The Structure of Unstructured Grids

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Unstructured grids are an efficient data structure in regards to reservoir flow simulation analysis. They offer several advantages including accurate representation of complex geological entities and near well flow, and the accuracy of simulation results are maintained at a fraction of the processing time required for regular grids. However, using unstructured grids for reservoir flow analysis is relatively new and not prevalent in industry. This paper gives a definition for unstructured grids and discusses considerations for their design from geologic and simulation perspectives. Grid generation techniques are also covered and a unified underlying grid structure, that being a triangular or tetrahedral grid, that is capable of representing practically any reservoir grid design is highlighted.

Introduction

Reservoir evaluation can be broken into two main categories: geologic modeling and flow simulation. The later will be referred to just as simulation. Modeling involves several stages including structural, facies, and rock property modeling. Resulting models are used to characterize simulation problems, which provide a means to evaluate the potential recovery of hydrocarbons from a reservoir for a particular recovery technique. A well known disparity between these two categories is grid resolution: the computational effort to model geological properties of a reservoir is much less than that for simulation. A high resolution grid is used to model geologic properties, which are then upscaled to a coarse grid used in solving the simulation problem.

Grids used for geologic modeling are almost always regular – it is intuitive and straightforward to use a coarse regular grid for simulation. Ideally, the upscaling process results in identical flow responses whether the fine or coarse scale grid are used for simulation. This concept raises questions surrounding the applicability of coarse regular grids to resolve flow through complex areas of a reservoir, for example through or around faults and lenses and near wells. Regular grids are inflexible and representing complex features such as faults and horizons is inaccurate. Accuracy is also lost in assuming regular grids capture the flow features for various recovery scenarios ranging from basic vertical producing or injecting wells to more complex enhanced techniques like SAGD. Resolving flow near wells is a particular problem because even the high resolution regular grid for geologic modeling may be too coarse.

Improving the accuracy of simulation is accomplished through use of unstructured grids in place of a coarse regular grid. The number of grid elements is not necessarily increased, but their shape and configuration is much more flexible. Unstructured grids are conformable to practically any geometry and flow scenario – it is possible to trace out faults, horizons, and well trajectories. A significant amount of research is being done in regards to the design, generation, and simulation of unstructured grids. Research is also targeting upscaling geologic properties from high resolution regular grids to unstructured grids. Virtually no research is targeting geologic modeling on unstructured grids. Advancing this area of research involves first gaining an understanding of the various styles of grids, their design considerations, how they are generated, and simulation considerations.

Background

Before delving into the definition and historical use of unstructured grids, the question – what qualities are we looking for in a reservoir simulation model? – will be addressed. Like reservoir evaluation, the qualities can be separated into two categories: geological and simulation-based. Resulting simulation models should accurately represent large scale geologic structures including the reservoir boundary, faults, and horizons (Kocberber, 1997). Geologic qualities at a smaller scale involve designing elements that do not contain significant permeability discontinuities and do not span structural surfaces like faults. Such occurrences are one source of numerical instability for simulation. Qualities in regards to simulation include the accurate representation of well trajectories, near-well flow, and any other area where complex flow patterns are anticipated, for example though highly permeable conduits or across faults. The overall grid should also be designed to meet requirements of a particular flow solver and for the most efficient solution of the flow equations. One last general quality is the process for building a reservoir simulation model should be user-friendly and not overly time consuming. Processes designed to generate unstructured grids should address these qualities.

An unstructured grid is a generalized geometric entity that partitions Euclidean space into a set of elements. There are no constraints on the geometry of elements or their configuration pattern apart from being non-intersecting. For reservoir modeling, grids partition the three dimensional space containing the reservoir into a finite set of elements. Because the targeted use of these grids is simulation, the design of elements is often done to optimize the quality of flow analysis: elements align with significant geologic features like faults and areas with potentially high volumetric flux, for example near wells. Grid design may also be constrained by the capabilities of the flow solver to be used and this will be discussed further.

