A Methodology for Modeling Vein-Type Deposit Tonnage Uncertainty

Michael J. Munroe and Clayton V. Deutsch

One of the main sources of uncertainty in vein type deposits is found in the calculation of the tonnage. The boundary limits often applied to a vein type deposit are calculated from sparse data using deterministic methods that offer no measure of uncertainty. The most common method used to calculate the tonnage of a vein-type deposit is to convert the volume of the deposit defined by a wireframe model into a tonnage. Wireframe models are deterministic in nature being created from the interpretation of level plans and cross sections. These types of models have no provision for the calculation of tonnage uncertainty. This paper presents a distance function (DF) approach that allows for the modeling of vein-deposit tonnage with uncertainty. The distance function approach uses individual drillhole samples coded with a distance calculated by the DF rather than a wireframe model to estimate the deposit tonnage. Therefore a considerable savings in time can be achieved by skipping the wireframe modeling process. Through standardization, tonnages corresponding to any probability interval can be extracted. This methodology is demonstrated using a simulated set of reference models created using SGS whose true tonnages are known. The tonnage uncertainty is assessed through the use of accuracy plots.

Introduction

In most vein-type deposits, the evaluation of deposit tonnage is usually calculated through the use of wireframe models. The models are usually built from level plans and sections interpreted using the expert knowledge of the geologist. The wireframe models have a fixed volume with no method for calculating the uncertainty associated with the tonnage. In most evaluations of vein deposits little attention is given to the uncertainty in the geometry of the deposit and in most cases any uncertainty is applied to grade interpolation within a predefined wireframe model based on a deterministic interpretation. The boundaries of these models are determined from an interpretation of the geology built from level plans and cross sections with no method incorporated to characterize uncertainty.

The uncertainty in the geometry and therefore tonnage of a vein-type deposit can be calculated using a distance function (DF). The method does not make use of wireframe models but rather the distances between drillhole samples are calculated and modified using a distance function. The simple kriging interpolator is used to construct 3D models using the modified drillhole sample data. The interpolated values are standardized which allow tonnages corresponding to any probability interval to be calculated. The methodology is tested here using a set of synthetic models built for this specific purpose. The results are analyzed using accuracy plots and the uncertainty assessed.

Vein Deposit Geology

Vein type deposits are one of five classes of hydrothermal deposits (Misra, 2000). Vein-type deposits are formed by hydrothermal fluids and have well defined zones of mineralization. They are generally inclined and discordant with local geology. Vein-type deposits come in all sorts of shapes and sizes with many different levels of complexity and occur in fault or shear related zones. Vein systems occur as groups of veins which exhibit similar characteristics which are related to the same structural event. Vein-type deposits vary from a single narrow structure to large brecciated zones and stockworks. The orientation of vein-type deposits varies from steeply plunging to flat lying. The synthetic orebodies created to test this methodology conform to the characteristics of vein-type deposits.

Methodology

The methodology was tested using simulated data. A set of 50 reference models were created that simulate the physical characteristics of a vein type deposit. Each simulated orebody is essentially a horizontal tabular orebody with finite dimensions. Each model has a known ‘true’ tonnage. The models were sampled to produce a data set that mimics drillhole data that would be available for an actual deposit. In the simple example presented here, the data represents vertical drillhole samples. Each sample in the dataset is given a vein indicator code that distinguishes it as vein (1), or non-vein (0).

At the heart of the methodology is the distance function (DF). The DF calculates the Euclidean distance from a sample with one indicator value, to the closest sample that has a different indicator value. The idea behind the methodology is to use the calculated distances to create a zone of uncertainty around the boundary of the vein deposit. Tonnages can then be extracted from the bandwidth for any specified probability interval. The width and projection of the uncertainty bandwidth is controlled by modifying the original Euclidean distances. Two modifying parameters C and β are applied to the Euclidean distances to control the uncertainty bandwidth. The bandwidth of uncertainty is controlled by specifying a constant C. The position and distribution of tonnages within the uncertainty bandwidth is controlled by β. Provision for geometric anisotropy is also built into the methodology. The distances are modified to account for geometric anisotropy by specifying the anisotropic ratios for the xyz axes.

