A Sensitivity Analysis for Equivalent Permeability Tensors Calculated from 2D Discrete Fracture Networks

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In practice, most current approaches for flow simulation of naturally fractured reservoirs rely on the continuum approach. This requires calculation of equivalent grid based permeability tensors from a discrete fracture network. This research reviews one such popular method for calculation of equivalent permeability. The effect of fracture length and density on the equivalent permeability of the fracture network is investigated. The box counting algorithm is used to determine the fractal dimension of the fracture network and its relationship with fracture network permeability is investigated. Finally, an experimental design approach is used to determine the discrete fracture network parameters that are most influential on the calculated equivalent permeability tensor. It is shown that fracture aperture is the most influential parameter in determining the equivalent permeability tensor.

Introduction

Research shows as much as 60% of the world's petroleum reserves are known to be in naturally fractured reservoirs (NFRs) (Waldren and Corrigan 1985, Beydoun 1998, Roxar 2009). As most convenient and economical reserves decline, producers are increasingly turning toward non-traditional sources of petroleum such as oil sands and NFRs.

Most current approaches for flow simulation of NFRs rely on either the continuum or the discrete fracture network (DFN) approach. Under the DFN approach flow is simulated directly on the fractures allowing incorporation of many of the characteristics of real fracture systems. Although the DFN approach can handle complex fracture geometry, its use has typically been limited to basic flow calculations and assumes zero permeability matrix rock. The continuum approach has seen far more use in actual practice. Under the continuum approach, the flow on fractures is not directly simulated. Instead, the field is represented by grid blocks and an equivalent permeability is calculated for each one. The continuum approach has the advantage that it can simulate complex recovery mechanisms such as capillary pressure and matrix-fracture interactions.

One approach to calculate an effective permeability tensor is based upon fininte-element simulation (Clemo and Smith 1997). For this approach, the discrete fractures in the DFN are discretized into a finite-element mesh. Then boundary conditions are specified on opposite faces of a box grid cell and no flow boundary conditions are imposed on the remaining faces. The steady-state flow through the box is calculated and used to derive an equivalent permeability based on Darcy's Law. This approach has the advantage of being able to account for fracture network connectivity but is sensitive to many assumptions that must be made for the flow simulation. One of the most popular methods for calculating an equivalent permeability tensor is that of Oda (1985). Oda's approach has the advantage that it is fast and does not require flow simulation and can obtain effective properties for grid cells based directly on the geometry and properties of the fractures within those cells. However it does not take fracture connectivity into account and is limited to well-connected fracture networks.

This study reviews Oda's (1985) approach for calculating equivalent permeability for input into a continuum flow simulator and performs a sensitivity study that determines the factors that are most influential on the equivalent permeability using an experimental design approach.

Oda's Method for Calculating an Equivalent Permeability Tensor

The continuum approach, where fracture permeability is represented by an equivalent permeability grid, works well when the rock mass has a sufficient amount of randomly oriented and interconnected fractures (Lee et al. 1995, Barla et al. 2000). In this case, Darcy's law can be used to calculate the seepage velocity as follows:

$$\overline{v}_i = -\frac{g}{\upsilon} k_{ij} \frac{\partial \phi}{\partial x_j} = \frac{g}{\upsilon} k_{ij} J_j \tag{1}$$

Where g is gravitational acceleration, υ is the kinematic viscosity and J_j is $-\partial\phi/\partial x_j$.

The Oda approach lays a specified grid on top of a DFN and derives effective properties based on the fractures contained in each cell. The orientation of fractures in each grid cell is specified as a unit normal vector *n*. Integrating the fractures over all of the unit normals N, Oda obtains the mass moment of inertia of fracture normals distributed over a unit sphere:

$$N = \int_{\Omega/2} n_i n_j E(n) d\Omega \tag{2}$$

Where N is the number of fractures in Ω, n_i and n_j are the components of a unit normal to the fracture n, *E(n)* is the probability density function that describes the number of fracture whose unit vectorsrs n are oriented within a small solid angle $d\Omega$ and Ω is an entire solid angle corresponding to the surface of a unit sphere.

