On the Challenge of Estimating Recoverable Reserves with Continuous Variable Sequential Indicator Simulation

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Indicator Kriging or Multiple Indicator Kriging received significant attention as a non-linear approach to estimate recoverable reserves. The basic idea is to discretize the range of variability and directly predict the conditional distribution at unsampled locations. These point-scale distributions are sometimes corrected to account for a selective block size, which provides a direct estimate of recoverable reserves. A related idea is sequential simulation, which is a well-established paradigm of simulation. A multivariate distribution is sampled via sampling a succession of conditional distributions. Sequential Indicator Simulation (SIS) was proposed in the 1980s as a flexible simulation approach to categorical and continuous variables. The conditional distributions are built with indicator kriging. Simulation is done at a point scale: no volume variance change is applied.

There are significant problems with SIS. The direct estimation of cumulative probability values does not necessarily lead to licit estimates – there are order relation deviations. These accumulate over grid nodes in SIS and lead to problems. The distributions must be extrapolated into grade ranges higher than the highest threshold, which introduces problems. There is no resolution within the different grade ranges, which leads to unwarranted noise. There is no correlation between different grade ranges, which also leads to unwarranted noise. The additional noise leads to SMU-scale block values that are smoother than they should be. The combination of these problems create a significant challenge in the use of SIS for recoverable reserves.

Introduction

The estimation of recoverable reserves is one of the most important problems faced by mining geostatisticians during the evaluation of a project. Consider a selective mining unit (SMU) size different based on the deposit type, mining equipment and data available at the time of mining for ore/waste discrimination. Recoverable reserves are the tonnage of ore (and waste) and the grade of the ore that will be mined at the SMU scale at some point in the future with more information than presently available. We would also like to predict the uncertainty in the recovered tonnage and grade.

Several techniques have been proposed to evaluate recoverable reserves. A common two-step procedure is to (1) build a block model by an estimation technique like kriging, and (2) apply a cutoff grade to the estimates to assess the number of ore blocks and their grade. This estimation-based approach is problematic with widely spaced exploration drillholes because of the smoothing effect of kriging. Sometimes, the search of kriging is limited so that the histogram of estimates is not overly smoothed. This leads to a global estimate that is more reasonable, but poorer local estimates.

Simulation may be performed with multivariate Gaussian techniques. The normal score or Gaussian transform of the variable is assumed to be multivariate Gaussian distributed in space. There are many techniques to sample this multivariate distribution. The point-scale simulated realizations can be scaled (averaged-up) to the SMU scale and recoverable reserves calculated. The realizations are useful to assess uncertainty. There are cases, however, where the multigaussian model is deemed inappropriate and alternative techniques are considered.

Indicator based estimation and simulation have been proposed as an alternative to traditional kriging and Gaussian simulation, respectively. Indicator kriging is used to build local conditional distributions at every location without a prior assumption about their shapes (Goovaerts, 1994). A change of support model can

be applied and recoverable reserves inferred. Sequential indicator simulation (SIS) applied to continuous variables, such as metal grades, is aimed at generating realizations that reproduce the global grade distribution (histogram) and the spatial continuity at different cut-offs, locally conditioned by the sample data. This procedure has advantages over Gaussian simulation since it permits the straightforward integration of secondary soft data and provides increased flexibility to account for different patterns of spatial continuity at different cut-offs (Deutsch and Journel, 1998), whereas parametric methods impose a particular spatial continuity for different thresholds given the spatial law.

These advantages have contributed to the popularity of indicator based techniques. In addition to the estimation of recoverable reserves, these techniques can be used for evaluating the probability of exceeding a particular threshold over the deposit. Indicator simulation has been deemed suitable for modeling deposits where grades at different classes have different spatial behaviour, such as a high spatial connectivity of high grades, and where mixed populations are present in conditions that do not allow a clear separation of these populations into different domains.

Although the indicator approach appears to have many advantages, there are several practical and theoretical difficulties associated to the indicator variograms modelling and the construction of the local distribution by indicator kriging, exist (Chilès and Delfiner, 1999; Christakos, 2000; Emery and Ortiz, 2004). In the context of the estimation of recoverable reserves the most relevant challenges of indicator simulation are: (1) uncontrolled transitions between classes, and (2) randomness within classes.

In this paper, we review the application of indicator simulation for continuous variables and discuss the main difficulties in its application. The next section presents a brief recall of the indicator approach, followed by a discussion on some of its issues. Potential solutions, particularly concerning the reserves estimation related difficulties, are proposed and their limitations discussed. The paper concludes with a discussion on the use of sequential indicator simulation for grade modeling.

Review of Sequential Indicator Simulation

The aim of indicator kriging is to estimate the conditional distribution at an unsampled location **u** from the available *n* samples in the neighbourhood. Then, by knowing the conditional distribution, estimates, confidence intervals and simulated values at point and block support can be generated.

Simulation of a continuous variable by the sequential indicator method is performed by drawing a value from a ccdf and using it as a conditioning value for subsequently simulated nodes, as is done in the Gaussian framework.

