

Fitting Local Anisotropy with Mass Moments of Inertia

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The direction and magnitude of geological continuity is often revealed by geological interpretation. These details of geological continuity/variability can be important for resource/reserve calculations. The connectivity of high and low values are critical for prediction. Geological interpretation is essential, but is often limited to large scale features and generalizations. An automatic approach is required for local refinement and repeatability. The variogram map (or volume in 3-D) is commonly used to assist in the determination of variogram directions. The correlation map (variance minus the variogram map) could be interpreted as “mass” and automatically fit by a tensor. The principal directions of this tensor could be used in subsequent modeling. A prototype FORTRAN subroutine is developed and different applications are examined.

Introduction

Geological continuity is a key factor for prediction of fluid flow responses in reservoir. The geological continuity and variogram continuity are direction dependent (Deutsch, 2002). Generally, the directions of continuity in variables are determined prior to geostatistical modeling. The fairly standard approach is to plot the variogram maps and detect the maximum and minimum continuity direction based on the range of variogram. The geological information and interpretations are another source to detect the continuity direction in lithofacies. This process is subjective and requires the user judgment. In cases that we are dealing with large number of variables and geological facies, applying this method could be frustrating. Using tensor of moment of inertia can be helpful.

Moment of inertia of a body is related to the distribution of the mass throughout the body and quantifies the rotational inertia of a rigid body:

$$I = \int_V m.r^2 dm \quad (1)$$

where m is mass and r is the perpendicular distance from the axis of rotation.

Generally, there are two forms of moment of inertia; scalar form which is used when the axis of rotation is known and the tensor form that summarizes all moment of inertia for different axes of rotation with one quantity.

For a rigid body consisting of N point masses m_i the moment of inertia tensor are defined as follows:

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \quad (2)$$

$$\begin{aligned}
I_{xx} &= \sum_{i=1}^N m_i (y_i^2 + z_i^2) & , & & I_{xy} = I_{yx} &= -\sum_{i=1}^N m_i x_i y_i \\
I_{yy} &= \sum_{i=1}^N m_i (x_i^2 + z_i^2) & , & & I_{xz} = I_{zx} &= -\sum_{i=1}^N m_i x_i z_i \\
I_{zz} &= \sum_{i=1}^N m_i (x_i^2 + y_i^2) & , & & I_{yz} = I_{zy} &= -\sum_{i=1}^N m_i y_i z_i
\end{aligned}$$

where x_i, y_i and z_i are the distances of point i from the coordinate axes. Here the physical meaning of I_{xx} is the moment of inertia around the x -axis when the objects are rotated around the x -axis and I_{xy} is the moment of inertia around the y -axis when the objects are rotated around the x -axis.

Principal directions of moment of inertia show the directions in which the rigid body is more closely distributed or less distributed. In this case the major direction is the one which related to the smaller moment of inertia. This interesting property leads us to determine the major and minor direction of continuity in the geological setting or define the principal direction of an unstructured grid block.

Methodology

Correlation indicates the strength and direction of a linear relationship between two random variables, U and V .

$$\rho_{U,V} = \frac{Cov\{U,V\}}{\sigma_U \cdot \sigma_V} = \frac{E\{(U - \mu_U)(V - \mu_V)\}}{\sigma_U \cdot \sigma_V} \quad (3)$$

where σ_U is standard deviation of U , σ_V is standard deviation of V , μ_U is mean of U and μ_V is mean of V .

Correlation varies between -1 to 1. Correlation of 1 shows the perfect linear relationship and -1 indicates an inverse linear relationship. The closer the coefficient is to either -1 or 1, the stronger the correlation between the variables. If the variables are independent the correlation is zero.

Correlation map shows the calculated correlation of a data set for different lag \mathbf{h} and directions. If we look at a map of correlation, we can find an ellipse which its major and minor radius are aligned with major and minor directions of continuity. See Figure 1 for examples of correlation map for isotropic and anisotropic medium. If we consider the correlation as a mass quantity, then the correlation map can be considered as a rigid body and we can determine the anisotropy ranges and directions analogous to the method of determination of principal axes for moment of inertia.

Given the components of a symmetric moment of inertia tensor in a coordinate system, we can find all components in any other coordinate system by coordinate rotation. Directions around which the moment of inertia tensor becomes diagonal are principal directions of tensor. Principal directions can be defined by eigenvalue and eigenvector decomposition (See Appendix A for details of calculation).

