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Unstructured Grid Generation Considering Reservoir Properties

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Accurately representing the structure of reservoirs is a non-trivial task and current practices utilize structured grids; however, these grids contain several features that are not amenable to commercial flow simulators. Unstructured grids are a very flexible data structure that can be designed to offer both accurate representation and unbiased flow simulation results. These grids are often designed using structured grid models and their properties are computed through various upscaling processes. Two approaches for design are developed to achieve grids for multipoint flux approximation simulators, that being tetrahedral grids. One technique uses the properties directly to define an element volume distribution for grid generation and the other uses flow based gridding. Results show that grid resolution can be controlled by these properties while honouring structural features in a reservoir. This research is developed to fit into a geostatistical modeling workflow for unstructured grids.

Introduction

Integrating unstructured grids into reservoir modeling and flow analysis requires further development of geostatistical algorithms and flow simulation algorithms. Currently, these grids are designed based on structured grid models and their properties are acquired via upscaling. This approach permits independence between geologic modeling and flow simulation workflows and this may not be the optimal approach to reservoir analysis. Considering both areas simultaneously for grid design purposes will result in a grid that is more appropriate for both – it allows an adequate level of detail for modeling heterogeneity and provides an appropriate discretization for the partial differential equations involved in flow simulation.

Merging geological and flow requirements for grid design requires a modified approach to reservoir analysis. Taking this step comes with a series of advantages: the grid is designed with a specific purpose, not just to provide a heterogeneous property model; resulting grids are fully compatible; and model resolution can depend on a variety of parameters. This first point addresses problems that occur because reservoir modeling and flow simulation are independent. Geologic models are not usable in commercial simulators since they are too high a resolution and inevitably contain non-orthogonal features and partial element connections. The second point addresses a problem with existing methods for populating unstructured grids – structured grids, where properties are modeled, are not compatible with unstructured grids making upscaling cumbersome.

This paper focuses on the grid design component of a modified workflow for geologic reservoir modeling. Preliminary mapping is used to generate property models that can be used in two ways: 1 – to control grid design directly by defining an element volume distribution; 2 – a permeability model is used for flow based gridding, from which streamline or particle tracking can be used to control element volumes. The purpose of such designs is to achieve higher grid resolution in areas where flow is anticipated to be more significant, i.e. where higher pressure gradients and velocities occur.

Background

A generally accepted workflow for geostatistical reservoir modeling involves the following (Deutsch, 2002): 1 – data preparation; 2 – modeling large scale features such as horizons and faults; 3 – choosing a grid; 4 – preliminary mapping; 5 – facies modeling; 6 – modeling continuous properties such as porosity and permeability; 7 – validation. It must be pointed out that grids chosen in step 3 are always structured or regular. Resulting models are later used in a reservoir flow analysis workflow. If the numbering is continued, these steps involve: 8 – coarse grid design; 9 – upscaling to effective properties; 10 – flow simulation. Step 8 is where unstructured grids are currently being considered, and their design is based on flow conditions, large scale features, and heterogeneity of geologic properties. Heterogeneity may be involved indirectly through flow based gridding schemes.

Unstructured grid design does not involve the choice made in step 3. Albeit the use of heterogeneity in the models or of the grid for flow based gridding indirectly involves that choice, a series of complications are introduced. The two grids do not conform. Structured grid elements will intersect unstructured grid element in a variety of ways, causing issues with upscaling for all types of reservoir variables. Consider the case shown in Figure 1 for a linear averaging variable like porosity. Current practice takes the value of the structured element and applies it to the fraction within the unstructured element when computing the average porosity. In terms of change of support theory from geostatistics this is not correct. Ideally, a downscaling process (Ren, 2007) would be required to obtain a porosity that reflects the volume fraction within. Even more problems are encountered by downscaling including the absence of both a unique solution and a methodology for permeability. Another related issue with non conforming grids occurs with flow based upscaling. Resolving volumetric flux across element interfaces and averaging pressure and pressure gradients within coarse elements when these quantities are available on the structured grid is not straightforward.

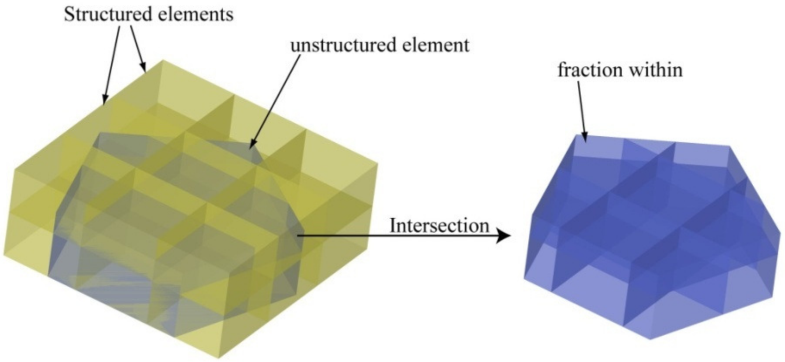

Figure 1: Intersection of structured elements with an unstructured element

How well effective properties are represented is another concern. Because unstructured grids involve a variety of geometries, each unstructured element may be represented by a different number of structured elements. Unstructured grids often have very small elements near wells and relatively large elements far from wells; it is possible to have less than one to greater than one thousand structured elements representing these extremes. This result is counterintuitive. Unstructured elements near cored or logged wells where a clear understanding of heterogeneity is available are represented with potentially only one structured element and far from wells, where uncertainty is higher, many structured elements represent the heterogeneity. Underrepresentation or smoothing of heterogeneity near wells is in itself a problem. The quality of effective properties is dependent on the number of elements involved.

Avoiding these and other issues is accomplished with a modified workflow, where the choice of grid in step 3 is made based on the grid design in step 8. This is possible under the notion that a structured grid is not mandatory to carry out the other steps in the workflow, and that much of the information required for unstructured grid design can still be made available. Large scale features are modeled prior to any gridding; preliminary mapping may be accomplished with a temporary grid to aid design; and existing wells are readily incorporated. These initial steps fall into the category of rough gridding for design purposes. Designing the unstructured grid is the following stage where grids are generated for a specific application. Some examples include evaluation of various injector-producer well configurations, the behaviour of different injection fluids, a SAGD application, and other enhanced recovery methods (Donaldson et al, 1985). Designing grids with a purpose at this stage will have a positive impact on later stages, including characterization. Contrast this with the existing workflow, where grid selection in step 3 is only intended for representing geologic heterogeneity and has unknown consequences on the related flow simulation workflow.