Indicator geostatistics for continuous variables requires a simple coding of the grades $Z(\mathbf{u})$ in a set of 0's and 1's depending if a particular cutoff z_k is exceeded or not (Journel 1983, Deustch and Journel 1998) :

$$I(\mathbf{u}; z_k) = \begin{cases} 1 & if\ Z(\mathbf{u}) \le z_k \\ 0 & if\ Z(\mathbf{u}) > z_k \end{cases}$$

The coding is performed for a set of thresholds $z_k, k = 1,...,K$ in order to discretize the conditional distributions. The same coding can be applied to take into account soft data, where the value is unknown for some ranges.

The direct experimental variogram of each indicator coded variable can then be calculated by the method of moments for a given sample separation **h** and considering $N(\mathbf{h})$ pairs of data approximately separated by such distance:

$$2\hat{\gamma}_I(\mathbf{h}; z_k) = \frac{1}{N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} \left[I(\mathbf{u}_\alpha; z_k) - I(\mathbf{u}_\alpha + \mathbf{h}; z_k) \right]^2$$

If the stationarity hypothesis is assumed, the expected value of the indicator transform is equivalent to the global cumulative distribution function (cdf) of the variable $Z(\mathbf{u})$, regardless the location **u** (Goovaerts, 1994):

$$E\left[I(u;z_k)\right] = \Pr ob\left\{Z(u) \pounds z_k\right\} = F_Z(z_k)$$

Using simple kriging and provided the models of the indicator direct variograms or covariances are known, this global cdf can be conditioned by the *n* surrounding data values at any location **u** and for several cut-offs, thus the conditional cumulative distribution becomes (ccdf):

$$F(\mathbf{u};z_k \mid (n)) = \text{Prob}\left\{Z(\mathbf{u}) \le z_k \mid (n)\right\} \quad k = 1....K$$

The simple indicator kriging (SIK) system of equations, for each cutoff $z_k, k = 1,...,K$, is expressed as:

$$\sum_{\beta=1}^{n} \lambda_\beta(\mathbf{u}_\alpha;z_k)\cdot C_I(\mathbf{u}_\alpha - \mathbf{u}_\beta;z_k) = C_I(\mathbf{u}_\alpha - \mathbf{u};z_k) \qquad \forall \alpha = 1,...,n$$

The resultant SIK weights $\lambda_\beta(\mathbf{u}_\alpha;z_k)$ are then used to estimate the conditional local probabilities as (Deutsch and Journel, 1998):

$$F^*_{SIK}(\mathbf{u};z_k \mid (n)) = I^*(\mathbf{u};z_k) = \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u};z_k)\cdot I(\mathbf{u}_\alpha;z_k) + \left[1 + \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u};z_k)\right]\cdot F(z_k)$$

Inference of these indicators at different cut-offs yields the discretized conditional distribution shown in Figure 1. Since the full distribution is needed for simulation, interpolation of the ccdf between thresholds and extrapolation below the lowest and above the highest threshold are needed.

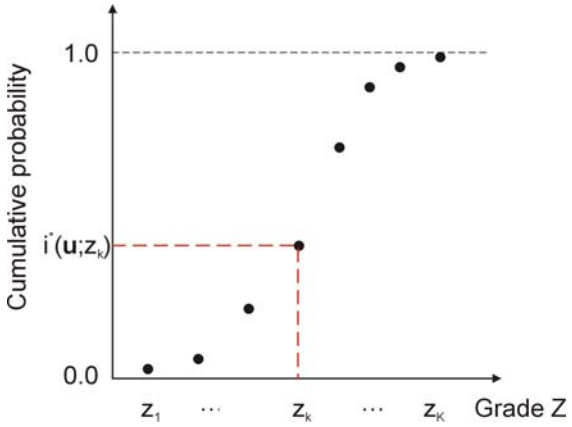

Figure 1: Discretized conditional cumulative distribution function at a given location u. The estimated indicator value for a cutoff z_k represents the estimated cumulative probability at that cutoff.

With the full ccdf, it is easy to calculate an estimate, infer a confidence interval and simulate a grade value. In the case of sequential indicator simulation, the procedure consists in visiting all uninformed nodes randomly within a grid and, at every location, inferring the distribution conditional to the available sample data and the previously simulated grid nodes. From this distribution a simulated value is drawn by Monte-Carlo simulation and its value is used to condition all subsequently simulated nodes. Multiple realizations can be obtained by changing the random order in which the nodes are visited and the drawing from each conditional distribution.

The conventional application of indicator simulation does not consider accounting for the correlation that exists between thresholds, generating realizations without the continuity that must exist between high and low valued zones. Furthermore, within a particular class, the drawing of the simulated value is done randomly, adding artificial noise to the final result.

Challenges of Sequential Indicator Simulation for Ore Reserve Estimation

Most textbooks and several papers discuss some of the problems that sequential indicator simulation has. A brief description of these problems and the correspondent proposed solutions is presented next

- The estimated indicators may not be consistent in that the estimated cumulative probability values may be less than zero, above one or decrease. These **order relation deviations** are explained by the independent estimation of each indicator and the unconstrained optimization in the kriging of each indicator. The problem of order relations deviations can be corrected by simple methods (Deustch and Journel, 1998).

