A Methodology for Geometric Modeling of Kimberlite Deposits with Uncertainty

Jeff Boisvert[1], Peter Oshust[2], and Clayton V. Deutsch[1]

[1]Centre for Computational Geostatistics
Department of Civil and Environmental Engineering
University of Alberta

[2]BHP Billiton Corporation
Kelowna, British Columbia

A significant source of uncertainty in kimberlite pipes is the volume of the deposit. Often all material within the pipe can be considered ore, leaving the actual geometry of the pipe as the largest source of uncertainty. Deterministic methods to assess uncertainty are time consuming and not reproducible. It is desirable to use stochastic techniques to model the pipe geometry as they will generate multiple realizations that are reproducible and can be used to assess the uncertainty in the pipe geometry. Once the realizations are completed, traditional geostatistical techniques can be used to assess the uncertainty in pipe volume. Generating stochastic realizations of pipe geometry is a difficult problem. The methodology proposed uses sequential Gaussian simulation of pierce points interpreted from drill holes through a kimberlite pipe. Kimberlite pipes are cylindrical in shape; therefore, the data will be transformed to cylindrical coordinates to facilitate the simulation of the pipe geometry. Rather than using xyz coordinates to define the pierce points, z, θ and the pipe radius are used. In cylindrical coordinates the radius variable can be simulated in z-θ space to generate multiple realizations of the pipe volume. The uncertainty in the volume of ore can then be evaluated. This methodology will be demonstrated with a data set from a kimberlite pipe at BHP Billiton's Ekati mine.

Introduction

There are many factors that must be considered in a standard geostatistical evaluation of an ore deposit. Often the uncertainty in the grade of a deposit can be quantified using standard approaches such as sequential Gaussian simulation (SGS). There are many deposits where characterizing the uncertainty in the grade and associated measures such as grade/volume above cutoff etc. is sufficient to characterize the uncertainty in the deposit; however, there are cases when the uncertainty in the geometry of the deposit will dominate the uncertainty in the volume of ore available and it may not be sufficient to only consider the uncertainty in the grade.

The uncertainty in the geometry of a deposit can be considered a boundary modeling problem where the ore-waste contact must be modeled. Perhaps the best modeling of this contact would be done by an experienced geologist with knowledge of the deposit of interest but it can be difficult to quantify the uncertainty with this approach. Geostatistical methods such as sequential indicator simulation and truncated plurigaussian do not consider a smooth contact between rock types and are often too erratic to be geologically realistic. Moreover, the qualitative knowledge of the pipes cylindrical shape cannot be captured using these techniques.

Consider modeling the ore waste contact of a kimberlite pipe. Often all material within the kimberlite pipe is considered ore. In this situation, a significant portion of the uncertainty in reserves will be in the geometry of the kimberlite pipe. The problem is how to quantify this uncertainty. A geologist could generate one interpretation of the geometry, but deterministic methods can rarely generate a range of uncertainty in the volume of the deposit.

Kimberlite pipe geometry is often continuous and can be approximated by a cylinder with a variable radius. For this reason we will consider a conversion from Cartesian coordinates to cylindrical coordinates to take advantage of the cylindrical geometry seen in many kimberlite pipes. The data available for an assessment of the kimberlite pipe geometry is in the form of pierce points from surface drilling. Often these drillholes are drilled at angles and intersect the pipe twice, once upon entry and once exiting the kimberlite. These pierce points will be converted to cylindrical coordinates (*r, θ, z*) where the radius (*r*) variable will simulated in *z- θ* space. There are some specific issues regarding simulation in *z-θ* space and the programs SGS_CYL and GAMV_CYL will be presented to permit simulation. A case study using a kimberlite pipe from BHP Billiton's Ekati mine will demonstrate the methodology.

Background

This section will present a brief review on coordinate transformations used in geostatistics as well as a review of cylindrical coordinates. Kimberlite pipe geometry will also be reviewed to demonstrate the validity of considering the kimberlite pipe as a cylinder with a decreasing radius with depth. The uncertainty in the histogram of the data is often the largest source of uncertainty in global parameters such as the tonnage calculated from geostatistical simulation; therefore, the spatial bootstrap will be presented as a method of considering this uncertainty. The SGS technique is well established in geostatistical modeling; the reader is referred to Isaaks (1990) or Deutsch (2002) for background.