Once an unstructured grid is designed, it is refined to a scale that captures an adequate level of heterogeneity. Refinement is the process of reducing an unstructured grid element into a set of smaller conforming and non-intersecting elements. The same pentagonal prism as in Figure 1 is refined into 111 approximately equal volume tetrahedra in Figure 2. Therefore conforming grids are the result, which overcomes previously mentioned issues: downscaling is not required; interfaces are represented exactly and resolving velocities is straightforward; the refinement process is flexible and it is possible to choose how well effective properties will be represented. Refinement is not discussed further in this paper. The final modified workflow is summarized as follows:

1. Data preparation
2. Structural modeling: horizons, faults, surfaces from seismic, and others
3. Grid design
    a. Incorporate flow related data: reservoir conditions, well test data, future well locations, and other pertinent information such as perforation intervals
    b. Rough gridding and preliminary mapping
    c. Unstructured grid design
4. Grid refinement
5. Facies modeling
6. Modeling continuous properties such as porosity and permeability
7. Upscaling to effective properties
8. Validation.
9. Flow simulation

Most of this workflow is not new; however, some of the steps require a new approach. Grid design is discussed in more detail in this paper.

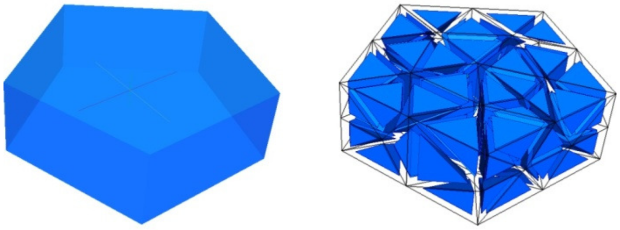

Figure 2: Unstructured element (left) and refinement into tetrahedra (reduced to show more detail)

Grid Design

In reservoir analysis, grids are generated for two main purposes: 1 – to describe the geology and geologic heterogeneity of the reservoir; and 2 – provide a suitable discretized space for solving the PDE's that are involved. In practice, grids are not designed for both purposes simultaneously. Grids are generated independently, or in series with the grid in 2 dependent on that in 1. In either case, grids are typically unsuitable for the other's purpose. Grids developed for geomodeling are not applicable to any commercial flow simulation tool and unstructured grids developed for flow simulation cannot be geologically characterized for a lack of tools all together. This is true even for grids designed in series. Many of the features of geologic grids are averaged out or removed due to simulator limitations.

Current tools for characterizing reservoirs with geologic properties are applicable only to regular or structured grids; therefore these grid types are exclusively used for the first purpose. In fact, to be scale consistent as geostatistical theory suggests, structured grids are not directly usable; they are mapped to a different space, sometimes referred to as chronologic or depositional space, where the grid is regular (Mallet, 2002 and 2004; Caumon et al, 2004). Transformation is controlled by large scale geologic features and depositional style. In most reservoir modeling applications, the grids that result from this limitation retain features that are not suitable for the second purpose. Structured grids are generated to align with large scale reservoir features including the boundary and internal layers and faults, see Figure 3. This results in a non-orthogonal grid, which for existing commercial flow simulators and others using the TPFA method causes grid orientation effects (Aziz, 1993; Aarnes et al, 2007).

Other problems with structured grids for simulators, and for their design in general, are summarized by Farmer (2005). Most problems are due to reservoir structure. Aligning a structured grid with surfaces and faults of varying orientation results in non-orthogonality and partial connections. Neither is amenable to the TPFA method. Particular grid design problems occur in the presence of overturned surfaces, intersecting faults, and thrust faults. To avoid non-orthogonality, reservoir structures are occasionally projected onto grid lines resulting in zigzagging features. This loss of detail has adverse effects on flow simulation, since flow within the vicinity of these structures is no longer representative. Local grid refinement can be applied to achieve better approximation of projected structures; however, this re-introduces non-orthogonality. To summarize, virtually all design complications for structured grids result in non-orthogonality, which is a disadvantageous feature for discretizations intended for solving flow equations using TPFA. Currently, all commercial simulators use this method.

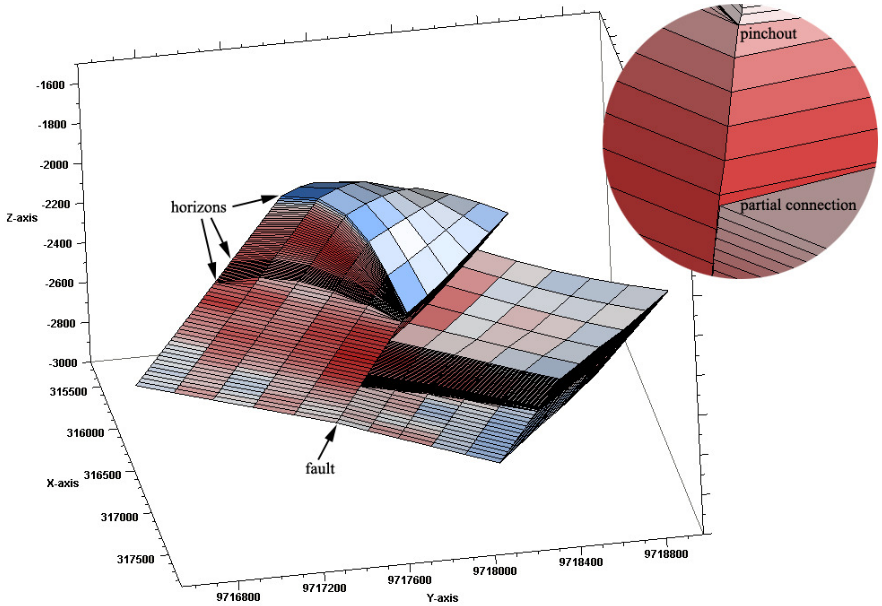

Figure 3: Structured grid aligned with horizons and faults.

For purpose 2, discretization for PDE's, a variety of discretizations are possible and they are constrained by flow simulation algorithms. Grids that do not violate these constraints for existing simulation techniques are regular, locally orthogonal, or composed of tetrahedral and parallelepiped finite elements. A variety of other unstructured discretizations are possible, for example isoparametric finite elements (Zienkiewicz, et al, 2005), but these have not been implemented for reservoir flow simulation. The TPFA method is applicable to grids that are orthogonal or at least locally orthogonal such as PEBI grids, and the MPFA method and finite element method are applicable to these and more complex unstructured grids including tetrahedral, general polygonal, and hybrid grids. Apart from the TPFA method, none of these simulation techniques have been made commercially available. However, they show potential and related grids do not result in a loss of detail.