The design capabilities of unstructured grids are practically unlimited and many forms of elements are possible for partitioning three dimensional space. Several have been used in the finite element method (Hrenikoff, 1941; Zienkiewicz, Taylor and Zhu, 2005), where elements range form tetrahedra to complex curvilinear elements (Figure 1). For reservoir simulation, however, elements are constrained to simple convex geometries due to the complexity of flow in porous media. Some examples are tetrahedra, and hexahedra from Figure 1, and voronoi elements (Figure 2).

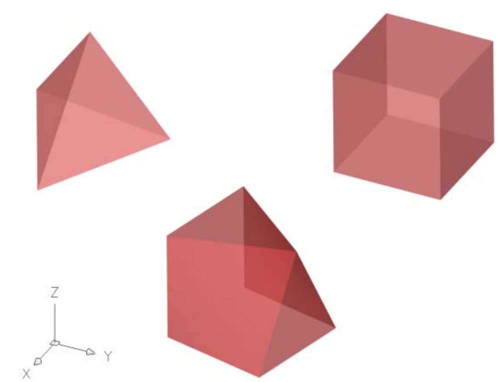

Figure 1: Clockwise from upper left: tetrahedron, hexahedron, degenerate curvilinear element

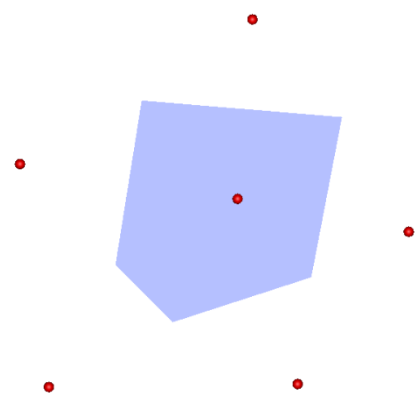

Figure 2: Voronoi element for central of six points

Historically, unstructured grids have been in use since the 1940's in various fields of continuum mechanics. However, they are a recent addition to reservoir simulation with research beginning in the 1980's (Pathak et al, 1980). Substantial attention was not seen until the 1990's with voronoi or perpendicular bisector (PEBI) grids (Heinemann et al, 1991; Palagi and Aziz, 1994). Research advancements in both simulation and grid design and generation from the 1990's to date are too extensive for this review. An extensive review can be found in the thesis work of Prevost (2004), pages 3-8. The majority involve convex elements of varying geometry designed to align with reservoir geology and to capture complex near-well flow or flow through highly permeable conduits. Some recent examples are automatic flow-based grid generation to represent reservoir structure and areas with high volumetric flux (Edwards, 2002; Prevost et al, 2005), see Figure 3 and 4, and a hybrid grid generation scheme for gaining accuracy near wells (Flandrin, Borouchaki, and Bennis, 2006), Figure 5.

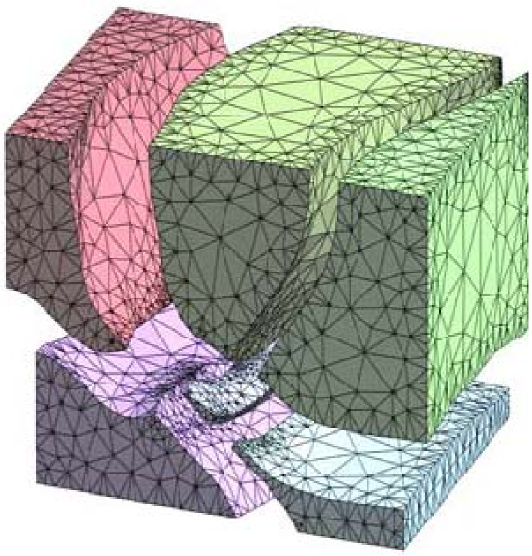

Figure 3: Exploded view of triangulated reservoir regions separated by faults, from Prevost et al (2005)

