of the copper data used for the example (top left), panel grades estimated for 40 x 40 x 12m^3 (top right), and two maps showing measures of dispersion within the panels: average variance of SMU grades with respect of the panel simulated grade over all realizations (bottom left) and variance of the simulated SMU grades with respect to the average panel value computed globally (bottom right).

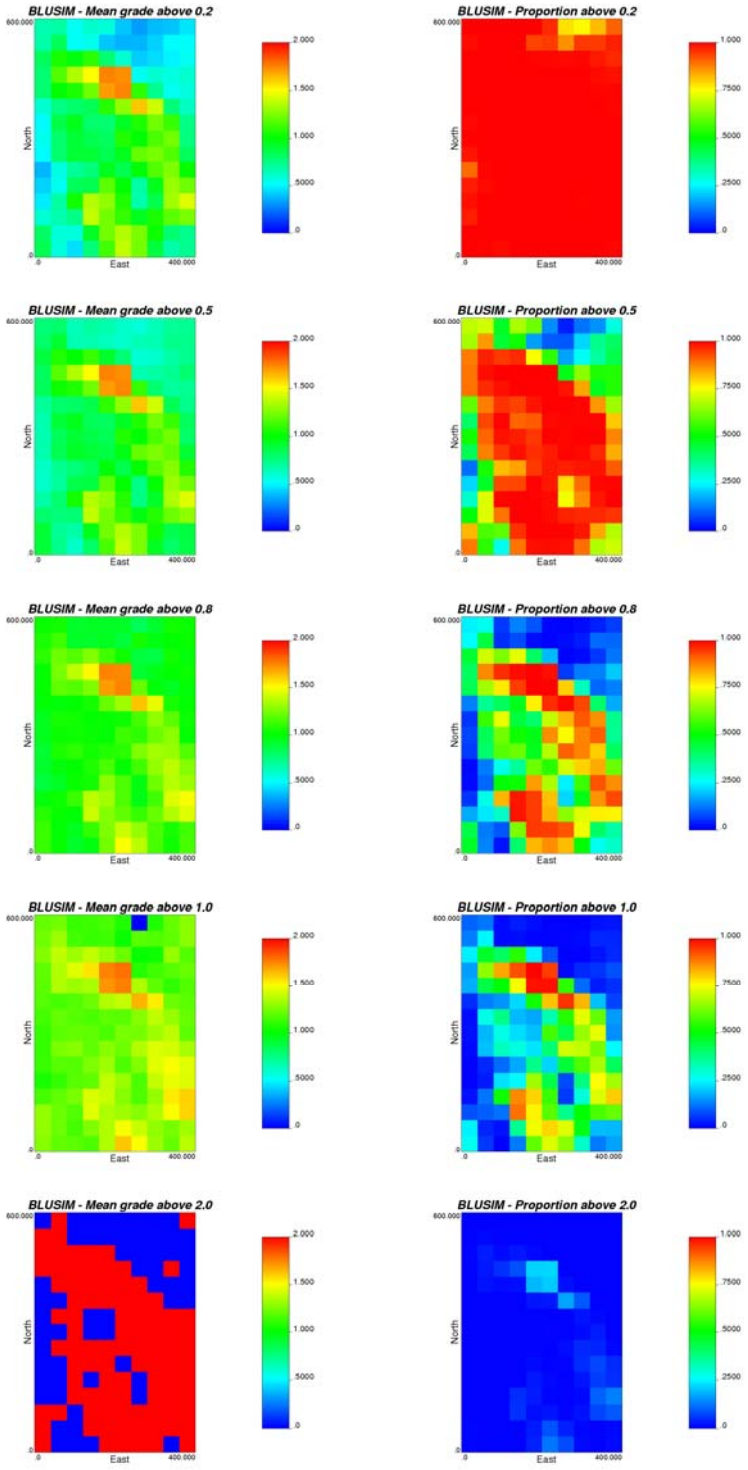

Figure 7: Mean grades and proportions of SMU within panels above increasing cutoffs.

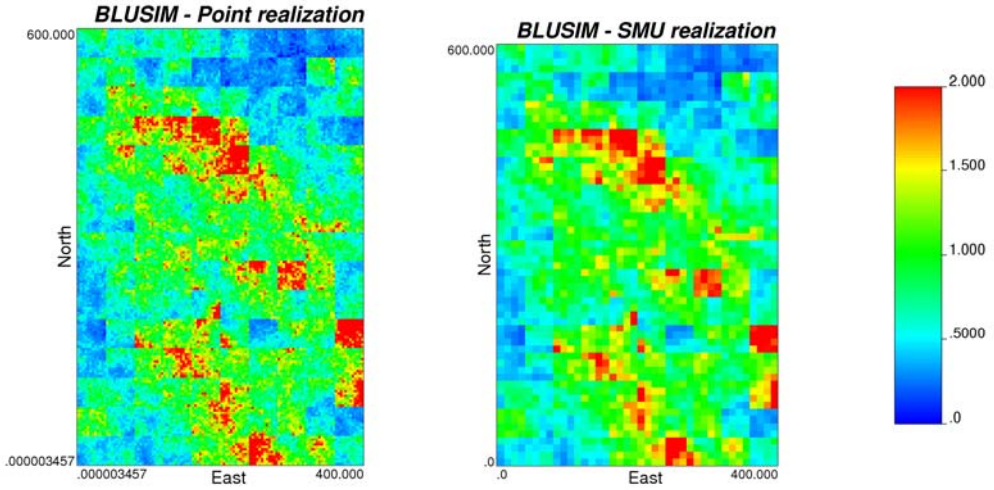

Figure 8: A point and SMU realization for checking.