The modified distances are then interpolated using simple kriging to produce a 3D model of the vein deposit and tonnage. Through calibration, discussed in a separate paper, the DF approach is capable of producing tonnage estimates with uncertainty that are unbiased and fair. The detailed methodology used, including the creating of synthetic reference models, is as follows:

Step 1: Simulate some vein deposits

To accurately measure uncertainty we need to know the true tonnage of the orebody being tested. To test the methodology a set of reference models were created for this purpose. The models reproduce the characteristics found in vein deposits. The true tonnage of the vein deposit is calculated and stored for use when assessing the tonnage uncertainty. The reference models were created using sequential Gaussian simulation. Two surfaces are created, the first, an unconditionally simulated model representing the footwall of the deposit, the second, a conditionally simulated model that represents the hangingwall of the deposit, see Figure 1. The simulated values represent the hanging wall and footwall elevations. The size, shape and thickness of the orebody are controlled by the conditioning data. The hanging wall is conditioned so that the centre of the grid is positive becoming negative towards the edges. Resetting all the negative values to zero and adding the surface to the footwall creates a closed three dimensional orebody. A cross section through the surfaces is shown in . The 'true' tonnage is calculated as the difference between the two surfaces. A total of 50 reference models were created using this procedure.

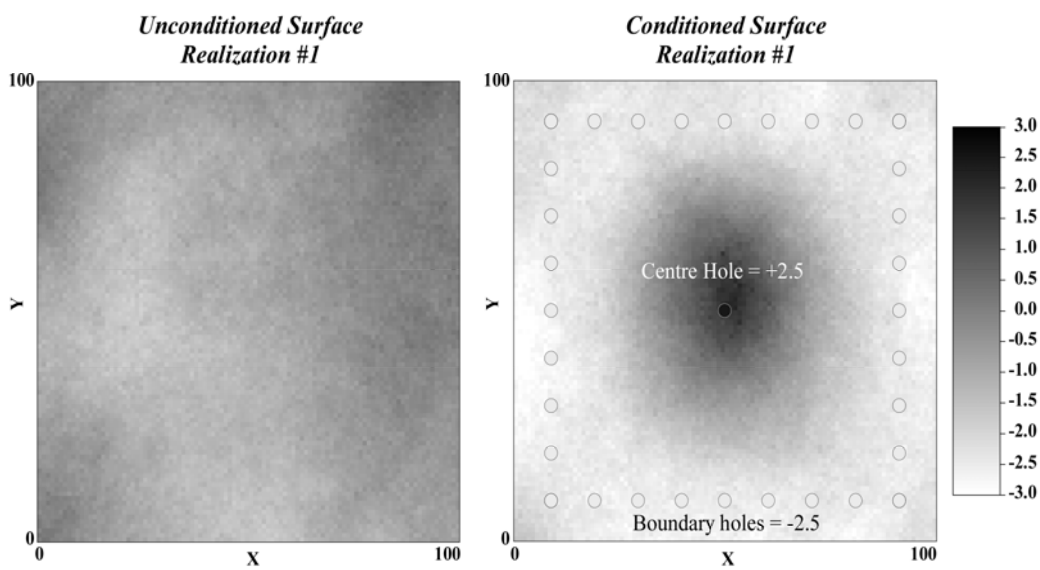

Figure 1: (left) unconditionally simulated surface, (right) conditionally simulated surface showing location of conditioning data.

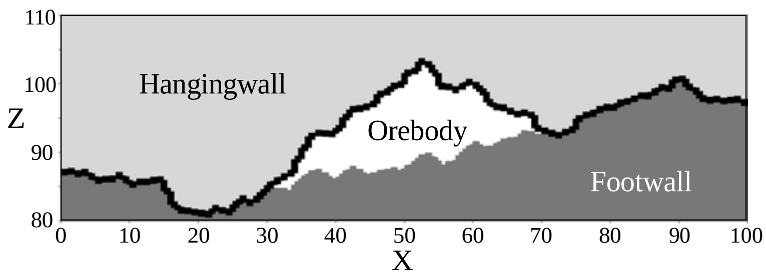

Figure 2: Vertical cross section through reference model.

Step 2: Drill the vein deposit

The second step is to 'drill' the reference orebody. Sampling on a rectangular grid was used to simulate drilling. Sample intervals are calculated through the model and each is assigned a vein indicator VI. The length of each hole corresponds to the model thickness. The drillholes are discretized into sample lengths of 1 unit. Samples located inside the orebody receive an indicator of 1, otherwise it is assigned 0. Figure 3 shows a vertical section through the orebody with simulated drillhole sample data. The final drillhole data set used for interpolation will have the xyz coordinates of each sample along with the vein indicator VI. Similar data would be used for an actual deposit.