Now, assuming that the flow in a joint is laminar, the so called parallel plate assumption is usually made. That is, the flow within a joint is modeled as the flow between two parallel plates with a separation equal to the fracture aperture. The flow between two parallel plates is:

$$\mathbf{q} = \frac{1}{12}\frac{e^3}{\mu}\nabla P \tag{3}$$

Where *e* is the fracture aperture and μ is the dynamic viscosity of the groundwater and ∇P is the pressure gradient.
Oda (1985) calculates a crack tensor as follows:

$$P_{ij} = \frac{1}{V}\sum_{k=1}^{N} l e^3 n_{ik} n_{jk} \tag{4}$$

Where *V* is the volume of the cell, *l* is the length of the fracture and n_{ik} and n_{jk} are the components of a unit normal to the fracture *k*. Note that equation (4) calculates the crack tensor for the 2D case where fractures are represented by lines (e.g. fracture traces on a 2D surface). Then, Oda's permeability tensor is derived from P_{ij} by assuming that P_{ij} expresses fracture flow as a vector along the fracture's unit normal. Assuming that fractures are impermeable in the direction parallel to their unit normal, P_{ij} must be rotated into the planes of permeability as follows:

$$k_{ij} = \frac{1}{12}\left(P_{kk}\delta_{ij} - P_{ij}\right) \tag{5}$$

Where k_{ij} is the permeability tensor, δ_{ij} is the Kroenecker delta and $P_{kk} = P_{11} + P_{22}$.

The permeability tensor is symmetric and can be decomposed into eigenvalues and eigenvectors. The eigenvectors represent the principal directions of permeability and the eigenvalues represent the magnitudes of the permeability in the corresponding principal direction. Two FORTRAN programs (EQUIVPERM2D and FRAC_EQUIV_PERM) have been constructed to calculate an effective permeability tensor from a discrete fracture network in either two or three dimensions.

Generation of Fracture Networks

The domain or area of interest for this study is a single 2D grid block that is 100 m in length and width. A number of 2D fracture networks were generated for this study. Each fracture network is composed of linear fractures with a centre, length, orientation (defined by dip direction) and aperture. The fracture centre locations are distributed randomly within the 2D grid block. The fracture length, density and aperture are considered as categorical variables. Fracture length is 20, 40, 60 or 80 m, fracture density is 50, 100, 150, 200 or 250. For the preliminary analysis, fracture aperture is kept as a constant (0.10 mm). Fracture orientation was considered by creating two fracture sets, one set had north-south and east-west striking fractures and another had northwest-southeast and northeast-southwest striking fractures. Figure 1 shows an example fracture network for the case with N/S and E/W striking fractures and a fixed length of 20 m. In total 40 data sets were created for this analysis, however, only the N/S and E/W striking fracture sets are presented here, reducing the number of data sets to 20 (every combination of length and density are tested here).

Preliminary Analysis of Results

In total, 20 scenarios with differing fracture parameters noted above were considered for preliminary analysis. For each of the 20 fracture sets, the equivalent permeability tensor was calculated along with the fracture network's fractal dimension using the box-counting technique.

Natural fracture patterns show fractal behavior (Babadagli 2001, Stach et al. 2001). A fractal is a rough or fragmented geometrical shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole. The fractal dimension of the same fracture network can be estimated using various techniques such as box counting, sandbox and scanline. Each technique yields different results since they each measure different fractal characteristics (Babadagli 2001). However, for this article, only the box-counting technique is considered. In general, the higher the fractal dimension that is measured, the more complex and dense the fracture network is.

The box counting method for determining fractal dimension is a standard technique proposed to quantify the spatial distribution of irregular objects (Mandelbrot 1982). In this method, a box size is selected. Then boxes with that size are laid over the domain of interest. Then the number of boxes of that size that contain fractures are counted. This process is repeated for various box sizes. The number of boxes containing fracture lines vs. box sizes is plotted on a log-log scale and the slope is the fractal dimension. The following equation shows the relationship between box size, fractal dimension and the number of boxes containing fractures:

$$N(r) \propto r^{-D} \tag{6}$$

Where N(r) is the number of boxes of size r containing fractures, r is the box size and D is the fractal dimension.

