Nonnet Size Determination with Mass Moments of Inertia

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The question of non-net size and connectivity determination is of great practical importance in petroleum engineering. It can be used to assist well placement and prediction of fluid flow responses in latter stages of reservoir modeling. The information on net/non-net intervals (thickness) collected from wells along with an indicator variogram of non-net facies can provide a hint to practitioner about the extent and connectivity of non-net intervals. There is still a great degree of uncertainty in the size of these intervals; therefore, a sound repeatable method able to quantify the relationship between the length and thickness of non-net facies and able to predict the length of non-net interval based on its thickness is needed. In this paper one such approach is proposed. The proposed approach is based on a novel idea of incorporating mass moments of inertia in calculating the size of non-net intervals.

Moment of Inertia

Moment of inertia, also known as mass moment of inertia or the angular mass of a body, is the rotational analog of mass; it related to the distribution of the mass throughout the body. Moment of inertia is the inertia of a rigid rotating body with respect to its rotation:

$$I = \int r^2 dm \tag{1}$$

where m is the mass and r is the perpendicular distance of the point mass to the axis of rotation. The moment of inertia has two forms: scalar form which is used when the axis of rotation is known and the tensor form which summarizes all moment of inertia for different axes of rotation with one quantity (Hassanpour and Deutsch, 2008). For a rigid body consisting of N point masses m_i, the moment of inertia tensor is defined as (Hassanpour and Deutsch, 2008):

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{2}$$

with

$$I_{xx} = \sum_{i=1}^{N} m_i \left(y_i^2 + z_i^2 \right) \quad , \quad I_{xy} = I_{yx} = -\sum_{i=1}^{N} m_i x_i y_i$$

$$I_{yy} = \sum_{i=1}^{N} m_i \left(x_i^2 + z_i^2 \right) \quad , \quad I_{xz} = I_{zx} = -\sum_{i=1}^{N} m_i x_i z_i$$

$$I_{zz} = \sum_{i=1}^{N} m_i \left(x_i^2 + y_i^2 \right) \quad , \quad I_{yz} = I_{yz} = -\sum_{i=1}^{N} m_i y_i z_i$$

where x_i, y_i and z_i are the distances of point i from the coordinate axes. I_{xx} can be interpreted as the moment of inertia around the *x*-axis when the objects are rotated around the *x*-axis and I_{xy} is the moment of inertia around the *y*-axis when the objects are rotated around the *x*-axis.

When a rigid body is an ellipsoid, then its moments of inertia tensor is diagonal. Then the moment of inertia of an ellipsoid around its major, medium and minor radius is found as follows (Hassanpour and Deutsch, 2008):

$$
\begin{aligned}
I_a &= \frac{1}{5} M \left(r_b^2 + r_c^2 \right) \\
I_b &= \frac{1}{5} M \left(r_a^2 + r_c^2 \right) \\
I_c &= \frac{1}{5} M \left(r_a^2 + r_b^2 \right)
\end{aligned}
\tag{3}
$$

where I_a, I_b and I_c are moment of inertia around major r_a, medium r_b and minor r_c radius, respectively and M is the ellipsoid mass.

Calculating the Thickness and Length of Non-Net Intervals with Mass Moments of Inertia

The idea behind the mass moments of inertia approach to calculating non-net size is as follows. Each non-net interval within a rock-type model can have an arbitrary assymetric shape and is characterized by its own moment of inertia tensor given in (2). It would be hard to parameterize all possible discontinuities/asymmetry of the shape of non-net interval using two parameters such as length as thickness directly. We replace the irregularly shaped non-net interval with a non-net ellipsoid which has 'equivalent' moment of inertia tensor. Then, calculation of the thickness and length of given non-net interval is done indirectly but in straightforward manner. Specifically, the thickness of a non-net interval is calculated as a vertical radius of the ellipsoid and the length (or width) of the non-net interval is calculated as the average of the two ellipsoid radii in the horizontal plane.

Note then ellipsoid with an equivalent moment of inertia tensor to given is just an ellipsoid such that its moment of inertia tensor has eigenvalues of the original moment of inertia tensor on the diagonal. Note that the largest eigenvalue (and the corresponding eigenvector) and the smallest eigenvalue (and the corresponding eigenvector) are related to the areal ellipsoid extent. The length and thickness of non-net intervals can be found from (2). It should be also noted that the moment of inertia approach works well only if enough data are available. In the case of small non-net interval, additional partitioning of non-net object may be required.