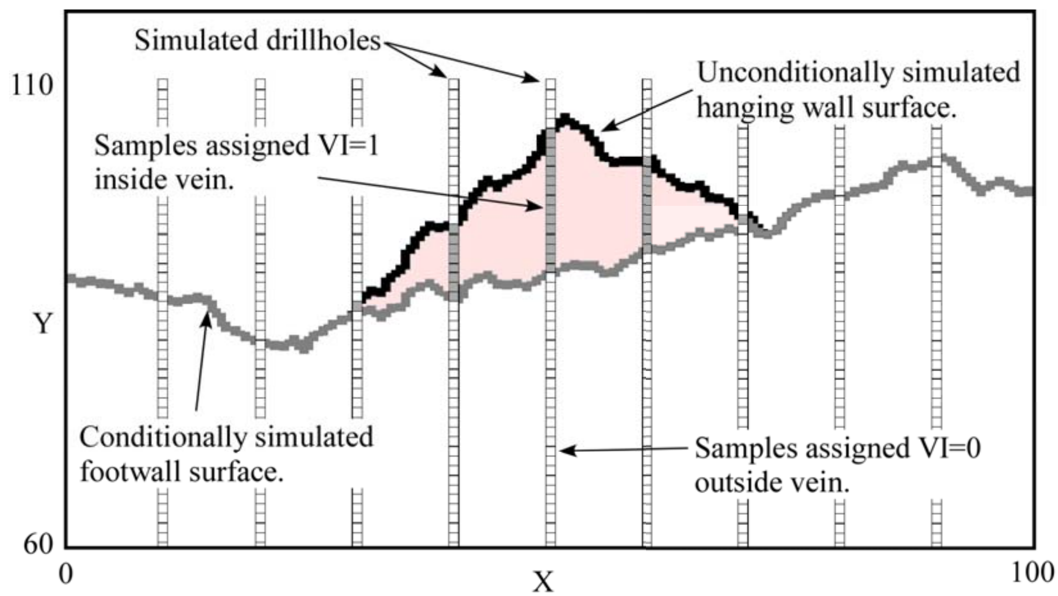

Figure 3: Vertical cross section through orebody showing drillhole samples.

Step 3: Calculate the DF values

For an actual deposit with drillhole samples the methodology would begin with Step 3, the calculation of the distances for each sample. A distance function (DF) is used to interpolate and map the boundary of a vein type deposit with uncertainty. The DF is based on the Euclidean distance between a sample and the nearest sample with a different indicator type. The distance values are given a sign according to the value of the vein indicator. Distances inside the vein are negative those outside the vein positive, see Figure 4 left. The distance function used is the Euclidean distance modified to account for geometric anisotropy, and is defined by the equation;

$$DF = \sqrt{\left(\frac{dx}{hx}\right)^2 + \left(\frac{dy}{hy}\right)^2 + \left(\frac{dz}{hz}\right)^2} \quad (1)$$

where ***d*** is the separation between two points in each of the x, y, and z directions and ***h*** is the geometric anisotropy defined for each of the x, y, and z directions. The idea is to adjust the distance function to favour the direction maximum continuity rather than treating all directions equally. The modification for geometric anisotropy allows the uncertainty bandwidth to have preference in the direction of maximum continuity. The left example in Figure 5 shows how the uncertainty bandwidth expands in a spherical envelope. Applying anisotropy to the DF Figure 5 right, allows the DF to expand horizontally from hole to hole while constraining the tendency to expand in the vertical direction.