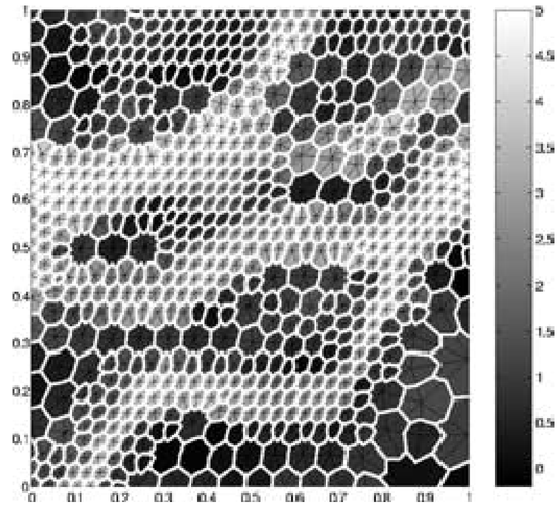

Figure 4: Flow based grid representing high permeable channels, from Edwards (2002)

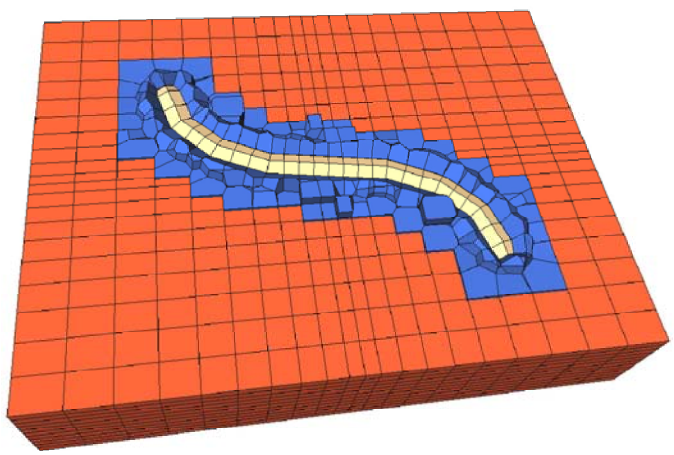

Figure 5: Radial-like voronoi grid embedded into a regular grid along the producing zone of a horizontal well, from Flandrin et al (2006)

Design

Designing an unstructured grid to represent the geology and flow of fluids within a reservoir is a significant and time consuming component of a reservoir modeling workflow. Two important considerations to account for are geological and simulation based. There is a balance between achieving accurate representation of geology while meeting conditions of a particular simulation algorithm.

Simulator Considerations

Several types of flow simulators that work with unstructured grids are available or being developed. Their capabilities dictate how flexible an unstructured grid design can be and the element geometry that is applicable. Several simulation techniques such as certain finite difference schemes and spectral methods are restricted to regular grids and will not be discussed. The finite element method (FEM) is an approach applicable for unstructured grids, but is unstable for complex porous flow problems (Farmer, 2005). The simulation technique that has received the most attention due to its use in industry and ability to handle unstructured grids is the finite volume method (FVM).

Most commercial simulators using FVM are limited to corner point grids (Ponting, 1992), which are structured, possibly irregular grids. The structure is logical: a well defined relationship between each point and its neighbours exists. Although corner point grids do offer some flexibility, there are complications for alignment with well trajectories, overturned surfaces, intersecting faults, and thrust faults (Farmer, 2005). These simulators are also based on the two point flux approximation (TPFA): flux from one element to another depends only on the pressures in those two elements. Transmissibility for the face separating the two elements is based on two points typically chosen as the element centers. It is calculated using the permeability tensors for each element, which are usually assumed diagonal. Use of TPFA with full tensors results in error (Edwards and Rogers, 1998; Eigestad and Klausen, 2005; Chen and Mallison, 2007), motivating the development of new simulation techniques.

There are three main types of next generation simulators that are being developed: FVM with multipoint flux approximation (MPFA), hybrid FE-FVM, and multiscale simulators. MPFA methods have been developed for both two dimensions (Aavatsmark et al, 1996; Aavatsmark et al[1,2], 1998) and three dimensions (Verma and Aziz, 1997) and are undergoing further development (Eigstad, Aavatsmark, and Espedal, 2002; Mlacnik and Durlofsky, 2006; Njifenjou and Nguena, 2008; Njifenjou and Mbehou, 2008). Most MPFA research appears to develop simulators for triangular or tetrahedral grids. Although this appears to be a constraint, practically any unstructured grid consisting of polygonal volumes can be reduced to a set of tetrahedra. Hybrid FE-FVM (Geiger et al, 2004; Paluszny, Matthai, and Hohmeyer, 2007; Maliska, Cordazzo, and Silva, 2007) are very similar to FVM using MPFA with the major difference being a finite element formulation is used to solve the fluid pressure field and finite volumes for the fluid transport phenomena. Tetrahedral meshes are the focus, but other possible element configurations are presented (Figure 6).