Coordinate Transformation in Geostatistics

It is common to use a Cartesian coordinate system when modeling a deposit. A location in space is defined by the distance from the three principal axes to a point of interest. Data collected in the field is often in this format, but it can be convenient to transform these coordinates into more 'model friendly' systems. One useful transformation is the rotation of the axes in Cartesian coordinates, Equation 1 (Deutsch 2002). This transformation is often done to help visualize a deposit that may not lie on the principal axes.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\varphi - \sin\alpha\sin\beta\sin\varphi & -\sin\alpha\cos\varphi - \cos\alpha\sin\beta\sin\varphi & \cos\beta\sin\varphi \\ \sin\alpha\cos\beta & \cos\alpha\cos\beta & \sin\beta \\ -\cos\alpha\sin\varphi - \sin\alpha\sin\beta\cos\varphi & \sin\alpha\sin\varphi - \cos\alpha\sin\beta\cos\varphi & \cos\beta\cos\varphi \end{bmatrix} \bullet \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad (1)$$

where α is the angle between the original azimuth and the new axis azimuth, β is the angle between the dips and φ is the angle between the plunges. *(x',y',z')* is the coordinates of the transformed point.

Other, more complex coordinate transformations simplify modeling and improve its effectiveness. One approach is to remove the curvilinear structure of the deposit, flatten it, model with traditional geostatistical techniques and transform back to the original coordinates. This is commonly done with vein type deposits, see Figure 1. After the vein has been digitized (black lines in Figure 1) control points are added to the center of the vein (blue line on Figure 1) and the segments are divided into an arbitrary number of sections between each control point and flattened. Because the variogram can only capture the linear relationships in data, unfolding the vein to be linear allows for better characterization of the variability in the grade within the vein. Traditional Gaussian based approaches can now be used in the flattened coordinates. After modeling, the coordinates are back-transformed to original space and further analysis completed.

A similar transformation is often employed when modeling petroleum reservoirs. Often the major direction of continuity in the deposit is along a curvilinear channel. In Figure 2 the dashed lines represent the domain that has a similar direction of continuity (along the direction of the channel). The area between the solid line and the dashed line has the same direction of continuity but is not considered reservoir grade material. The area between the dashed lines is flattened and the major direction of continuity becomes horizontal. Now geostatistical modeling can proceed with variogram fitting and SGS in the transformed coordinates. After simulation, a back transformation will provide a model of the reservoir in original coordinates. Often multiple transformations are necessary; consider the channel structure in Figure 3 (again with the major direction of continuity along the channel). Making the two successive transformations of coordinates, the channel becomes much easier to model as the direction of continuity is now constant and along the y-axis. For more details on stratigraphic coordinate transformation see Deutsch 2002.

Cylindrical Coordinates

It is not common in geostatistics to use cylindrical coordinates, mainly because there are few deposits that would be simplified with such a transformation. Kimberlite pipes, however, lend themselves to this transformation. Cylindrical coordinates are defined by the horizontal distance from the point to a centerline (r), the angle to this point (θ) and the depth (z), see Figure 4.

The z coordinate is identical to the Cartesian z coordinate. The location in the xy plane is then defined by r and θ rather than by x and y as in Cartesian coordinates. In order to convert from xyz coordinates to cylindrical coordinates a centerline is required. This centerline is the location of the z-axis on Figure 4. This does not have to be linear, as shown in Figure 4, and for a kimberlite pipe it will roughly follow the center of mass for the pipe. To convert from Cartesian to cylindrical coordinates the equations (Figure 4) can be used where x and y are defined as the horizontal distance from the centerline (x_c, y_c) to the point in question (x_p, y_p). The centerline coordinates x_c and y_c will change with depth.

Kimberlite Pipe Geology

The specifics behind the formation of a kimberlite pipe are not completely agreed upon but the general mechanisms that produce the pipes are known. The diamonds contained in kimberlite are formed at great depths under high temperature and pressure and brought rapidly to the surface by magma. The diamond bearing magma rises through the earth's mantle through crack propagation at speeds between 10 and 30km/hr (Eggler, 1989). This violent assent through the earth's mantle causes a very characteristic 'carrot' shaped deposit once cooled, see Figure 5. Near the 'root' of the deposit there are often more irregularities (Guilbert and Park, 1996), but the upper zone where mining (and thus modeling) occurs is characterized by a cylindrical shape with a decreasing radius with depth. Because of the explosive nature of the magma rushing to the surface the vertical walls of the pipe are often smooth and cylindrical. It is this characteristic of kimberlite pipes that will be exploited in the conversion to cylindrical coordinates.