As stated previously the idea is to create an uncertainty bandwidth around the contact between vein and non-vein based on the distances defined by equation 1. This is accomplished through the application of two parameters, C and β, to the sample dataset. The parameter C is the uncertainty constant. The parameter β is the bias correction constant. The constants are applied differently for each indicator value VI as shown in equations 3 and 4. The uncertainty constant *C*, is specified as a percentage of drill spacing. The value of *C* ranges from a value of 0, where the uncertainty bandwidth has a zero thickness and therefore no uncertainty, to a value of 1 where the uncertainty bandwidth has a width equal to the specified drill spacing. The actual value of the constant added to the *DF* is a function of drill spacing (*DS*) and *C* and is calculated as:

$$\frac{1}{2} C \cdot DS \tag{2}$$

Since the modified DF is made up of separate equations for vein and non-vein, equations 3 and 4, half the uncertainty constant is added to the positive non-vein samples and half to the negatively signed vein samples. The uncertainty parameter *C* must be calibrated so that the width of uncertainty to neither too large nor too small. Calibration is addressed in a separate paper.

$$DF_{\text{mod}} = \left(DF + C \right) / \beta \qquad \forall \text{ VI=0} \tag{3}$$

$$DF_{\text{mod}} = -\left(DF + C \right) \cdot \beta \qquad \forall \text{ VI=1} \tag{4}$$

The bias correction parameter β is is designed to control the position of the uncertainty bandwidth and allows the shifting of the interpolated set of estimates towards an unbiased distribution. An unbiased distribution will fall along the 45° line on an accuracy plot as illustrated in Figure 6. The β parameter is a positive number greater than zero and is dependent on drillhole spacing. When the VI is equal to 1, the DF is multiplied by β. When the VI is equal to 0, the DF is divided by β. If drillhole spacing tends to overestimate the tonnage, then β values less than 1 are used to shift the distribution to the left towards the 45° line as in Figure 6 right. On the contrary, if there is a tendency to underestimate the tonnage, then a β value greater than 1 is used to shift the distribution to the right as shown in Figure 6 left. The closer the set of estimates are to the 45° line, the more unbiased they are.

The contact between vein and non-vein defines a boundary called the iso-zero boundary. Recall that distance values outside the vein are positive and negative inside the vein therefore the contact between the two is zero. The zero point is known in the drilling and will be honored by the interpolation method. At unknown locations away from sampled locations however, there will be uncertainty as to where the actual position of the iso-zero boundary is located. The uncertainty in the location of the vein boundary away from known locations is determined by; 1) the *C* parameter which defines the width of the zone of uncertainty centred on the iso-zero boundary, and 2) β which defines the position of the zone to ensure unbiasedness.

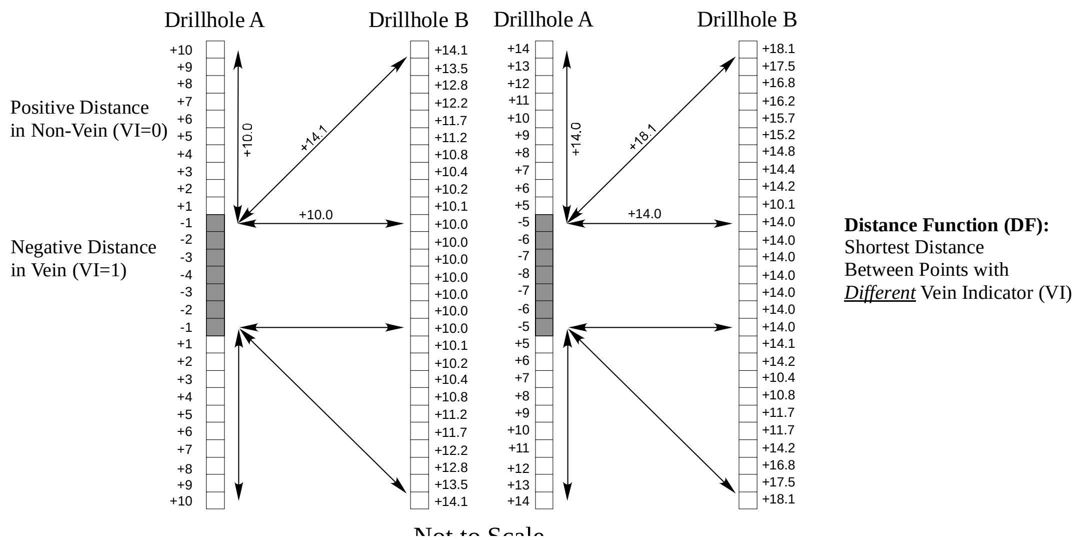

Figure 4: (left) Euclidean distances to samples with different VI values, negative values are VI =1, Positive values are VI=0. (right) The distances applied by the DF when *Cp*=4 and $\beta = 1$.