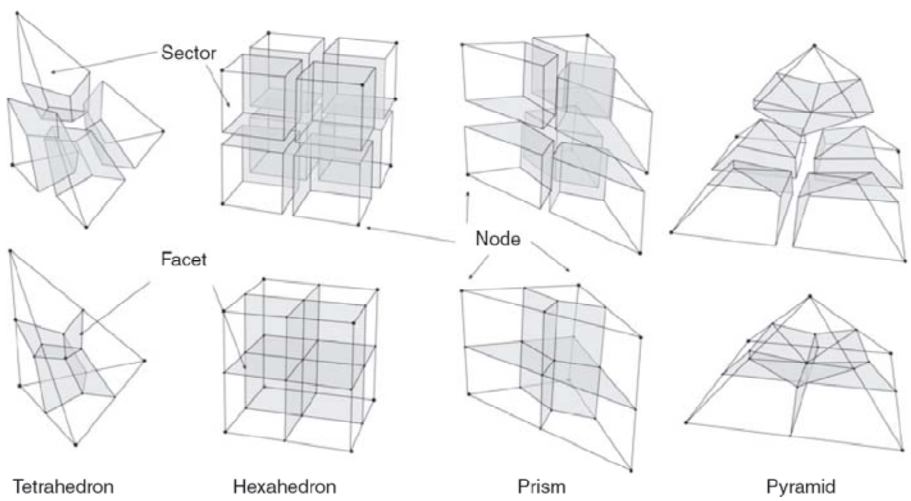

Figure 6: Various elements for grid construction, from Paluszny, Matthai, and Hohmeyer (2007)

Multiscale methods (Hou and Wu, 1997; Kippe, Aarnes, and Lie, 2008; Pavliotis and Stuart, 2008) are a recent class of simulation technique that assumes each element is characterized by pressures and fluxes that are functions of position within the element, rather than a constant. Fine scale information is incorporated into the solution of coarse scale equations without having to resolve the fine scale, thus the methods are more efficient than directly simulating at the fine scale. Although still highly in the research and development stage, multiscale simulation does show promise for complex grid geometries. Aarnes, Krogstad, and Lie (2008) discuss a multiscale mixed/mimetic finite element method that is extensible to general polyhedral cells. They also identify three considerations for generating coarse grids: 1 - Grid geometry should minimize the possibility of bidirectional flow across element interfaces; 2 - Element faces should follow geological layers; 3 - Elements should adapt to flow obstacles. Extending multiscale methods to more complex multiphase flow regimes is ongoing.

In general, the design of an unstructured grid should be done to maximize the accuracy of the targeted simulator. Properties that may be of importance include:

- Element geometry: this may be limited to a specific format as with corner point grids, or to a pre-defined set of elements as with finite element and finite volume methods. Many techniques are applicable to geometries that can be reduced to a set of tetrahedra.
- Orthogonality: certain solvers achieve maximum accuracy with grids having local orthogonality such as PEBI grids.
- Convexity: elements should be convex for any solver and this greatly simplifies geometric operations including intersections, volume calculations, and refinement.
- Orientation: elements should align with structures that will have a significant impact on flow and be designed to reflect anticipated flow behaviour, for example radial flow near wells. Grid orientation effects may be realized in simulation results otherwise.

Geological Considerations

Several geological considerations have been identified and generally involve accounting for structure within a reservoir. Geological models are becoming increasingly complex with the advent of more sophisticated modeling software and visualization tools and with the acquisition of 3D seismic data. Detailed models may consist of highly faulted stratigraphic systems (Figure 7). Unstructured grids should be designed to align with significant stratigraphic layers, fault surfaces, and other important interfaces such as impermeable shale boundaries and highly permeable channels. Discontinuities in permeability within elements should also be avoided.