The Spatial Bootstrap

Uncertainty in global statistical parameters such as the histogram and input mean for SGS is an important component of uncertainty quantification. The bootstrap is an application of Monte Carlo simulation and was developed by Efron (1982). The spatial correlation between the data will be accounted for using the variogram. To consider the correlation between data in the bootstrap technique a spatial bootstrap will be used. Multiple 'realizations' will be simulated by drawing n values, one at each data location, from the distribution of n original data. The simulated values are unconditional and are only required at the data locations. The LU method is often used because (1) it is simple and efficient for a large number of realizations, and (2) the number of data would typically not exceed 10000; sequential methods are not warranted. A 3-D variogram model is required to establish the covariance values between each pair of data. The spatial bootstrap procedure leads to multiple realizations of the input statistic that considers the correlation structure of the data (i.e. the variogram). Correlation in the n values will lead to greater uncertainty than the assumption of independence. For details on the spatial bootstrap see Deutsch 2006.

Methodology - Sequential Gaussian Simulation in z-θ Space

The methodology for simulating with cylindrical coordinates will be very similar to simulating in Cartesian coordinates. The main difference is that the θ coordinate is allowed to wrap as 360° is equivalent to 0°. Specifically, the methodology will be:

Step 1: Consider a trend in the radius of the pipe with z and subtract the mean radius from all data values.
Step 2: Transform the pierce point data to cylindrical coordinates. Consider r as the variable in z-θ space. The data is now two dimensional with r as the variable of interest.
Step 3: Decluster and debias the data if necessary. Transform r to normal score units.
Step 4: Variogram analysis considering that θ is permitted to wrap.
Step 5: Perform a spatial bootstrap on the distribution of r.
Step 6: Simulate r with traditional SGS with the multiple histograms obtained from the spatial bootstrap and back transform r to original units.
Step 7: Calculate the volume of the pipe.

The majority of this methodology is commonly applied in a geostatistical analysis. The major difference is the wrapping of the θ coordinate. Consider a set of available data in the form of pierce points in cylindrical coordinates, plotted in Figure 6. The θ coordinate is continuous, i.e. an angle of 360° is equivalent to 0°. When searching for pairs to generate a variogram for this data and when searching for nearby data for SGS, this wrapping effect must be considered. To this end, the program GAMV and SGSIM, as implemented in GSLIB, were modified to search beyond the 360° limit, see Figure 6.

Simulation of r in z-θ space will define the pipe geometry. Currently simulation of continuous attributes within the pipe, such as grade, has not been considered and only pipe geometry is modeled. Simulation is performed in 2D (z-θ space) with r as the variable.

Example

This methodology will be demonstrated on a kimberlite pipe investigated by BHP Billiton. The modified programs GAMV_CYL and SGS_CYL will be used to account for wrapping. These programs are identical in how they function as well as the parameters they take compared to GAMV and SGSIM in GSLIB. There are a total of 112 pierce points available for the kimberlite pipe. These 112 points are converted to cylindrical coordinates and represented on the z-θ plot in Figure 6. The r variable will be simulated in z-θ space to fully define the geometry of the kimberlite pipe. The data will be declustered, normal scored, fit with a variogram and simulated to generate the final pipe models. Within SGS a spatial bootstrap will be considered to account for the uncertainty in the pipe radius and a trend model will be used to account for the decreasing r with depth. A short uncertainty analysis will follow for the volume of the pipe. For this example, θ will be in degrees measured from north, r and z will be in meters.

Trend and Declustering

There is a significant trend in r with depth, see Figure 7. The trend will be considered by subtracting the trend from the variable, r, and modeling the residual, Equation 2. The mean radius at all locations will be determined by the trend modeled in Figure 7 and the residual can be calculated by subtracting the mean from the data values, Equation 2. See Journel and Rossi (1989), Isaaks and Srivastava (1989), or Leuangthong and Deutsch (2004) for more trend modeling techniques.

$$Z(\mathbf{u}) = m(\mathbf{u}) + residual(\mathbf{u}) \tag{2}$$

The residual will be declustered with cell declustering, modeled with a variogram and then simulated. After back transforming each simulation, the modeled mean will be added to the simulations. Declustering of the residual in z-θ space will be accomplished with cell declustering.

Variogram Analysis

A variogram in z-θ space is difficult to visualize as the units of the variogram are not consistent. Normally in a Cartesian coordinate system the x, y and z variables are in units of meters, thus the variogram will also have units of meters; however, in cylindrical coordinates the θ variable is in degrees while the z variable is in meters, making the units on the variogram inconsistent. Regardless, the variogram can still be used to represent the spatial relationship within the dataset. Anisotropy in the dataset, due to the differing units and the heterogeneity of the data, will be captured by the variogram.

