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3D Trend Modeling by Combining Lower Order Trends

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Large scale trends are important features that should be accounted for in geostatistical modeling, however, unfortunately there is no geostatistical technique to account for the trend in an implicit way. A typical way for 3D trend modeling could be divided into three steps: (1) model the areal trend, (2) model the vertical trend against vertical coordinate, and (3) merge 2D areal and 1D vertical trend into 3D trend. Conditional independence assumption between the modeled 2D and 1D trends has been used for the combining lower order trends. The main drawback of the simple method is that the combined trend value can be unfairly larger or lower than both input aerial and vertical trend value leading to a large variability in 3D trend model of which smoothness is an intrinsic characteristic. This work advances a combination method with introducing weights on lower order trends. The resulting 3D trend is adjusted to reasonably reproduce the input trends and not to be very high or very low. The related program is implemented with several options in combination models.

Introduction

A typical workflow of geostatistical reservoir modeling would start from large scale trend modeling which will be used either for brief reservoir interpretation such as a quick-look of reservoir property distribution or for a deterministic value that the detailed variations of simulated properties are added into. For the latter purpose, automatic fitting of a trend within the kriging formalism, such as ordinary kriging, could be considered; however, it only works in presence of many data. A far better approach is to model the trend in an explicit way (Deutsch, 2002).

In practice, we consider the vertical trend first and then map areal trends. Vertical proportion curve (VPC) for facies is constructed from plotting the vertical coordinate and averaged proportions of facies. It should be constructed to look for and quantify vertical trends. Areal trend is separately modeled from vertical trends. Vertically averaged proportions at well locations are used for areal trend modeling. Moving window average or kriging with moderate nugget effect provides a smoothed 2D trend map. Soft secondary information such as seismic map often helps for identifying areal variations.

For full 3D proportion modeling, combining lower order proportions has been considered since it is easier to fit the vertical and areal proportion than direct modeling a 3D proportion. In this approach, the elementary 2D and 1D proportion are standardized and multiplied by a global proportion. There is a risk; however, that the combined proportions are toward extreme values of 0 or 1 which is far smaller or larger than the used areal and vertical proportions. This *non-convexity*, falling outside the input values, leads to a high variability in the final 3D trend of which smoothness is an intrinsic characteristic. Besides, this property may prevent the modeled 3D trend from reasonable reproduction of input trends.

This work advances a weighted combination method and its related program for building 3D trend. Weights are introduced to adjust the influence of proportions used to combine. By imposing weights, the combined proportion in 3D is adjusted not to be very high or very low. The related program is implemented with several options in weighting scheme. Examples are tested with the program. The effectiveness of the chosen combination methods is evaluated in terms of how well input information is reproduced from the modeled 3D trend.

Methodology

The theoretical background of combining proportions is based on the probability combination schemes that approximate the probability of geologic event jointly conditioned to diverse data sources through combining the calibrated probabilities conditioned to individual data source (McConway, 1981; Journel, 2002; Krishnana, 2004). Integrating the 2D and 1D proportion that may be modeled by different data sources can be viewed as a probability combination problem. Consider the proportion of facies k in (x,y,z) location $p_k(x,y,z)$ given the areal proportion $p_k(x,y)$ and the vertical proportion $p_k(z)$, k=1,…,K where K is the number of facies. Of course, areal and vertical proportions meet closure property such as:

$$\sum_{k=1}^{K} p_k(x,y)=1 \;\; and \;\; \sum_{k=1}^{K} p_k(z)=1 \tag{1}$$

The permanence of ratios (PR-model in short) models $p_k(x,y,z)$ through combining elementary proportions $p_k(x,y)$ and $p_k(z)$ such as (Journel, 2002):

$$p_k(x,y,z)=\frac{\left(\dfrac{1-p_k}{p_k}\right)}{\left(\dfrac{1-p_k}{p_k}\right)+\left(\dfrac{1-p_k(x,y)}{p_k(x,y)}\right)\left(\dfrac{1-p_k(z)}{p_k(z)}\right)}\in[0,1] \tag{2}$$

p_k is a global proportion of facies k. The estimated proportion $p_k(x,y,z)$ always lies within [0,1] . The PR-model allows us to combine areal proportion $p_k(x,y)$ and vertical proportion $p_k(z)$ under independence assumption. Notice that the estimated proportion in equation (2) meets closure condition only when binary facies is considered, k=1 and 2. If multiple categories are of interest, the closure condition $\sum_{k=1}^{K} p_k(x,y,z)=1$ is not guaranteed, and thus the PR-model is no longer valid in case of multiple categories. Let us show a counterexample as following:

	p_k	$p_k(x,y)$	$p_k(z)$	$p_k(x,y,z)$
k=1	0.2	0.3	0.1	0.16
k=2	0.3	0.4	0.3	0.4
k=3	0.5	0.3	0.6	0.3913
marginal	1	1	1	**0.9513**

The marginal sum of $p_k(x,y,z)$ over k is not equal to 1. One can make any counterexample for showing the violation of closure condition.
In a different form of combination model, the $p_k(x,y,z)$ can be approximated as following:

$$p_k(x,y,z)=\frac{p_k(x,y)}{p_k}\frac{p_k(z)}{p_k}p_k C\in[0,1] \tag{3}$$

In this form, the 3D proportion is estimated from the multiplication of areal and vertical proportion standardized by global proportion. This approximation also adopts an independence assumption between 2D and 1D proportions conditioned to the estimated 3D proportion. *C* is an unknown normalizing term. By enforcing the closure condition on the equation (3), the *C* term is:

$$C = 1\Big/\left(\frac{p_{k=1}(x,y)}{p_{k=1}}\frac{p_{k=1}(z)}{p_{k=1}}p_{k=1}+\cdots+\frac{p_{k=K}(x,y)}{p_{k=K}}\frac{p_{k=K}(z)}{p_{k=K}}p_{k=K}\right) \tag{4}$$

$$\text{since}\left(p_{k=1}(x,y,z)+\cdots+p_{k=K}(x,y,z)=C\left(\frac{p_{k=1}(x,y)}{p_{k=1}}\frac{p_{k=1}(z)}{p_{k=1}}p_{k=1}+\cdots+\frac{p_{k=K}(x,y)}{p_{k=K}}\frac{p_{k=K}(z)}{p_{k=K}}p_{k=K}\right)=1\right)$$

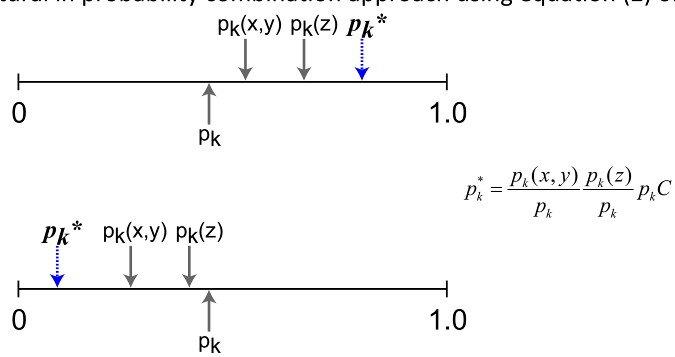
and insert the *C* term into equation (3) then:

$$p_k(x,y,z)=\frac{\dfrac{p_k(x,y)}{p_k}\dfrac{p_k(z)}{p_k}p_k}{\sum_{k=1}^{K}\left(\dfrac{p_k(x,y)}{p_k}\dfrac{p_k(z)}{p_k}p_k\right)}\in[0,1] \tag{5}$$

If binary facies (*K*=2) is of interest, the equation (5) is exactly same as the PR-model in the equation (2). The equation (5) is valid regardless of the number of facies and thus it can be regarded as a general form of combination model.