Inputs

An abundance of data sources are possible in reservoir analysis, many of which can be used to guide grid design. Variables for both geologic modeling and flow simulation provide information about the structure, heterogeneity, and conditions of a reservoir. Input variables that have an impact on grid design are summarized in Table 1. Variables in the table are organized in a potential order they would be incorporated for grid generation, starting with determining the type of grid to use, constraining it to geologic boundaries, and finally using other parameters to determine interior grid resolution and geometry.

Table 1: Variables and their influence on grid design

Variable	Influence
Choice of simulator	Determines how the flow equations, and thus the grid, are discretized to achieve convergence to the correct solution. For example, use of the TPFA technique should limit grids to regular or locally orthogonal, whereas use of the MPFA method would permit general polygonal or tetrahedral grids.

Variable	Influence
Purpose of the grid	The type of production the grid is intended to simulate will influence grid parameters such as the type of elements to use and their volume and resolution. For example, a large scale application may use a fairly coarse hybrid grid, whereas a SAGD application may use a high resolution tetrahedral grid.
Large scale structure	Horizons, faults, and other surfaces, geologic objects, or boundaries that are expected to influence flow should be honoured by the grid. Flow across or along these structures is captured more accurately and it is possible to incorporate additional information about the surfaces. For example, faults may be conductive, transmissive, or sealed and impermeable. Structural modeling is also important for transformation to chronological or depositional space where covariances are evaluated.
Existing wells	Wells that are involved in the production process, such as those used for injection and those for production, will influence the discretization locally and between communicating wells. Unstructured grids commonly incorporate radial grids around wells to better represent flow in those regions. Well trajectories and perforated intervals can also be used to constrain near well discretization.
Future wells	If future well sites for production are known in advance, they can be incorporated into the grid in a similar manner to existing wells.
Single well test data	RFT/DST tests and drawdown/buildup tests provide information about the effective permeability within the vicinity of a well, which can be used to condition the near-well permeability field (Wen et al, 2005). They also provide information about the area of influence of the test, which can be used for controlling grid design.
Multiple well test data	Interference tests and tracer data can provide information about the occurrence and flow character of faults, effective permeability and connectivity between wells, and flow paths and pressure distributions. This data can be used to characterize surfaces with flow parameters such as transmissibility multipliers, to condition permeability fields between wells based on connectivity, and to provide a rough idea of pressure gradients and streamlines for flow based gridding.
Historical production data	Provides similar information as multiple well test data. Additional information includes well drainage volume and interwell communication. This can be used to identify regions that are undergoing flow and those that are relatively stagnant, which can be used to control grid resolution. Coarse gridding can be used where no or limited flow is anticipated and fine gridding where higher pressure gradients and flow may exist.
Seismic data	Aids the identification and modeling of structures and gives some idea of heterogeneity. The influence of structure was already mentioned. Heterogeneity can be used to guide grid resolution, especially in regions where flow is known to occur from various well test data.
Well log and core data	Provide information about distributions and variography of facies, porosity, and permeability. Preliminary mapping of these variables helps to identify the spatial distribution and level of uncertainty which can be tied to grid resolution. Generation of rough permeability models can also be used in flow based gridding.

Grid Choice

Selecting the type of grid to use is dependent on the simulator, the application, and on knowledge about the reservoir including the depositional environment and fluids involved. Simulators using TPFA should be limited to regular or locally orthogonal grids, but the later does not necessarily lead to an optimal solution. It may not be appropriate to use for example a PEBI grid to discretize the entire domain for a flow simulation study. PEBI grids can lead to an exaggerated amount of dispersion where grid elements have a high number of interfaces or connections with other elements leading to delayed predictions of flow events such as breakthrough time in water injection applications. It may be reasonable to use a hybrid grid that is regular where possible and unstructured only along boundaries and near wells. The advantage of this approach is a reduced number of element connections, which increases efficiency.

More flexibility in terms of grid choice and design is possible with MPFA simulators. It is also possible to incorporate tensor permeability. However, finding a solution for the pressure field is sensitive to the magnitude and anisotropy of permeability. In extreme cases, it can be shown that the system matrix for MPFA is not an M-matrix, resulting in non-physical pressure oscillations (Aavatsmark et al, 1998; Eigstad and Klausen, 2005; Mlacnik and Durlofsky, 2006). Even if oscillations are relatively small, they can be detrimental to applications involving both fluids and gasses. Oscillations that reduce pressure can lead to dissolution of gasses where this would normally not occur. If it is possible that the environment and fluids will lead to these conditions, either the MPFA method may not be the best choice, or care must be taken in subsequent grid design steps.

Grid selection is also impacted by the targeted application and scale of the problem. Full field scale conventional production processes may effectively be discretized using coarse hybrid grids. If little structure is involved, a regular grid may even suffice. Enhanced recovery processes such as immiscible and miscible displacement may require more accurate representation of structure and higher resolution to better describe the geometry of the frontal region. The use of miscible displacement will also affect simulator choice based on the previous mention of pressure oscillations. Different discretizations may be required for thermal applications such as cyclic steam injection and SAGD. Describing the distribution of temperature and viscosity and simulating the gravity drainage process demands a higher resolution model than conventional production for example.

Incorporating Structure

In terms of the workflow suggested in this paper, as well as in existing reservoir modeling workflows, structural modeling is carried out prior to any gridding. After a grid is decided upon, these structures can be used to constrain resulting discretization; however, this is a complicated task even for structured grids. In many cases, incorporating faults and stratigraphic surfaces into a structured grid cannot be accomplished while maintaining orthogonality, resulting in the distorted grid designs and partial element connections commonly seen for geological modeling, see Figure 3. If orthogonality was a condition of grid choice, then a more appropriate discretization is a hybrid PEBI grid; structured everywhere except within the vicinity of faults and other surfaces where voronoi elements are required to maintain orthogonality and reproduce such features, see Figure 4. Generating such a grid is non-trivial and is a current area of research (Flandrin et al, 2006; Katzmayr and Ganzer, 2009). The problem is to specify a point distribution such that the resulting voronoi vertices align with constraining surfaces.

