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Generating LVA Fields for use in Nonstationary Geostatistics

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Recently there has been interest in incorporating locally varying anisotropy (LVA) into geostatistical models. The traditional assumption is that a deposit is stationary, that is, that the mean and variogram are constant in the modeling domain. Many natural deposits display non-stationary features and the assumption of stationarity is questionable. This paper does not discuss how the LVA field is used to build geostatistical models, the interested reader is revered to (Boisvert 2008). To implement the methodology in Boisvert (2008) it is necessary to parameterize the LVA field by the magnitude (anisotropy ratios) and orientation (major direction of continuity); an exhaustive LVA field is required for the deposit. This paper presents 6 different types of data that can be used to infer the LVA field as well as 5 methodologies to determine the LVA field from the data types (magnitude and direction). This paper provides practical methodologies to generate the LVA field from potential sources of data. Real data sources are used in this paper to highlight the proposed LVA field generation methodologies.

LVA Field Parameterization

The LVA field delineates the direction and magnitude of anisotropy in a modeling domain. The direction of anisotropy is defined by three angles (Deutsch 1998). The magnitude of anisotropy is taken relative to the major direction of continuity, thus, two ratios are necessary to define the anisotropy in 3D, minor/major, and vertical/major. The LVA field is used to calculate the similarity between two points (Boisvert 2008) and can be defined by the matrix of rotation (Equation 1) and Equation 2.

$$R = \begin{bmatrix} \cos\alpha\cos\varphi - \sin\alpha\sin\beta\sin\varphi & -\sin\alpha\cos\varphi - \cos\alpha\sin\beta\sin\varphi & \cos\beta\sin\varphi \\ \frac{1}{r_1}\sin\alpha\cos\beta & \frac{1}{r_1}\cos\alpha\cos\beta & \frac{1}{r_1}\sin\beta \\ \frac{1}{r_2}(-\cos\alpha\sin\varphi - \sin\alpha\sin\beta\cos\varphi) & \frac{1}{r_2}(\sin\alpha\sin\varphi - \cos\alpha\sin\beta\cos\varphi) & \frac{1}{r_2}\cos\beta\cos\varphi \end{bmatrix} \qquad \mathbf{1}$$

where two points are separated by the vector (**h**). The direction of anisotropy is defined in 3D by three angles, strike (α) dip (β) and plunge (ϕ). The magnitude of anisotropy is defined by two ratios: r_1 is the ratio between the minor and major directions and r_2 is the ratio between the vertical and major direction. The calculation of the anisotropic distance between two points becomes:

$$d^2(\mathbf{h}) = \mathbf{h}^T \; R^T R \; \mathbf{h} \qquad \mathbf{2}$$

Thus, the LVA field is an exhaustive set of five parameters (α, β, ϕ, r_1 and r_2) that vary locally. Calculating the anisotropic distance between points separated by **h** is accomplished using the local rotation matrix. Note that in a 2D case only α and r_1 are required.

Sources of Data for LVA Field Inference

A common justification for ignoring locally varying anisotropy is the lack of available data to infer the LVA field. This section explores the diverse range of sources commonly available for LVA field generation (Table 1). Each data source is described and an example data set with a calculated LVA field is provided. Real data is shown where available, otherwise synthetic data is generated.

Table 1: Possible sources of information that can be exploited when generating the LVA field. 'Potential availability' represents this authors subjective appraisal of the availability of each data source based on past experience.

Source	Brief Description	Potential Availability
Drill hole data	Data based on the variable of interest.	High
Direct angle measurement	Direct measurements of the strike, dip and/or plunge of the deposit at drill hole locations.	Low

Remote sensing	Remote methods including seismic, magnetic, ground penetrating radar, and other geophysical measurements.	Medium
Structural models	Stratigraphic deposits are often modeled by layer. Layer orientation can indicate the local direction of anisotropy.	High
Facies/rock type models	Categorical variable models generated by any technique can be used to generate models of anisotropy.	High

Normally, it is necessary to merge multiple sources of information from Table 1 to generate a single LVA field. Some areas of the deposit may be extensively sampled and the drillhole data can reliably characterize the LVA field. Sparsely sampled areas may require geological interpretation, inference from analogue deposits, exploitation of remote sensing information or even a stochastic description of the anisotropy field.

Drill hole data

If a deposit has been sufficiently sampled, the drill hole data of the variable of interest can be used to provide a reliable inference of the LVA field. The available sample data must be interpreted to extract information with respect to the underlying LVA field. The following example is based on the well known Walker Lake Data set (Isaaks and Srivastava, 1989). In this data set there are two exhaustive variables U and V. Consider the U variable to be the variable of interest and the more extensively sampled V variable is used to generate the LVA field for U. This is possible under the assumption that U displays the same spatial features as V.