Examples

Let us now illustrate the proposed methodology to non-net size calculation with several examples. All examples will be based on unconditional Sequential Indicator Simulation (SIS) of binary codes, that is, net (0) and shale (1), based on different proportions of net-non-net facies and different indicator variogram models of non-net (and net). Note that continuity of net and non-net is the same due to binary coding.

Each simulation study is conducted as follows. A volume of 100 by 100 by 100 cells of size 10 by 10 by 1 cubic distance units is populated using SIS (10 SIS realizations are generated). Six cases for different net/non-net proportions and indicator variograms are considered.

$$
\begin{array}{ll}
1) & p_{net} = 0.8; \quad p_{non-net} = 0.2; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=1000 \\ a_{vertical}=40}}(h); \\
2) & p_{net} = 0.8; \quad p_{non-net} = 0.2; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=400 \\ a_{vertical}=10}}(h); \\
3) & p_{net} = 0.8; \quad p_{non-net} = 0.2; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=200 \\ a_{vertical}=5}}(h); \\
4) & p_{net} = 0.5; \quad p_{non-net} = 0.5; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=1000 \\ a_{vertical}=40}}(h); \\
5) & p_{net} = 0.5; \quad p_{non-net} = 0.5; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=400 \\ a_{vertical}=10}}(h); \\
6) & p_{net} = 0.5; \quad p_{non-net} = 0.5; \quad \gamma(h) = 0.2 + 0.8 Sph_{\substack{a_{areal}=200 \\ a_{vertical}=5}}(h);
\end{array}
\tag{4}
$$

Reproduction of the target proportions is carefully checked.

After simulation is completed, a light image cleaning using MAPS (see software catalog) is performed. This is done to avoid pixelation inherent to SIS and to make clouds of non-net produced by SIS more continuous. In MAPS, a window size (template) of 5 by 5 by 3 is used in image cleaning (more cleaning is performed in areal directions). Then the cleaned SIS realizations are analyzed and all non-net geo-objects are found. This is achieved using program called geo_obj (see software catalog). This program can scan through multiple net/non-net realizations (can also use porosity, permeability realizations) and calculate connected regions. Connected regions are defined by face-connected blocks. Each connected region of non-net will be defined as a separate non-net geo-object. After each geo-object is found, a moment of inertia approach can be applied to find the thickness and length of non-net intervals.

Figure 1 shows one example SIS realization for each of the 6 simulation studied. The realizations were cleaned with MAPS. It can be noted from Figure 1 that depending on the net/non-net spatial continuity structure characterized by the indicator variogram and the proportions of each net and non-net facies, intervals of different thickness and length are obtained for net and non-net facies. In particular, the largest (thickest and longest) fairly continuous non-net intervals are observed in simulation studies 1 and 4 due to long range of continuity of the variogram used to generate simulated realizations. The thinnest non-net intervals are observed in simulation study 2. With increase in the global proportion of non-net (net) facies, the thickness of non-net (net) intervals increases.

Figure 2 shows results for the thickness vs. length of non-net intervals for each of the 6 simulation studies zoomed to thicknesses up to 10 units. Figure 2 also shows the P_{25}, P_{50}, P_{75} probability curves for length as a function of thickness. These curves can be used to predict the length or give a probability interval for the length of non-net intervals for a given observed thickness.

When analyzing Figure 2, it can be easily noted that with increase in the proportion of non-net facies, for the same non-net thickness interval, longer non-net intervals are usually observed. Moreover, it is also interesting to note that despite the variograms 1 and 3 used in simulation studies are very different. That is, first variogram is characterized by very long ranges of continuity in each principal direction; the third variogram is characterized by very short ranges of continuity in each principal direction. However, results of simulation studies obtained based on these two different variograms are quite similar (see case 1 and case 3). This is because, in the case 1 the simulation produces very long quite continuous intervals; while in the case 3 the simulation produces a lot of smaller intervals which oftentimes overlap with each other due to short ranges of continuity.