Figure 2 through Figure 4 show the relationship between fracture length, density and fractal dimension and grid block equivalent permeability. Note, only the results using permeability in the x-direction are shown to save space. As is shown in the figures, the equivalent permeability increases with increases in each parameter. Figure 2 shows a near-linear relationship between fracture density and permeability. In fact, the relationship between fracture density and equivalent permeability is linear due to the method used to calculated it with Equation (4). Any deviation from linearity shown in Figure 2 is due to minor differences in the number of E/W vs. N/S fractures since only E/W fractures contributed to equivalent permeability in the x-direction using Oda's method. Figure 3 shows a perfectly linear relationship between fracture length and equivalent permeability, which is expected given the linear relationship between fracture length and crack tensor in equation (4). Figure 4 shows the relationship between fractal dimension calculated using the box-counting technique and effective permeability in the x-direction for various fracture lengths. In general, there is a very predictable relationship between fractal dimension and calculated effective permeability using Oda's method.

Sensitivity Analysis Using Experimental Design

In order to perform a sensitivity analysis to determine the relative importance of fracture network properties an experimental design technique was applied. The experimental design technique can also capture the effect of interaction of variables. The full-factorial and fractional-factorial designs are often used where the objective of the study is to determine which variable has the largest impact on the result of the process. In this analysis only two levels of each variable are considered.

Sensitivity Analysis 1

In this study, four variables are considered (length, density, orientation and aperture) for two level full-factorial design. Optimistic and pessimistic values for each variable were selected and are shown in Table 1, respectively. For length, density and aperture the optimistic value is twice the pessimistic value to minimize the influence of the value of the variable on the response. The orientation of the fractures refers to the strike of the fracture in degrees from north. Both the pessimistic and optimistic orientation scenarios are for orthogonal fracture sets. Fractures oriented in the NW/SE-NE/SW direction are considered optimistic. This is because, under Oda's method, all fractures in the NW/SE-NE/SW case contribute to equivalent permeability in the x-direction. However, for fractures in the N/S-E/W directions, only the fractures in the E/W direction actually contribute to permeability in the x-direction.

All sixteen combinations of possible optimistic and pessimistic data are used in the experimental design. Each individual variable is considered: length, density, orientation and aperture (L, D, O, A, respectively). In addition, the interaction between the variables is also considered with all possible combinations such as: LD, LO, LA, DO, DA, OA, LDO, LDA, DOA, LOA and LDOA.

The factor effect shows the influence of each variable (or combination of variables) on the response of the experiment (here the response is the effective permeability calculated using Oda's method). The factor effect is calculated as follows:

$$\text{F.E.} = \text{Avg. response at one level} - \text{Avg. response at second level} \tag{7}$$

For example, there are sixteen runs in total for sensitivity analysis 1. There are eight runs using a length of 40 m and 8 runs using a length of 80 m. The average equivalent permeability using 40 m length is subtracted from the average equivalent permeability using 80 m length to obtain the factor effect. A Pareto chart showing the relative influence of each parameter and their interactions is shown in Figure 5. As is shown in the figure, fracture aperture has the biggest influence on the calculated effective permeability by far. The factors with the next biggest influence are length and density. Fracture orientation plays an even smaller role than length and density. This result is not surprising since the permeability of a fracture is related to the cube of aperture and only linearly to length and density (see Equation (4)). This result appears to contradict those of Jafari and Babadagli (2009) who conduct a similar sensitivity analysis. However, Jafari and Babadagli use optimistic values that range between 1.5 and 5 times their pessimistic values and study the effect of fracture conductivity instead of fracture aperture. For example, in a similar sensitivity analysis Jafari and Babadagli use values of fracture conductivity of 1000 and 1500 mD for their pessimistic and optimistic cases. However, since the conductivity of a fracture is related to the cube of fracture aperture, the difference between their pessimistic and optimistic conductivity is analogous to a pessimistic and optimistic fracture aperture of 0.100 and 0.114 mm, respectively. Since distributions of fracture aperture are difficult to define in practice, a larger difference between pessimistic and optimistic fracture apertures was used in this study.

