Testing for the Multivariate Gaussian Distribution of Spatially Correlated Data

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Most geostatistical simulation is based on an assumption that the variable is multivariate Gaussian after a univariate normal scores transform. It is interesting to test how far the data depart from the multivariate Gaussian distribution; the decision of stationarity could be reconsidered or a different multivariate distribution considered. Tests for multivariate Gaussianity, however, require data independence, which is rarely the case in geostatistical modeling. Different techniques are reviewed and a new testing methodology is developed. This test identifies the correct number of failures in a multivariate Gaussian setting and provides a measure of how far real data depart from being multivariate Gaussian.

Introduction

Geostatistical simulation is a useful tool for modeling variables that cannot be described deterministically due to their inherent complexity. The most common simulation approach is Gaussian simulation. Each variable is transformed to a Gaussian distribution. This ensures a univariate Gaussian distribution of each variable; then, an assumption of multivariate Gaussian distribution is made. Real multivariate distributions are not likely multivariate Gaussian and show such non-Gaussian features as non-linearity and heteroscedasticity. In this case, Gaussian simulation may not reproduce important aspects of the spatial variability of the phenomenon under study. This could result in biased predictions. Therefore, a procedure for testing how far the data depart from a multivariate Gaussian distribution is of great practical interest.

A number of tests for departures from the multivariate Gaussian distribution in a spatial data context are reviewed. The theory behind each test is developed and practical implementation is discussed. A test is proposed that is based on a univariate test after data orthogonalization. This test is shown to be fair, that is, the number of falsely rejected tests matches perfectly the confidence level of the test. The performance of the tests is illustrated using real petroleum reservoir data.

Testing for a Multivariate Gaussian Distribution

Consider n data values $Y(\mathbf{u}_1), Y(\mathbf{u}_2), \ldots, Y(\mathbf{u}_n)$ at locations $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ that have been normal score transformed. We would like to test the assumption of a multivariate Gaussian distribution between these data. Specifically, we would like to determine if an n by 1 vector of data $Y = [Y(\mathbf{u}_1)\ Y(\mathbf{u}_2)\ \ldots\ Y(\mathbf{u}_n)]^T$, where $Y(\mathbf{u}_i),\quad i = 1,\ldots,n,$ are normal scores, is n-variate Gaussian with mean of zero and n by n variance-covariance matrix $\mathbf{C}$ is given below

$$\mathbf{C} = \begin{bmatrix} Var(Y(\mathbf{u}_1)) & \cdots & Cov(Y(\mathbf{u}_1), Y(\mathbf{u}_i)) & \cdots & Cov(Y(\mathbf{u}_1), Y(\mathbf{u}_n)) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Cov(Y(\mathbf{u}_i), Y(\mathbf{u}_1)) & \cdots & Var(Y(\mathbf{u}_i)) & \cdots & Cov(Y(\mathbf{u}_i), Y(\mathbf{u}_n)) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Cov(Y(\mathbf{u}_n), Y(\mathbf{u}_1)) & \cdots & Cov(Y(\mathbf{u}_n), Y(\mathbf{u}_i)) & \vdots & Var(Y(\mathbf{u}_n)) \end{bmatrix}.$$

Under the assumption of stationarity the variance-covariance matrix $\mathbf{C}$ is calculated through the normal score transformed data variogram model $\gamma(\mathbf{h})$ and can be rewritten as follows

$$\mathbf{C} = \begin{bmatrix} C(\mathbf{u}_1, \mathbf{u}_1) & \cdots & C(\mathbf{u}_1, \mathbf{u}_i) & \cdots & C(\mathbf{u}_1, \mathbf{u}_n) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C(\mathbf{u}_i, \mathbf{u}_1) & \cdots & C(\mathbf{u}_i, \mathbf{u}_i) & \cdots & C(\mathbf{u}_i, \mathbf{u}_n) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C(\mathbf{u}_n, \mathbf{u}_1) & \cdots & C(\mathbf{u}_n, \mathbf{u}_i) & \vdots & C(\mathbf{u}_n, \mathbf{u}_n) \end{bmatrix},$$

where $C(\mathbf{u}_i, \mathbf{u}_j)$ is the stationary covariance between $\mathbf{u}_i, \mathbf{u}_j \quad i, j = 1, \ldots, n$ (Goovaerts, 1997). Now let us note that

$$Y = [Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{C}),$$

if and only if

$$\tilde{Y} = \mathbf{L}^{-1}[Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{I}),$$

where $\tilde{Y} = [\tilde{Y}(\mathbf{u}_1)\, \tilde{Y}(\mathbf{u}_2) \ldots\ \tilde{Y}(\mathbf{u}_n)]^T$; **I** denotes the identity matrix of size *n* by *n*; $\mathbf{L}^{-1}$ stands for inverse of the lower triangular matrix **L** in the Cholesky decomposition (Golub and Van Loan, 1996) of the variance-covariance matrix **C,** that is, $\mathbf{C} = \mathbf{L}\mathbf{L}^T$.

Thus, to test if

$$Y = [Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{C}),$$

we need to test if variables corresponding to different elements in $\tilde{Y}$ are (1) standard normally distributed and (2) independent (Lehmann, 1999).