Other grid choices, such as tetrahedral, make the inclusion of structure more straightforward. Specifying a point distribution defines the vertices and faces of tetrahedral elements directly. Vertices can be placed directly on the surfaces thereby reproducing them. Since the set of element faces along the surface form a triangulation, it would be logical to use triangulations for initial structural modeling. They can be reproduced exactly by the final tetrahedralization. Constrained tetrahedral grid generation is largely solved for a variety of problems (Du and Wang, 2006).

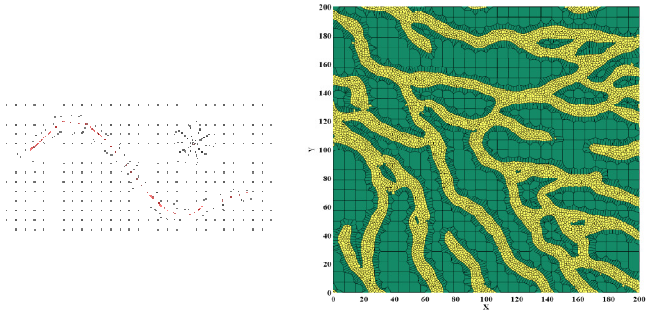

Figure 4: Hybrid PEBI grid showing a single fault and one well (left) and representing a fluvial system

Property Distributions

Knowledge of the density function and spatial distribution of properties can be used to constrain the discretization process. These types of data are used as soft constraints on design, unlike reservoir structures that are used as hard or discrete constraints. In terms of numerical optimization, these constraints are similar to equalities and inequalities. Reservoir structures specify exactly where grid element interfaces must lie, and this can be expressed as an equality constraint; whereas density functions and spatial distributions might be used to specify limits on the variability within elements, element volumes, or other inequalities. Use of property distributions requires some degree of preliminary mapping and modeling, which was identified as step 3.b. of the modified workflow.

Preliminary mapping can provide an idea of the spatial distribution of many variables that have some correlation with flow such as facies, porosity, permeability, net to gross, and storativity. In geostatistics, mapping is accomplished with one of several forms of kriging, which provides an estimate of the expected value of a property, as well as a homoscedastic estimation variance that serves as a measure of local uncertainty. Resulting maps can be used to control grid resolution and element geometry in two ways: 1 – values are transformed into a spatial element size distribution that is used to distribute grid element vertices or centroids; 2 – values are incorporated into a flow based gridding scheme that provides information such as streamline density to control grid resolution. The later is an active area of research.

Certain discretization criteria selected for the first method will also require knowledge of the density function of a property. Suppose results from kriging are used to provide a measure of anticipated heterogeneity, which will depend on the estimation variance. For highly skewed variables such as permeability it is possible to grossly underestimate the variance in areas where high permeability is expected and overestimate it in areas where low permeability is expected leading to a suboptimal discretization. The phenomenon for such distributions is known as the proportional effect (David, 1977; Manchuk et al, 2008), and various techniques are available to obtain a more appropriate estimation variance including distribution mapping (Oz et al, 2003), sample variance calculation (Yamamoto, 2000), and indicator kriging methods (Goovaerts, 1994).

One characteristic of data that complicates the preliminary mapping stage and geostatistical modeling in general is data dependence. For example, permeability is dependent on variables such as facies, occasionally porosity, depth, and other factors. It may be inappropriate to generate a grid based on permeability when such dependencies are overlooked. Therefore, modeling must be done jointly and several techniques are available for this. Typical applications simplify the problem by modeling facies, then within each facies category modeling continuous properties using collocated cokriging to account for dependence. Other transformation techniques such as the stepwise conditional transform (Rosenblatt, 1952; Leuangthong, 2003) have been used for relationships that are non-Gaussian and some research is targeting the use of non-parametric distribution modeling (Manchuk and Deutsch, 2008). In regards to the modified workflow previously mentioned, it is best to use the same methods during preliminary mapping that will be used in generating the final models. Also, any structural modeling from step 2 should be used to map data to a space where covariances are deemed meaningful.

Several pieces of information are made accessible through preliminary mapping including local distributions of uncertainty from kriging and various realizations if geostatistical simulation is used. When facies and complicated relationships are involved, local distributions of uncertainty may only be accessible from averaging realizations, rather than from kriging. For designing a grid, it is important to consider this local uncertainty or equivalently all realizations. Design of this type is referred to as designing in expectation, and the objective is to generate a grid to account for a set of realizations, rather than for a specific realization. Generating designs based on one realization is not advocated because it will lead to a bias in expected flow response over other realizations and will not account for the uncertainty involved.

To provide examples showing the two uses of preliminary mapping in grid design, a basic geostatistical workflow is followed to generate a facies, porosity, and permeability model. Results are converted into an element size distribution property and used to control grid design, and also used in a flow based gridding approach. When preliminary mapping takes place, data preparation and structural modeling would already be completed, and this is assumed in the following.

Preliminary Mapping

For this example, three stratigraphic units are considered along with two faults. The top and base units are permeable while the central unit is assumed impermeable shale. These structures are shown in Figure 5. They control transformation to a depositional space where covariance calculations are done. For simplicity, the space may be discretized using a regular grid; however, elements along the faults would not be ideal for flow based gridding purposes. Figure 6 shows the gridded depositional space, characterized with a facies realization which is explained later, and the effect of transformation to the existing space. In this example, faulting was assumed to occur last so the grid was not skewed as a whole; rather the faces of elements straddling the fault were translated to reproduce the fault plane.