- The resulting simulated models are highly **sensitive to the extrapolation** of the higher tail. Biased results are easily obtained and there is a risk of transferring this bias when a change of support is performed (Emery and Ortiz, 2004). Good practice is to check and validate the resulting numerical models to ensure they properly reproduce the sample data, global histogram and indicator variograms. The definition of the extrapolation of the higher tail can be addressed by using some parametric shape of the tail up to a reasonable maximum (e.g. hyperbolic or power model) and finding by trial and error the value that better reproduces the global distribution and the relevant statistics of the population (declustered mean and variance).

- The indicator coding carries a **loss of information** since, rather than knowing the exact value of a sample, after the coding, only the class at which it belongs is known. This loss is not high if the variable is Gaussian, and it is compensated by the superior resistance of this approach to outliers (Solow 1993). However it can be demonstrated that this loss of information can be considerably high if samples with values close to a cut-off are very close to the estimated location. Moreover, the local ccdf can change if a new sample location is added even when its value is not known (Emery and Ortiz, 2004).

In addition to these problems, an important weakness is that models often do not look geologically realistic. This can be explained because (1) indicator kriging does not take into account the correlations between cut-offs and (2) within classes the values are independent from each other.

By not taking into account the correlation between cut-offs and due to the random drawing of the simulated values within the corresponding class, sequential indicator simulation realizations show a spatial persistence of unstructured short scale intermixing of low and high values. Geologically unrealistic adjacent patches of high and low values are characteristic of indicator simulation realizations. Rather than just being a visually and geologically unappealing feature, it leads to a reduced variability when block averages are calculated to Selective Mine Units (SMU) support. This is caused by the homogenization of the intermixed low and high values when they are averaged to larger volumes and it can considerably underestimate the uncertainty in the recoverable reserves, as it is demonstrated in the next section.

If simple or ordinary kriging is used to build the ccdf no information regarding the interclass spatial correlation is introduced. Thus from one location to another the ccdf can change abruptly according to the proximity of samples or previously simulated values corresponding to different classes. This results in the characteristic uncontrolled adjacent presence of different class values in SIS realizations. An obvious solution is to use an indicator co-kriging to calculate the ccdfs. However, deriving the full matrix of indicator direct and cross variograms required for indicator co-kriging leads to other difficulties related to the validity of the coregionalization model needed; this problem is presented and discussed below.

The second unwarranted issue, the randomness within classes, is produced by the independent drawing of simulated values on the indicator kriging ccdf.

Case study: the effect of abrupt transitions and randomness within classes in uncertainty assessment of SMU grades

As explained above, the excess variability in SIS realizations yields to an underestimation of the uncertainty in the recoverable reserves when the simulated nodes are upscaled to the SMU support. A real data example is presented in order to illustrate this.

The data subset was taken from a diamond drilling exploration campaign of a Chilean copper deposit; it is comprised of 180 samples corresponding to a single bench and within a single lithotype domain (Tourmaline Breccias). Figure 2 presents the sample location and histogram.

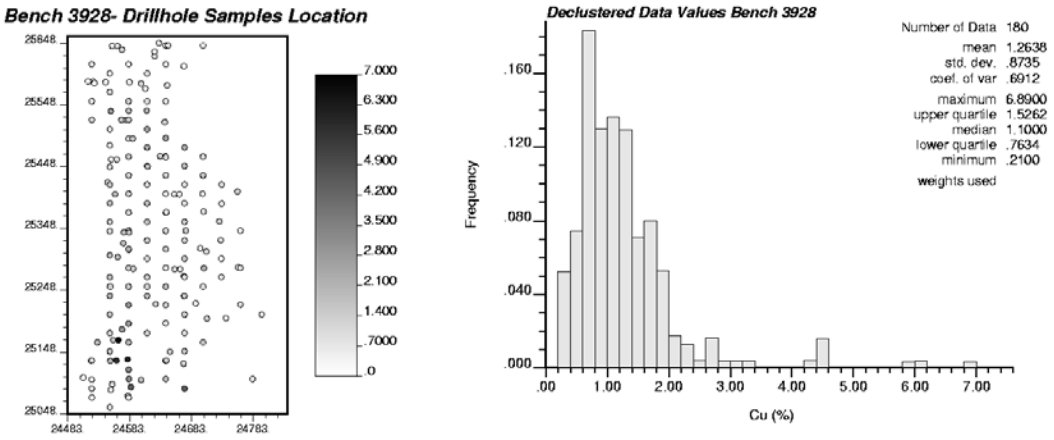

Figure 2: data locations and distribution used for illustrate the challenges of SIS

Nine thresholds were defined coinciding with the deciles of the distribution. The experimental indicator variograms for all these cut-offs were generated and, in order to account for variogram uncertainty, each one was fitted with three different variogram models: a low continuity model, a high continuity model and a best fit model with mid continuity. These correspond, respectively, to the highest nugget effect with the shorter range model the lowest nugget effect with the longest range model and the average model that can be reasonably fitted to the experimental indicator variograms generated. Additionally three variograms models are fitted to the continuous variable under these same criteria.