Unfortunately, it is impossible to test if each of the elements in vector $\tilde{Y}$ is standard normal because we have only one multivariate observation (spatially correlated sample). We can, however, test if there is strong departure from the multivariate Gaussian distribution. Specifically, if

$$Y = [Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{C}),$$

then vector $\tilde{Y} = [\tilde{Y}(\mathbf{u}_1)\, \tilde{Y}(\mathbf{u}_2) \ldots\ \tilde{Y}(\mathbf{u}_n)]^T$ is univariate standard normally distributed. This follows directly from the fact that if $Y = [Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{C}),$ then each element in $\tilde{Y}$ is standard normally distributed.

Therefore, to test if the data departs strongly from the multivariate Gaussian we can apply Komogorov-Smirnov test (Berry and Lindgren, 1990) to test if $\tilde{Y} = [\tilde{Y}(\mathbf{u}_1)\, \tilde{Y}(\mathbf{u}_2) \ldots\ \tilde{Y}(\mathbf{u}_n)]^T$ is univariate standard normal. If the transformed data $\tilde{Y}$ fails the Komogorov-Smirnov test, then we conclude that our data is strongly non multivariate Gaussian (level I departure from a multivariate Gaussian distribution). However, if the null hypothesis of the standard normality of the $\tilde{Y}$ is not rejected, then we cannot conclude that the data is multivariate Gaussian. It only means that there is no strong departure from the multivariate Gaussian distribution at the particular level of significance. In particular, if our data fail to reject the univariate test for multivariate Gaussianity, we can devise a test for a bivariate Gaussian distribution. We know that if

$$Y = [Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T \sim N(\mathbf{0}, \mathbf{C}),$$

then matrix

$$Y_M = \begin{bmatrix} Y(\mathbf{u}_1) & Y(\mathbf{u}_2) \\ Y(\mathbf{u}_3) & Y(\mathbf{u}_4) \\ \vdots & \vdots \\ Y(\mathbf{u}_{n-1}) & Y(\mathbf{u}_n) \end{bmatrix}$$

is bivariate standard normal. A test for a bivariate Gaussian distribution amounts to a Komogorov-Smirnov test if Y_M is bivariate standard normal. Then if the data fails this test, then it is not multivariate Gaussian normally distributed (level II departure from a multivariate Gaussian distribution); however if the null hypothesis is not rejected it does not mean mean that our data is multivariate Gaussian, it only means that additional testing is required. Higher order tests can be designed in similar fashion.

Depending on the size of the spatially correlated data set at hand, multivariate tests of different orders can be conducted to identify evident departures from a multivariate Gaussian distribution. In general, however, because real spatial data are not multivariate Gaussian in nature, conducting a univariate test for a multivariate Gaussian distribution is likely sufficient. A logical conclusion from the statement that "real data are not multivariate Gaussian" is that testing for a multivariate Gaussian distribution is aimless. However, test results are interesting because they provide a measure of how far the data depart from a multivariate Gaussian assumption. This measure could be used to identify problematic data. Data that depart a great deal from an assumption of multivariate Gaussian distribution could be investigated further. There may be problem data, there may be trends or other non-stationary features, and there may be other techniques better suited to the data at hand. This application will be further developed in Section 5.

Univariate Test of a Multivariate Gaussian Distribution (UTMG)

The test proposed above requires knowledge of the stationary variogram model $\gamma(\mathbf{h})$; however, the true underlying variogram is not known. To account for the fact that the variogram/covariance of the normal score transformed data is unknown, the following modification is considered. Instead of testing if

$$\tilde{Y} = \mathbf{L}^{-1}[Y(\mathbf{u}_1)\,Y(\mathbf{u}_2)\ldots Y(\mathbf{u}_n)]^T \text{ is standard normal}$$

we will test

$$\tilde{Y} = \hat{\mathbf{L}}^{-1}[Y(\mathbf{u}_1)\,Y(\mathbf{u}_2)\ldots Y(\mathbf{u}_n)]^T \text{ is normally distributed}$$

where $\hat{\mathbf{L}}^{-1}$ stands for inverse of the lower triangular matrix $\hat{\mathbf{L}}$ in the Cholesky decomposition of the variance-covariance matrix $\hat{\mathbf{C}}$ obtained based on modeled experimental variogram. No restriction is made on the mean and variance of the normal distribution. The Lilliefors test is (Lilliefors, 1967) used rather than the conventional Kolmogorov-Smirnov test because it accounts for the fact that the underlying statistical parameters are not known.

*The recommended test in this paper will be denoted UTMG for a **U**nivariate **T**est for a **M**ultivariate **G**aussian distribution. The test accounts for spatial correlation and the fact that the variogram is unknown. In summary, (1) normal score transform the data, (2) calculate and fit a variogram, (3) orthogonalize the data according to the covariance matrix, and (4) perform the Lilliefors test on the result.*

A number of small studies are documented to show that the proposed corrections are necessary. Consider 500 samples of 500 data spaced a unit distance apart from a multivariate Gaussian distribution to with spatial correlation. The following isotropic spherical variograms were used:

$$\gamma_1(\mathbf{h}) = Sph_{a=50}(\mathbf{h});$$
$$\gamma_2(\mathbf{h}) = Sph_{a=200}(\mathbf{h});$$
$$\gamma_3(\mathbf{h}) = 0.25 + 0.55Sph_{a=10}(\mathbf{h}) + 0.2Sph_{a=20}(\mathbf{h});$$
$$\gamma_4(\mathbf{h}) = 0.25 + 0.55Sph_{a=50}(\mathbf{h}) + 0.2Sph_{a=100}(\mathbf{h}).$$