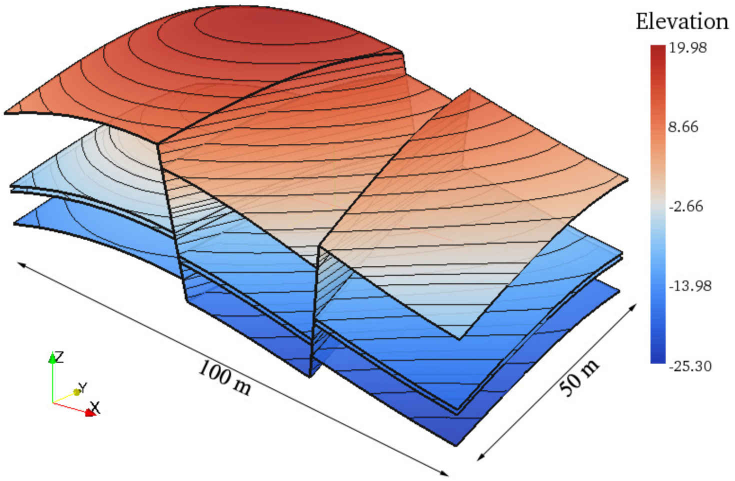

Figure 5: Structures for preliminary mapping example

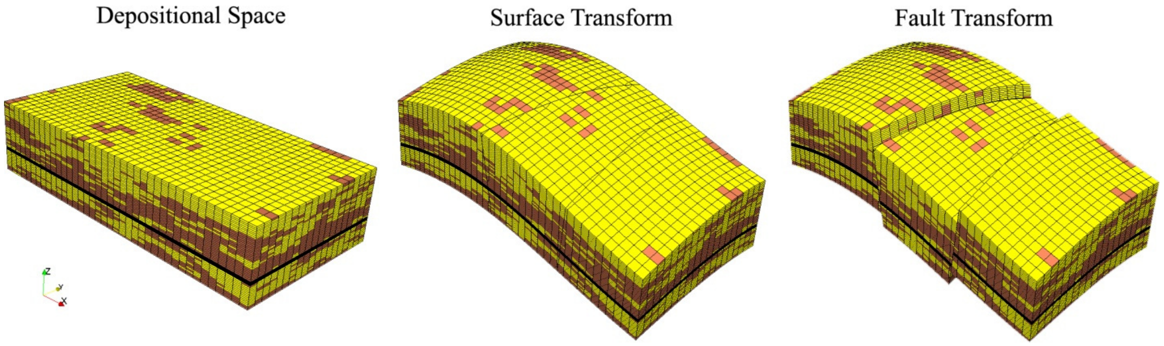

Figure 6: Transformation process from depositional to existing space (left to right)

In typical applications, each stratigraphic unit is modeled independently, which assumes they were deposited as independent events through geologic time. Facies are modeled first, followed by continuous variables including porosity and permeability. Although more variables and input data are often available, this example only involves these three. The final use of models will be similar regardless of the complexity of preliminary mapping. Three facies are considered: sand, shale, and mud. Sand and mud are present in the lower and upper units, while shale is present in the central unit. A coarsening upwards trend from mud to sand is assumed within the upper and lower units.

Synthetic conditioning data for this example was generated by creating an unconditional realization of facies, porosity, and permeability and sampling it at twelve locations, which represent twelve vertical wells. Sequential indicator simulation and sequential Gaussian simulation were chosen to generate the data. Porosity and permeability were generated with a correlation coefficient of 0.5. In depositional space, a regular grid with dimensions 40 by 20 by 40 in x, y, and z was used. Conditioning data were then used to generate a larger set of realization that was averaged to provide expected values and variances to drive grid design, see Figure 7 and Figure 8.

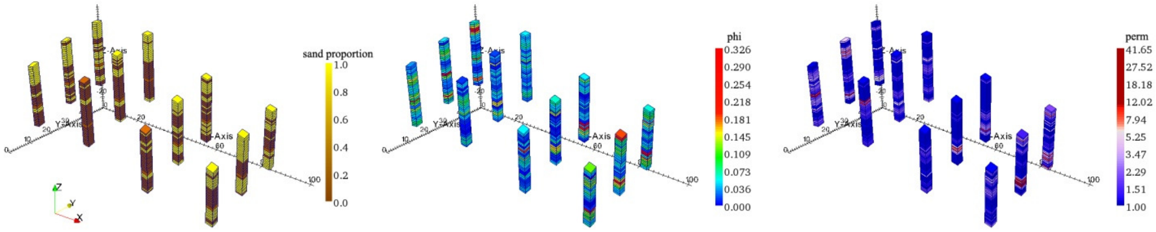

Figure 7: Sample data extracted from an unconditional realization

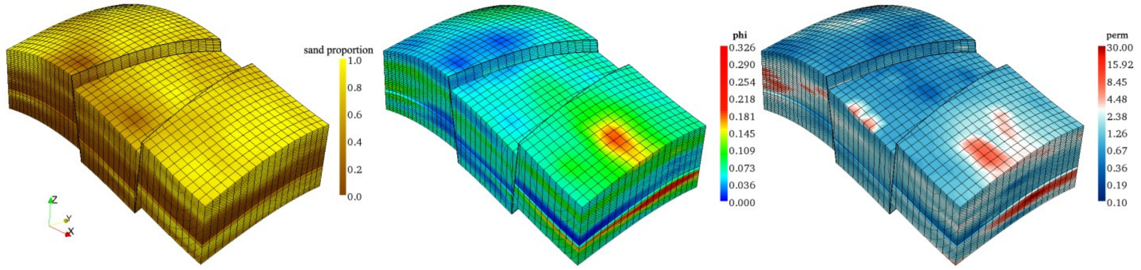

Figure 8: Expected values of 50 realizations

Expected value models are used in the following sections to control grid generation. Sand proportion, porosity, and permeability are converted into an element size distribution for one approach, and permeability is used in flow based gridding as well.

Element Size Distributions

A notable advantage of unstructured grids is their adaptability to user defined criteria. Practically unlimited transformations can be devised to provide an element size distribution from a set of preliminary maps. However, care must be taken to target properties that are significant to flow simulation and to the final purpose of the grid. Towards designing in expectation, element size distributions are developed in one of two ways: either directly from expected values of one or more properties, or averaged from element size distribution calculated for several realizations. The later is more appropriate if non-linear relationships between properties and element size are used.

One intuitive method to derive an element size distribution is using property to volume relationship curves. A particular proportion of sand, porosity, and permeability are associated with a specific element volume through a lookup table. In this example, sand proportion and facies are tied to volume by linear relationships while permeability is related by an approximate logarithmic relationship, see Figure 9. These curves were generated arbitrarily, but are reasonable: smaller volumes are targeted where more sand, higher porosity and higher permeability are anticipated. Volumes were chosen to range between 0.5 m^3 and 20 m^3. Final element volumes are calculated using a weighted combination of the three functions, with the weights summing to one to preserve the specified volume range. Weights can be chosen by variable importance. In this example, sand proportion and porosity were given less importance with weights of 0.2 and 0.3 respectively, and permeability was assumed most important with a weight of 0.5. The resulting volume model is shown in Figure 10.