One interesting aspect of the combination equation (5) is that the combined proportion depends on the ratio of areal and vertical proportion to the global proportion. If two modeled proportions $p_k(x,y)$

and $p_k(z)$ are large relative to the global proportion p_k, then the ratio $p_k(x,y)/p_k$ and $p_k(z)/p_k$ will be greater than 1 and consequently the multiplication of two ratios (these ratios are greater than 1) produces much larger value. The resulting proportion $p_k(x,y,z)$ will be larger than the used proportion $p_k(x,y)$, $p_k(z)$ and p_k. Similarly, if two ratios become less than 1 then the multiplication of ratios (ratios being less than 1) produce much smaller value. This property falling outside the input values is termed as *non-convexity*. This appearance is natural in probability combination approach using equation (2) or equation (5):



In case that one tries to build the full trend in 3D through combining lower order trends, one should notice that:

(1) The modeled 3D trend is smooth in distribution
(2) The modeled 3D trend reasonably reproduces the used lower order trends, $p_k(x,y)$, $p_k(z)$ and global proportion p_k
(3) Closure condition of $p_k(x,y,z)$ over k=1,…,K should be met

Normalizing enforces the closure condition and the third comment is satisfied. Under non-convex property, the first and second expectations may be violated.

As a way of reducing the possibility of falling outside the input proportion, a weighted combination approach is advanced:

$$p_k(x,y,z) = \left(\frac{p_k(x,y)}{p_k}\right)^{w_1}\left(\frac{p_k(z)}{p_k}\right)^{w_2} p_k C \in [0,1] \tag{6}$$

where w_1 and w_2 are weights imposed on each proportion ratio. The weighted model reverts to the conditional independence model with letting w_1=w_2=1. The proportion ratio has a minimum bound of 0, but theoretically it has no maximum bound. The normalizing term C is disappeared by enforcing the closure condition ($\sum_{k=1}^{K} p_k(x,y,z) = 1$). The figure-1 below shows how the proportion ratio (denoted as X in the figure) changes according to the change of weight. By imposing weights lying within [0,1], the ratio is decreased if it is larger than 1, and the ratio is increased if it is smaller than 1. In other words, proportion ratio is limited when they tend to be extreme.

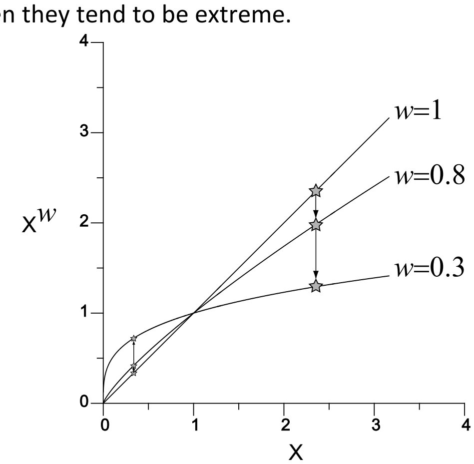


Figure-1: An illustration of how the proportion ratio change according to weight in [0,1]. As smaller weights are used, the weighted proportion ratio is bounded in shorter range. The curves pass at (1,1) regardless of weight values in which case areal and vertical proportion are all equal to the global proportion and the combined proportion reverts to the global proportion.

In a weighted combination approach, we can prevent the multiplication of proportion ratios, finally the combined proportion, from being too high or too small, and consequently, it may lead to better reproduction of input trends. Weights w_1 and w_2 have a different effect in the calculation of combined proportion. w_1 adjusts the influence of the areal proportion (precisely ratio of areal proportion to the global proportion). As smaller w_1 is used, vertical proportion becomes relatively influential. w_2 adjusts the influence of the vertical proportion (precisely ratio of vertical proportion to the global proportion). The areal proportion becomes relatively influential as smaller w_2 is used. For instances, if w_1=0 and w_2=1 in the equation (6) then the resulting 3D proportion is completely dependent on the vertical proportion in which case the input vertical trend is perfectly reproduced and areal trend is poorly reproduced. Conversely, if w_1=1 and w_2=0 then the resulting 3D proportion is completely dependent on the areal proportion in which case the input vertical trend is poorly reproduced and areal trend is perfectly reproduced. It is difficult to obtain the 3D trend model simultaneously honoring the input areal and vertical trends. One may observe a severe bias in the reproduced vertical proportion when areal proportion is more emphasized (using small w_2), and severe bias in the reproduced areal proportion when vertical proportion is more emphasized (using small w_1).

Universal weights w_1 and w_2 over entire locations might be undesirable. As the proportion ratios $p_k(x,y)/p_k$ and $p_k(z)/p_k$ are different at locations, different weights can be considered. To find appropriate weights, weighting curve depending on the proportion ratios is used. Figure-2 shows a smooth Gaussian weigh curve. As the proportion ratio (denoted as X in the figure) is far way from 1, weights become smaller and it has an effect of diminishing the input proportion ratio. The related program gives an option to choose the slope of the weight curve (denoted as wf in the figure-3). Large wf in the figure-3 makes steep weight curve and it causes strong weighting effect. Similarly, small wf makes gentle weight curve and it causes weak strong weighting effect. No weighting effect happens when wf=0 which is a conditional independence combination model.

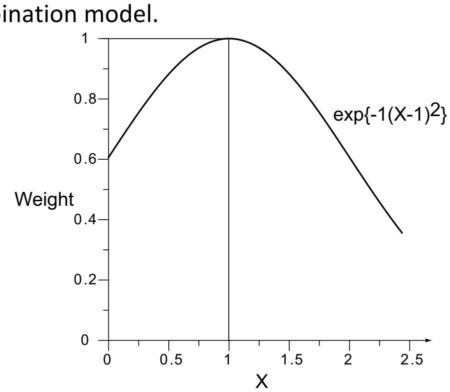


Figure-2: A smoothed weight curve is function of the proportion ratios denoted as X.