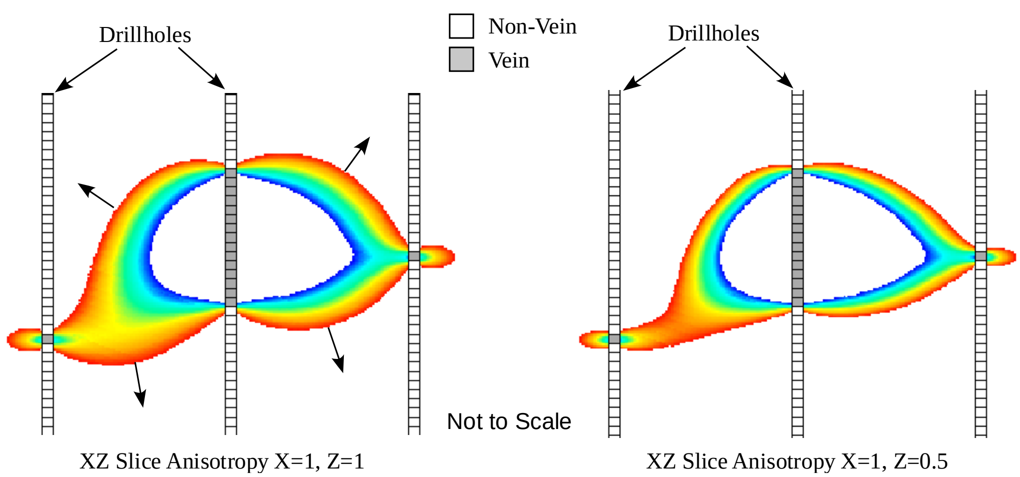

Figure 5: Correction for geometric anisotropy; (left) No correction, (right) Corrected

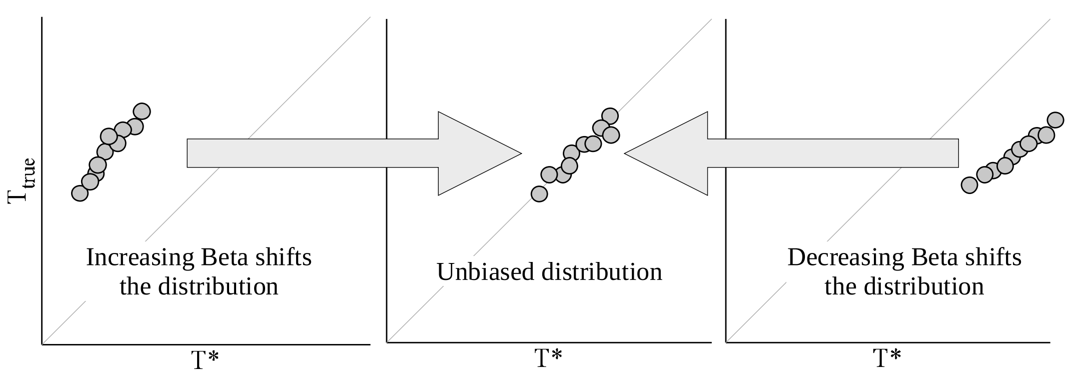

Figure 6: Effect of β on the distribution of a set of interpolated estimates. Increasing β shifts the distribution to the right. Decreasing β shifts the distribution to the left.

Step 4: Mapping the distance function values

Kriging is a commonly used interpolator which is a linear combination of the weighted data. The kriging weights are designed to minimize the error variance and therefore the kriging equation is often referred to as a Best Linear Unbiased Estimator (BLUE). With respect to mineral estimation, kriging is considered to be too smooth an interpolator incapable of reproducing short scale variability, however, for mapping the distance function it is ideal. Interpolating a vein contact capitalizes on the smoothness of kriging and is essential in calculating uncertainty of the vein boundary. The basic form of the simple kriging equation is:

$$d^*(u) - m = \sum_{\alpha=1}^{n} \lambda_\alpha \cdot \left[d(u_\alpha) - m \right] \tag{5}$$

where $d^*(u)$ is the kriged estimate at location *u*, *m* is the stationary mean over the area of the deposit, and for each location α, $d(u_\alpha)$ is the data and λ_α is the weight assigned to that data.