The major direction of continuity is chosen as 0° and the minor direction will be 90°. This choice of angles simplifies the interpretation of the variogram. The variogram in the major direction will be in the z direction with units of meters while the minor direction will be in θ with units of degrees. The directions of continuity need not coincide with the z or θ directions. A nugget effect of 0 is used to ensure there are no irregularities in the final pipe model; however, a nugget effect could be used to model pipes with erratic geometries. The model used is shown in Figure 8 and has the following equation:

$$\gamma(\mathbf{h}) = 1.0 sph_{\substack{ah1=470 \\ ah2=220}}(\mathbf{h})$$

Where $ah1$ is in the 0° direction and $ah2$ is in the 90° direction. Sph is a spherical variogram structure.

SGSIM in Cylindrical Coordinates

Using the variogram modeled above, 200 SGS realizations are generated for the residual in z-θ space with wrapping in the θ direction. The standard GSLIB program SGSIM was modified to consider the wrapping past 360°. Some details on simulation used for this case study: a maximum of 24 original data used; 180 blocks in θ and 110 in z with a cell size of 2° and 5m respectively; a maximum search radius of 470 and 220 was used in the major and minor directions respectively. A typical simulation is shown in Figure 9, note the continuity between 360° and 0° because of the wrapping effect. The original SGSIM program with the above parameters requires 0.96 minutes while the modified program takes 1.4 minutes, an increase of 46%. This difference is due to the extra time it takes to calculate the distance between the points as well as the wrapped distance between the points. The minimum distance is required so that the correct covariance between points can be determined.

Using the spatial bootstrap the uncertainty in the distribution of the residual can be quantified. For each of the 200 realizations, the spatial bootstrap is used to generate a different reference distribution to transform the 112 pierce points to normal score units. For each realization the normal score transform and the resulting back transformation will be different because a different distribution is considered.

Uncertainty in Pipe Volume

The goal of modeling the pipe geometry stochastically is to be able to assess the uncertainty in the volume of the pipe. To this end, 200 realizations of r in z-θ space are created and transformed back to xyz coordinates to calculate the volume of each realization. Because of the nature of the transformation from cylindrical coordinates to Cartesian coordinates, the pipe is defined by a set of polygons for each of the z slices. For example, in the model considered here there is a polygon every 5m (the z block size modeled). To calculate the volume of the pipe, the area of each polygon is calculated and given a thickness of 5m in the z direction. Adding all slices gives the overall volume as shown in Table 1. There is more variability in the pipe volume when considering the uncertainty in the pipe radius with the spatial bootstrap. This can be visualized in Figure 10 where the boundaries of the pipe with the spatial bootstrap are more variable and the p_{10} is consistently smaller than the p_{90} where as without the bootstrap the randomness of SGS generates pipe boundaries that are more erratic and the global uncertainty is averaged out. Moreover, the cdf in Figure 11 highlights effect of considering uncertainty in the input distribution.

Table 1: Total volume of the kimberlite pipe for select realizations.

Realization	Pipe Volume without bootstrap (m^3)	Pipe Volume with bootstrap (m^3)	% Difference
p_{10}	8,434,000	8,012,000	-5.0 %
p_{50}	8,503,000	8,633,000	1.5 %
p_{90}	8,580,000	9,651,000	12.5 %

Future work

There are inherent secondary data that arise because of the geometric nature of this problem. Often drillholes are drilled at an angel to the kimberlite pipe to generate intersections. In the proposed methodology only the pierce points were considered; however, there are areas in the model where it is known that the pipe exists (the gray blocks in Figure 12) and where the pipe does not exist (the white blocks in Figure 12). This inequality data could be incorporated by modifying SGS to ensure these locations are inside or outside the pipe as appropriate.

SGSIM and GAMV were modified to allow wrapping in the x direction for cylindrical coordinates as well as wrapping in the y direction if spherical coordinates were desired. The application of this methodology would be similar for spherical coordinates where r could be modeled in θ - ϕ space and would depend on the availability of a suitably spherical ore deposit.

The simulation of attributes, such as grade, within the pipe boundary would not be possible in cylindrical coordinates. It may be reasonable to generate a locally varying mean (LVM) model in cylindrical coordinates and use the LVM model in simulating the grade. In this way the LVM model could consider a trend in the *r* and/or θ directions. This type of LVM model of grade would be difficult to generate in Cartesian coordinates.

Another feasible method to construct the boundary would be to use a volume function (McLennan, 2007). This function labels all blocks in the model as either inside or outside a boundary. The drillhole data is recoded as integer values from $-c\ldots0\ldots+c$ in such a way as to reproduce a realistic boundary. The drill holes would be populated with values near 0 at the kimberlite contact and will have increasingly negative values within the pipe and increasingly positive values outside the pipe. SGS is performed with the integer coded drillholes and, by definition, the boundary is taken to be the zero contact in the model. Repeated simulations will span the uncertainty in the modeled pipe volume. This method has the advantage of being able to consider any geometry as it is not limited to a cylindrical shape.