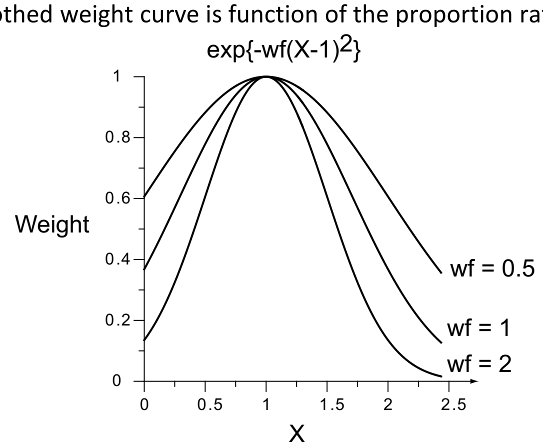


Figure-3: Various weight curves depending on the coefficient (wf) inside the exponential function. Large wf makes steep weight curve and it results in a strong effect of weighing.

It must be noted that the considered multiplicative form shown in equation (5) and (6) is dominated by a zero proportion from either $p_k(x,y)=0$ or $p_k(z)=0$ or both $p_k(x,y)= p_k(z)=0$. For example, when areal proportion at a particular location $p_k(x,y)$ is zero, the combined proportion $p_k(x,y,z)$ becomes zero regardless of $p_k(z)$ and weights. This dictatorship is a drawback of the multiplicative form and thus areal and vertical trends should be carefully modeled with possibly non-zero proportions (McConway, 1981; Benediktsson et al., 1992).

Program Description and Examples

The program pcsTM.exe (pcsTM: probability combination schemes for Trend Modeling) implements the combination of areal and vertical proportions into building 3D proportion model with some options in combination method. Figure-4 is a screen capture of the parameter file.

```
                Parameter file for PCSTM

line 1:  2Dtrend.out       -2D areal trend
line 2:  VPC.out           -1D vertical trend
line 3:  50 50 30          -nx,ny,nz
line 4:  3                 -Number of facies
line 5:  0.1 0.4 0.5       -Global proportions
line 6:  0.8 0.8           -slope for weight curve
line 7:  1.0  1.0  1.0     -overall modification factor
line 8:  3Dtrend.out       -3D trend output file
```

Figure-4: A screen capture of the parameter file

2D areal and 1D vertical proportion files are required input files in the program. Areal trend file contains only proportion values corresponding to the facies k=1,…,*K* not including *x*,*y*,*z* coordinate. The first column of the vertical proportion file represents a relative Z coordinate that is a transformed coordinate after stratigraphic layer flattening (Deutsch, 2002). Vertical proportions of each facies are followed by vertical coordinate. Number of grids in x,y,z direction are defined in line 3. Number of facies and global proportions are defined in line 4 and 5. Line 6 specifies the slopes of weight curve for the calculation of weight. Real positive values should be used. First and second values in line 6 specify the slope of weight curve applied to the areal and vertical proportion, respectively. Weighted combination model becomes independence model when line 6 has zeros. Line 7 is for overall raise or lowering the obtained 3D trend model. Despite of limiting the proportion ratios by weights, the reproduced proportions may not match the input proportions; they can be still departed from the input trends. Line 7 is an additional option to better match the input trends through overall shifting the obtained $p_k(x,y,z)$ by multiplying factors defined in the line 7. Number of these factors should be same number of facies. Notice that factor values far away from 1 distort the reproduced global proportion.

A test data is prepared as shown in the figure-5. Reservoir extends over 50m×50m×30m separated by 1m for x,y,z direction. The areal and vertical trends are modeled with 13 wells: vertically averaged proportions from 30 samples at each well are used for 2D trend modeling, and horizontally averaged proportions from 13 wells at each rel-Z coordinate are used for 1D trend modeling. 1D and 2D trend modeling was performed for each three facies. Figure-5 demonstrates well location map, modeled areal and vertical trend. Averaged facies proportions from each 2D and 1D trend are close to the global proportions. Vertical solid line shown on the VPC represents global proportion of each facies.

Given the areal and vertical trend, full trend model in 3D is conducted with the described program. Conditional independence model was first tested by letting the line 6 as 0 in the parameter file. Figure-6 shows the reproduced areal trend from the modeled 3D trend. Locations of high and low proportion in the input areal trend map still have high and low proportion in the reproduced map, respectively. Proportion value itself; however, seems to be systematically biased. Reproduced proportions tend to be unfairly too high and too low. This appearance is less prominent in facies 3 because the combined proportions are affected not only by areal proportions but also vertical proportions and less variable vertical trend of facies 3 (almost constant $p_k(z)$ over rel-Z coordinate as shown in the figure-5) may mitigate the phenomenon. Figure-7 illustrates more details at a particular location marked as star in the areal trend map. Areal proportion at marked location and vertical proportion curve are standardized by global proportions; input VPC, standardized VPC and multiplied curves (not normalizing yet) are shown at each row. Since the multiplied curve of facies 1 has larger value than other curves of facies 2 and 3, the combined proportion of facies 1 becomes higher and thus vertical average of them is also high at the specified location.

Figure-8 shows the cross plots of the input trends versus reproduced trends from the modeled 3D trend. Reproduced areal and vertical trends of facies 1 are higher than the input ones. This is because large amounts (72%) of nx×ny×nz grids have larger areal and vertical proportions than the global proportion, and consequently, those locations have very high proportion of facies 1. Reproduced trends of facies 2 and 3 are lower than the input ones because 77% of total grids have smaller areal and vertical proportions than the global proportion.

Depicting the results from simple combination model provides that the combined proportions are systematically higher or lower than the input proportion. Limiting the upper and lower proportions could mitigate the systematic departures. The following parameters are defined for a weighted combination.

```
line 6:  1.0  1.0             -slope for weight curve
line 7:  0.8  1.15  1.15      -overall modification factor
```

Setting line 6 as 1 represents that areal and vertical proportions use same weight curve with moderate slope. Overall modification factors in line 7 are defined to more closely match the input trends by overall shifting the obtained 3D proportions. Figure-9 shows cross plots of input versus reproduced 2D and 1D proportions. Due to applying weights, the combined proportions do not extend into very high or low proportions. Table below summarizes the reproduced global proportions.

	Facies 1	Facies 2	Facies 3
Reproduced	0.21	0.46	0.34
Input	0.28	0.42	0.31

Another example with real data is demonstrated through figure-10~figure-12. Reservoir extends over 1600m×1600m×23m separated by 1m for all directions. 39 wells are drilled over the reservoir. For three facies, areal and vertical trends are modeled. Different combination methods are tested and results are evaluated in terms of reasonable reproduction of lower order trends. Independence combination model causes departures from the input trends. Parameters were defined for a weighted combination:

```
line 6:  0.3  0.3           -slope for weight curve
line 7:  1.3  1.0  0.7      -overall modification factor
```

Cross plots of input and reproduced proportions are shown in the figure-13. Reproduced global proportions using weighted combination are summarized in the table below.