Because the data are already multivariate Gaussian, no normal score transformation was done. Tests were conducted at four different levels of significance α, $\alpha = 0.2, 0.1, 0.05$ and 0.01. The number of Komogorov-Smirnov tests that rejected the null hypothesis for each value of α was recorded. The same procedure based on Komogorov-Smirnov tests but with variance-covariance matrix C calculated using modeled experimental variograms was also calculated for comparison. Note that experimental variogram for each of 500 sampled was calculated and modeled separately based on gamv from GSLIB (Deutsch and Journel, 1998) using two spherical variogram structures. Results of the test for each cases are given in Tables 1-4 for all four different variogram models. Tables 1-4 also show respective results of the corrected UTMG (correction is made for the unknown covariance). The results of the corrected univariate test of multivariate Gaussian distribution shown in Tables 1-4 were obtained using Lilliefors test of normality (Lilliefors, 1967) conducted using modeled experimental variograms.

Note that in the case of a known variogram model, the Komogorov-Smirnov test results are very close to that predicted by theory; however, when the variogram model is unknown, then results of Komogorov-Smirnov test are usually very different from the theoretical expectation. Results of the Lilliefors test, on the other hand, are very close to the theoretical expectation for the unknown variogram model. The results of the Komorov-Smirnov test and Lilliefors test were similar when modeled experimental variogram were close to true variogram models used in simulation, see Tables 3 and 7.

Tables 5-8 show results of the simulation study for 500 samples of 200 data each. The same variogram models as before were used in simulation. The conclusions of the analysis of Tables 5-8 are the same.

Table 1: Number of rejected tests for 500 samples of size 500 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_1(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	111	317	106
90%	50	66	252	58
95%	25	33	211	30
99%	5	3	121	3

Table 2: Number of rejected tests for 500 samples of size 500 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_2(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	112	415	102
90%	50	47	380	55
95%	25	20	345	27
99%	5	3	288	1

Table 3: Number of rejected tests for 500 samples of size 500 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_3(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	108	77	117
90%	50	49	41	65
95%	25	23	23	40
99%	5	4	3	7

Table 4: Number of rejected tests for 500 samples of size 500 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_4(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	101	170	114
90%	50	54	105	62
95%	25	30	65	29
99%	5	7	27	3

Table 5: Number of rejected tests for 500 samples of size 200 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_1(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	106	287	111
90%	50	54	226	65
95%	25	31	167	29
99%	5	4	104	4

Table 6: Number of rejected tests for 500 samples of size 200 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_2(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	102	415	101
90%	50	55	380	57
95%	25	29	344	21
99%	5	7	288	1

Table 7: Number of rejected tests for 500 samples of size 200 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_3(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	116	91	103
90%	50	55	52	63
95%	25	30	27	26
99%	5	7	8	2

Table 8: Number of rejected tests for 500 samples of size 200 data each from multivariate Gaussian distribution. Results were obtained based on $\gamma_4(\mathbf{h})$.

Confidence level	Theoretical	Komogorov-Smirnov test with known $\gamma(\mathbf{h})$	Komogorov-Smirnov test with unknown $\gamma(\mathbf{h})$	Lilliefors test (with unknown $\gamma(\mathbf{h})$)
80%	100	86	204	79
90%	50	47	141	45
95%	25	25	101	25
99%	5	3	57	2

The Multivariate Aspect of the UTMG

The data being tested are univariate Gaussian by design (Deutsch, 2002). The univariate test of a multivariate Gaussian distribution (UTMG) truly tests the multivariate distribution. Consider a small example to illustrate this fact.

A string of 613 data is available for analysis. The data in the string are equally sampled with a distance of 0.1 meters between the samples. The histogram of the data is shown in Figure 1. The univariate distribution of the data is non-Gaussian. Figure 1 also shows the histogram of the normal score transformed data; the distribution of the normal score transformed data is perfectly Gaussian (p-value of the

Komogorov-Smirnov test is 1). The variogram of the normal score transformed data is shown in Figure 2. The modeled experimental variogram is given below:

$$\gamma(\mathbf{h}) = 0.25 + 0.55 Sph_{a=1}(\mathbf{h}) + 0.2 Sph_{a=20}(\mathbf{h}).$$

Figure 2 also shows the variogram of the $\tilde{Y}$ data obtained by removing the spatial structure from the normal score transformed data. It can be clearly seen from Figure 2 that observations $\tilde{Y}$ are independent; the variogram model of $\tilde{Y}$ is pure nugget. Figure 3 shows the histogram of $\tilde{Y}$. It is clear from Figure 3 that these modified values are not univariate Gaussian. The Lilliefors statistic for testing univariate normality at significance level $\alpha = 0.01$ is 0.1028 (critical value is 0.0446), p-value is <<0.0001. Thus, the data are strongly not multivariate Gaussian; however, the univariate distribution of the normal score transformed data is perfectly Gaussian.