In simple kriging there is no constraint placed on the kriging weights. Under the assumption of stationarity, the mean is known and the same for all locations over the domain. This assumption is useful when modeling vein-type deposits. The unbiased simple kriging equation can then be written as;

$$d^*(u) = \sum_{\alpha=1}^{n} \lambda_\alpha \cdot d(u_\alpha) + \left[1 - \sum_{\alpha=1}^{n} \lambda_\alpha \right] \cdot m \tag{6}$$

where, $d^*(u)$ is the estimate at location u, $d(u_\alpha)$ is the data at location u_α, *n* are the number of data, *m* is the global mean and finally, λ_α is the weight assigned to the data at location α. Equation 6 is important in that is shows the relation between the weights, λ_α, and the mean *m*. As the estimate becomes more distant from the data, the weights assigned approach zero, along with the influence of the data, and the value of the estimate approaches the value of the mean. A very useful property when applied in conjunction with the distance function. Therefore, specifying a simple kriging mean can be used to control the estimate in areas where there is little or no information. Finally, since kriging is an exact interpolator, the values at all data locations are honored.

The variogram is a necessary and essential part of the mapping process. The variogram supplies the spatial relationship between data pairs used by the kriging algorithm. The spatial relationship is quantified using the variogram defined by;

$$2\gamma(h) = E\left\{ \left[Z(u) - Z(u+h) \right]^2 \right\} \tag{7}$$

It is the expected value of the squared difference between a sample $Z(u)$ and a second sample $Z(u+h)$, separated by a lag vector **h**. When applying the variogram to the mapping algorithm, the semivariogram, one half the variogram is used. When modeling the DF the interest is in the short scale, distances that are close to the vein boundary and within the range of the drillhole spacing. Since the idea is to map the boundary, samples that are located more than a distance equal to the drill spacing away become redundant. The idea is to provide a variogram that will produce a smooth zone of uncertainty from drillhole to drillhole as illustrated in Figure 7.

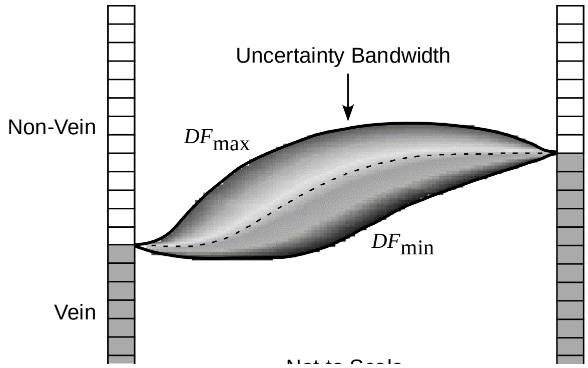

Figure 7: Schematic of the smooth uncertainty boundary between drillholes.

Step 5: Calculate Vein Tonnage

Calculate the vein tonnage within predefined probability thresholds defined from the DF. The tonnage for each model is calculated from the uncertainty bandwidth, the limits of which is determined by the uncertainty constant *C*, and the bias component β .

The inner limit of the uncertainty band, $df_{\min}$ is calculated as;

$$DF_{\min} = -\frac{1}{2} C \cdot DS \cdot \beta \tag{8}$$

where *DS* is the drill spacing. The outer limit uncertainty band, $df_{\max}$ is calculated as;

$$DF_{\max} = \frac{1}{2} \frac{C \cdot DS}{\beta} \tag{9}$$

In short, the minimum is defined as one half the modified drill spacing applied to the vein side of the iso-zero, and the maximum as one half the modified drill spacing applied to the non-vein side of the iso-zero as shown in Figure 8.

The probability thresholds within the bandwidth are defined as a probability value. The bandwidth interval is recalibrated to [0,1] so that $DF_{\min} = 0$, and $DF_{\max} = 1$. The iso-zero, the solid line in Figure 8, is the p_{50} and has a *p* value of 0.5. The *p* values are used to extract tonnages for defined probability intervals by converting cell values into *p* values.

The *p* value is calculated as;

$$p = \frac{z - DF_{\min}}{DF_{\max} - DF_{\min}} \tag{10}$$

where *z* is the estimated value. The total tonnage for a particular probability interval p_i , is the total number of cells where $p \le p_i$. Recall that the zone of uncertainty is located between $DF_{\min}$ and $DF_{\max}$. If $z < DF_{\min}$ then *z* is certainly located within the vein structure. If $z > DF_{\max}$ then *z* is most certainly located outside the vein structure. Rescaling the space between $DF_{\min}$ and $DF_{\max}$ into a [0,1] interval allows tonnages from a mapped distance function to be extracted for any probability interval.