	Facies 1	Facies 2	Facies 3
Reproduced	0.484	0.272	0.244
Input	0.437	0.269	0.293

Continuous Variable

In case of trend modeling of continuous variable, similar approach has been considered. For building 3D trend $m(x,y,z)$, the lower order trends and global mean, $m(x,y)$, $m(z)$ and m, are combined by the following equation:

$$m(x,y,z) = \frac{m(x,y)}{m}\frac{m(z)}{m}m \tag{7}$$

Similar to the categorical variable modeling, the above equation may result in unfairly very high or very low 3D trend, and consequently, the modeled trend in 3D does not reasonably honor the input trends. The described program was applied to the building 3D trend of continuous variable. Figure-13 shows areal and vertical trend used to combine. Simple independence model was first applied and top cross plots in the figure-14 shows the comparison of input and reproduced trends from the modeled 3D trend.

Reproduced trends are systematically higher than the input trends for both 2D and 1D. Weighted combination with overall lowering by 7% reasonably matches the input trends (see bottom cross plots in the figure-14). Global means are compared in the table below. Numbers in the parenthesis is difference in % based on the input mean.

Reproduced from independence model	Reproduced from weighted model	Input
3.12 (+4%)	2.88 (-4%)	3.0

Conclusions

Large scale feature of reservoir properties is important and it should be accounted for in the final reservoir model. A typical way of modeling full 3D trend is to combine 2D areal trend and 1D vertical trend that are individually modeled and possibly using different data sources such as well data for vertical trend and seismic map for areal trend. Probability combination scheme is a basis for building higher order trend by combining lower order trends. Conventional combination method with independence assumption causes unfairly very high or very low trend values, which makes it difficult not only to reproduce the input information but also to preserve the smoothness in the modeled 3D trend. In this work weighted combination method is advanced. The method changes the influence of the elementary lower order trends by introducing exponential weights and consequently it adjusts the combined 3D trend not to be very high or very low. The related program is implemented with several options: independence or weighted combination. User can choose the slope of weight curve in the weighted combination option. Examples are tested with the program and results are evaluated in terms of reasonable reproduction of input information and preservation of smoothness in the final 3D trend.

In the demonstrated examples, weighted combination method shows better performance than the simple method with independence assumption. Simple combination approach; however, could be better in a certain case such that trends in areal and vertical direction are not significant. Several attempts are good practice to find the appropriate 3D trend model by applying from simple to weighted combination ways depending on the problem at hand

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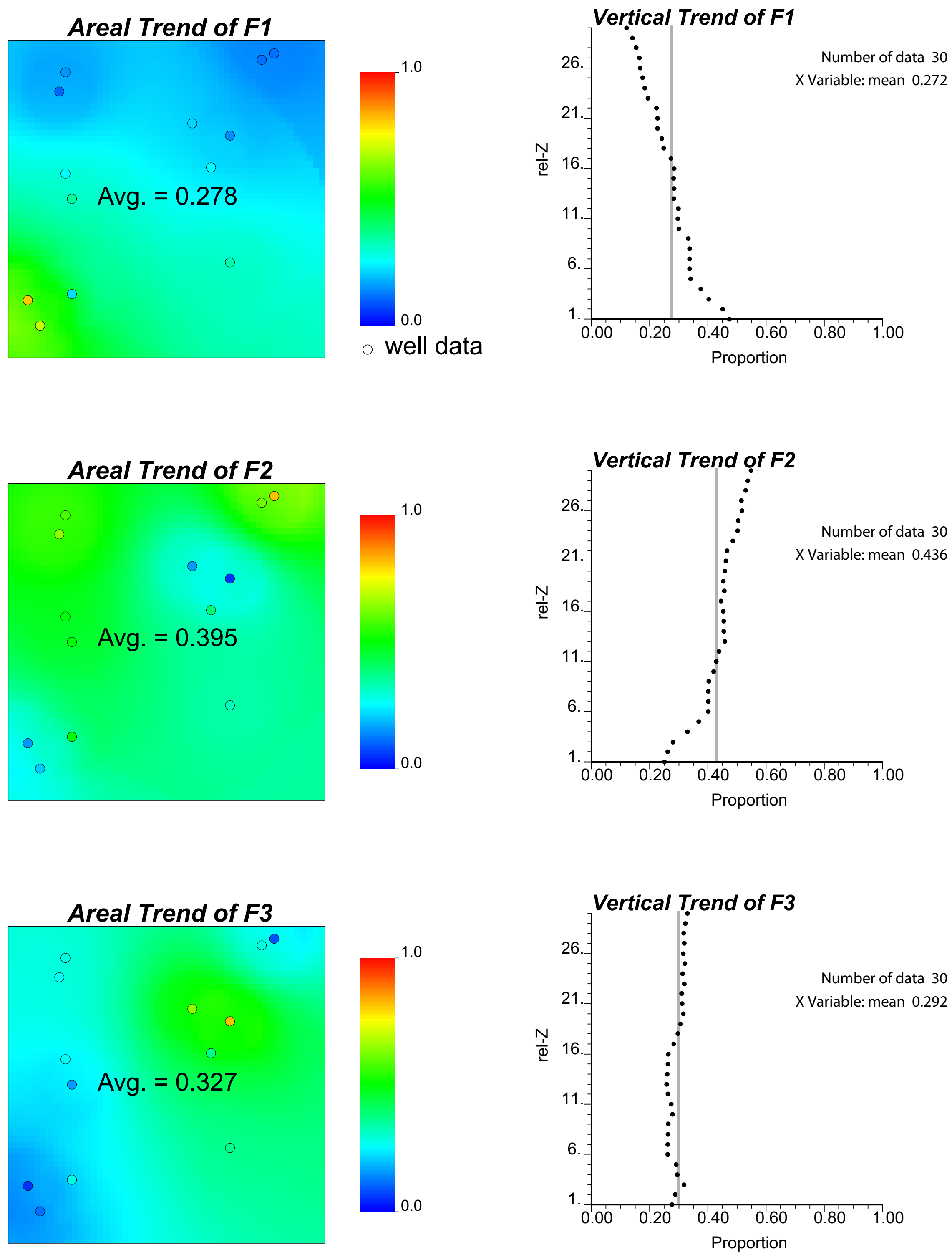


Figure-5: A synthetic test data for 3D trend modeling. Areal and vertical trends are modeled with 13 well samples for three facies. Global proportions are represented by a solid line on the VPC plot. Averages of 2D and 1D proportions are close to the global proportion.