Three sets of 100 Sequential Gaussian Simulation (SGS) realizations and three sets of 100 SIS realizations are generated using the corresponding minimum, middle and maximum continuity variogram models in a 2m x 2m x 2m grid. These point support realizations are averaged to a SMU size of 12m x 12m x 12m. Figure 3 presents point support sample realizations maps and histograms for both SGS and SIS and Figure 4 presents the same graphs for sample SMU scale average realizations. Only the realizations using the mid continuity model are shown.

Both SIS and SGS reproduce the input histogram very well at the point support scale, and honour the low and high grade zones, however the SIS realizations present the characteristic patches of different class values mentioned above.

When averaged to a larger SMU size, SGS realizations clearly preserve the distribution and structure of high and low grade areas in concordance with the original sample values, but this differentiation is much less clear in the SIS realizations, which show an increased smoothing of values.

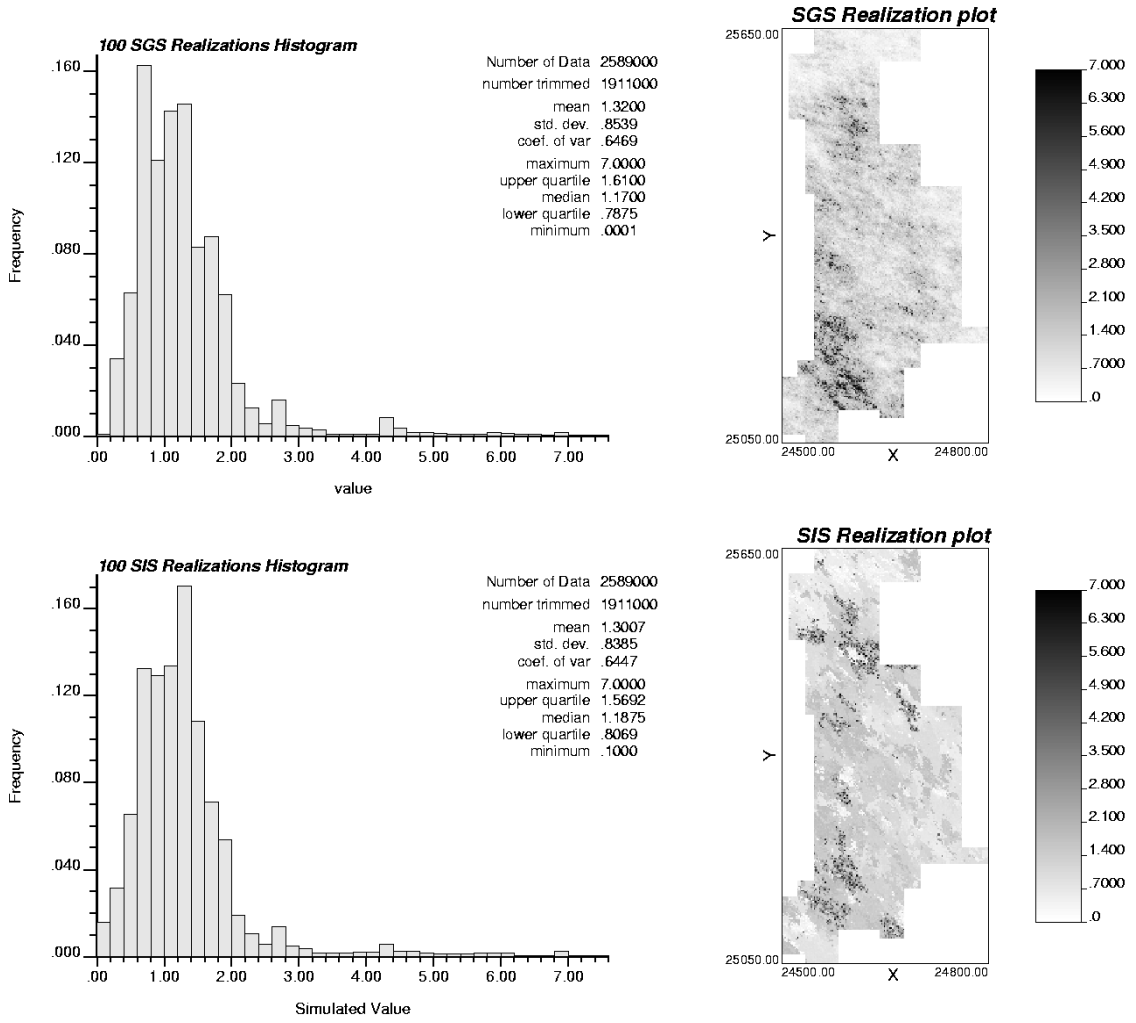

Figure 3: SGS (on top) and SIS (on bottom) realizations maps and histograms at a 2m x 2m x 2m grid size.