Software Implementation

A program called testing_MG was specifically prepared to test strong departures from multivariate Gaussianity using UTMG. The software implementation of this program is consistent with all the FORTRAN programs of GSLIB group (Deutsch and Journel, 1998). The parameter file for program testing_MG is presented below:

```
                  Parameters for Testing MG
                  *************************

START OF PARAMETERS:
100data.dat                       -file with normal score transformed data
1  2  0  3                        -columns for X,Y,Z,var
-1.0e21   1.0e21                  -trimming limits
0.05                              -level of significance for the test
1    0.2                          -nst, nugget effect
1    0.8  0.0   0.0   0.0         -it,cc,ang1,ang2,ang3
         20.0  20.0  10.0         -a_hmax, a_hmin, a_vert
```

A run of the program testing_MG creates an output directly in the output window with the following information: reject or do not reject the null hypothesis; critical value of the Lilliefors test and Lilliefors statistic. Note that the data inputted to the program testing_MG must be normal score transformed.

Selecting Data with the UTMG

The UTMG has one very interesting application. This application allows us to measure how real data depart from a multivariate Gaussian distribution. We do not expect any of the data (even after normal score transformation) to be multivariate Gaussian; however, the UTMG can rank data according to the Lilliefors statistic. The data with lowest value of the Lilliefors statistic can be thought of most closely resembling the multivariate Gaussian distribution. Consider a small case study.

Five strings of 900 data each are available for analysis. The data in the strings are equally sampled with distance of 0.1 meters between the samples. The histograms of the data are shown in Figure 4. Looking at Figure 4 we can note that all data sets exhibit different non-Gaussian features, therefore normal score transformation is employed to make each data set univariate Gaussian. The experimental variograms and their fits for the five normal score transformed data sets are shown in Figure 5.

In order to rank the data with respect to departure from a multivariate Gaussian distribution, the Lilliefors test of normality for $\tilde{Y} = \hat{\mathbf{L}}^{-1}[Y(\mathbf{u}_1)\, Y(\mathbf{u}_2) \ldots Y(\mathbf{u}_n)]^T$ will be employed for each normal score transformed data set. Table 9 shows results of the Lilliefors test for all five normal score transformed data sets.

Table 9: Result of Lilliefors test for all 5 data sets considered in the case study.

	Lilliefors statistic	Rank	Critical Value at $\alpha = 0.2$	Decision	Critical Value at $\alpha = 0.1$	Decision	Critical Value at $\alpha = 0.01$	Decision
1	0.1466	2	0.0245	Reject H_0	0.0268	Reject H_0	0.0368	Reject H_0
2	0.1974	4	0.0245	Reject H_0	0.0268	Reject H_0	0.0368	Reject H_0
3	0.2227	5	0.0245	Reject H_0	0.0268	Reject H_0	0.0368	Reject H_0
4	0.1185	1	0.0245	Reject H_0	0.0268	Reject H_0	0.0368	Reject H_0
5	0.1928	3	0.0245	Reject H_0	0.0268	Reject H_0	0.0368	Reject H_0

Note that all five data sets are univariate Gaussian after normal score transformation; however, none of them are multivariate Gaussian. They all depart strongly from a multivariate Gaussian distribution. This result holds true for any commonly accepted level of significance α. Note also that each of the five data sets has a different value of the Lilliefors statistic. The data sets can be ranked with respect to the value of the Lilliefors statistic. Data with the smallest value of the Lilliefors test statistic will receive rank 1 and is considered to be closest to the multivariate Gaussian among all data sets. In the particular case considered above data set 4 is the closest data set to being multivariate Gaussian, while data set 3 is the furthest away from being multivariate Gaussian.

Subsets of different sizes were considered to see if the test would preserve the rank order departure from a multivariate Gaussian distribution. Table 10 shows results for the average Lilliefors test statistic calculated based on 500 data sets of sizes 899 data points, 500 data points and 200 data points from the five data sets. To calculate the Lilliefors test statistic each of the selected data sets was normal score transformed independently of other data. The variogram models for the selected data sets were not recalculated. This was done mainly to avoid artifacts that could be observed in automatic fitting of the variograms. Figure 6 shows the variogram models obtained for 900 normal score data (all data) of data set 3 and to normal score transformed 899 first data in data sets 3. Note that despite the fact that experimental variograms are virtually identical, there is a clear mismatch in the variogram fits produced by automatic fitting program `varfit`. The difference is especially significant at short lag distances that has the most impact on the testing procedure.

Table 10: Result of Lilliefors test for all 5 data sets considered in the case study.

	Lilliefors statistic	Rank	Average Lilliefors statistic for 500 data sets of 899 data	Rank based on average Lilliefors statistic	Average Lilliefors statistic for 500 data sets of 500 data	Rank based on average Lilliefors statistic	Average Lilliefors statistic for 500 data sets of 200 data	Rank based on average Lilliefors statistic
Data 1	0.1466	2	0.1496	2	0.1078	2	0.0784	2
Data 2	0.1974	4	0.1948	4	0.1621	4	0.1178	4
Data 3	0.2227	5	0.2205	5	0.2016	5	0.1415	5
Data 4	0.1185	1	0.1161	1	0.1032	1	0.0736	1
Data 5	0.1928	3	0.1925	3	0.1322	3	0.1031	3

The preservation of the rank position in Table 10 allows us to conclude that the univariate test of a multivariate Gaussian distribution is robust.