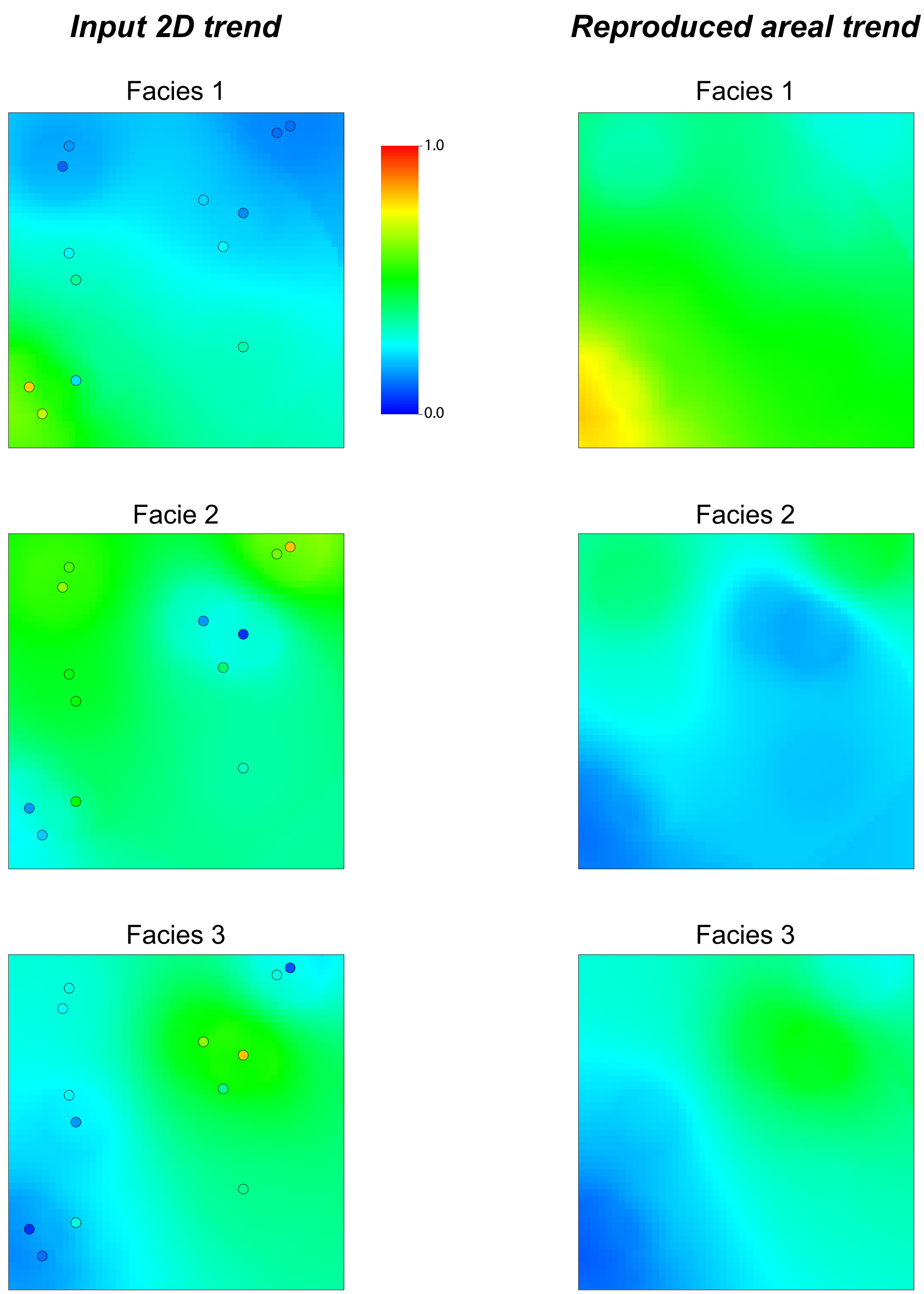


Figure-6: The comparison of the input areal trend and reproduced areal trend from the modeled 3D trend. For building 3D trend, conventional method with independence assumption was used.

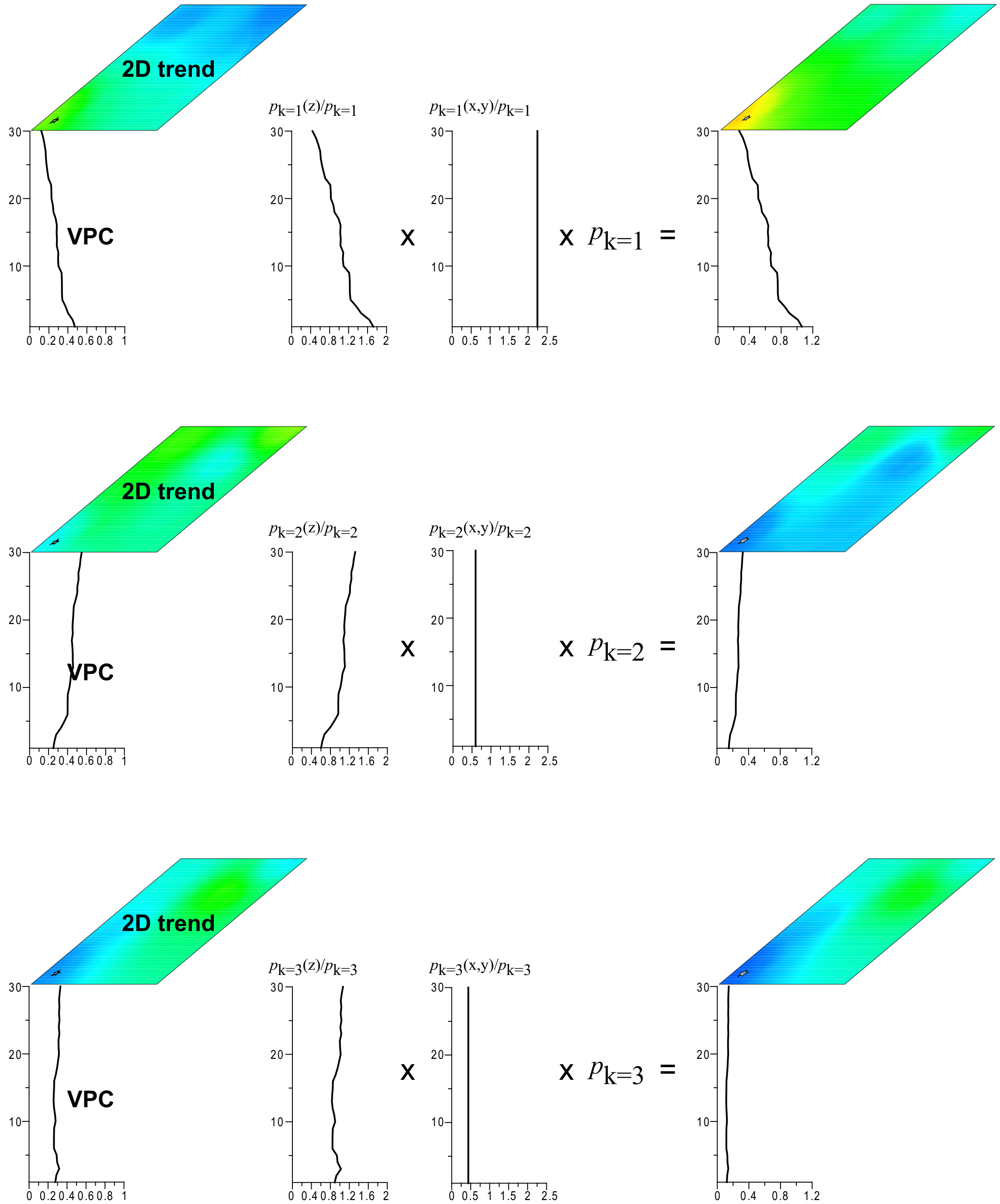


Figure-7: Detailed illustration about combing 2D and 1D trend at a particular location.