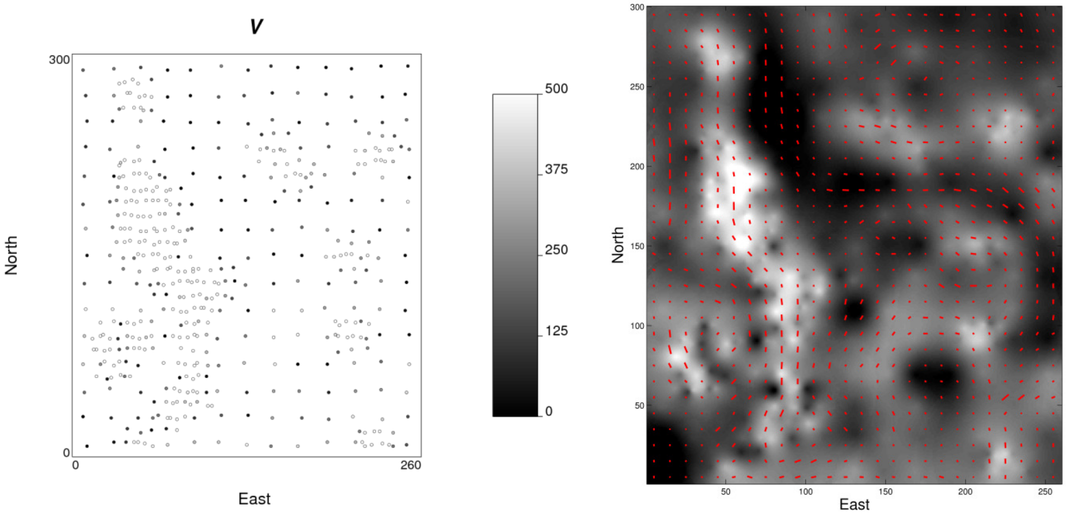

Figure 1: Left: Location and values of the available V sample data. Right: Using the more extensively sampled secondary V data to generate the LVA. The V data were first kriged using block kriging to obtain a smooth map before applying the moment of inertia method to generate the LVA field.

Direct angle measurement

It may be possible to measure the orientation of a deposit at a sample location. This type of data is a direct measurement of the LVA field. Direct angle measurements can be obtained from outcrops, exposed underground workings, down the hole cameras, FMI data or the formation dip can be measured with a dipmeter. Typically, only the local orientation is measured by these devices.

When direct angle measurements are available they are often sparsely sampled; however, an exhaustive LVA field is required. This can be obtained by modeling the LVA field between the sparse samples with a traditional technique such as kriging or inverse distance. Consider Figure 2, where there are 22 direct measurements of the strike of the LVA field in a 2D domain. Each angle is decomposed into its X and Y components and these components are modeled independently. The X and Y components are then recombined to produce an exhaustive LVA field orientation.

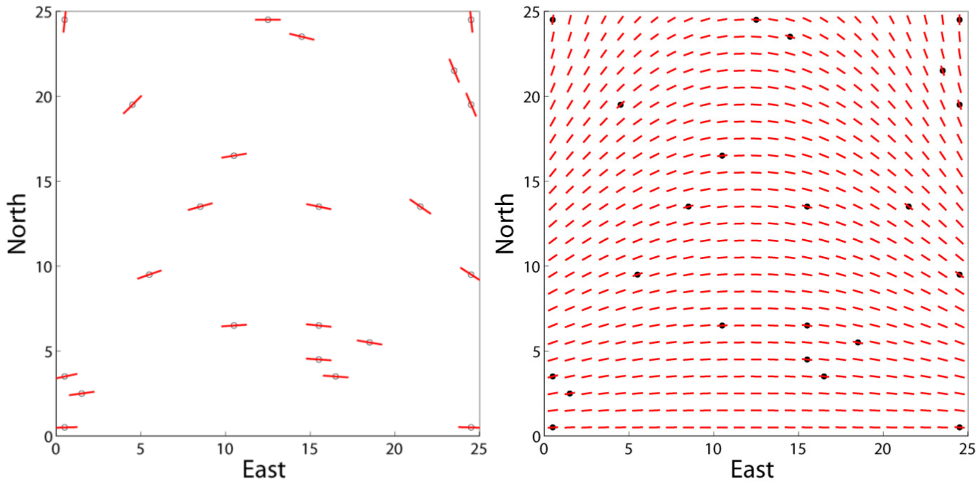

Figure 2: Left: Synthetic dipmeter data. Direction at the locations (circles) is indicated by the orientation of the line through each circle. Right: LVA field after inverse distance interpolation of the X and Y components of each angle measurement.

Remote Sensing

Geophysical remote sensing data can take the form of seismic, magnetic, electromagnetic, gamma-ray or gravitational surveys. When the bulk property measured by the remote sensing technique is related to the variable of interest, it can be used to infer the LVA field. For example, in petroleum applications seismic surveys are often available and are sensitive to porosity changes in the reservoir. It is desirable to first reduce the seismic survey down to a single attribute, such as acoustic impedance, that is calibrated to porosity (Deutsch, 2002). From this single attribute, the LVA field for the variable of interest can be inferred. One difficulty when inferring the LVA field is the use of sparse samples to infer an exhaustive field; remote sensing surveys are attractive because of their exhaustive nature. They provide much needed information where direct sampling of the variable is sparse. Consider the 62 wells in the Amoco data set. An exhaustive 2D map of an interpreted seismic attribute is also available. There is a high correlation between porosity and the seismic attribute, ρ=0.62. The seismic attribute is used to infer the LVA field for porosity (Figure 3).

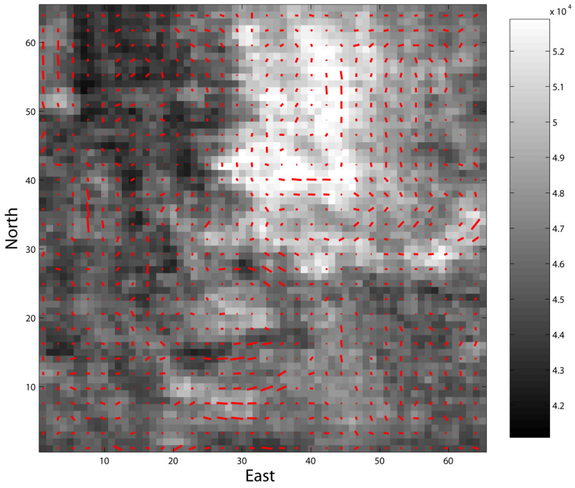

Figure 3: LVA field built from an exhaustive seismic survey. The gray scale image is the filtered seismic attribute. The moment of inertia method was used to generate the LVA field. The length of the line is proportional to the magnitude of the anisotropy.