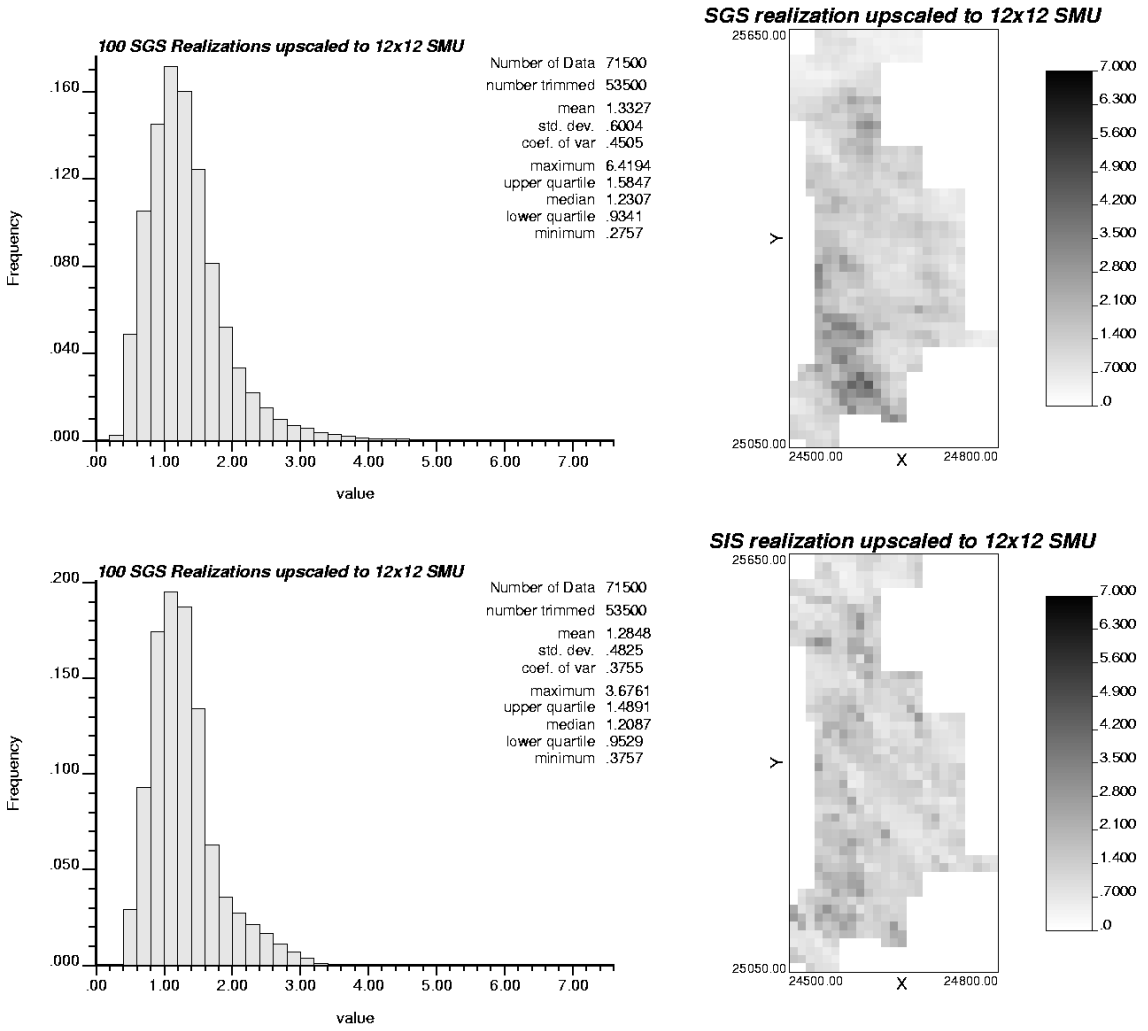

Figure 4: Upscaled SGS (on top) and SIS (on bottom) realizations maps and histograms to a 12m x 12m x1 2m SMU size.

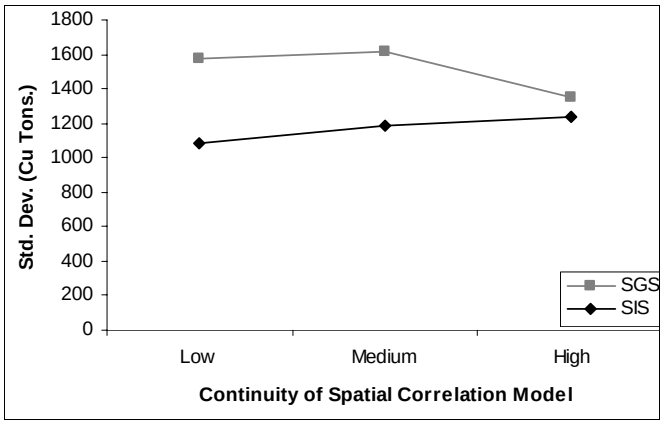

Figure 5: Comparative uncertainty in the total metal content for SGS and SIS realizations.

Overall uncertainty can be quantified by the standard deviation of the average grade above cut-off, for the proportion of recoverable tonnage or the recoverable metal content obtained from the multiple realizations. Here only the standard deviation of the recoverable metal content is analysed. Figure 5 presents the standard deviation of the recoverable metal content for the SMU scale SGS and SIS realization at different levels of spatial continuity in the variogram model. The uncertainty in the recoverable metal content is lower for the SIS realizations; it can be as much as 30% lower for this particular case and slightly increases as the variogram continuity increases due to a gain in the structure and continuity of SIS realizations as the nugget effect diminishes and the variogram range becomes larger.