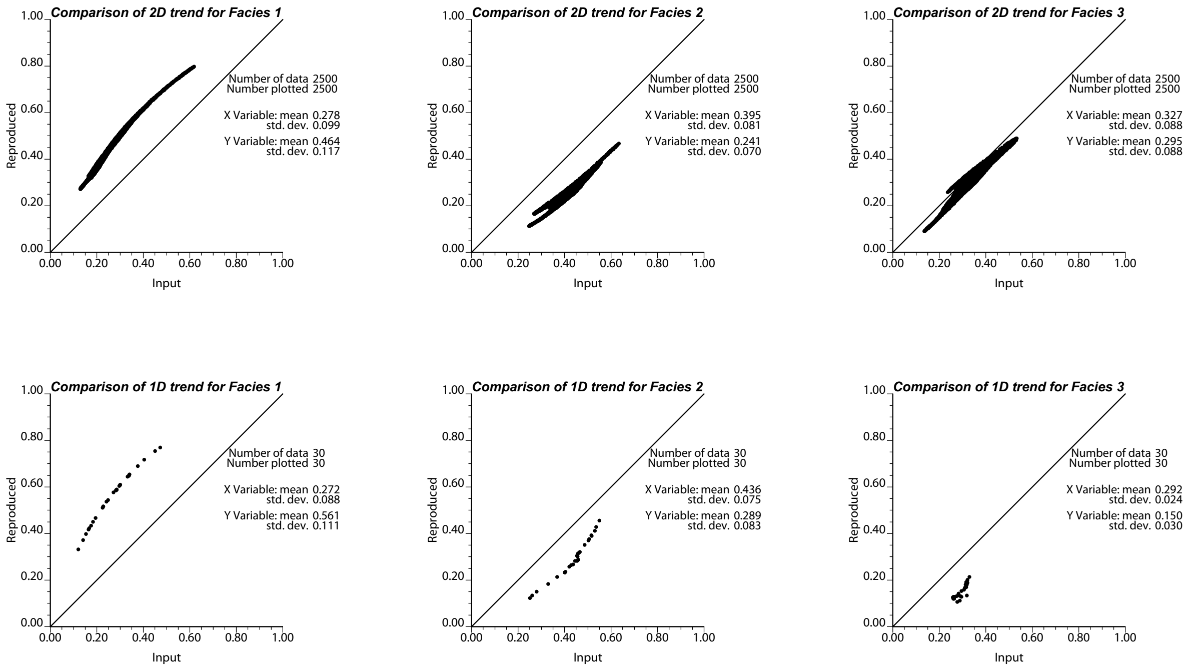


Figure-8: Cross plots of input and reproduced proportions from the modeled 3D trend using independence assumption.

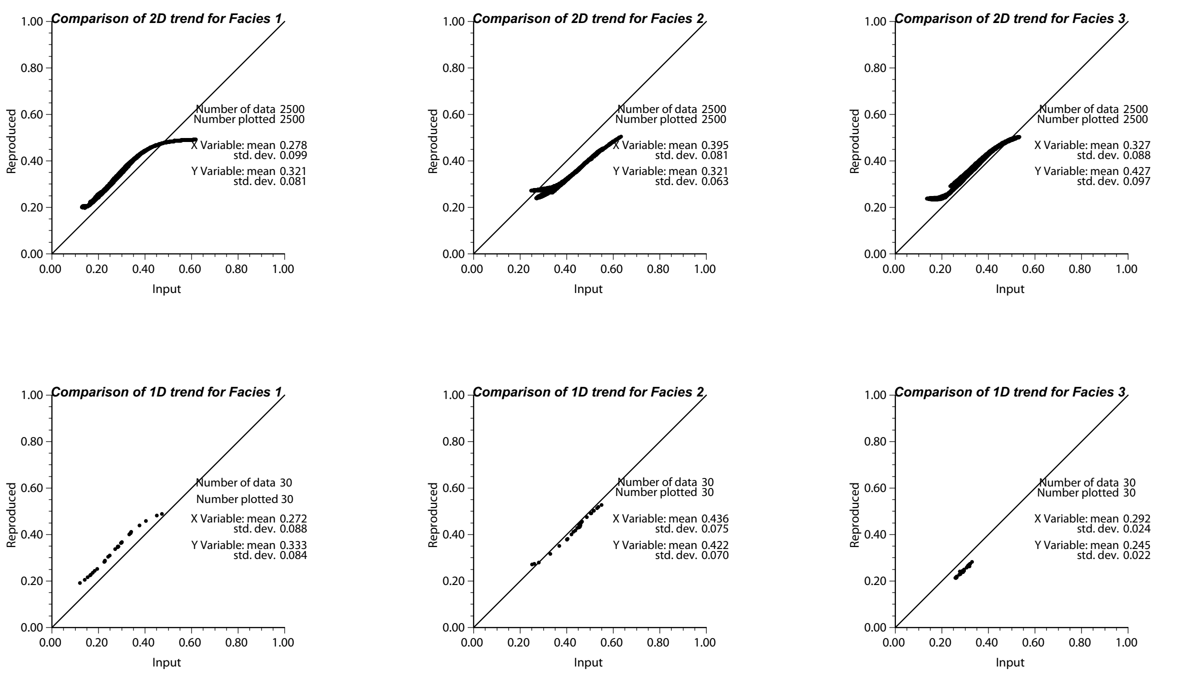


Figure-9: Cross plots of input and reproduced proportions from a weighted combination model.