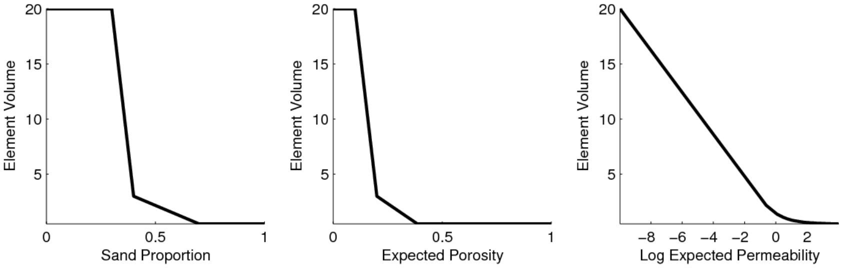

Figure 9: Relationships between element size (m^3) and properties

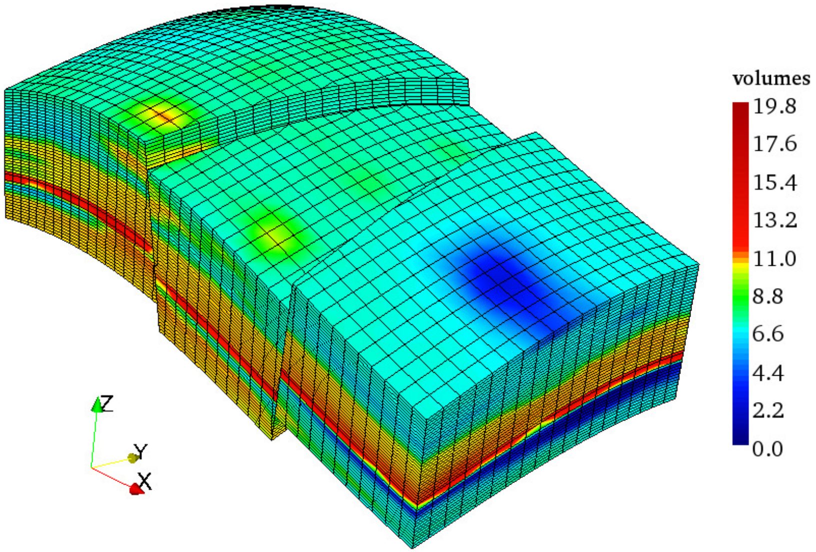

Figure 10: Element volume distribution, m^3

Using the element volume distribution to control grid generation can be accomplished in several ways, two of these include: distributing points that will ultimately become part of the final grid design (Hale, 2002); and starting with an initial coarse grid and refining elements that are larger than the underlying volume distribution. Points generated in the first method may become vertices of tetrahedral elements or centers of PEBI elements. The second method is applied to this example for generating a tetrahedral grid. TetGen (Si, 2006) was utilized to generate an initial quality tetrahedralization, which was adapted by refining tetrahedra having volumes larger than the underlying volume distribution. Quality is measured by the radius-edge ratio (Miller et al, 1995): the radius of the circumsphere of the four vertices to the shortest edge. A resulting grid is shown in Figure 11. To maintain a quality measure of 2, the volumes are not distributed exactly according to the distribution of Figure 9; however, the goals of achieving smaller elements where the proportion of sand, porosity, and permeability are higher have been obtained. These properties have been interpolated using inverse distance squared onto the resulting grid in Figure 12 for comparison purposes with Figure 8.

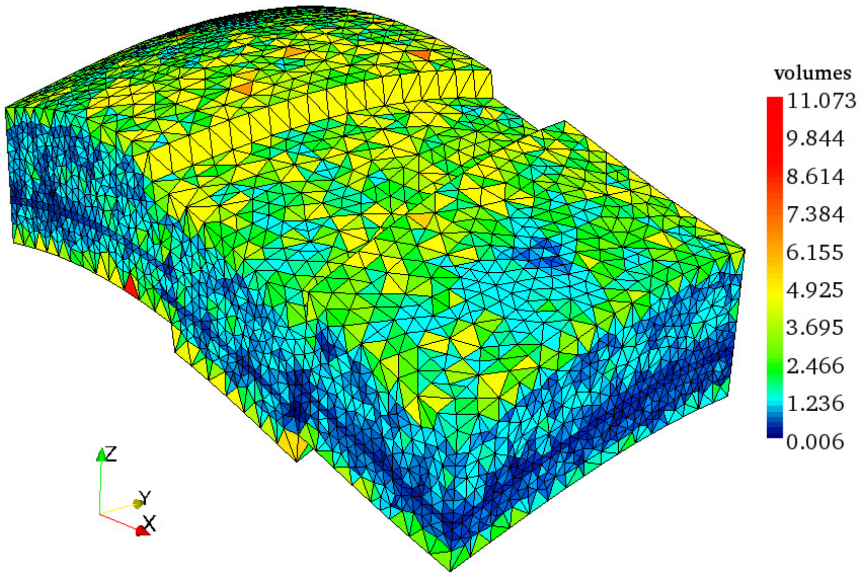

Figure 11: Tetrahedral grid generated with volume control

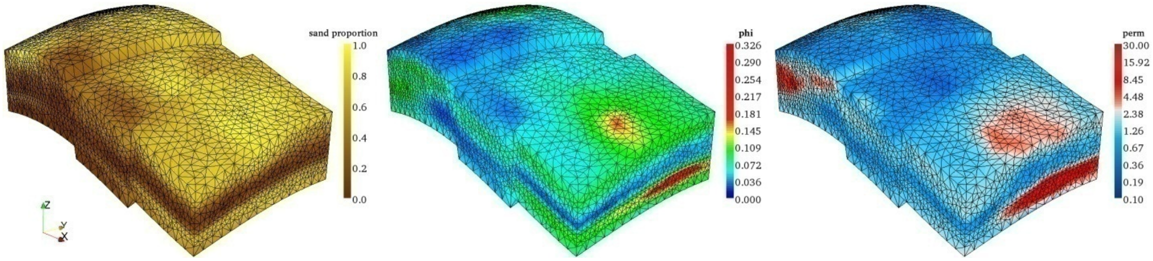

Figure 12: Expected values from Figure 8 interpolated to tetrahedral vertices

Flow Based Gridding

Element size and geometry can be controlled by flow attributes such as velocity or streamline density, which are derived by solving the flow equations for pressure and velocity. Depending on the final application of the grid, and on the fluid components of the reservoir, different flow equations may be involved. Single phase incompressible flow is implemented here. TPFA is used to solve for pressure and velocity, so to avoid grid orientation effects, permeability is resampled to a regular grid in the existing space of the reservoir. Some of the problems with structured grids become apparent here: the price of achieving orthogonality is a loss of detail around faults and surfaces that are no longer represented exactly. Resampling results for permeability along with faults and the top and base surfaces are shown in Figure 13.