Figure 7 shows the histograms of the values of Lilliefors test statistic for 500 data sets of size 500 data points obtained for each of the five data sets. Looking at Figure 7 we can note that values of Lilliefors test statistic change from data set to data set selected from the same data. The critical value of the Lilliefors test for data sets of size 500 at significance level $\alpha = 0.01$ is 0.4930. None of the data sets are 500-variate Gaussian even at such low level of significance. Note, however, that as size of the data sets decreases to 200, we can observe that some of the chosen data sets do not continue to exhibit strong departures from multivariate Gaussianity. In particular, majority of data sets selected from data set 1 (50.2%) and great majority of data sets from data 4 (65%) can be considered 200-variate Gaussian at significance level

$\alpha = 0.01$ (critical value of Lilliefors test for data sets of size 200 at $\alpha = 0.01$ is 0.0780). Note also that none of the data sets from data 3 and only 4% of data sets from data 2 and 8.6% of data sets at significance level $\alpha = 0.01$ can be considered as not exhibiting strong non multi-Gaussian features.

Further, to investigate if the ranks calculated based on Lilliefors test statistic are preserved for data selected with the same configuration from 5 different data sets another small analysis was performed. For each of 500 data sets of size 500 we calculated how many times of the 500 data sets selected from data 1 was rank as number 1, 2, 3, 4 and 5. The same was done for 500 data sets selected from data 2, data 3, data 4 and data 5. Results of this analysis are given in Table 11.

It is apparent from Table 11 that the rank of Lilliefors test statistic is preserved for majority of data sets selected from each of the 5 data sets considered in a case study. Moreover, looking at the results presented in Table 11, we confirm that the univariate test of a multivariate Gaussian distribution is robust and therefore applicable for ranking multiple datasets with respect to departures from a multivariate Gaussian distribution.

Table 11: Results for ranking of 500 data sets of 500 data each selected for each of the 5 data sets considered in the case study.

	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
Data 1	174	**294**	32	0	0
Data 2	0	0	10	**490**	0
Data 3	0	0	0	0	**500**
Data 4	**319**	162	19	0	0
Data 5	7	44	**439**	10	0

Conclusions

The longstanding problem of testing for a multivariate Gaussian distribution of spatially correlated data is considered. A number of simple tests for testing strong departures from multi-Gaussian distribution have been developed. The proposed UTMG test is fair; the number of falsely rejected tests matches perfectly the confidence level of the test. Performance of the tests was illustrated using real petroleum data. An interesting new approach for ranking data according to closeness to a multivariate Gaussian distribution was proposed. This approach uses the Lilliefors statistic as a measure of departure from a multivariate Gaussian distribution was shown to be very robust in a case study.

References

Berry, D. A., and Lindgren, B. W., 1990, Statistics: Theory and Methods. Brook/Cole, Pacific Grove, CA.

Deutsch, C.V., 2002, Geostatistical Reservoir Modeling. Oxford University Press, New York.

Deutsch, C.V., Journel, A.G., 1998, GSLIB: Geostatistical Software Library and Users Guide. Oxford University Press, New York

Goovaerts, P., 1997, Geostatistics for Natural Resources Evaluation. Oxford University Press, New York.

Golub, G.H. and Van Loan, C.F., 1996, Matrix Computations. Johns Hopkins University Press

Lehmann, E.L., 1999, Elements of Large-Sample Theory. Springer-Verlag.

Lilliefors, H., 1967, On the Kolmogorov-Smirnov test for normality with mean and variance unknown. Journal of the American Statistical Association, 62: 399-402.

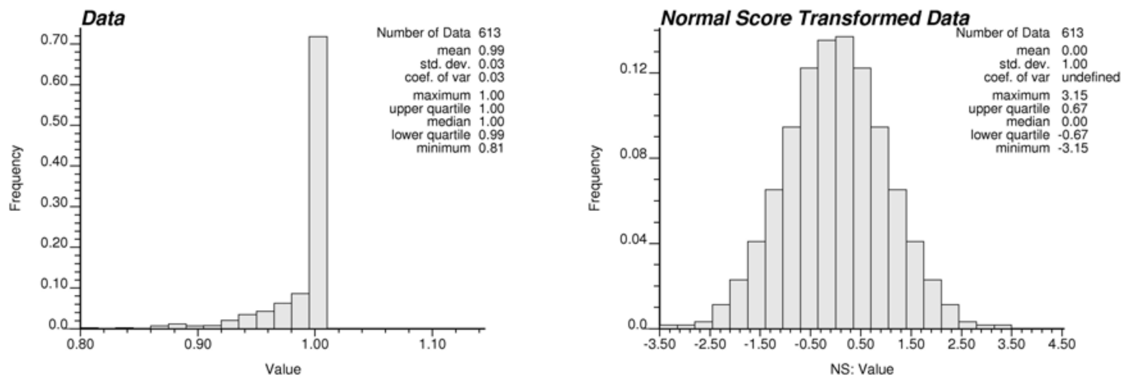

Figure 1: Histogram of the 613 data in original units (left) and normal score units (right).