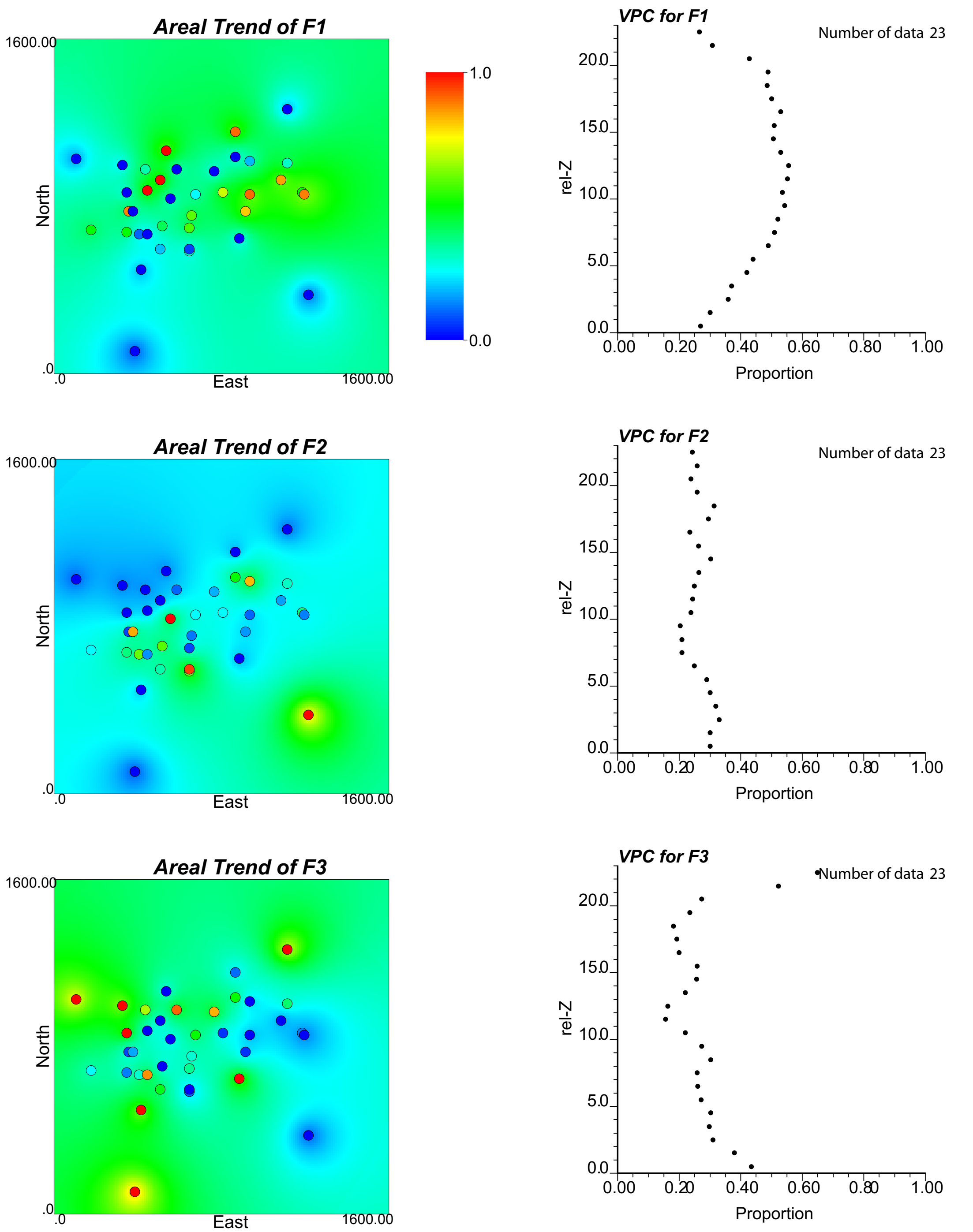


Figure-10: Another real example for test. Three facies are identified over the area, and areal and vertical trends for three facies are modeled.

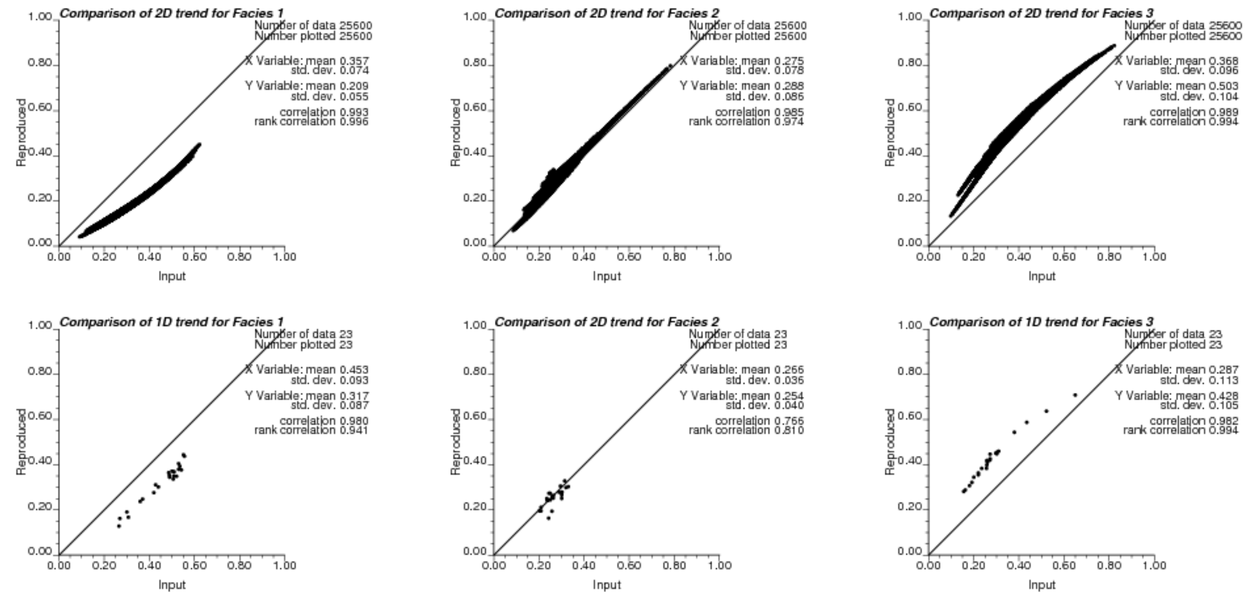


Figure-11: Cross plots of input and reproduced proportions from the 3D trend using independence assumption.