In this example, grid element size is controlled by flow velocity in the *x* direction. This attribute is derived using general boundary conditions on the *yz* model faces, with influx at the minimum face and outflux at the maximum face. The model is sealed along all other interfaces by encasing the reservoir by very low permeability (1×10^{-7} md) regular grid elements, which is shown in Figure 14 along with the resulting velocity in the *x* direction. To be similar with the previous example involving grid control by reservoir properties, velocity in *x*, q_x, was converted to a volume distribution with a range from 0.5 to 20 m^3 using the following equation: $V = 20/(q_x^2+1)$. Results are shown in Figure 15. There are some similarities between this grid and that shown in Figure 11 since higher flow velocities will be observed where higher permeability exists along the flow path.

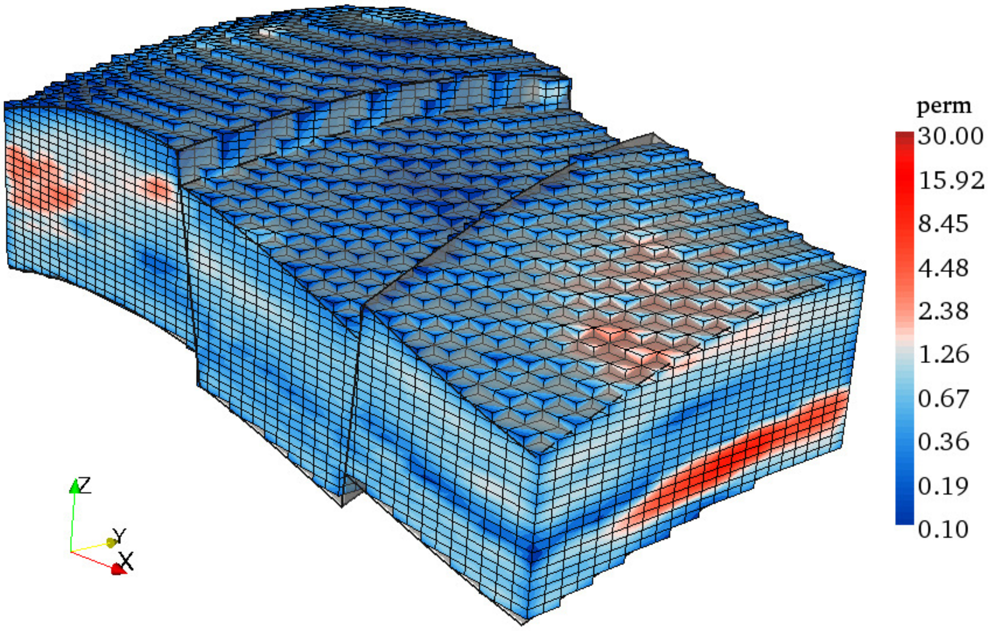

Figure 13: Resampled permeability and approximation of surfaces by a regular grid

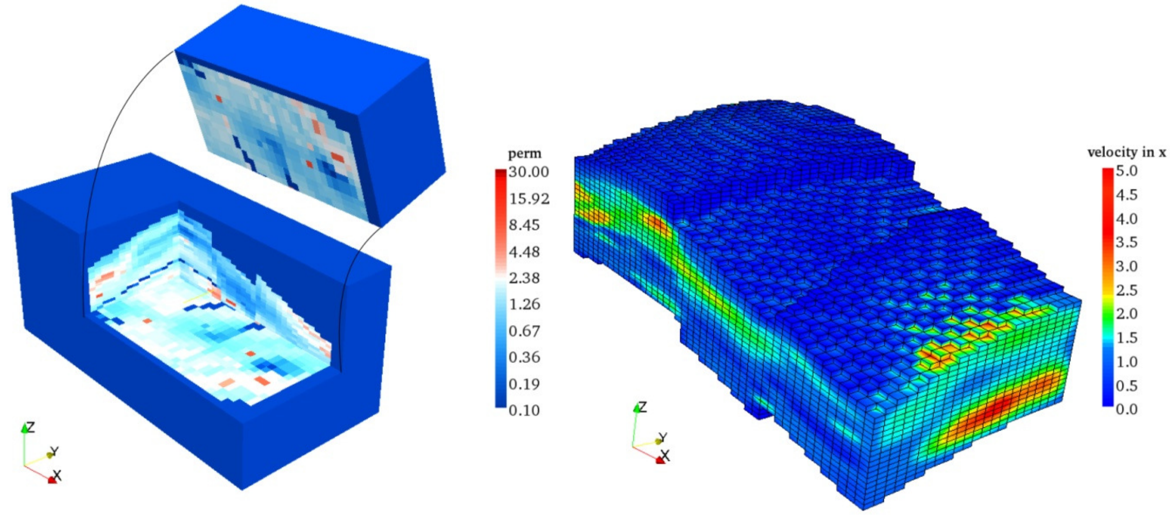

Figure 14: Enclosure of reservoir by low permeability elements (left) and velocity in x direction

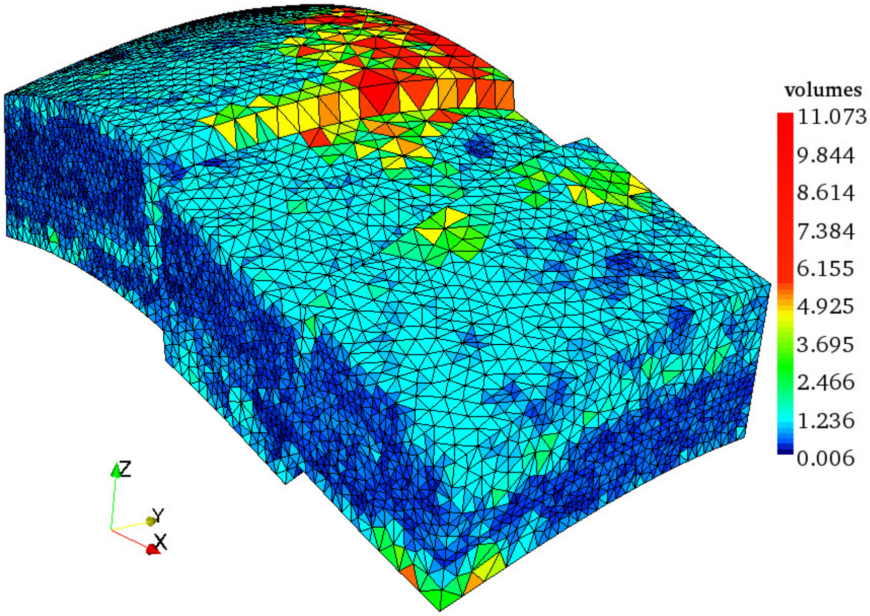

Figure 15: Tetrahedral grid generated with volume control using flow velocity in the *x* direction