Thus this lower uncertainty in the SIS at the SMU scale does not correspond to a better representation of the grades distribution, but to the loss of spatial structure in the short scale, which translates in the increased smoothness at the SMU scale, lowering the uncertainty on the recoverable metal for SIS results. Intuitively, one would expect the opposite: The higher the spatial continuity, the lower the uncertainty in the transfer function. SGS results suit better this expected behaviour: uncertainty in the recoverable metal content decreases slightly as variogram continuity increases.

The full indicator Co-kriging approach

In order to mediate the uncontrolled interclass transitions and its effects, the most direct procedure would be to include the interclass correlation information via indicator cross-variograms and implementing the indicator co-kriging for building the ccdfs required by SIS. However this approach leads to serious difficulties related to the validity of the coregionalization model required as shown next.

Gaussian based methods, as sequential gaussian simulation, require the assumption of multigaussianity, that is, not only the univariate (one-point) distribution must be Gaussian, but also the bivariate (two-point) distribution and all higher order distributions must be muli-Gaussian. A common check for the assumption of bigaussianity is to compare the experimental indicator variograms of the original or, equivalently, Gaussian transformed grades, to the theoretical indicator variogram derived from a perfectly multigaussian distribution that corresponds to the variogram of the Gaussian transformed grades. This check is usually performed only on the direct indicator variograms, however it is worthwhile to assess if the full co-kriging approach is applicable to a perfectly bigaussian variable.

Given two different Gaussian transformed cut-offs, y_k and $y_{k'}$ the experimental indicator cross variogram can be calculated as:

$$2\hat{\gamma}_I(\mathbf{h}; y_k, y_{k'}) = \frac{1}{N(\mathbf{h})}\sum_{\alpha=1}^{N(\mathbf{h})}\left[I(\mathbf{u}_\alpha; y_k) - I(\mathbf{u}_\alpha + \mathbf{h}; y_k)\right]\left[I(\mathbf{u}_\alpha; y_{k'}) - I(\mathbf{u}_\alpha + \mathbf{h}; y_{k'})\right]$$

The theoretical Gaussian indicator cross variogram can expressed analytically as:

$$\begin{aligned} 2\gamma_I(\mathbf{h}; y_k, y_{k'}) = {} & 2\min(p, p') - 2p \cdot p' \\ & - \frac{1}{2\pi}\int_0^{\arcsin C_Y(\mathbf{h})} \exp\left[-\frac{y_k^2 + y_{k'}^2 - 2y_k y_{k'}\sin\theta}{2\cos^2\theta}\right] d\theta \\ & - \frac{1}{2\pi}\int_0^{\arcsin C_Y(-\mathbf{h})} \exp\left[-\frac{y_k^2 + y_{k'}^2 - 2y_k y_{k'}\sin\theta}{2\cos^2\theta}\right] d\theta \end{aligned}$$

Where p and p' are the quantiles corresponding to the y_k and $y_{k'}$ cut-offs, respectively, and $C_Y(\mathbf{h})$ is the covariance of the Gaussian variable for the vector **h**. The analytical solution of this equation is quite complicated, therefore a FORTRAN program based in a Monte Carlo simulation of the bigaussian distribution was developed to derive the theoretical indicator cross variograms.

If K cut-offs are considered, the K^2 experimental indicator direct and cross variograms must be simultaneously fitted in order to construct the $K \times K$ variogram matrix:

$$\Gamma_I(\mathbf{h}) = \begin{bmatrix} \gamma_I(\mathbf{h}; y_1, y_1) & \cdots & \gamma_I(\mathbf{h}; y_1, y_K) \\ \vdots & \ddots & \vdots \\ \gamma_I(\mathbf{h}; y_K, y_1) & \cdots & \gamma_I(\mathbf{h}; y_K, y_K) \end{bmatrix}$$

This matrix must be positive semi-definite to ensure that the variance of any linear combination of the transform $I(\mathbf{u}; y_k)$ is non negative (Journel and Huijbregts, 1978). The Linear Model of Coregionalization (LMC) is the only model available that fulfills this condition by modelling all the direct and cross variograms as a linear combination of a limited number of variogram functions (Goovaerts; 1994, 1998):

$$\Gamma_I(\mathbf{h}) = \sum_{l=1}^{L} B^l \cdot g_I(\mathbf{h})$$

Where $g_I(\mathbf{h})$ is a permissible variogram model with standardized sill, and B^l is a $K \times K$ matrix containing the sill contributions, $b^l_{pp'}$, of the model $g_l(\mathbf{h})$ for each indicator direct and cross variogram. In order to assure the positive definiteness of the LMC, the matrix B^l must be positive semi definite and the same permissible variogram models, $g_l(\mathbf{h})$, must be fitted to direct and cross variograms.