The largest eigenvalue and the corresponding eigenvector and the smallest eigenvalues and eigenvector are related to minor and major directions respectively. Anisotropy ranges can be defined by using the analytical formula for calculation of moment of inertia around major and minor axes of an ellipse/ellipsoid.

In 2D, moment of inertia of an ellipse around its major and minor radius is defined by the following formula:

$$I_a = \frac{\pi}{4} r_a r_b^3$$

$$I_b = \frac{\pi}{4} r_a^3 r_b$$
(4)

where I_a and I_b are moment of inertia around major radius r_a and minor radius r_b , respectively. Since are available in right hand side of both equations, ranges can not be calculated directly and iteration can be considered.

in 3D, moment of inertia of an ellipsoid around its major, medium and minor radius is defined by the following formula:

$$I_a = \frac{1}{5} M (r_b^2 + r_c^2)$$

$$I_b = \frac{1}{5} M (r_a^2 + r_c^2)$$

$$I_c = \frac{1}{5} M (r_a^2 + r_b^2)$$
(5)

where I_a , I_b and I_c are moment of inertia around major r_a , medium r_b and minor r_c radius, respectively and M is the ellipsoid mass.

Applications

The anisotropy in the geological setting varies from one point to another due to heterogeneity. In some complex geological setting such as fluvial setting which contains channel features, the continuity direction significantly changes from one region to another. Geostatistical modelling techniques such as sequential Gaussian simulation or sequential indicator simulation with a global variogram orientation and search parameters can not reproduce these local changes in continuity direction. In complex cases where the anisotropy varies locally, locally varying anisotropy (LVA) can be defined and transferred into modeling to improve performance (Leuangthong et. al., 2006; Biosvert et. al., 2007). The locally varying anisotropy parameters can be defined by using seismic information, post-processing facies models (training Images) or directly from data.

Seismic

Seismic attributes are widely used in petroleum exploration to better characterize the reservoir and make some good and educated guesses about reservoir features. Seismic models contain important information about spatial variability in large scale and can be used to determine the anisotropy in reservoir prior to geostatistical modeling. This can be more investigated by the following example.

Figure 2 shows an example seismic attribute model with resolution of 560 by 320. In order to find the local anisotropy we first need to define some windows. Then the correlation map is calculated within each window and the local anisotropy is defined. Three different window sizes (small, medium, large) are considered. Figure 3 shows result of LVA determination for three different window sizes.

Facies Model or Training Image Post Processing

Facies model and any reliable training image can be post-processed to determine the local anisotropy prior to modeling of petrophysical properties such as porosity and permeability. This has been shown in the following example

A 2D training image is considered. The model has 256 cells in both x and y direction. There are three facies in the model. In order to locally determine the continuity direction some windows are considered which contain 16 fine grid cells in x and y direction. Figure 4 shows the training image and all windows considered for LVA determination. Correlation map is calculated for the major facies within the specified windows and the anisotropy directions and ranges are defined locally. Result of local anisotropy determination is shown in Figure 5.

Local Correlation with Scattered Data

This is a fairly standard approach in geostatistical modeling to define the major continuity directions before variogram calculation. This can be done by plotting variogram map based on available data. However, in most cases variogram map is noisy and it is not trivial to detect major directions. Indeed, local anisotropy may be required for simulation and estimation method in presence of locally varying anisotropy.

In order to validate our methodology, we considered a known reservoir model and sampled 310 data point from that true reservoir model. Figure 6 shows location of scattered data which are sampled from reservoir model. Area is divided into 15 windows and anisotropy directions are determined within each window.

Considerations and Limitations

The methodology described in this paper works for 2D and 3D cases. But there are always some considerations. Choosing the correct window size is very important in local anisotropy determination. Using very small windows may lead to isotropic results and loosing the large scale variability. If we choose a fairly large window then the results are not locally precise and the variability will be averaged out. Using negative correlation values in calculation may destroy the methodology and lead to wrong directions and ranges. The correlation map should be trimmed for negative correlations.