Sensitivity Analysis 2

The second sensitivity analysis was conducted using the same data as the first sensitivity analysis except that the orientation of the fractures was changed. For this analysis, the pessimistic scenario is two fracture sets with a strike of 10 and 170 degrees from north, respectively. The optimistic scenario is two fracture sets with a strike of 80 and 100 degrees from north, respectively. Recall, that the sensitivity study considers the factor effect on the effective permeability in the x-direction (along a strike of 90 degrees). The results are shown in Figure 6 as a Pareto chart. In this case, fracture orientation is the most influential factor followed by aperture. The reason for this is that there is a very high contrast between the pessimistic and optimistic fracture orientation. The fractures with strikes of 10 and 170 degrees contribute very little to permeability in the x-direction while fractures at strikes of 80 and 100 degrees contribute a great deal to permeability in the x-direction. Unfortunately it is not possible to simply choose an optimistic fracture orientation that is twice that of the pessimistic orientation (as was done for the other three variables). While the orientation was the most influential parameter in this sensitivity study, in practice this would rarely be the case. Fracture orientation is usually known with reasonable accuracy compared to variables such as length and aperture.

Sensitivity Analysis 3

The third and final sensitivity analysis was designed to take into fracture connectivity into account. The data used in the analysis is shown in Table 2. Oda's method does not take fracture connectivity into account in the calculation of effective permeability. However, it is assumed here that fractures which do not intersect any other fractures or the borders of the grid block have no practical effect on grid block permeability. The code to calculate the effective permeability of the grid blocks using Oda's method was modified to remove all fractures which do not intersect other fractures or the borders of the grid block.

Pessimistic and optimistic fracture lengths were shortened to 20 and 40 m and fracture densities were reduced to 50 and 100 since the data presented in Table 1 usually result in 100% of all fractures being connected to other fractures or the grid block border. All fractures are oriented to strike at 90 degrees from north to remove any orientation effect.

The result of the sensitivity analysis is shown in Figure 7. As is shown in the figure, the two most important factors are the interaction of connectivity and aperture as well as length and connectivity. At present it is unknown why the single fracture attributes such as aperture and length are less important than their combined effects, and why connectivity is less important than when it is combined with other parameters.

Conclusions

This paper first reviews Oda's approach to calculating an effective permeability tensor from a discrete fracture network. Oda's approach is fast and does not require flow simulation to obtain equivalent permeability. However, it does not take fracture connectivity into account and is limited to well-connected fracture networks. A computer program has been constructed to calculate effective permeability tensors from discrete fracture networks using Oda's approach.

Many synthetic discrete fracture networks were generated. The relationship between fracture length, density and the fracture network equivalent permeability was investigated. As expected, as fracture length and density increase, so does the equivalent permeability for the grid block. A box counting algorithm was used to determine the fractal dimension of the DFNs. A strong relationship exists between the fractal dimension calculated using the box-counting algorithm and the equivalent permeability of the grid block.

A sensitivity analysis was conducted using experimental design techniques. The sensitivity analysis shows that fracture aperture is the most influential parameter on the equivalent permeability, followed by length and density. The influence of orientation is more difficult to assess because its influence depends on the values chosen for the sensitivity study. Fracture connectivity also appears to be an important parameter although it is unknown why the interaction of connectivity and other parameters is more important than connectivity itself.

References

Babadagli, T. 2001. Fractal analysis of 2-D fracture networks of geothermal reservoirs in south-western turkey, Journal of Volcanology and Geothermal Research, **112**(1-4): 83-103.

Barla, G., Cravero, M., and Fidelibus, C. 2000. Comparing methods for the determination of the hydrological parameters of a 2D equivalent porous medium, International Journal of Rock Mechanics and Mining Sciences, **37**: 1133-1141.

Beydoun, Z.R. 1998. Arabian plate oil and gas; why so rich and so prolific? Episodes, **21**(2): 74-81.