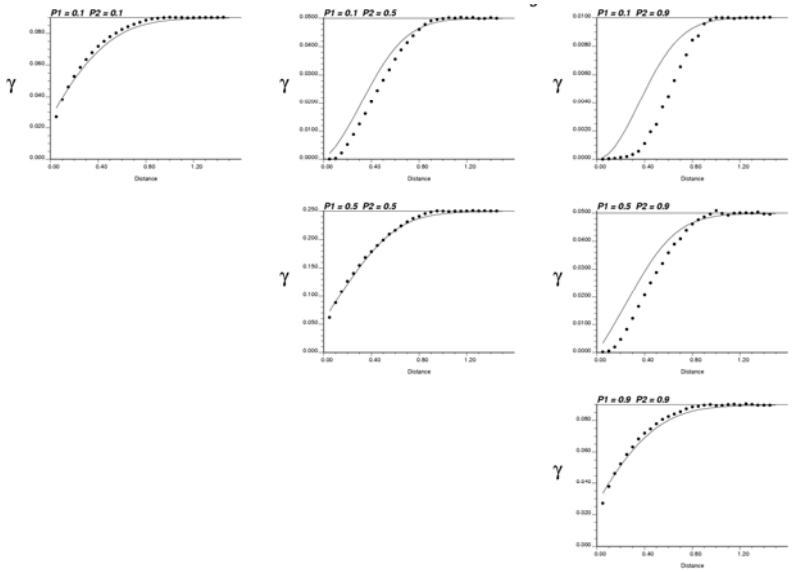

Figure 6: Theoretical bigaussian derived indicator direct and cross variograms (dots) and an attempt to fit them with a LMC (continuous lines)

Figure 6 presents the upper side of theoretical indicator direct and cross variograms symmetrical matrix derived for three cut-offs corresponding to the deciles p10, p50 and p90 of a variable with multigaussian distribution and a spherical variogram of range 1 without nugget effect, in the same graph the attempt to fit a permissible LMC is also presented.

The most important feature in this matrix is the extreme continuity at short scale that indicator cross-variograms of the most divergent cut-offs present. This extreme continuity cannot be modeled by any permissible variogram model, and can be explained by the fact that the indicator cross variogram increases only if a simultaneous class transition from the tail to the head of the separation vector **h is** registered for both cut-offs. This event becomes rarer as cut-offs diverge, and at short distances.

The lack of a valid variogram model to fit the extreme thresholds indicator cross-variograms is the major hindrance to the adequacy of the LMC to fit the complete variogram matrix. The second difficulty is to apply the same set of variogram models to the changing shapes of the indicator direct and cross variograms as the cut-offs diverge.

It could be argued that this extreme continuity for divergent cut-offs is characteristic only for a multigaussian variable, where extremely large and small values are spatially uncorrelated (Goovaerts, 1997), but, as it is shown in figure 7, it also can appear in the indicator cross variograms of real cases non-Gaussian variables such as the copper grades of the referred Chilean deposit. Therefore, in many cases, the LMC is not suitable to fit the complete matrix of non-gaussian real data indicator direct and cross-variograms.

Adjacent cutoff indicator kriging

As the LMC is inadequate for fitting the full indicator variogram matrix, a practical, but partial, alternative is to use the adjacent Cut-offs Indicator Cokriging (acoIK) for introducing the interclass correlation information in the SIS algorithm.

For each one of the *K* cut-offs, the correspondent ccdf value estimation by acoIK requires only the LMC of the indicator direct and cross variograms corresponding to the adjacent thresholds y_{k_0-1}, y_{k_0} and y_{k_0+1}, that is, two 2 x 2 variogram matrices for the first and last cut-offs, and *p*-2 3 x 3 matrices for the intermediate cut-offs. The modeling of each one of these matrices is performed independently, although some consistency between the resultant *K* LMC should be kept. A satisfactory fitting of the LMCs can be obtained, without the hindrances of fitting a single full variogram matrix.

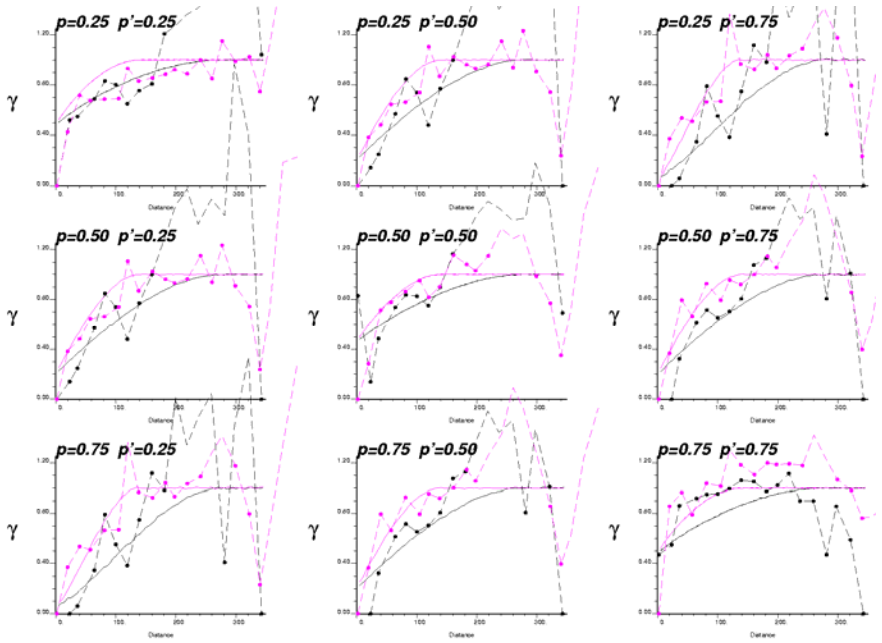

Figure 7: Experimental indicator direct and cross variograms (dashed lines) and the bigaussian derived variograms (continuous lines)