References

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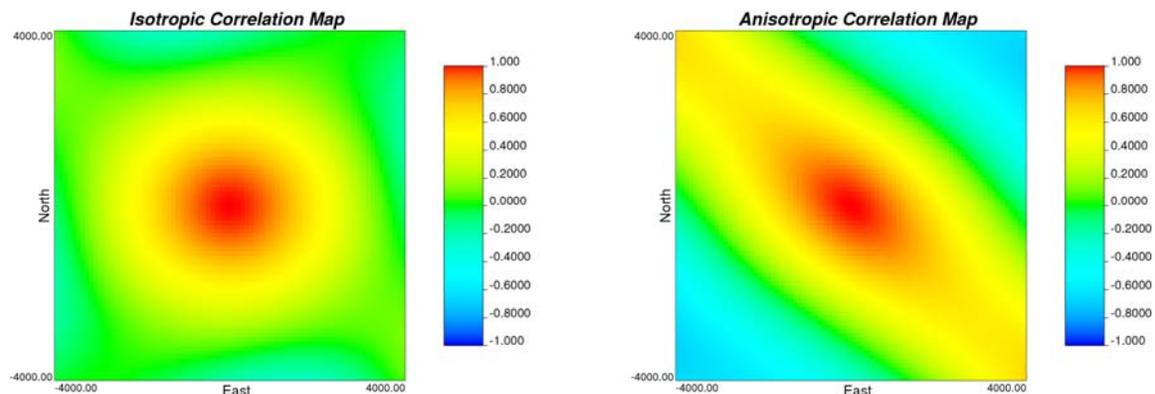


Figure 1. Example of isotropic (left) and anisotropic correlation map.

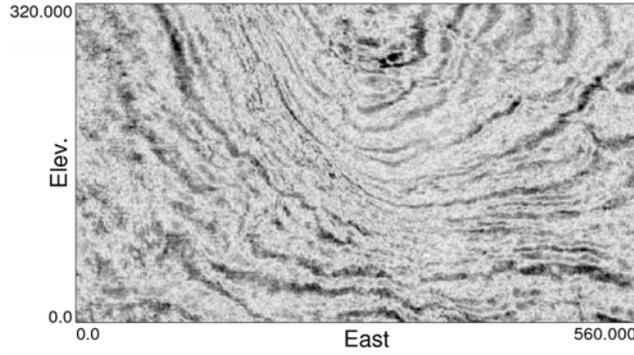


Figure 2. A seismic model used for local anisotropy determination example.

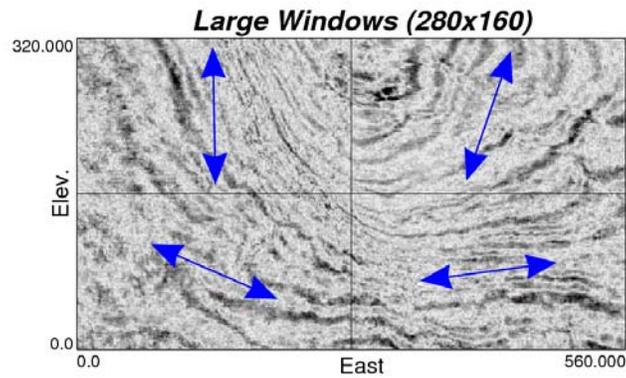
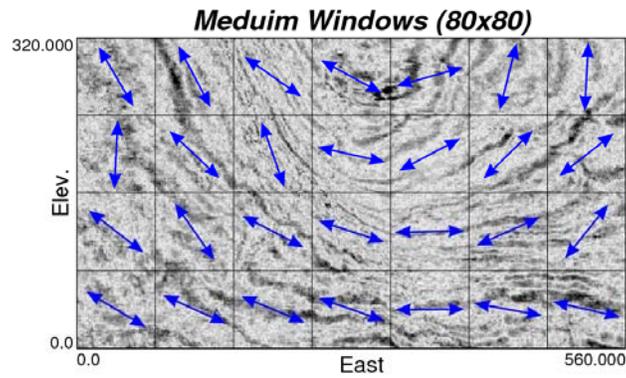
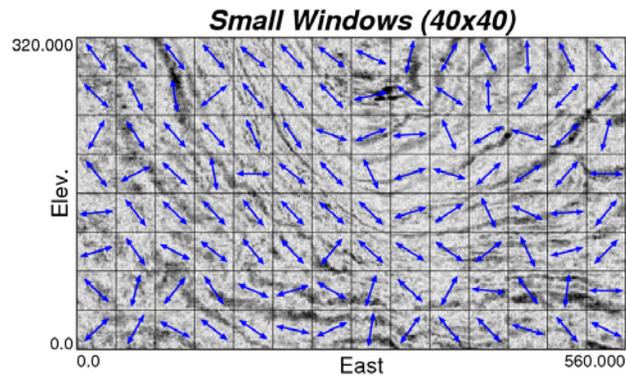


Figure 3. Local Anisotropy determination for example seismic model with three different window sizes.