Conclusions

The uncertainty in the volume of resources can be significant. In the case of a kimberlite pipe, the uncertainty in the geometry of the pipe may be larger than the uncertainty in the actual grade of the ore. A variant of SGS was implemented to perform boundary modeling of the pipe radius in cylindrical coordinates. It was found that the uncertainty in the pipe volume is mainly due to the uncertainty in the mean pipe radius. The uncertainty in the kimberlite volume was quantified.

The methodology presented here for modeling a kimberlite pipe in cylindrical coordinates is specific to the modeling of a pipe-like geometry. Cylindrical coordinates can not consider geometries beyond a simple cylinder (although it was demonstrated how a variable pipe radius could be considered). Moreover, if the kimberlite pipe wraps back on itself the methodology would not be feasible. In this case there would be multiple values of *r* for a given z-θ and this is not possible in the present implementation.

This methodology works well for quantifying the uncertainly inherent in the pipe geometry. Traditional methods of digitizing are too time consuming to generate multiple realizations and are often deemed too subjective. Most geostatistical rock type modeling techniques, such as SIS, cannot consider the regular pipe boundary. Transformation of the coordinates to cylindrical coordinates permits the evaluation of the uncertainty in the pipe geometry.

References

Deutsch, C. V., 2002. Geostatistical Reservoir Modeling. New York, Oxford University Press, 376 pp.

Deutsch, C. V., 2004. A Statistical Resampling Program for Correlated Data: Spatial_Bootstrap. CCG Report 2004.

Efron, B., 1982. The jackknife, the bootstrap, and other resampling plans. Society for Industrial and Applied Math, Philadelphia. Vol. 38.

Eggler, D. H., 1989. Kimberlites: how do they form? In Proceedings of the Fourth International Kimberlite Conference, Kimberlites and Related Rocks, Ross, J. (ed.). Geological Society of Australia, Special Publication 14, Blackwell Scientific Publications, Oxford, Vol. 1, 323-342.

Guilbert, J. P. and Park, C. F. Jr., 1996. The Geology of Ore Deposits. New York, W. H. Freeman and Company, 985 pp.

Isaaks, E. H., 1990. The application of Monte Carlo Methods to the Analysis of Spatially Correlated Data. PhD thesis, Stanford University, Stanford, CA.

Isaaks, E.H. and Srivastava, R. M., 1989. An Introduction to Applied Geostatistics. Oxford University Press, New York, 561 pp.

Journel, A.G. and Rossi, M.E., 1989. When do we Need a Trend Model in Kriging? Mathematical Geology, vol. 21 (7), 715-739.

Kjarsgaard, B. A., 1996. Kimberlites. In Searching for Diamonds in Canada, LeCheminant, A. N., Richardson, D. G., DiLabio, R. N. W., Richardson, K. A. (eds.). Geological Survey of Canada, 29-38.

Leuangthong, O. and Deutsch, C. V., 2004. Transformation of Residuals to Avoid Artifacts in Geostatistical Modeling with a Trend, Mathematical Geology, vol. 36 (3), 287-305.

McLennan, J., 2007. Boundary Modeling. PhD thesis, University of Alberta, Alberta, Canada.

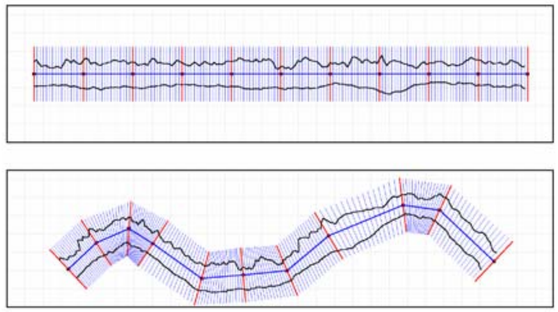


Figure 1: Unfolding of a vein. Above - unfolded vein. Below - original vein.

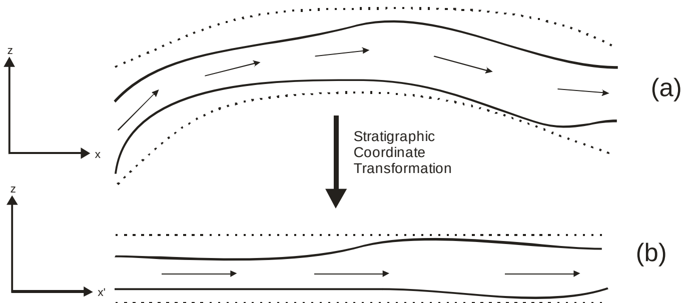


Figure 2: Stratigraphic coordinate transformation in section (after Deutsch 2002).