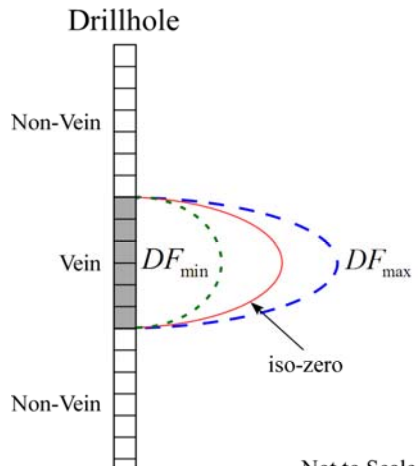

Figure 8: Illustration of uncertainty bandwidth limits, $DF_{\min}$ inner limit (dotted line), $DF_{\max}$ outer limit (dashed line), iso-zero (solid line).

Step 6: Assess Uncertainty

There are a few simple guidelines for good uncertainty: (1) the result needs to be unbiased, (2). the uncertainty must be fair, and (3) the uncertainty must be as small as possible.

The β parameter controls bias. A set of estimates is unbiased when the expected value of the estimates is the expected value of the truth.

$$E\left\{Z^{*}\right\} = E\left\{Z\right\} \tag{11}$$

A schematic illustration showing a plot of the estimate, T^*, versus the true value, T, is shown in Figure 6. If the points fall around the 45° line, Figure 6 centre, the estimates are unbiased.

The *C* parameter controls accuracy. The accuracy of a set of estimated values is based on the fraction times the true value falls within a symmetric probability interval based on the estimated values.

$$E\left\{Z \in Z^{*} \pm \frac{p}{2}\right\} \quad \forall \text{ p } \in \left[0,1\right] \tag{12}$$

An estimate is considered accurate if it falls within or exceeds the interval of true values specified by the experiment. For example, for the 80th percentile, it is expected that 80% of the estimates fall within the mean centred probability interval defined by;

$$\begin{aligned} & p^{*} \pm \frac{p}{2} = p^{*} \pm \frac{p_{80}}{2} \\ & p^{*} \pm p_{40} \end{aligned} \tag{13}$$

For the 80th percentile that defines a probability interval from *p10* to *p90*.

To assess the uncertainty in a kriged model, the tonnage for a range of probability intervals is calculated.

Example

A set of 50 synthetic vein deposits were created using SGS. Each deposit has a known true, calculated tonnage. To test the methodology, the grids are sampled on an equal spaced regular grid equal to 10 units. The grid sampling simulates drilling by producing strings of sample through the orebody as illustrated in Figure 9. The sample data is modified using specified values for C and β and then used to create gridded models using simple kriging.

Tonnages are calculated for each of the 50 kriged models based on the uncertainty bandwidth defined by C and the uncertainty assessed.

Figure 10 left shows a scatterplot of the 50 tonnage estimates versus the true tonnages. The models were estimated using an uncertainty constant C of 0.173 and a β value of 1.124. The actual values used for C and β are determined through calibration which is discussed in a separate paper. The plot indicates that the global tonnage estimates are within an acceptable tolerance.