Structural Model

The continuity within a stratigraphic deposit often follows the form of the original deposition of the layer, that is, the direction of continuity is parallel to the deposited layer (Figure 4). The structural model for a stratigraphic deposit can provide insight into the LVA field. Consider a reservoir with multiple stratigraphic layers (Figure 4). If the continuity of properties within each layer follows the stratigraphy, the surfaces of the interface between each layer provides the direction of the LVA field. LVA orientation can be extracted directly from the structural model; in this example the direction is obtained by averaging the slope of the top and bottom surfaces defining the layer.

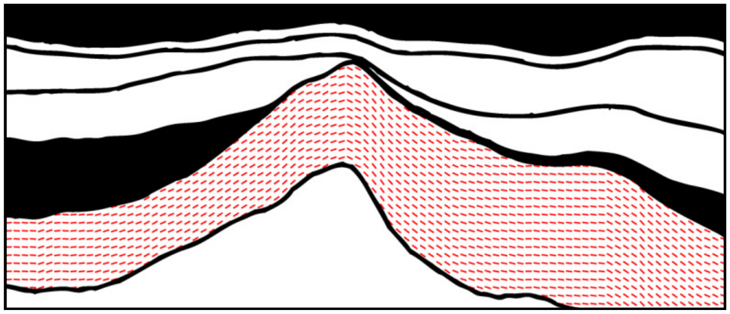

Figure 4: A cross section through a reservoir with 6 different stratigraphic layers (Deutsch, 2002). NTS.

Facies or rock type models

The generation of geostatistical models that respect known geological features is important. Once the facies models have been generated that contain the desired 'geological features' the LVA field can be inferred if the variable of interest follows the geological objects. Consider a 3D channel facies model (Figure 5). If the property of interest follows the modeled channels, this can provide valuable information when inferring the underlying LVA field.

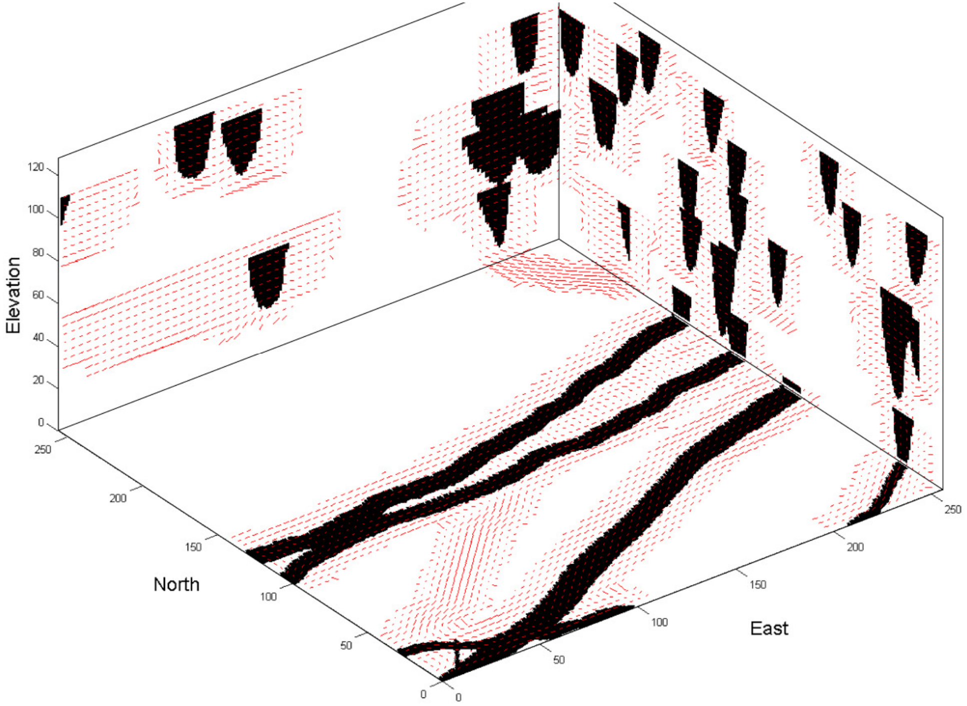

Figure 5: LVA field built from a 3D channel facies models using the moment of inertia method. Where the LVA field is not indicated, the anisotropy ratio is 1:1 (i.e. no channels present). Models taken from Pyrcz, Boisvert and Deutsch (2007).

LVA Field Inference from Available Data

The previous section discussed potential sources of data for LVA field inference. This section presents methodologies for generating the LVA field from the relevant data (Table 2).

Table 2: Methods for LVA field generation. 'Difficulty of application' represents this authors subjective appraisal of the difficulty involved when implementing each technique. Some techniques can only be used to determine LVA orientation.

Technique	Orientation/ Magnitude	Brief Description	Difficulty of Application
Hand Drawn Directions	Orientation and Magnitude	A geologist can often generate a single, subjective LVA field. Such methods have the benefit of incorporating expert knowledge and experience.	Medium
Moment of inertia	Orientation and Magnitude	A moving window moment of inertia calculation can give the direction and magnitude of the LVA field based on the measurements of the variable of interest.	Medium
Automatic feature interpolation	Orientation only	Data values similar in magnitude are joined to automatically generate an exhaustive LVA field orientation.	Low
SGS or Kriging	Orientation and Magnitude	If direct measurements of the LVA field exist at sparse locations, the field can be inferred with traditional methods.	Low

Hand Drawn Directions

Perhaps the most straightforward methodology to generate the LVA field is to manually assign the direction and magnitude of the LVA field based on professional expertise. Most likely this would be done by an experienced geologist with knowledge of the deposit of interest. Taking into consideration all available types of data, an expert could generate a subjective LVA field that considers many different sources of information such as the depositional environment, available sample data, available dipmeter data and personal experience with similar deposits. The expert could assess the LVA field at a number of discrete locations and fill in the exhaustive field automatically with estimation or simulation (Figure 6). This would reduce the professional time required to determine the exhaustive LVA field and also reduce the difficulty inferring an exhaustive field.