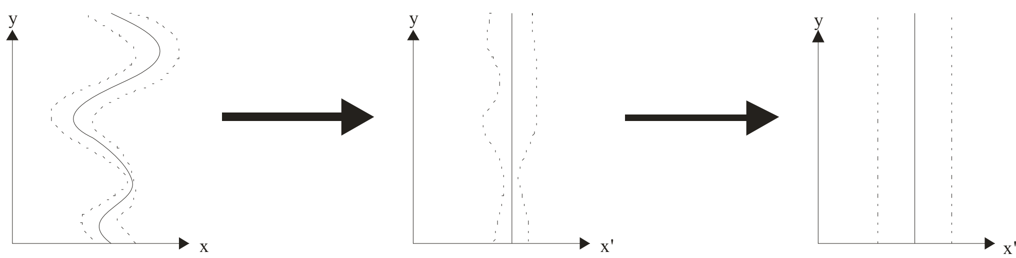
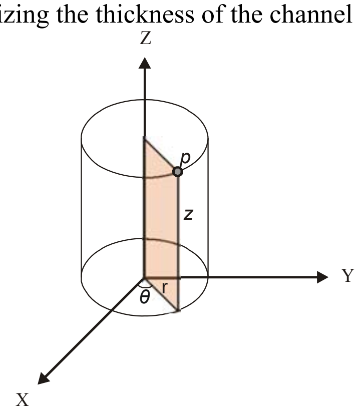


Figure 3: Stratigraphic coordinate transformation in plan. First the curvilinear features are removed, followed by standardizing the thickness of the channel (after Deutsch 2002).



Converting to (r, θ, z)

$$x = x_c - x_p$$

$$y = y_c - y_p$$

$$r = \sqrt{x^2 + y^2}$$

$$z_{Cylindrical} = z_{Cartesian}$$

Figure 4: Defining a point in space (p) using cylindrical coordinates (r, θ, z).

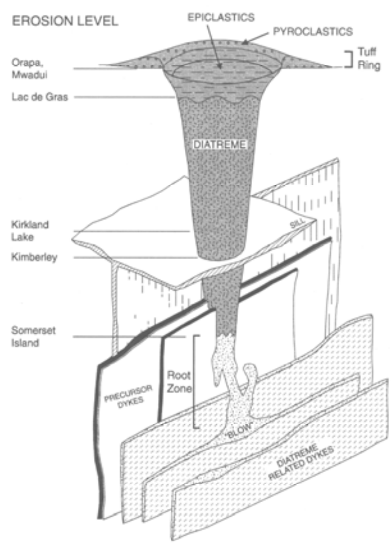


Figure 5: Typical kimberlite system. The depths of erosion of Canadian pipes are shown (Kjarsgaard, 1996).

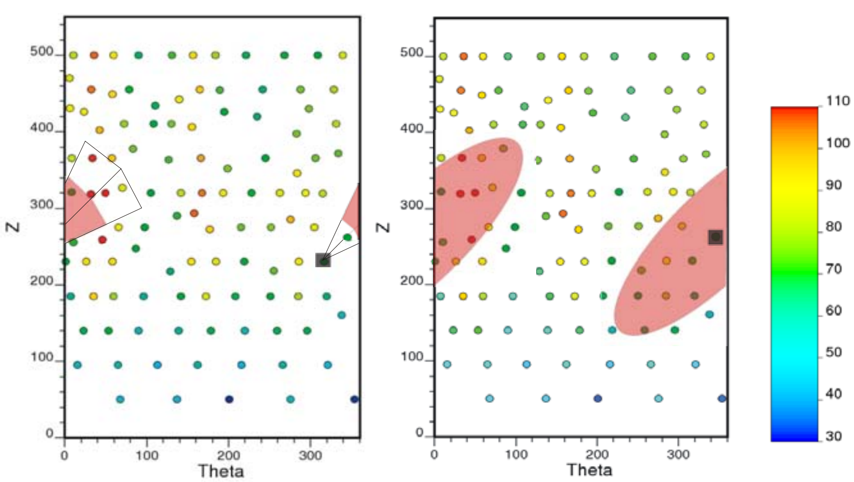


Figure 6: The variable plotted is r for each pierce point in the Ekati data set. Left – searching for variogram pairs (GAMV_CYL). Points falling in the red areas would be paired with the boxed point. Right – searching for conditioning data (SGSIM_CYL).