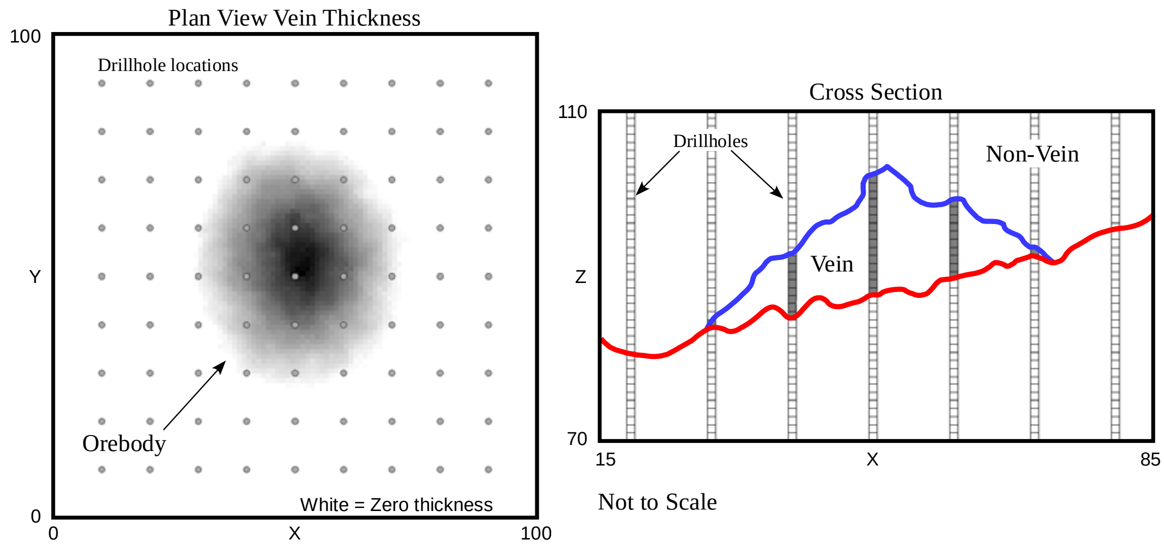

Figure 9: (left) Plan view showing drillhole layout; (right) Cross section through orebody

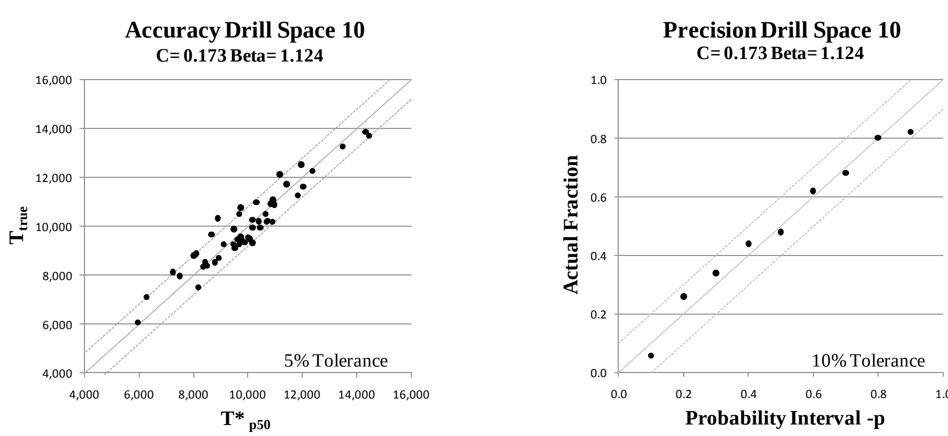

Figure 10: Accuracy plots

Future work

The methodology presented here was tested using synthetic reference models created using SGS. The method must be tested using real data from an actual vein type deposit. There is a need to develop the methodology to deal with multiple intercepts or structures as well as drilling on an irregular grid.

Conclusions

The tonnage of vein type deposits can be significant source of uncertainty. Tonnages of vein deposits are commonly calculated using wireframes built from the interpretation of geologic level plans and sections. The construction of wireframe models is a deterministic process that is often time consuming. The wireframe models created provide a single tonnage estimate with no provision for the determination of the uncertainty associated with the estimate.

The purpose of modeling the boundary using a distance function is to provide a measure of the uncertainty in the tonnage of the given vein deposit. The method provides an estimate of the tonnage with uncertainty without the need to create a wireframe model.

The methodology was tested using 50 synthetic reference models with a calculated known true tonnage. The modeling process creates an uncertainty bandwidth that is calibrated to allow the calculation of tonnages corresponding to any probability value. Tonnages are calculated by summing the tonnages from all grid cells whose values are less than or equal to the probability value.

The method when calibrated properly is shown to provide an estimate of vein deposit tonnage uncertainty that is both fair and accurate.

References

Deutsch, C.V., A Short Note on Prediction of Uncertainty in Tonnage and Grade of Vein Type Deposits, paper #305. In Centre for Computational Geostatistics, Report 7, 2005.

Deutsch, C.V., Direct Assessment of Local Accuracy and Precision, In Geostatistics Wollongong '96, Volume 1, 115-125, E.Y. Baafi and N.A. Schofield, eds.

McLennan, J. and Deutsch, C., Implicit Boundary Modeling (Boundsim), paper #102. In Centre for Computational Geostatistics, Report 8, 2006.