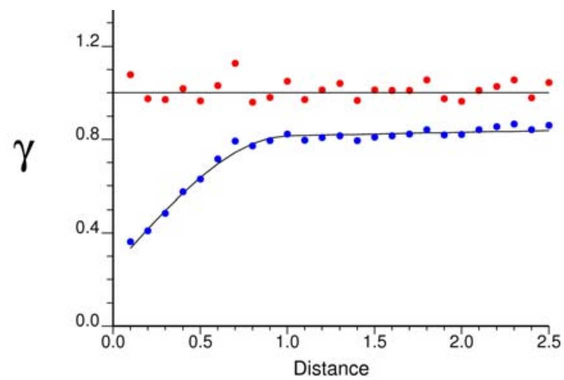

Figure 2: The experimental variogram of the normal score transformed 613 data (dark dots) and its variogram model (line). Experimental variogram of the $\tilde{Y}$ data obtained by removing the spatial structure from the normal score transformed data are shown in light dots.

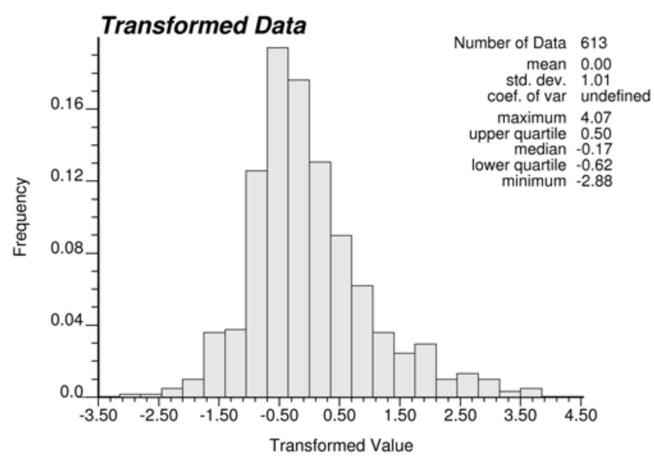

Figure 3: Histogram of $\tilde{Y}$.

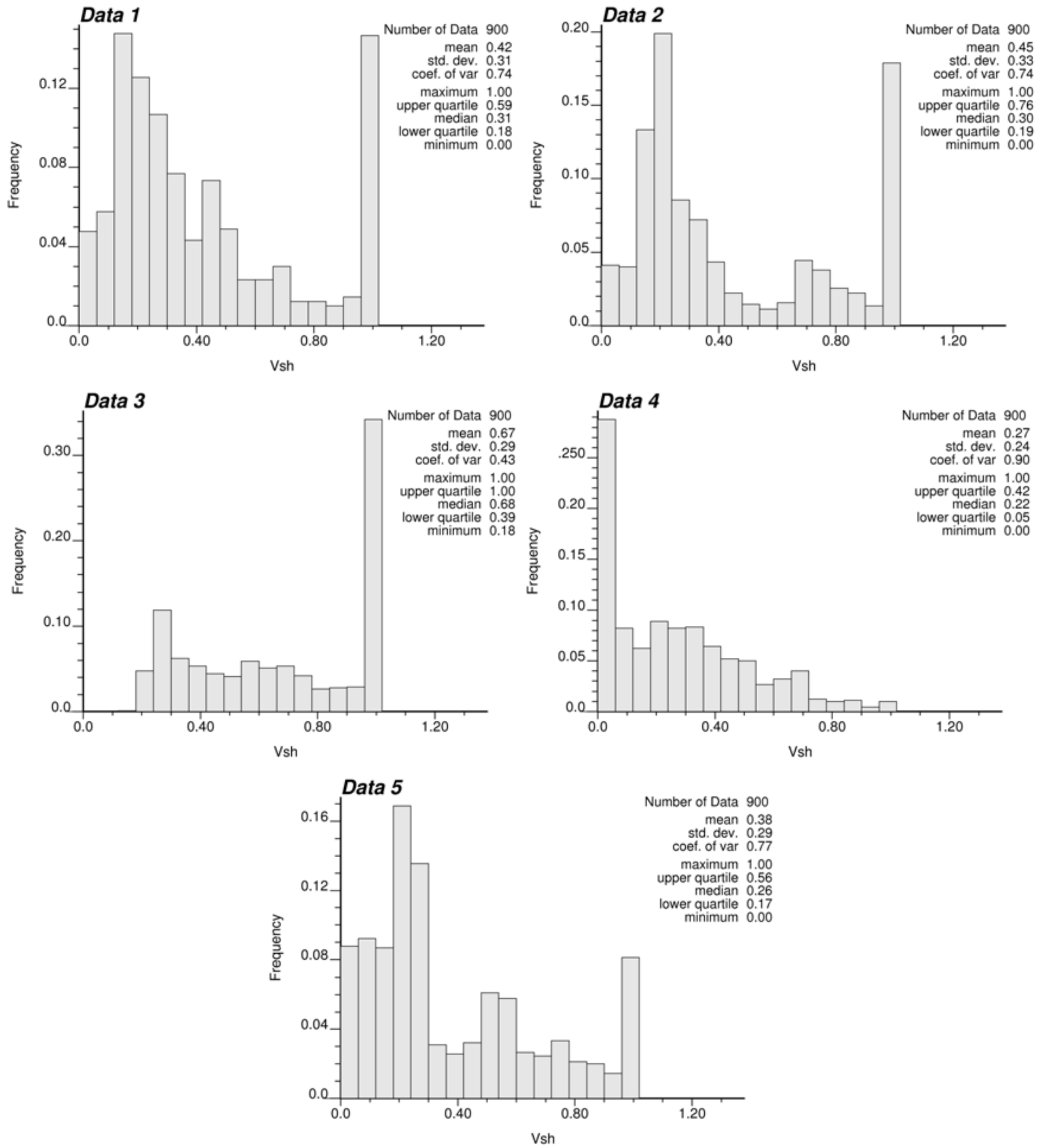

Figure 4: Histograms of the five data sets of 900 data each used in a case study.