More specific boundary conditions can be involved in flow based gridding as well. This example showed general boundary conditions applied in *x* such that pressure and velocity could be determined. In cases where a well configuration is known or is to be tested, more appropriate boundary conditions can be applied. Using the same models, an injector-producer well pair might be chosen to test the effectiveness of water flooding. Flow boundary conditions can be applied at the targeted injection and production sites. Particle tracking is used to assess the potential flow between the wells, which is then converted to an element volume distribution and used in grid generation. Figure 16 shows locations of an injector and producer, and the resulting tetrahedral grid. Several shells depicting element volumes below 0.15, 0.25, and 0.35 m^3 are shown in Figure 17; they coincide with the regions that will be affected most by water flooding activity, although some of the small elements along the faults occur to meet grid quality criteria.

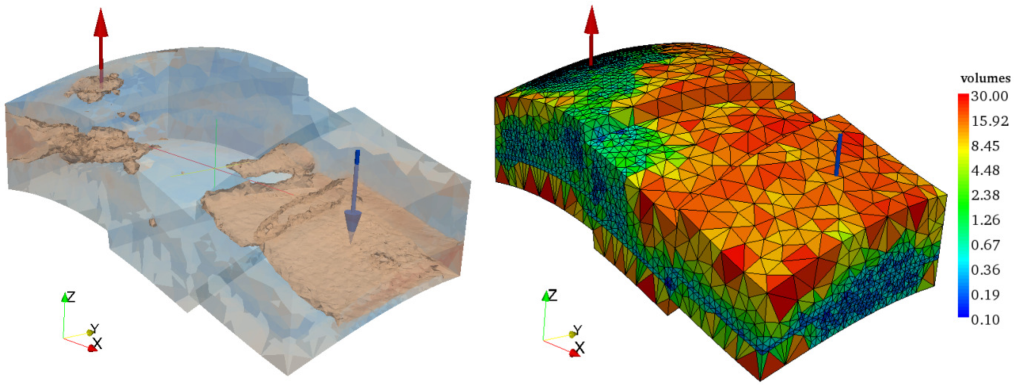

Figure 16: Injector producer pair shown with permeability > 3 md shell (left) and resulting tetrahedral grid volumes using particle tracking

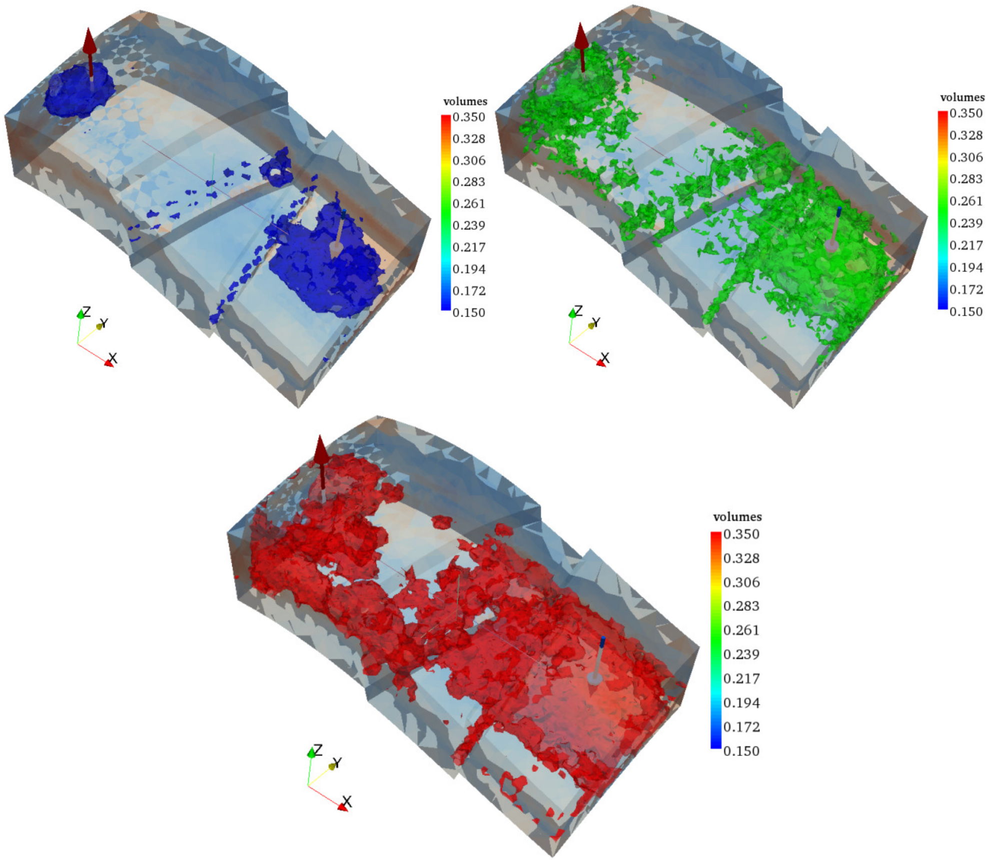

Figure 17: Shells enclosing element volumes less than 0.15 m^3 (top left), 0.25 m^3 (top right), and 0.35 m^3

Conclusions

Grid design is an important aspect of the reservoir analysis workflows. It depends on many features ranging from the type of simulator that is accessible to reservoir properties. This paper placed it into a modified workflow that offers several advantages for integration of unstructured grids. One of the more significant advantages is designing a grid with a specific purpose in mind. Examples showed grids created from volume distributions defined strictly from properties, from velocities using general boundary conditions, and from particle tracking in water flooding. Each design is better suited for particular uses than other designs. For example, grids generated from velocity fields using general boundary conditions would be more appropriate for choosing production well locations, since a general sense of connectivity and flow is achieved; however, this grid may not be optimal for choosing well pairs involved in water flooding, or when the well configuration is known in advance.

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