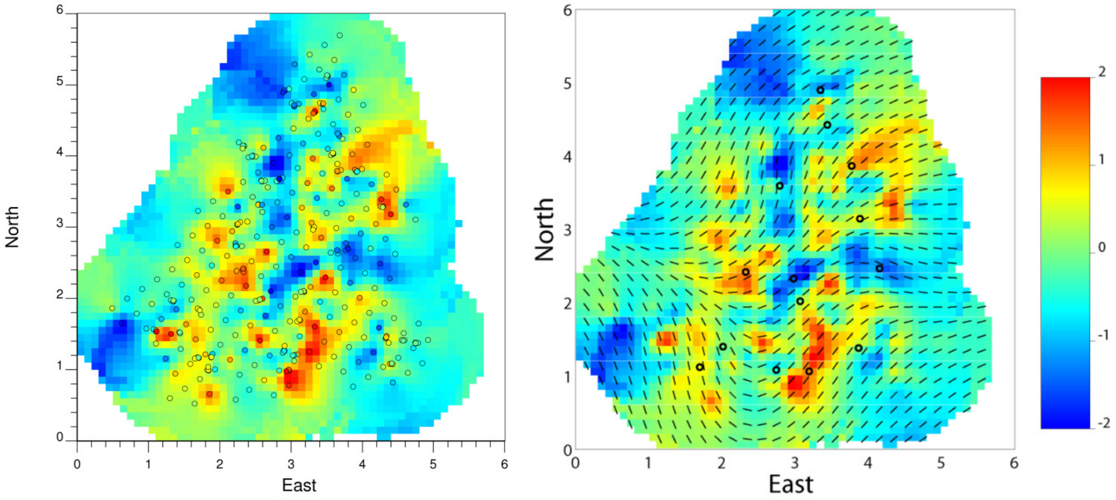

Figure 6: Left: The original data have been normal scored and kriged to produce the underlying map. Right: An expert has defined the LVA field orientation at 23 locations (circles). The exhaustive field was generated by kriging the components of the 23 measurements.

Moment of Inertia

Manual LVA field inference of many locally varying directions of continuity for potentially many variables of interest is time consuming and subjective. An automatic method to generate the LVA field directly from available data is required. The moment of inertia method (Mohammadhassanpour, 2007) can be used as an automatic technique for LVA field generation. A summary of the method is to:

1) Generate the local covariance maps by considering all data within a local window.
2) Calculate the moment of inertia tensor for the local covariance map.
3) Determine the eigenvalues and eigenvectors of the tensor. The eigenvector with the largest eigenvalue corresponds to the major direction of continuity. Anisotropy magnitude is calculated based on the magnitude of the moment of inertia in each direction.

In this example a kriged map is used to generate the locally varying directions as it is smoother than the actual data and results in an LVA field that does not contain abrupt changes due to noisy data. First, locally varying covariance maps are generated at the desired LVA field grid resolution (Figure 7). The moment of inertia tensor is then calculated for the covariance map at each grid location and the LVA field direction and magnitude is determined by the moment of inertia of the covariance map (Mohammadhassanpour 2007). Many sources of data for the LVA field are not direct measures of the magnitude or the direction of continuity. The moment of inertia method provides a valuable tool for determining the LVA field from measurements of intrinsic properties of the deposit such as grade, concentrations, porosity, seismic attributes, etc.