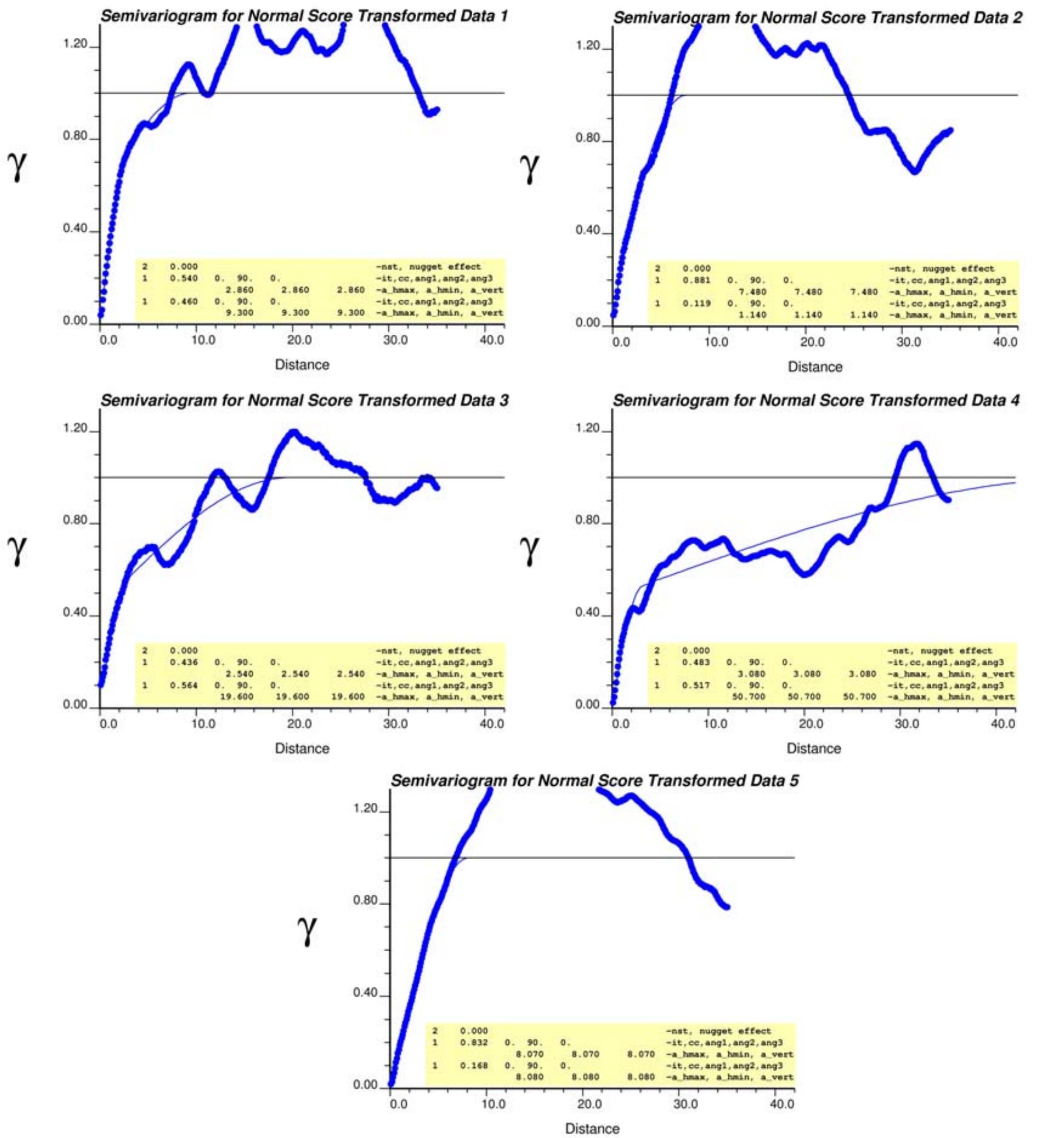

Figure 5: The experimental and model variograms for the five normal score data sets.