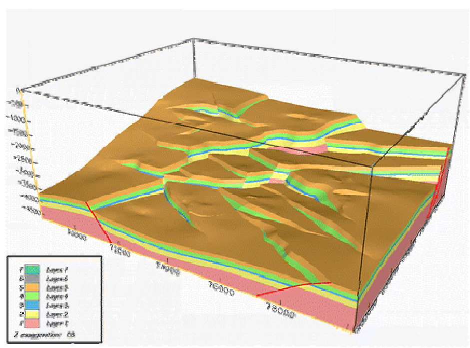

Figure 7: Faulted stratigraphic system

An important consideration with geological structures is uncertainty (Deutsch, 2002; Groshong, 1999; Mallet, 2002). The location of horizons and fault planes is only precisely known along wells that intersect the surfaces. They are only approximately known through seismic data (Milton and Myers, 1996). In between wells and where seismic is unavailable, the geometric and positional characteristics of structural surfaces are uncertain. Accounting for uncertainty involves stochastic modeling approaches to generate several possibilities for the geometry and position of structures. Each possible configuration would lead to different grid geometry. Quantifying this level of uncertainty is only feasible if automatic grid generation techniques are available.

For a given reservoir interpretation, one could argue that accounting for the uncertainty in horizons and faults would not significantly alter the grid. Assuming that the number of surfaces and faults is kept constant as well as their positions in the geologic time-line, grid elements will also maintain a consistent configuration. Element geometry would scale with relative surface positions, but the total number of element would not necessarily change. On the other hand, given two or more different interpretations of a reservoir, for example its depositional origin, it is reasonable to expect completely different grid designs for each setting. Assessment of uncertainty would be carried out for each interpretation.

At a higher level of detail involving inter-layer geology, more complicated structural information is encountered, for example with fluvial deposits. A system of channels may be represented with an unstructured grid as in Figure 4; however, if uncertainty is introduced, the sinuosity of channels may be drastically different from one possible model to the next. This would lead to significantly different grid geometry within layers containing such features. Although this is not necessarily a problem for simulation, it adds to the complexity of research targeting automatic grid generation and geologic property modeling on unstructured grids.

Generation

Characterizing reservoir structure and constructing a good quality grid, or mesh, is often a time consuming exercise. The terms grid and mesh are both commonly used in reference to generation and will be used interchangeable in this section. With the complexity of reservoir models, automatic mesh generation methods are a necessity. Many mesh generation techniques exist and several are described by Frey and George (2000). They describe triangulations and tetrahedralizations as being the most flexible for representing complex geometries. In the context of reservoirs, complex geometries include faults, horizons and stratigraphic layers, channels, etc., and the possible intersection and combination there of. Three of the main types of unstructured mesh generation algorithms are aimed at generating triangular or tetrahedral meshes and include spatial decomposition, advancing-front, and Delaunay methods. Further processing such as triangle or tetrahedron merging can be utilized to construct quadrilateral or hexahedral meshes if required. Several hybrid methods also exist which involve combining multiple mesh types such as shown in Figure 5. For a more complete account of mesh generation techniques, the survey by Owen (1998) can be reviewed. A common series of steps for unstructured mesh construction is given in Frey and George (2000): 1 - Boundary definition; 2 - Definition of element size distribution function; 3 - Mesh generation; 4 - Optimization.

For reservoir mesh generation, step 1 includes both external and internal boundaries. External would involve an aerial extent and extreme upper and lower horizons containing the reservoir. Internal boundaries are any other horizons, faults, and other structural surfaces to be explicitly included in the discretization. All boundaries should be honoured in the final mesh. For example, if a tetrahedral mesh is constructed, one face of each element contacting a boundary must coincide with that boundary. The collection of faces on each boundary forms a triangulation.

Step 2, defining an element size distribution function, is an open ended aspect of reservoir mesh generation. Reservoir models consist of multiple scales of information ranging from the very fine scale of geological properties to the very large scale of structural surfaces and well production data. It would be ideal to use the scale of geological properties; however, flow simulation at this scale is not practical. This is one motivation for unstructured meshes. The notion of generating good quality meshes was mentioned at the beginning of this section. Flow simulation is the targeted problem to be solved; thus a good quality mesh is one providing convergence of the process and accuracy of results. Achieving convergence and accuracy of any reservoir flow solver is dependent on the element size distribution.