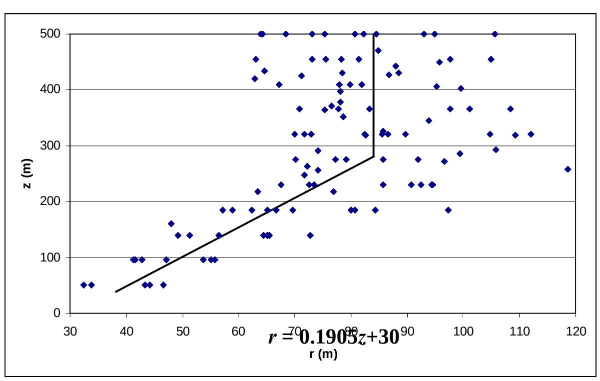


Figure 7: Modeled trend for r.

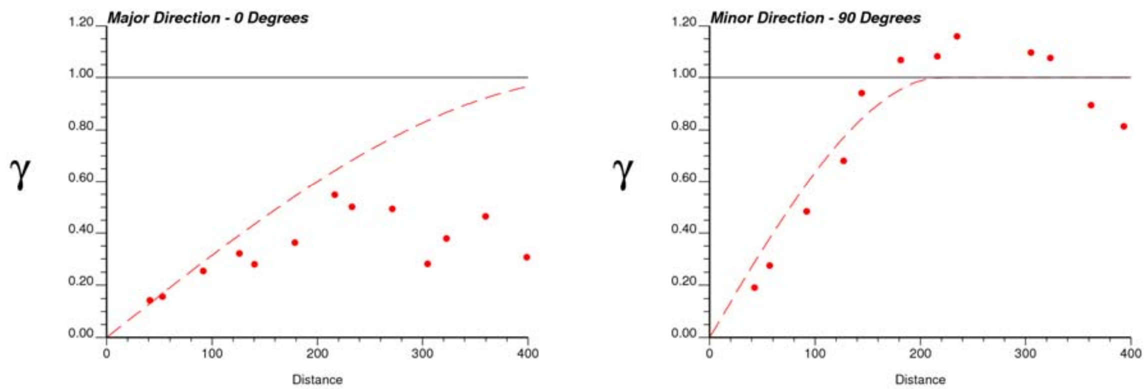


Figure 8: Experimental variogram (points) and modeled variogram (lines) for the major and minor directions of anisotropy. The x-axis in the major direction will be in units of meters whereas the x-axis in the minor direction will be in units of degrees because of the inconsistent units in z-θ space.

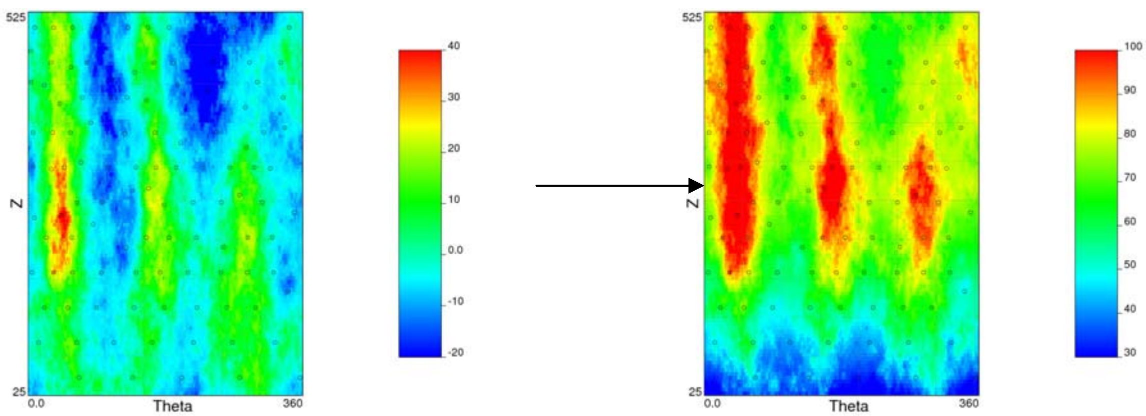


Figure 9: One SGS_CYL realization. Left: SGS_CYL realization of the modeled residual. Right: SGSIM realization after adding the trend back into the simulation.