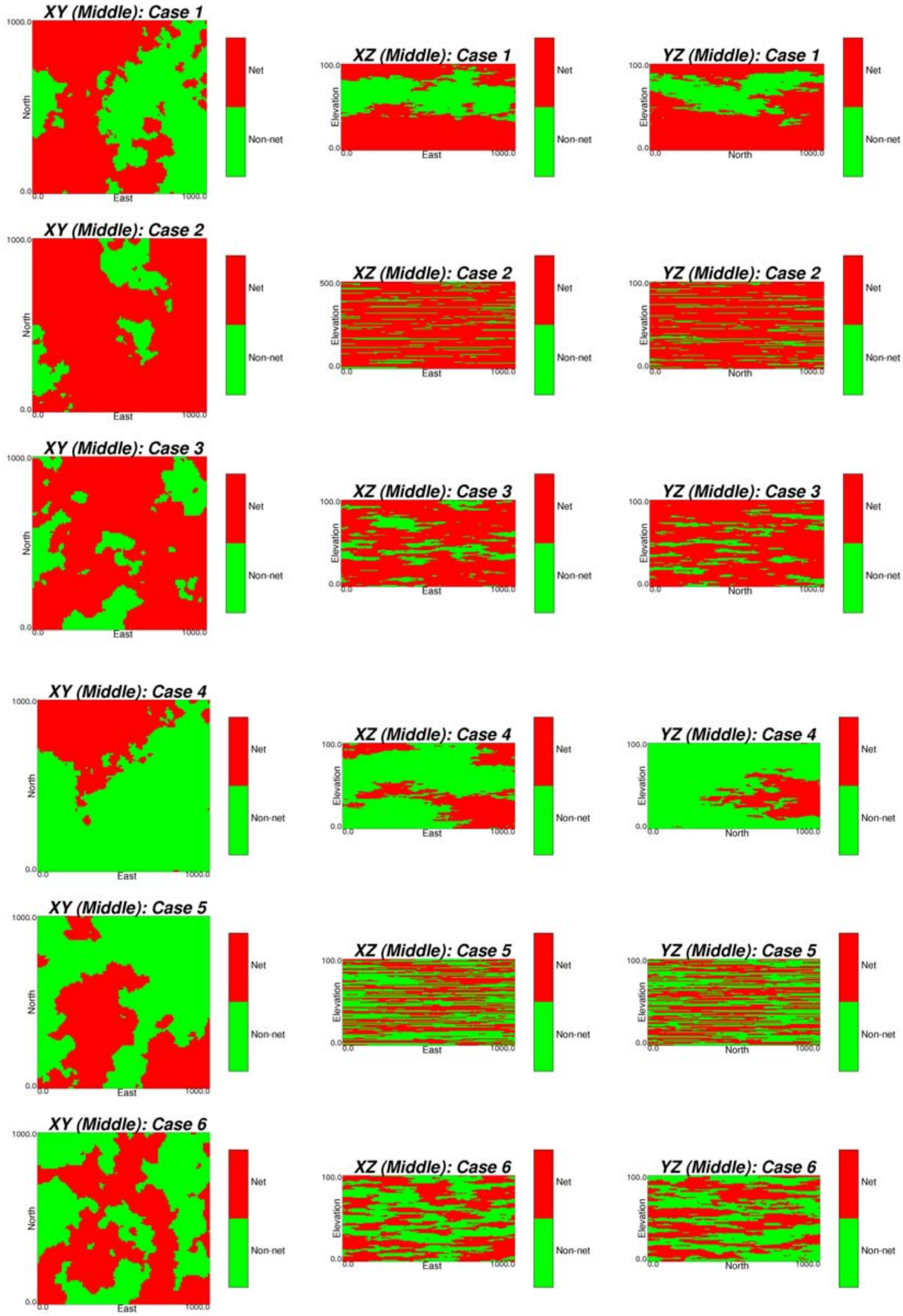
Figure 3 shows the histograms of the ratio of thickness to length obtained in each of the six simulation studies. Note that the distribution of the ratio depends on both variogram and proportion of net/non-net used in simulation. Also note that the ratio of thickness to length is variable; and can not be assumed to be a constant (e.g., taken as a ratio of vertical to horizontal continuity in variogram model).

Conclusions

In this paper an interesting new approach for calculating the size and connectivity of non-net intervals was proposed. The developed approach uses the mass moments of inertia calculation to predict the length of non-net interval based on thickness and to quantify the relationship between the length and thickness of non-net facies intervals. The proposed approach was illustrated using different simulation studies. The results of each study were analyzed and documented.