Clemo, T. and Smith, L. 1997. A hierarchical model for solute transport in fractured media, Water Resources Research, **33**(8): 1763-1783.

Jafari, A. and Babadagli, T. 2009. A sensitivity analysis for effective parameters on 2D fracture-network permeability, SPE Reservoir Evaluation & Engineering, **SPE 113618**: 455-469.

Lee, C., Deng, B., and Chang, J. 1995. A continuum approach for estimating permeability in naturally fractured rocks, Engineering Geology, **39**: 71-85.

Mandelbrot, B.B. 1982. The fractal geometry of nature. W.H. Freeman and Company, New York.

Oda, M. 1985. Permeability tensor for discontinuous rock masses, Geotechnique, **35**(4): 483-495.

Roxar. 2009. Naturally fractured reservoirs: An introduction to their appraisal and management [online]. Available from http://www.roxar.com/category.php?categoryID=2141 [cited April 22 2009].

Stach, S., Cybo, J., and Chmiela, J. 2001. Fracture surface — fractal or multifractal? Materials Characterization, **46**(2-3): 163-167.

Waldren, D. and Corrigan, A.F. 1985. An engineering and geological review of the problems encountered in simulating naturally fractured reservoirs. *In* SPE Middle East Oil Technical Conference and Exhibition, Bahrain, pp. 311-316.

Table 1: Levels for four fracture parameters (sensitivity analysis 1)

Variable	Pessimistic (-1)	Optimistic (1)
Length (m)	40	80
Density	100	200
Orientation (strike of fracture)	N/S – E/W (90/0)	NW/SE – NE/SW (45/135)
Aperture (mm)	0.1	0.2

Table 2: Levels for four fracture parameters (sensitivity analysis 3)

Variable	Pessimistic (-1)	Optimistic (1)
Length (m)	20	40
Density	50	100
Orientation (strike of fracture)	E/W (90)	E/W (90)
Aperture (mm)	0.1	0.2

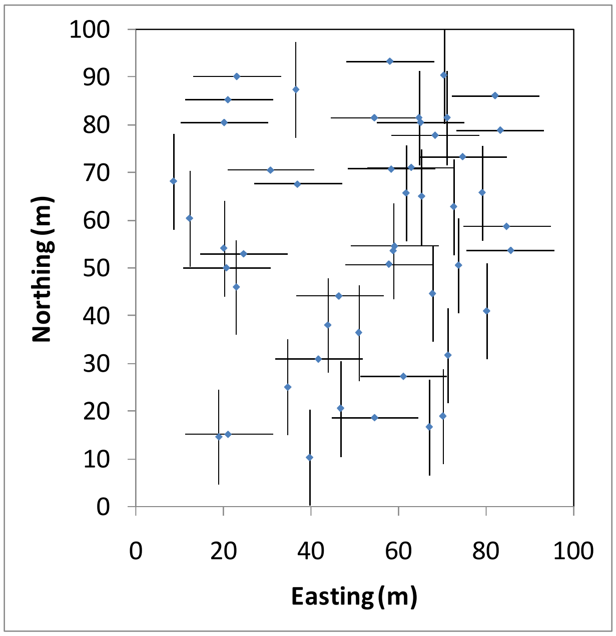

Figure 1: Example fracture network with fracture sets N/S and E/W. Fractures have a fixed length of 20 m.