A good quality mesh, which will be referred to as the coarse mesh from here on, is one that provides flow simulation results identical to those that would be obtained using a finer scale mesh representative of the heterogeneity throughout the reservoir, even though obtaining a result for such a fine mesh is infeasible. Several developments have been made in generating coarse meshes for reservoirs that maintain accuracy. Most are based on smaller models for which simulation of the respective fine mesh is manageable for comparison purposes. One of the earliest automatic unstructured meshing schemes involved PEBI grids and was developed as an advancing front method. Points were distributed through space based on certain criteria, and then connectivity was determined resulting in a voronoi mesh (Palagi and Aziz, 1994). Generating PEBI grids remains an active research topic (Hale, 2002; Mlacnik, Durlofsky, and Heinemann, 2006). Some other grids as previously described are generated for finite volume MPFA methods and multiscale finite element methods and are typically triangulations or tetrahedralizations.

Two primary methods of generating unstructured meshes for reservoirs exist: hybrid techniques and flow based techniques. In hybrid techniques, reservoirs are initially partitioned by a background mesh, which is then refined in areas with complex geology and anticipated high saturation and flow gradients (Aziz, 1993). The background mesh is commonly structured so does not perfectly align with faults and horizons. Automatic methods are employed to create elements to align with these structures (Kocberber, 1997;

Flandrin, Borouchaki, and Bennis, 2006; Mallet, 2002). High saturation gradients are typical near injection and production wells where the background mesh is refined to conform to well paths. This refinement is done by removing elements within the vicinity of a well path, then filling the void space with a radial-like mesh (Figure 8).

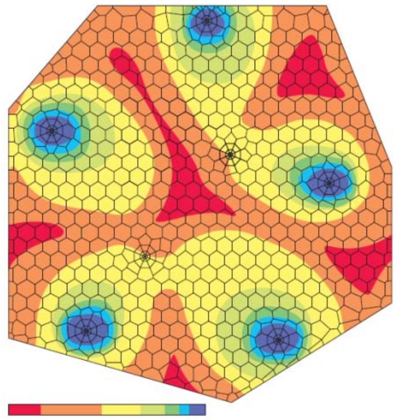

Figure 8: Radial mesh refinement around vertical wells, from Adamson et al (1996)

Flow based mesh generation techniques require the solution to a fine scale flow problem from which mesh elements are derived. Mesh vertices may be distributed based on streamlines (Verma and Aziz, 1996), streamlines and isopotentials (Mlacnik, Durlofsky, and Heinemann, 2006), streamtubes and isobars (Portella and Hewett, 2000), or other flow information. The fine scale problem is usually a basic incompressible, single phase pressure equation, although a two phase equivalent has also been used. It characterizes the pressure and velocity fields throughout the fine scale mesh, which is often a structured regular grid: a permeability model is required as input to the pressure equation and geologic property modeling techniques are currently designed for regular grids.

Both static (Edwards, 2002) and dynamic (Mlacnik, 2003) flow based generation techniques are possible. In the static case, a mesh is generated once and remains fixed for the duration of time dependent simulation runs. The mesh may be generated using no wells and arbitrary global boundary conditions so that streamlines etc. are influenced mainly by reservoir structure and heterogeneity. Element size and density varies according to characteristics of the flow solution. A mesh may also be generated for a specific flow scenario, a production well configuration for example, where boundary conditions are imposed globally and at well locations. Simulation runs using the resulting grid are only valid if there are no significant changes made over the time of the run. For example shutting in a well or converting a producer into an injector would have a significant impact on flow and thus on the optimal flow based mesh.

To alleviate the influence of changing reservoir operating conditions over time, some work has been done with dynamic flow based grids. Each alteration to reservoir conditions requires the solution to a fine scale simulation problem, grid generation, grid optimization, and permeability upscaling stages, providing the grid and properties for the next series of time steps in an overall simulation run. Depending on the size of the fine scale model, this series of stages can become quite time consuming.