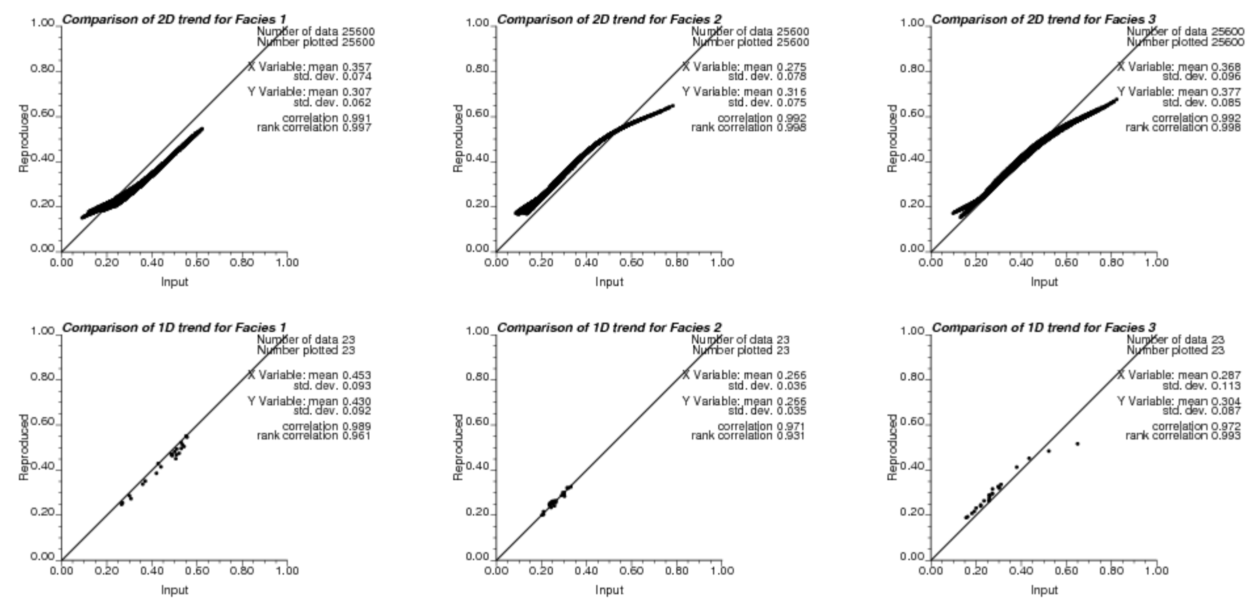


Figure-12: Cross plots of input and reproduced proportions from the 3D trend using a weighted combination model.

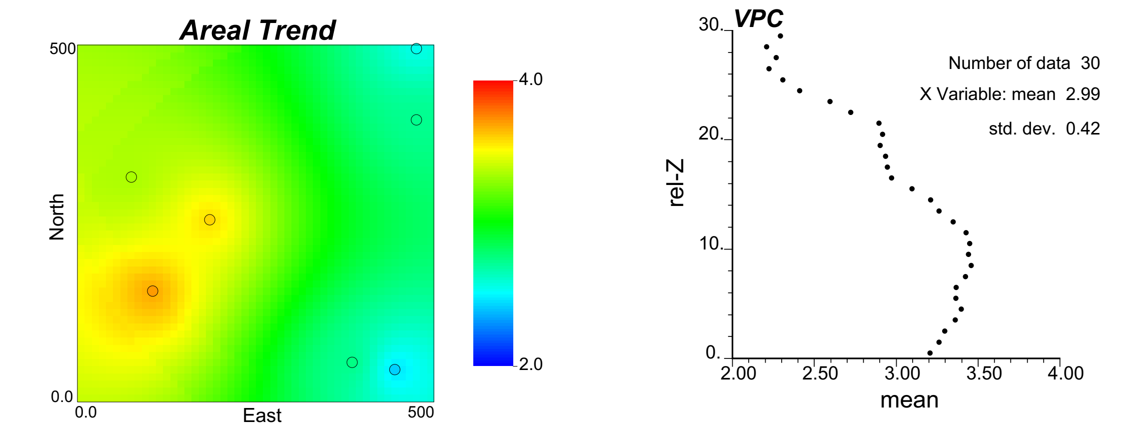
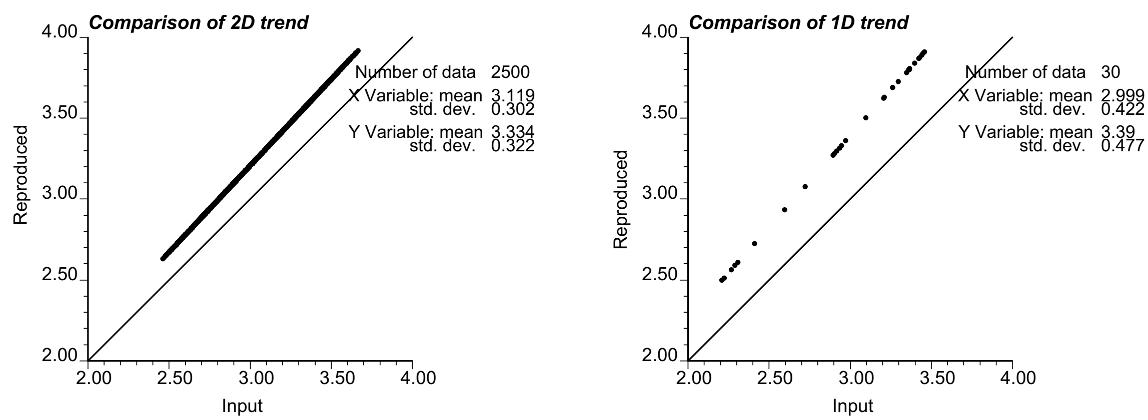


Figure-13: Areal and vertical trend for the continuous variable

Results with simple combination model



Results with weighted combination model

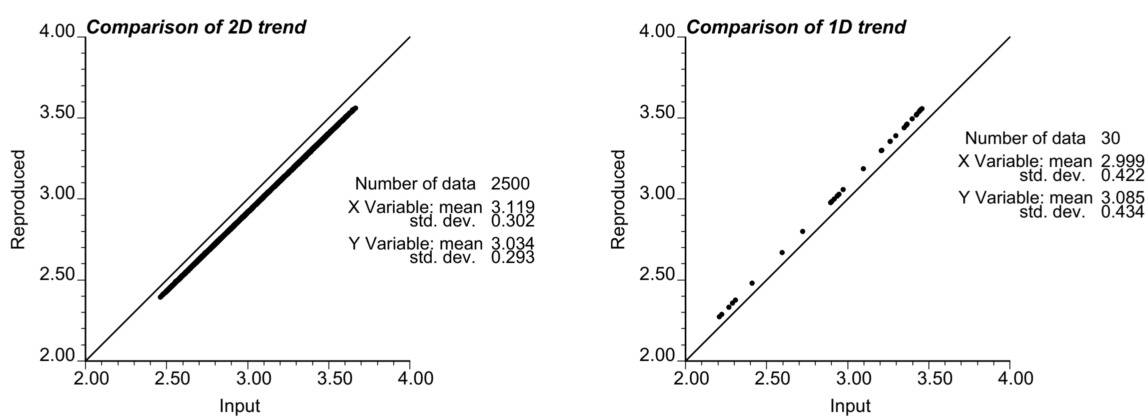


Figure-14: Evaluation of the reproduced lower order trends for the continuous variable.