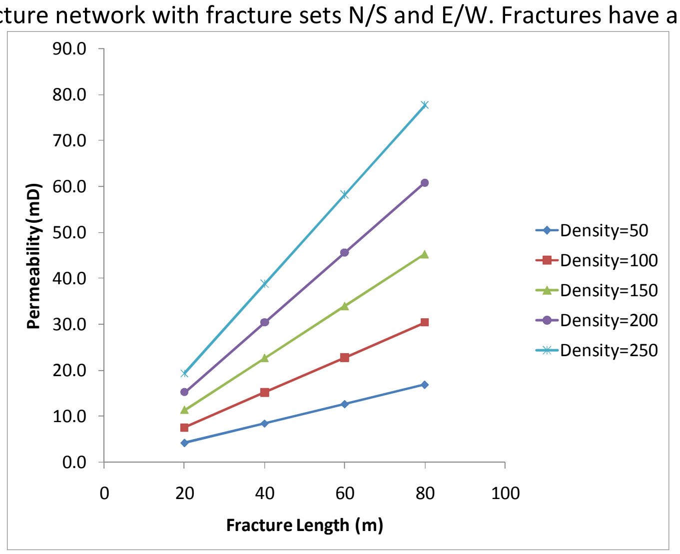

Figure 2: Fracture length vs. equivalent permeability in x-direction

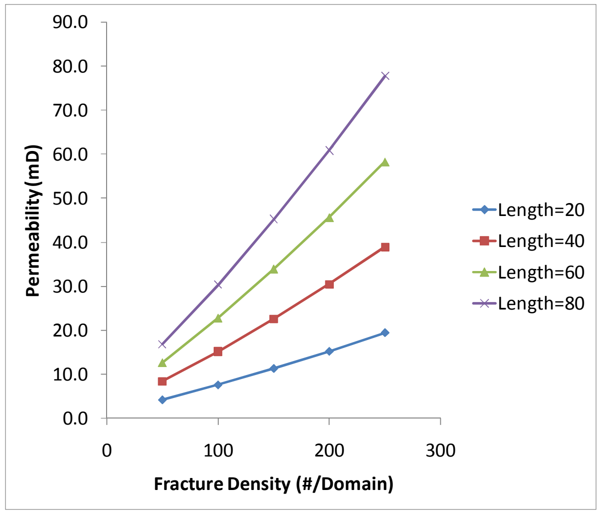

Figure 3: Fracture density vs. equivalent permeability in x-direction

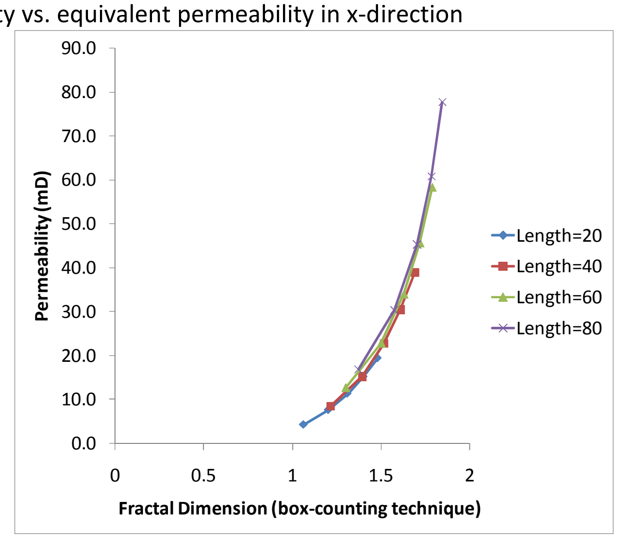

Figure 4: Fractal dimension vs. equivalent permeability in the x-direction

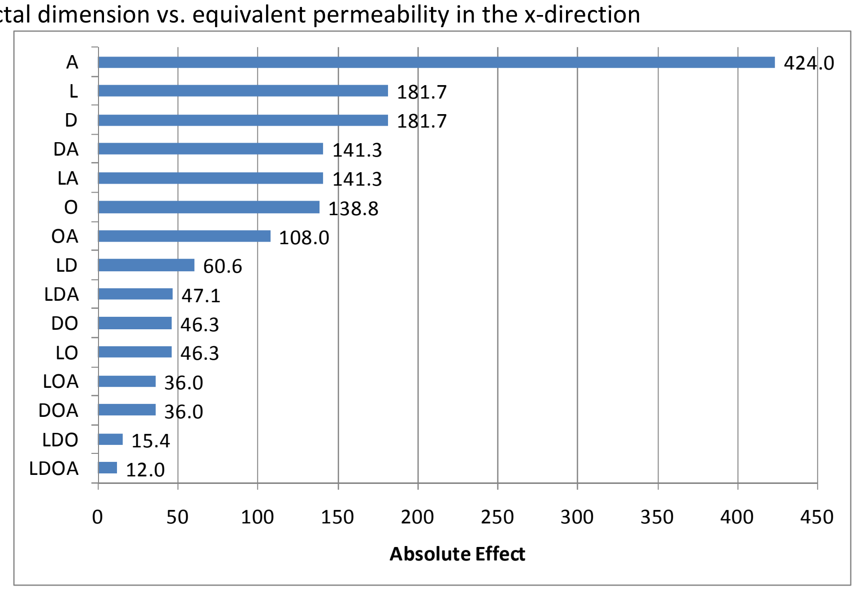