References

Hassanpour, M. and Deutsch, C.V. (2008) Fitting Local Anisotropy with Mass Moments of Inertia. Centre for Computational Geostatistics Report 10.



a) b) c)

Figure 2: Cleaned 3D SISIM realizations of Figure 1. Shown are middle slices in a) XY; b) XZ; and c) YZ direction.

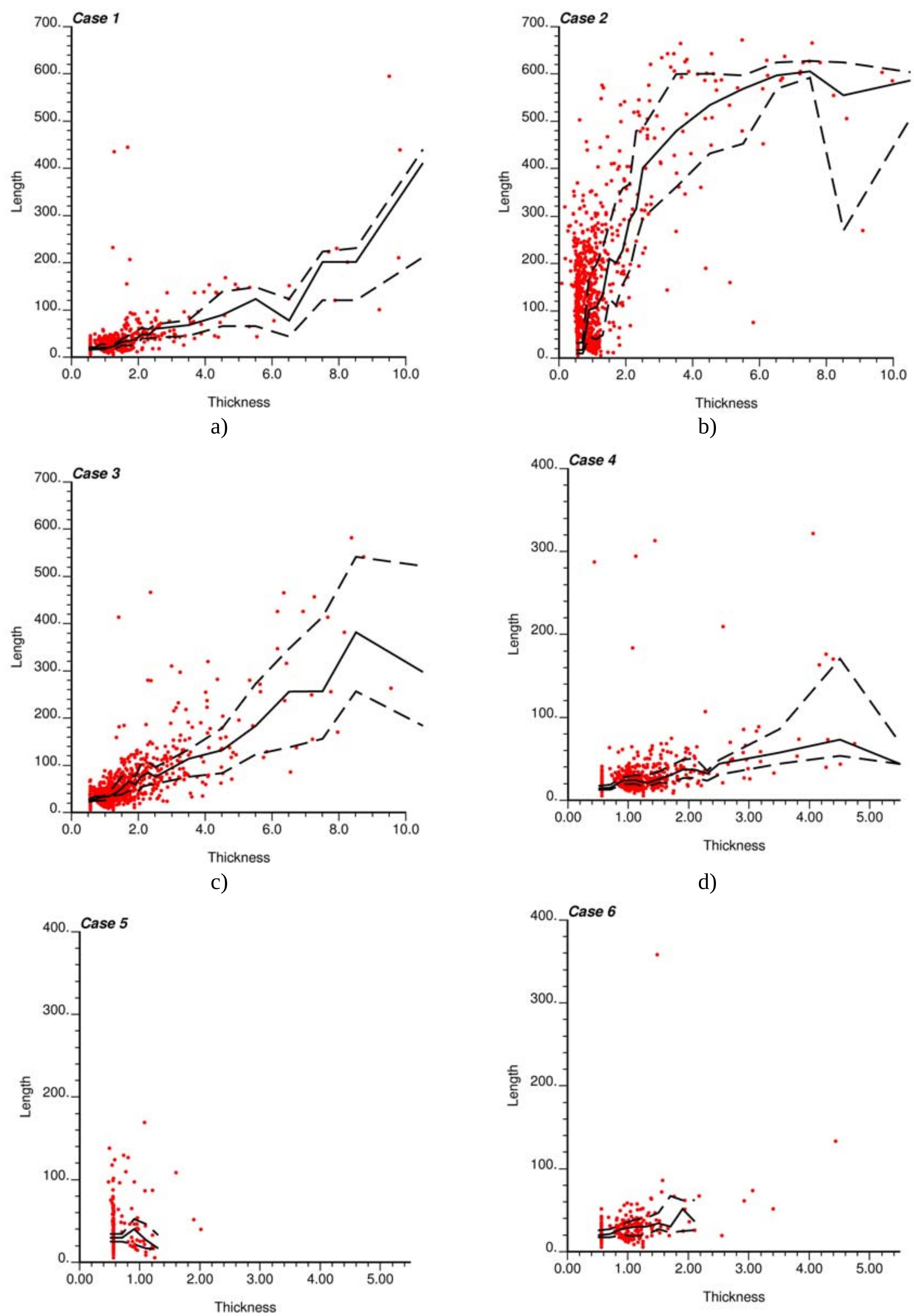


Figure 2: Length as a function of thickness for non-net intervals (zoomed): a) case 1; b) case 2; c) case 3; d) case 4; e) case 5; and f) case 6. Results are shown only for thicknesses up to 10 units for cases 1-3 and up to 5.5 units for cases 4-6. Solid lines denotes smoothed P50 length curves for different thicknesses; dashed lines show smoothed (P25, P75) length intervals.