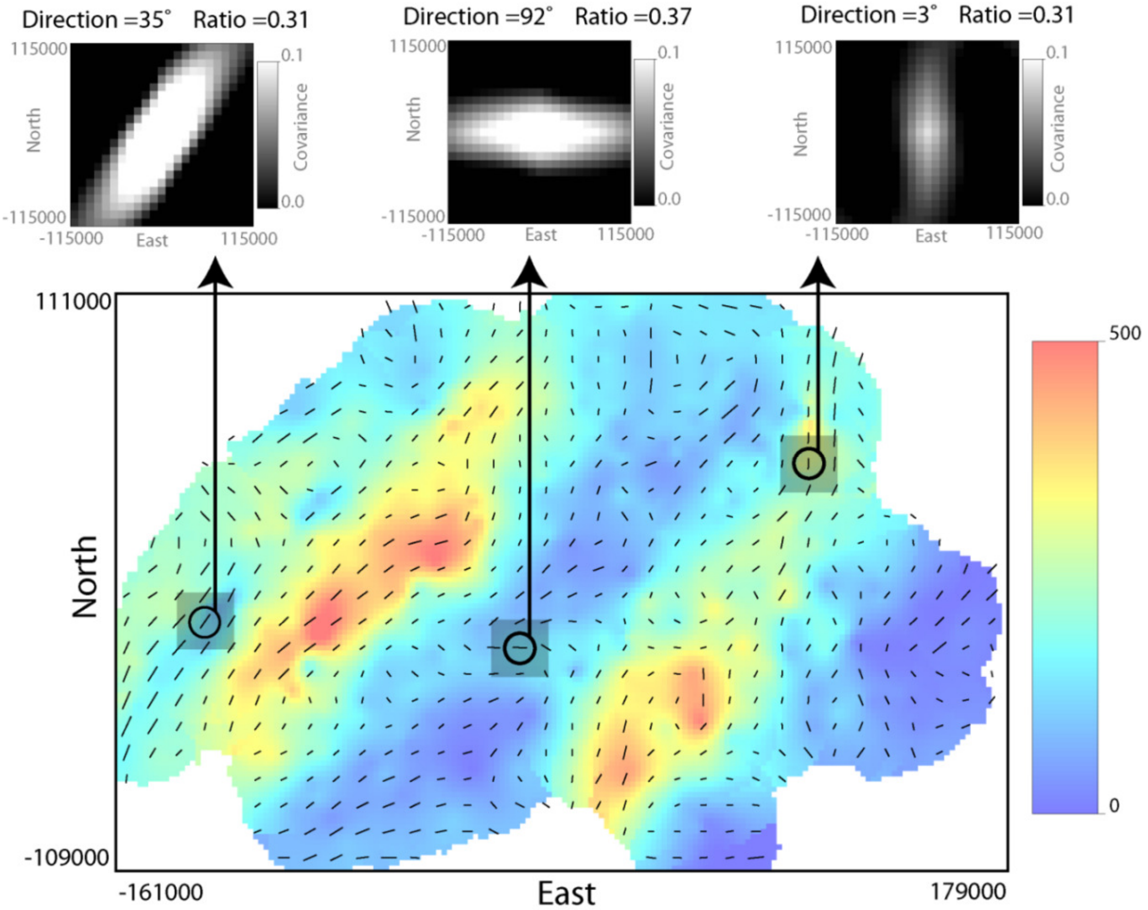

Figure 7: LVA field generated by considering a moving window of +-10 cells. The size of the moving window is indicated by the shaded regions at the three highlighted locations; local covariance maps are also shown for these three locations.

Automatic Feature Interpolation

An alternative semi-automatic LVA field generation methodology is based on connecting nearby sample values that have similar values. The orientation of the LVA field is estimated as the azimuth of the line connecting a sample location with a nearby sample location of similar magnitude. Consider generating an LVA field for the data set presented in Figure 6. Data values have been joined to nearby data of similar magnitudes (Figure 8 above). The methodology used to connect data points in Figure 8 requires three tolerance parameters (1) a distance parameter, Δ, (2) an azimuth parameter, δ, and (3) a magnitude parameter, ξ. Each sample location is visited in a random order. The nearby data within a circular search radius, Δ, are found. The sample data that is most similar in magnitude to the current location is connected to the original location. This process is repeated to generate pseudo-stream lines connecting the data. The polygon is terminated when either (1) there are no data nearby within ξ to connect to or (2) the polyline intersects an existing polygon, in which case the proposed polyline is clipped. This procedure is highlighted in Figure 9.

Once the data have been connected, the orientation of the LVA field can be determined at each data location. The azimuth of the polyline at each data location (Figure 8) is taken to be the orientation of the LVA field at the data location. This procedure is repeated many times and the average azimuth value at each data location is found (Figure 8 below).

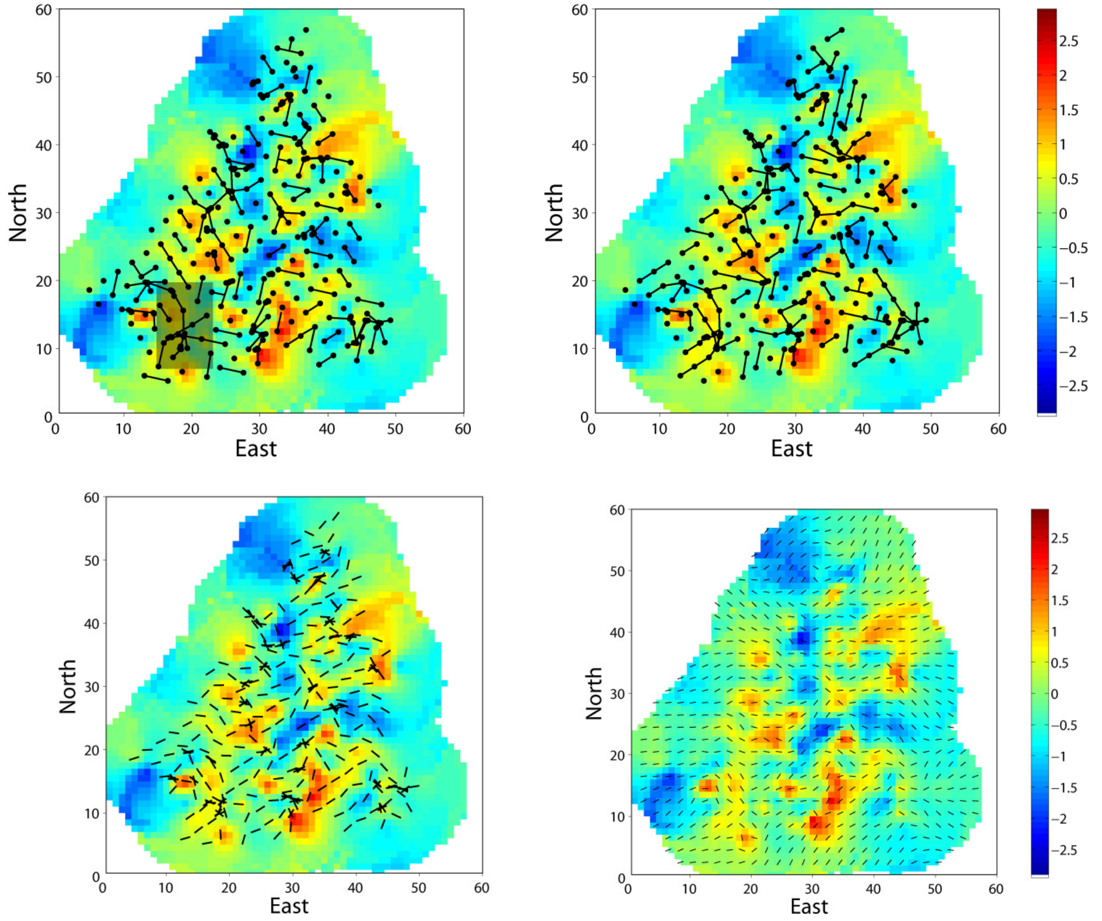

Figure 8: Above: Data points of similar magnitude (+-0.5) are joined. There is no unique way to join data points. Two possible configurations are shown. The shaded region on the left plot is used to highlight the connection methodology in Figure 9. Below left: The orientation of the LVA field at each data location obtained by averaging 1000 realizations. Below right: The exhaustive LVA field generated by kriging the components of the LVA field at each data location.