Figure 5: Pareto chart of the absolute effect of different variables and their combinations for sensitivity study 1 (data from Table 1).

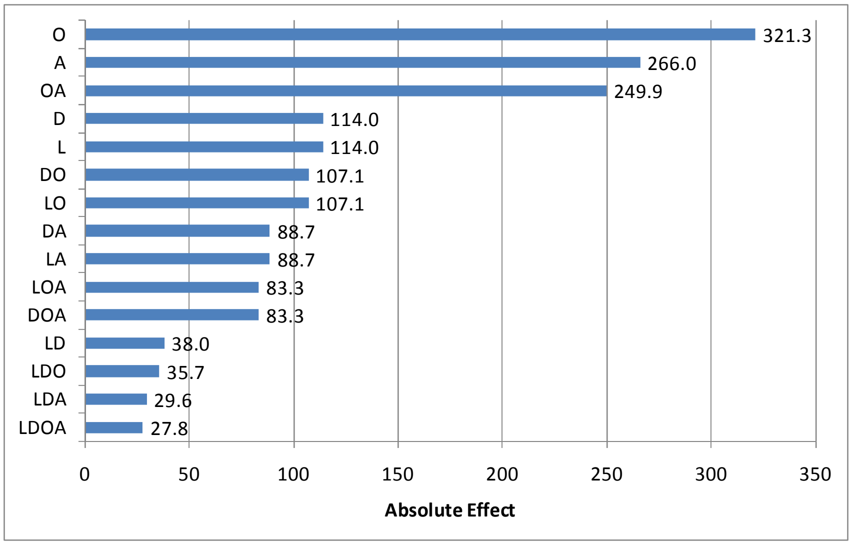

Figure 6: Pareto chart of the absolute effect of different variables and their combinations for sensitivity study 2.

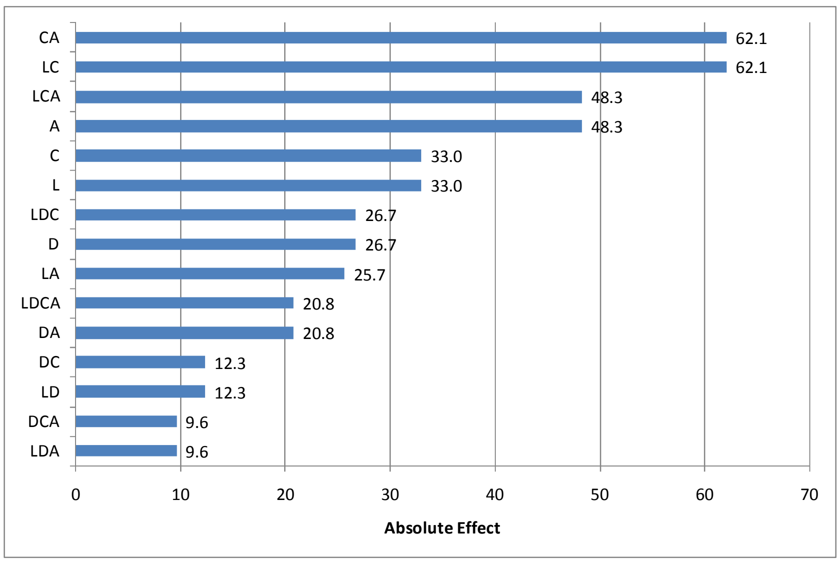

Figure 7: Pareto chart of the absolute effect of different variables and their combinations for sensitivity analysis 3 (data from Table 2).