Structure

Defining one grid structure that is applicable to all reservoirs, flow scenarios, and simulators is not a realistic proposition. However, a common property across all grid types discussed is that elements are linear and convex. Regardless of the unstructured grid designed for reservoir simulation, this property enables it to be represented by a triangular or tetrahedral mesh in two or three dimensions. These simplicial meshes are straightforward to store, traverse, and manipulate. Several data structures have been designed for these purposes (Frey and George, 2000; Alumbaugh and Xiangmin, 2005). Another advantage is triangular and tetrahedral elements can be easily mapped across coordinate systems – this is typical of finite element analysis.

For reservoir analysis, several mesh related operations must be possible: traversal, search, range queries, element refinement and merging, and property storage and recall. Traversal involves having access to the neighbours and geometric components including vertices, edges, and faces, of a particular element (Figure 9). Calculation of transmissibilities for example requires an element and its neighbour and their common face. Searching for an element given some criteria is also important. Determining which elements a well path intersects is important for upscaling well properties. A well log might contain thousands of sample points so an efficient search is essential.

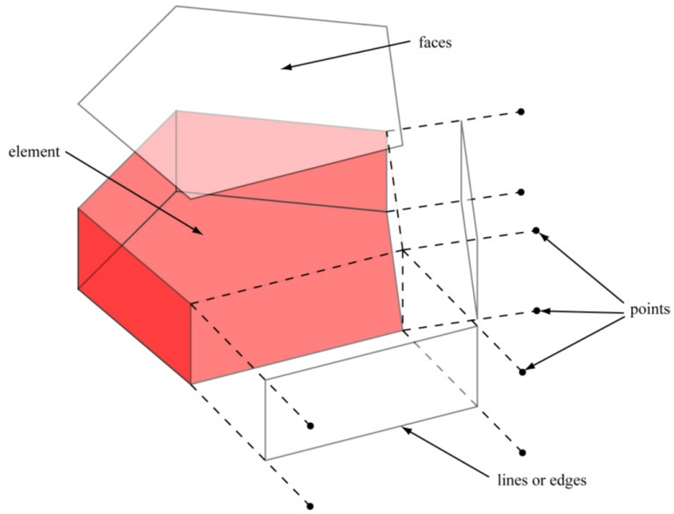

Figure 9: Element geometry

Range queries are more applicable to property modeling and interpolation and involve determining all elements that are within a specified region. Various data structures are designed for optimal querying of a specific type of region be it rectilinear, spherical, or ellipsoidal. Element refinement is typically carried out to improve the accuracy. A mesh designed for reservoir simulation is used to approximate the continuous flow function. If poor accuracy of the function is achieved in certain areas, the mesh may be refined. Similarly, if the mesh is to approximate the heterogeneity of reservoir properties, refinement may be required in areas that are characteristic of higher variability. The process of merging involves recovering some initial mesh prior to refinement. Property storage and recall is straightforward, but it should be noted that elements will contain multiple properties including indicator, various scalar, and tensor properties. Storage and recall is efficiently done with arrays and indexing.

Summary

Unstructured grids are an efficient structure for simulation since it is possible to achieve results similar to those using a much higher resolution regular grid and in much less time. Several next generation simulators designed for unstructured grids are being researched and developed – it is likely that commercial simulators will be equipped for unstructured grids in the near future. Another related area of research is grid generation. Creating a reservoir model and designing a grid are time consuming processes. Generation tools must be easy to use, flexible, and efficient for industry to incorporate unstructured grids into their workflows. Several grid design considerations imposed by geology and simulators must be taken into account as well.

After discussing the qualities of a grid design, various geologic and simulator considerations, and generation techniques, it was concluded that practically all grids for reservoir simulation will involve convex, linear elements. A unified structure for such elements is the triangular or tetrahedral grid, which has undergone a substantial amount of development in finite element analysis. All three next generation simulators discussed can operate on these grids, either with a straightforward conversion to finite volumes, or directly. Grids may involve other elements such as hexahedra, which can be represented as sets of tetrahedral elements.

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