Steps to generate a single realization of the connections (Figure 9):

Step 1: A location is randomly selected to begin the polyline (point A). The location with the most similar data value within a search radius, Δ, is selected to be the next point in the polyline (point B). 'Similar magnitude' is simply the difference between the sample value at A and the value at B and must be smaller than ξ or the polyline is terminated

Step 2: The connection to point B is made.

Step 3: The azimuth tolerance, δ, and the search radius, Δ, are used to determine potential points to extend the polyline. Within the search area, point C is the location with the most similar magnitude to point B.

Step 4: The polyline is extend to point C.

Step 5: The polyline crosses an existing polyline. The polyline (ABC) is clipped to the existing polyline (123).

Step 6: Return to step 1 until all locations have been visited.

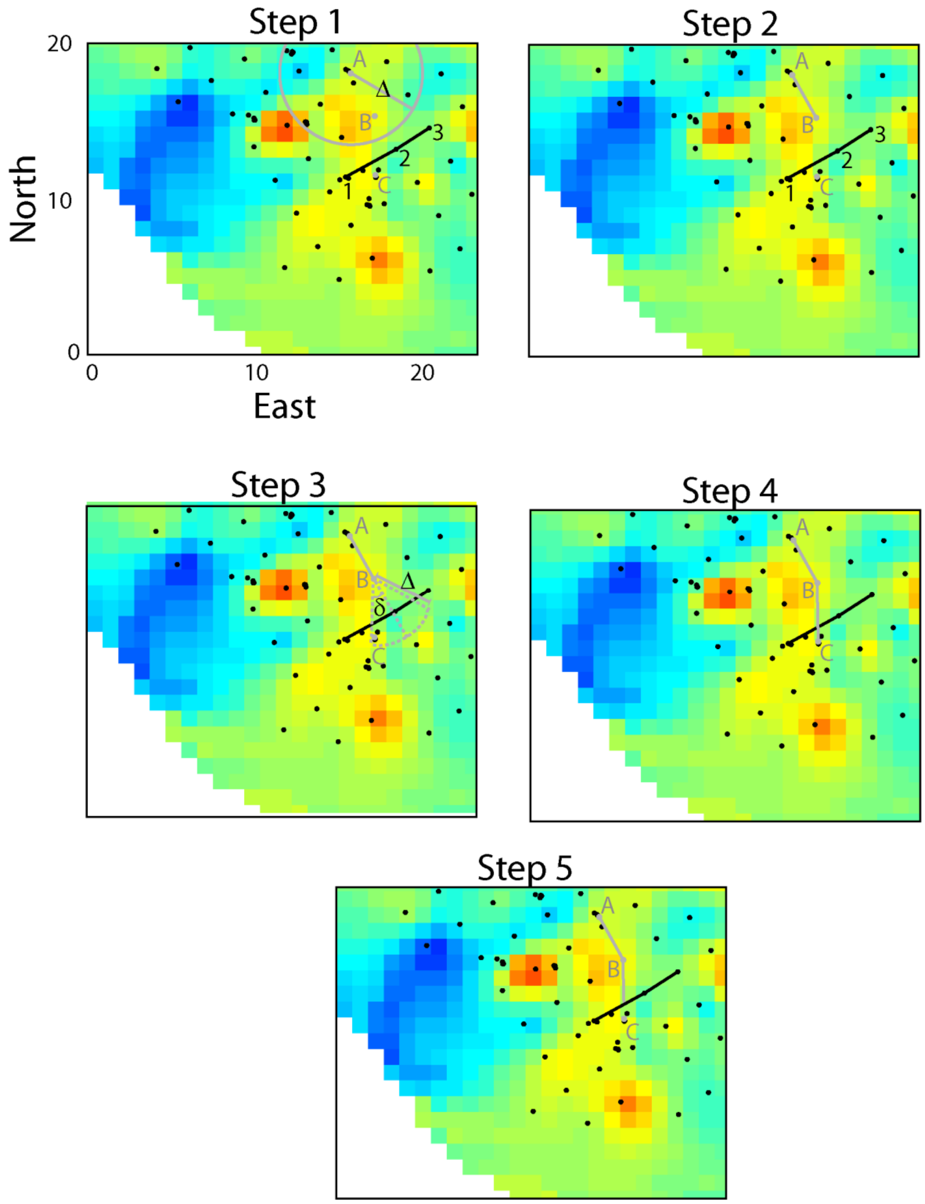

Figure 9: Methodology to connect sample data. The region shown is the highlighted area on Figure 8. Polygon 1-2-3 was constructed before starting polygon A-B-C.

Simulation and Estimation

When direct measurements of the LVA field are available, traditional estimation techniques can be used to generate the exhaustive field. The nature of the available data requires some unique preprocessing before the LVA field can be directly estimated. Because angles are continuous from 360˚ to 0˚, the angles cannot be directly estimated from the available data. The nature of how angles 'wrap' between 360˚ – 0˚ must be explicitly accounted for by decomposing the angles into their X, Y and Z components. In this methodology the X, Y and Z lengths are estimated and then recombined to generate the resulting LVA field. Consider the synthetic dipmeter data from Figure 2 reprinted in Figure 10. The X and Y components of the angles can be calculated from Equation 3. The X and Y components (Figure 10 above right) can be estimated independently and then recombined to generate the LVA field (Figure 10 below). In this example inverse distance was used to interpolate the X and Y components, but kriging could also have been implemented.

$$\mathrm{X} = \sin\alpha \qquad \mathrm{Y} = \cos\alpha \tag{3}$$

where α is the strike angle measured clockwise from North.