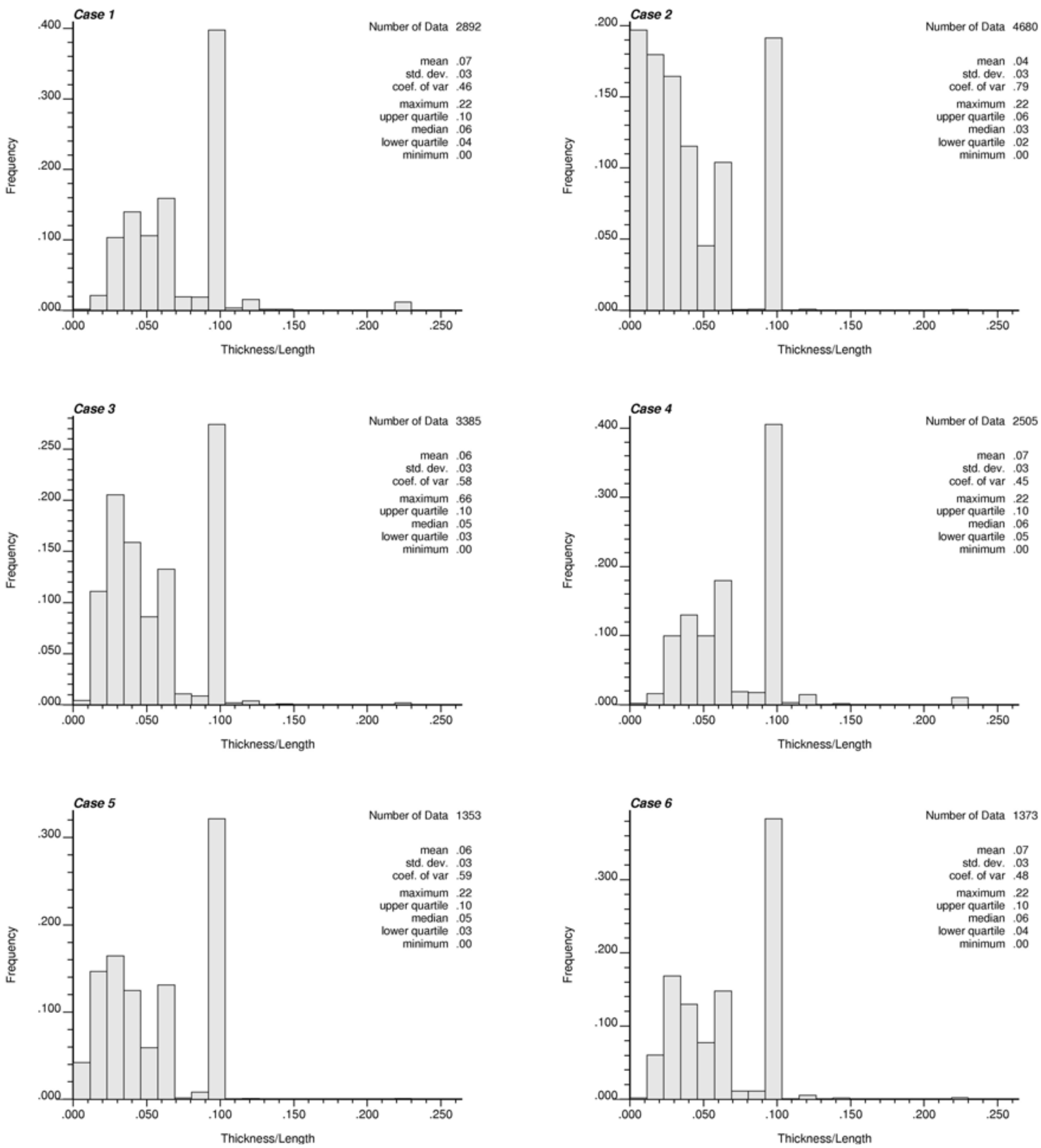


Figure 3: Histogram of the ratio of thickness to length of non-net intervals obtained in each of the six simulation studies.