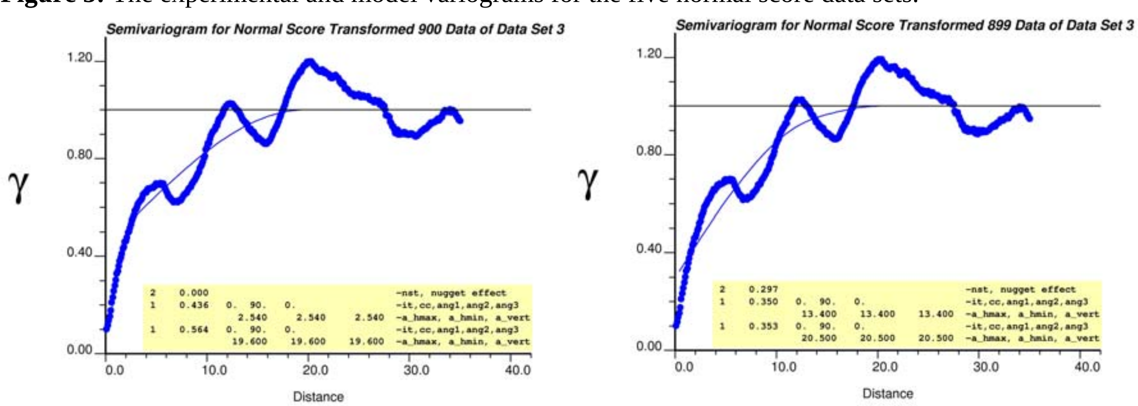

Figure 6: Variogram models fitted by varfit to 900 normal score data (all data) of data set 3 (left) and to normal score transformed 899 first data in data sets 3 (right).

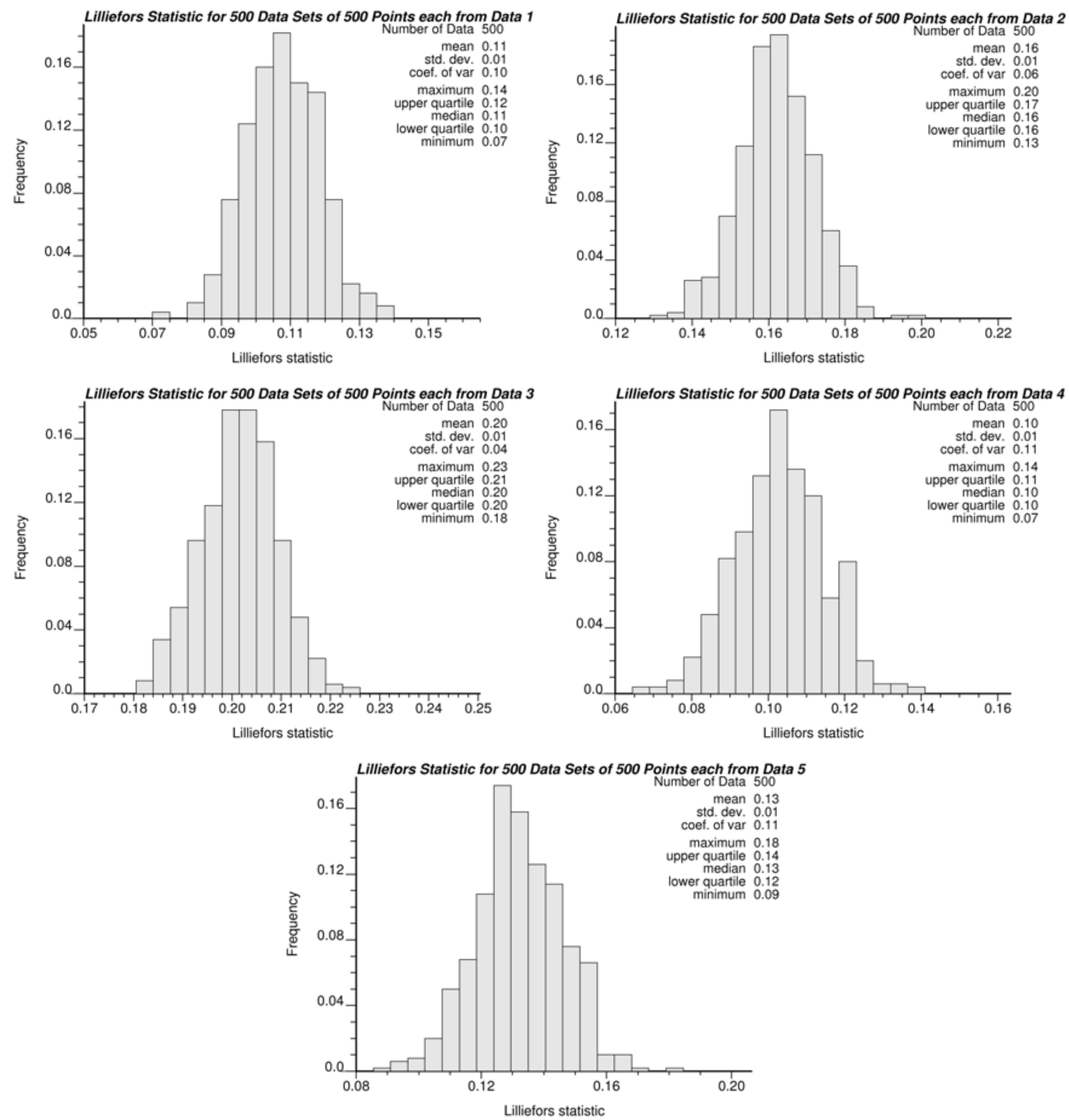

Figure 7: Histograms of the values of Lilliefors test statistic for 500 data sets of size 500 data points obtained for each of the five data sets considered in a case study.