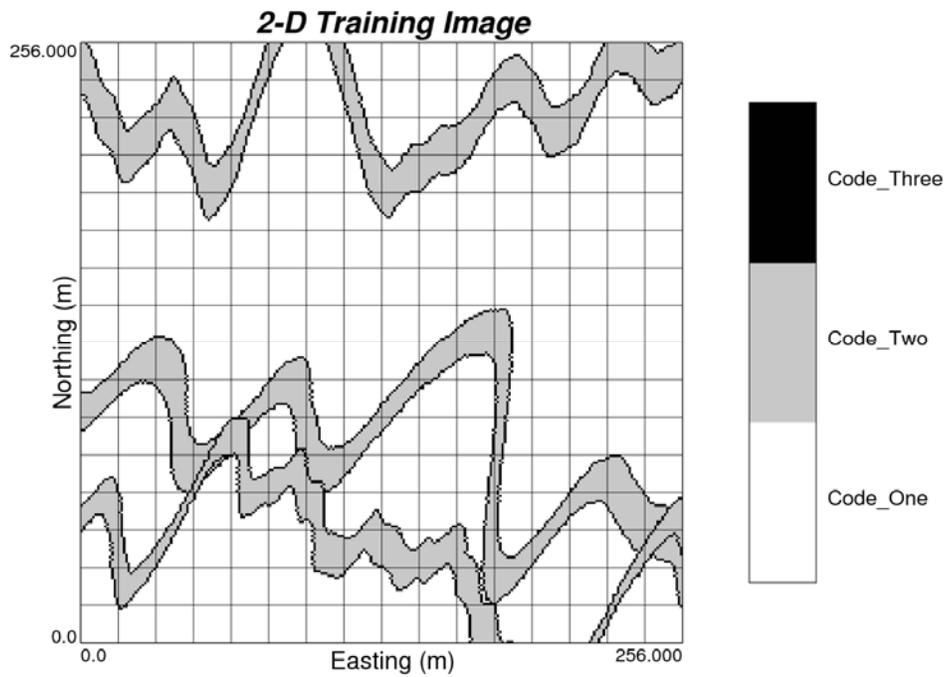


Figure 4. A training image considered for determination of locally varying angle. The squares show the windows.

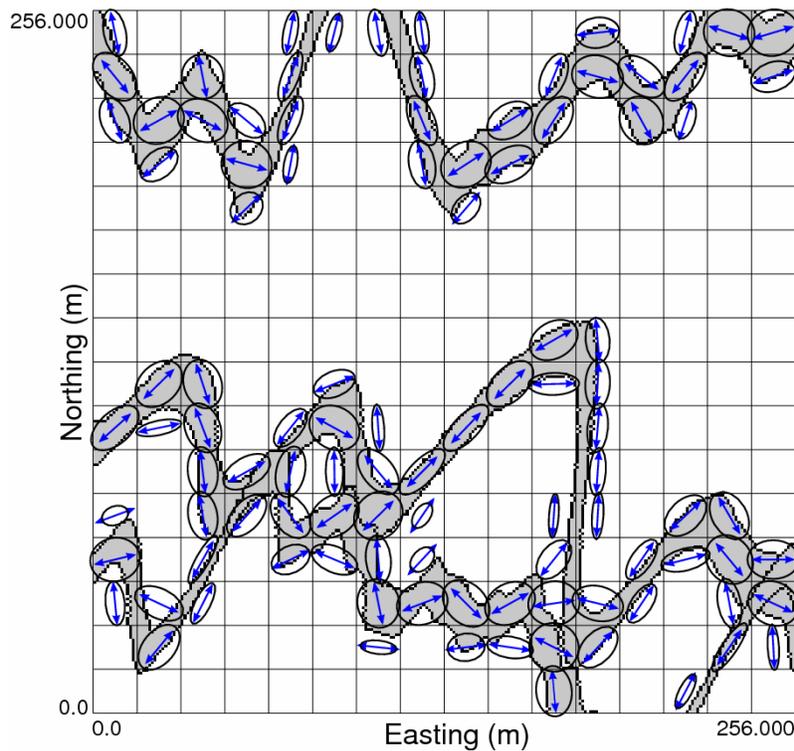


Figure 5. Results of locally varying anisotropy determination. Ellipses show the anisotropy direction and ranges within each window.