The acoIK system is similar to the full coIK (Goovaerts, 1994):

$$\sum_{k'=k_0-1}^{k_0+1} \sum_{\beta=1}^{n} \nu_{\beta,k'}(\mathbf{u}_\beta; y_{k_0}) \cdot C_I(\mathbf{u}_\alpha - \mathbf{u}_\beta; y_k, y_{k'}) = C_I(\mathbf{u}_\alpha - \mathbf{u}_0; y_k, y_{k_0})$$

$$\forall \alpha = 1 \text{ to } n, \quad k = k_0 - 1 \text{ to } k_0 + 1$$

And the estimate is calculated by:

$$F^*_{acoIK}(\mathbf{u}_0; y_{k_0} \mid (n)) - F(y_{k_0}) = \sum_{k'=k_0-1}^{k_0+1} \sum_{\alpha=1}^{n} \nu_{\alpha,p}(\mathbf{u}_\alpha; y_{k_0}) \cdot \left[I(\mathbf{u}_\alpha; y_{k'}) - F(y_{k'}) \right]$$

Order relation issues are expected after the ccdf estimation for all cut-offs, but they can be corrected in a similar way as with indicator Kriging. Once a valid ccdf is built, it can be used to draw a random number, which should carry the information of the inter-class correlation if the acoIK is implemented in the indicator simulation algorithm.

Although the restricted adjacent cut-offs matrix of indicator direct and cross variograms can be fitted satisfactorily with a LMC, the adjacent cut-offs kriging do not improve considerably the disordered interclass transitions observed in SIS.

Discussion

The indicator approach can be a useful alternative when dealing with variables that do not fulfill the multigaussian assumption, for variables that exhibit different patterns of spatial correlation at different cut-offs, and for incorporating secondary information. However, indicator simulation suffer not only of the hindrances related to non parametric ccdf built by indicator Kriging, but the use of Monte Carlo simulation on this ccdf, brings also other difficulties that yield to a considerable underestimation of the uncertainty in SIS realizations.

Issues like order relation deviations and the extrapolation of the ccdf lower and upper tails can be solved reasonably. The loss of information related to the indicator coding can be palliated using a large number of thresholds; however this requires an increased effort in generating and modeling the indicator variograms for each of these cut-offs.

Using a full indicator cokriging approach could introduce some structure to the interclass transitions by informing about the correlation between all cut-offs, but this solution faces the inadequacy of the LMC, the only model of coregionalization available, for fitting the full matrix of indicator direct and cross variograms, which shows an unusual high continuity for indicator cross variograms of extreme cut-offs. The adjacent cut-offs approach provides a partial solution, since close cut-offs yield to indicator direct and cross variograms that can be satisfactorily fitted by the LMC .

The randomness within classes remains unaddressed, the introduction of the distance to the class boundary for each simulated node has been proposed to solve this problem, but its practical implementation is still pending.

In view of these challenges, the application of the SIS algorithm, despite its advantages over parametric methods, remains very limited in Recoverable Reserves evaluation, being the mayor issue the excess variability present in SIS realizations, this leads to an increased smoothness when the point support simulations are upscaled to the SMU size causing a misleading underestimation of the uncertainty in the reserves.

However, adjacent cut-offs indicator simulation still has a place in mineral reserves evaluation, it can be used for discriminating between different populations with a locally complex spatial distribution, previously to their separate simulation with a Gaussian based algorithm (Journel and Kyriakidis, 2004)

Finally the indicator approach is still useful for categorical variables mapping, as it can introduce different continuity patterns for different lytotipes, and no upscaling is needed. The irregularities the high short scale variability in categorical variables realizations can be easily eliminated using image cleaning methods.

Conclusions

Sequential indicator simulation is affected by the difficulties carried by the Indicator Kriging built ccdf. Although most of these drawbacks can be reasonably solved or minimized for the Kriging framework,

when translated into the simulation framework two main challenges arise: the uncontrolled interclass transitions and the randomness within classes.

These challenges cause the geologically unappealing “patchiness” present in the SIS realizations at point support, an increased smoothness at SMU size and the misleading low uncertainty in the reserves.

The attempt to solve the first of these challenges using a full indicator co-kriging approach are restricted by the inadequacy of the lineal model of correlation to fit the complete matrix of indicator direct and cross variograms.

The adjacent cut-off’s alternative provides only a partial solution. And the randomness within classes still unaddressed.

Thus, the application of indicator simulation is limited for recoverable reserves evaluation, despite its several advantages over parametric methods.

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