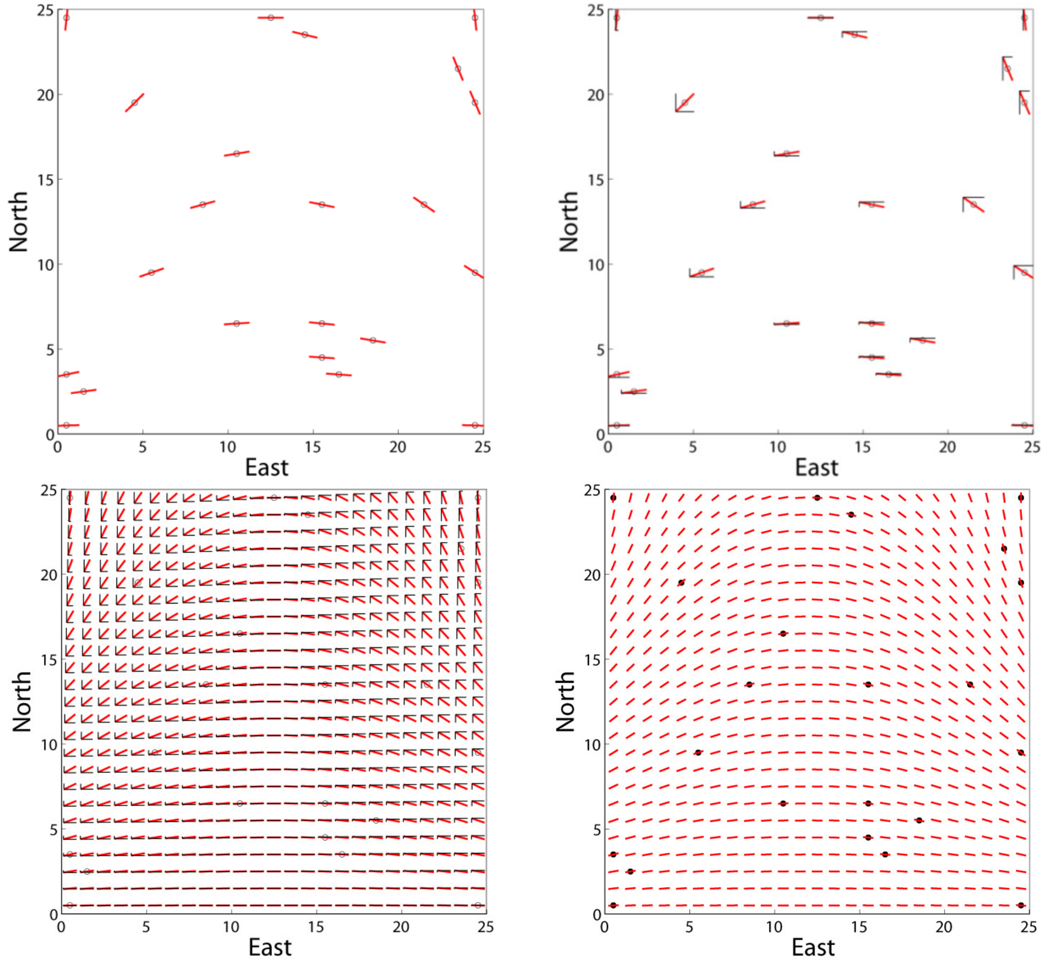

Figure 10: Above Left: Synthetic dipmeter data. Above Right: Each measurement has been decomposed into X and Y components. Below: recombining the simulated components to create the final LVA field.

Estimation of the LVA field from the available azimuth measurements generates a single LVA field that can be used to capture the desired geological features (Figure 10). However, in some practical applications it may be important to quantify the uncertainty in the LVA field and carry that uncertainty though the geostatistical analysis into flow simulation (petroleum), reserve calculation (mining) or contaminate classification (environmental). This can be accomplished by considering multiple realizations of the LVA field by simulating the X, Y and Z components.

Selecting an LVA Field Generation Method

A number of methodologies to generate the LVA field were presented. This section presents recommendations to help determine when to apply each method. There are four available techniques:

1) Manual LVA Inference
2) Moment of Inertia
3) Estimation or Simulation
4) Automatic Feature Interpolation

The choice of method influences the resulting LVA field. Consider applying the methods to the Jura data set (Figure 11).