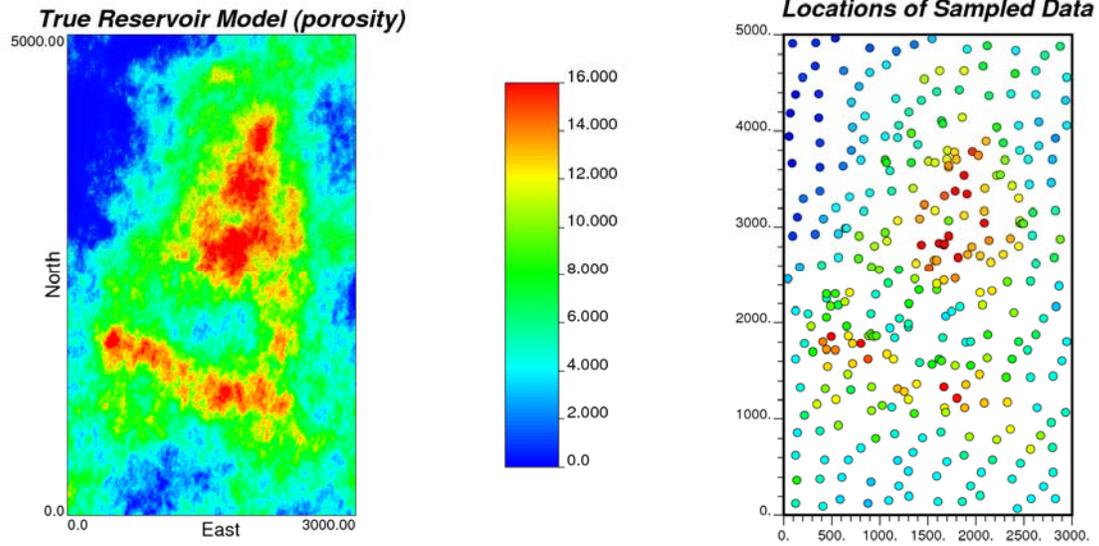


Figure 6. True reservoir model (left) and location of data sampled from true model.

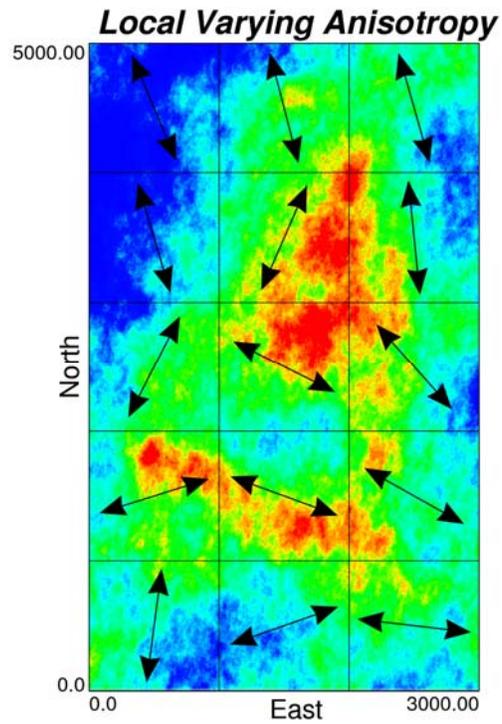


Figure 7. Result of local varying anisotropy direction based on scattered data.

Appendix A: Eigenvalue and Eigenvector Determination

Eigenvalues and eigenvectors are numbers and vectors associated to square matrices, and together they provide the eigen-decomposition of a matrix which analyzes the structure of this matrix. Consider a second order 3D matrix. Eigenvalues and eigenvectors determination of this matrix requires solving the following equation:

$$\begin{bmatrix} I_{xx} - \lambda & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} - \lambda & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} - \lambda \end{bmatrix} \cdot \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{0} \quad (1)$$

where v_x, v_y and v_z are the components of the eigenvector in the principal direction and λ is the corresponding eigenvalue.

The three eigenvalues of equation (1) are the roots of the following cubic equation:

$$\begin{vmatrix} I_{xx} - \lambda & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} - \lambda & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} - \lambda \end{vmatrix} = \lambda^3 - a\lambda^2 + b\lambda - c = 0 \quad (2)$$

where the coefficients are:

$$\begin{aligned} a &= I_{xx} + I_{yy} + I_{zz} \\ b &= I_{xx} \cdot I_{yy} + I_{yy} \cdot I_{zz} + I_{xx} \cdot I_{zz} - I_{xy}^2 - I_{yz}^2 - I_{xz}^2 \\ c &= I_{xx} \cdot I_{yy} \cdot I_{zz} + 2I_{xy} \cdot I_{yz} \cdot I_{xz} - I_{xx} \cdot I_{yz}^2 - I_{yy} \cdot I_{xz}^2 - I_{zz} \cdot I_{xy}^2 \end{aligned} \quad (3)$$

Once the eigenvalues are calculated, the eigenvectors are determined by substituting them into equation (1).