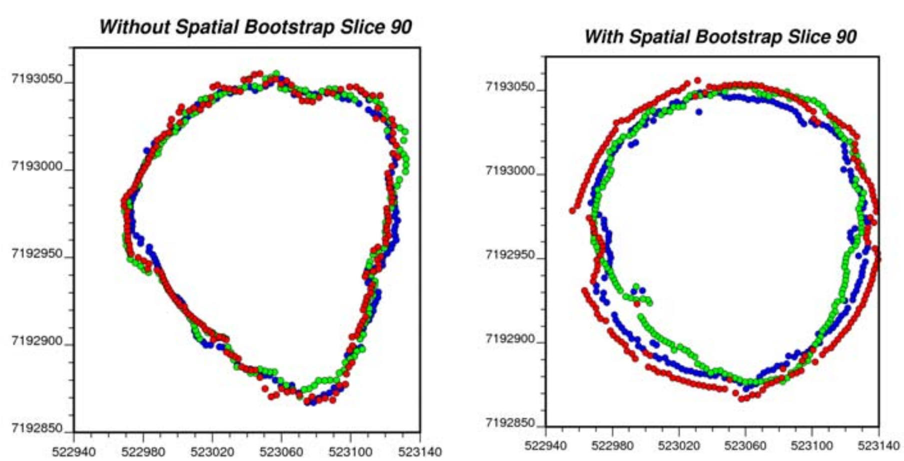


Figure 10: Horizontal slice through realizations. Red = p_{90} realization. Green = p_{50} realization. Blue = p_{10} realization.

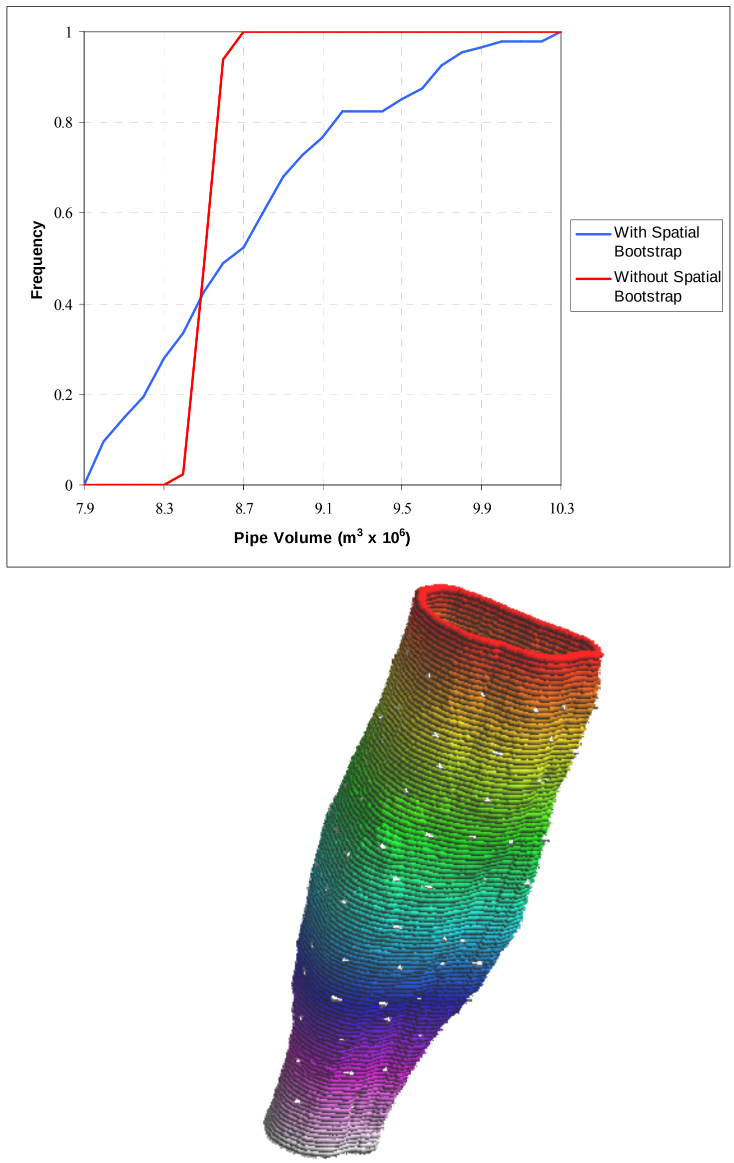


Figure 11: Left CDF of 200 realizations of the pipe volume. There is a much larger range when considering the spatial bootstrap. Right: 3D view of the average pipe. All 200 realizations of *r* are averaged to obtain this model. The data locations (pierce points) are shown gray.

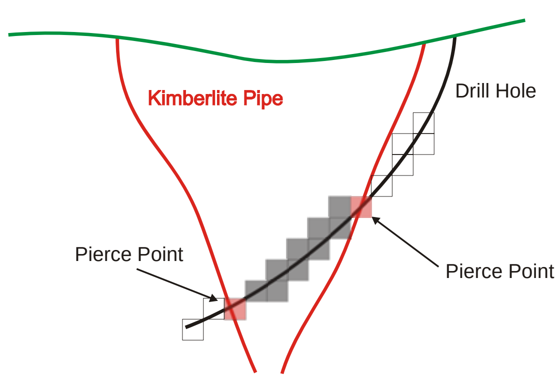


Figure 12: It is known that the white blocks are outside the pipe and the gray blocks must be kimberlite.