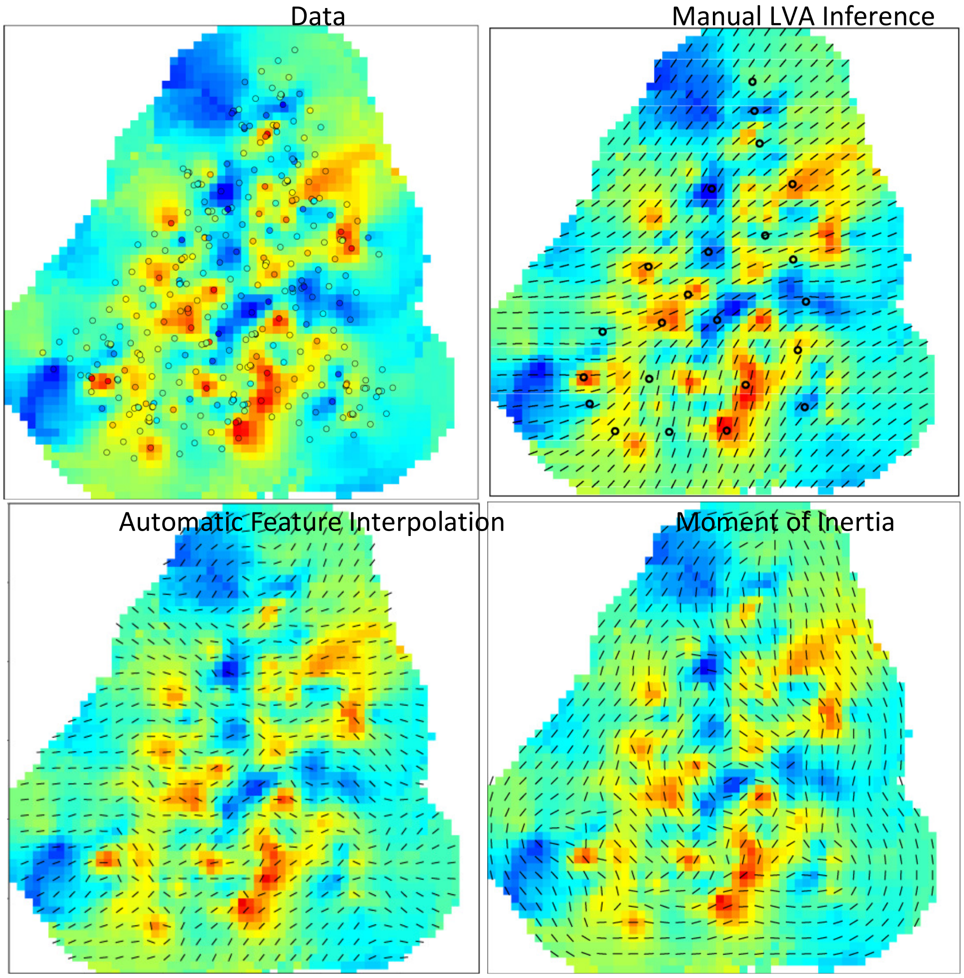

Figure 11: Above Left: Data used for LVA field generation. Above Right: Manual LVA inference at 22 locations, exhaustive field determined from inverse distance estimation. Below Left: Applying the automatic feature interpolation method. Below Right: Applying the moment of inertia method. Color scale varies from -2 (blue) to +2 (red).

Of the methods proposed for LVA field generation in Figure 11, all seem plausible and consistent with the data with the exception of the moment of inertial method. In this case the data are too erratic for the local covariance maps to provide meaningful information about the LVA field. Visual assessment and a comparison with the available data should always be performed on the potential LVA field. Beyond the visual inspection of the LVA field, the nature of the available data is the driving factor in determining which method to select. A decision tree is presented to summarize (Figure 12). When direct measurements of the LVA field orientation or magnitude are available, an estimation or simulation technique is used to generate the exhaustive LVA field. Such discrete measurements are available if the

LVA field has been (1) interpreted by an expert, (2) measured from an outcrop or (3) measured by a dipmeter. If the available data is exhaustive the only automatic technique available is the moment of inertia method. In this situation the moving window size can be varied to obtain the desired features in the LVA field; a large moving window results in a smooth LVA field with few abrupt changes while a small moving window displays more of the local features of the exhaustive data. If a 3D LVA field is required the manual inference of the LVA field can be difficult, although inferences could be made on a number of horizontal plans and vertical sections. This results in a number of 3D discreet measurements which can be estimated or simulated to generate the exhaustive 3D LVA field.

If direct measurements of the variable of interest are the only available data source (as in Figure 11) the choice is to use either (1) the automatic feature interpolation method or (2) the moment of inertia method. From experience, the automatic feature interpolation method works well when the data are not highly clustered in areas of similar magnitude (i.e. high or low values). Where data are clustered, the method may produce unreliable and erratic LVA fields.

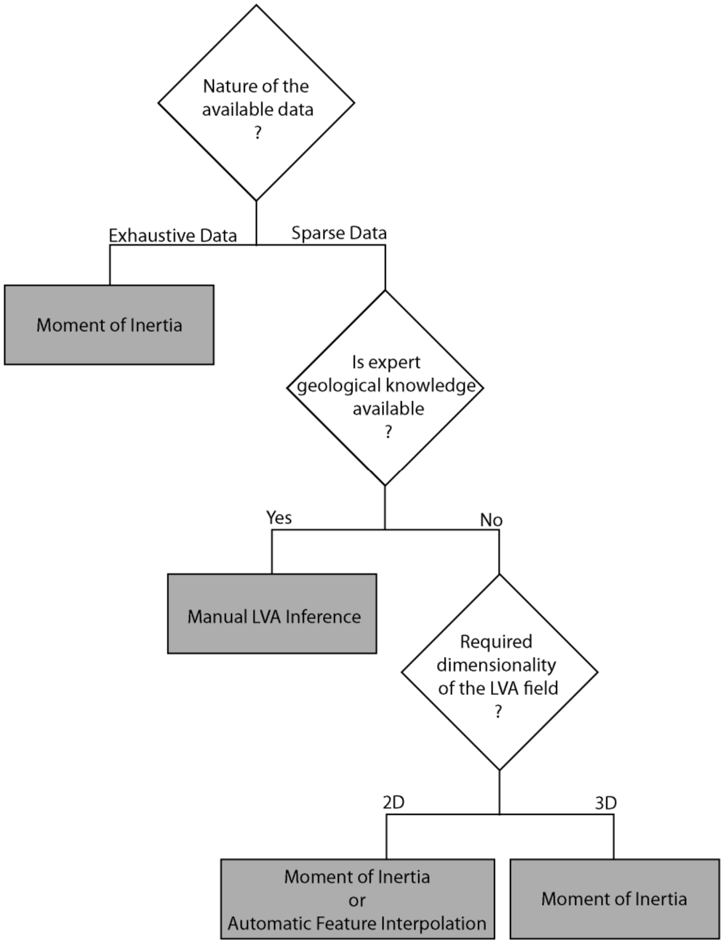

Figure 12: Decision tree for the selection of an appropriate LVA field generation methodology.

Conclusions

This paper provides a critical step in the incorporation of LVA into geostatistical modeling. The difficulty in inferring the LVA field is often the largest impediment to the incorporation of LVA into numerical modeling. This paper provided a number of techniques that can be used to generate the LVA field from a number of different data sources. Situations exist when it is difficult to accurately infer the LVA field because of the sparsely of data available. In such cases, it is necessary to generate multiple LVA fields to carry forward into the overall modeling workflow.

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