Developments Toward Multiscale Modeling in a Gaussian Framework

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Scale is an important issue in reservoir modeling. The variety of data available for reservoir modeling are all at drastically different scales. The data scale must be taken into account when integrating the data into a unified numerical reservoir model. Data measured at different scales reflect different degrees of heterogeneity and have different degrees of accuracy. This paper presents in general approach for multi-scale reservoir heterogeneity and uncertainty modeling. The goal is a reservoir model that reproduces all multiscale data in a way that creates no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect. The Gaussian framework will be expanded with ideas from disjunctive kriging and uniform conditioning technique to incorporate data collected from different scales. The Hermite polynomials coefficients are adjusted according to scale

Introduction

Predicting future reservoir performance is an important goal of reservoir flow models. Performance forecasting permits optimization of the economic recovery of the oil and gas resources. Reservoir simulation is an established approach to forecast the performance of a reservoir for a particular development strategy. Data is expensive and sparse. Geostatistical models are used with the available data to build numerical models for reservoir simulation. Petroleum reservoirs are heterogeneous. Reservoir properties such as facies, porosity, permeability, faults, fractures and fluid saturations vary in space. The heterogeneity comes from variability in the depositional environment and subsequent events such as compaction, solution and cementation. An important goal of geostatistics is to build numerical models of heterogeneity that can be used in flow simulation. A central premise of geostatistics is to represent realistic spatial variability. Flow simulation is more reliable using geostatistical models that take into account heterogeneity. Historical geological models built using different techniques such as inverse distance led to less accurate flow forecasting.

Scale is an important issue in reservoir modeling. The aim is to describe a reservoir volume of 10^5-10^7 cubic meters of rock with few data. The data are gathered from different sources often at a much smaller scale. Accounting for the data scale is essential for accurate forecasting. For example, porosity values may be determined from cores or well logs that have significantly different scale than the grid blocks in flow simulation. The difference in scale should be accounted for when assigning properties to flow simulation grid blocks of an even larger scale. **Table1** shows some of the available measurements at different scales. Geostatistical models can be produced at different scales. The resulting models should be consistent when upscaled or downscaled: however, they will not be if the models are constructed by conventional techniques. **Figure1** illustrates the upscaling and downscaling concept. The scale is in cubic metres. There have been attempts to construct scale consistent models. Several methods for multi-scale modeling are available including conventional techniques such as cokriging, sequential gaussian simulation with block kriging and bayesian updating of point kriging.

Direct simulation is a recent proposal. The direct simulation proposal is difficult to implement because of practical problems such as the proportional effect. High valued areas often show more variability than low valued areas. The proportional effect is a natural phenomenon; it is a fundamental fact that needs to be dealt with. The proportional effect can be seen on the variogram and in the prediction of local uncertainties. Relative variograms can be used to address the issue of the proportional effect on the variograms; however, there is no clear methodology on how to tackle the proportional effect issue in the prediction of local uncertainties. Transferring the data to Gaussian units mitigates the proportional effect issue, however, multi-scale data cannot be transferred directly to Gaussian units as data from different scale do not average linearly which can lead to biases and inconsistencies in the results. A common practice is to perform multi-scale modeling with direct simulation techniques, that is, using the data in their original units. This practice can handle the difference in scale, but the proportional effect issue still exists as direct simulation techniques assume that the variance is independent of the

mean, while in reality the variance is indeed a function of the mean. A consequence of this assumption is that uncertainty in low valued areas is overestimated and uncertainty in high valued areas is underestimated. This paper presents in very general way the attempts to develop a methodology for multi-scale reservoir heterogeneity and uncertainty modeling. The goal is a reservoir model that reproduces the multi-scale data in a way that encounters no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect. The ideas behind the Uniform Conditioning technique (which is one of the Gaussian techniques) will be considered to build geostatistical models that incorporate data collected from different scales. The essential idea is to fit the transform with Hermite polynomials and adjust coefficients according to scale.

Recent previous work

Direct simulation techniques such as the exact down scaling technique [Ren,2007] are useful tools to predict uncertainty profiles without strong dependence on the Gaussian distribution. One problem with such techniques is that they ignore the proportional effect issue by assuming the variance is independent of the mean [Manchuk, Leuangthong and Deutsch, 2007]. Uniform Conditioning technique is another way to build geostatistical models that incorporate data collected from different scales. The name uniform conditioning comes from the conditioning of one scale estimation to preliminary other scale estimation. Estimation is first performed at a large scale, and then the Discrete Gaussian model is fit to the data. The Discrete Gaussian model is used to estimate the change of support from one scale data to other scales by introducing a change of support coefficient. The distribution of data at the small scale is fit with hermite polynomials. [Neufeld, 2005]. Uncertainty distributions cannot be generated using Uniform conditioning technique. However, Uncertainty distributions can be generated by generating multiple realizations using simulation. There are different ways to introduce the change of support coefficient [Machuca, Babak and Deutsch, 2008].

Framework

This research is aimed at developing a methodology for multi-scale reservoir heterogeneity and uncertainty modeling. The goal is a reservoir model that reproduces the multi-scale data in a way that encounters no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect .The concept of scale-dependent transformations will be investigated. The ideas behind the Uniform Conditioning technique (which is one of the Gaussian techniques) will be considered to build geostatistical models that incorporate data collected from different scales. The name uniform conditioning comes from the conditioning of one scale estimation to preliminary other scale estimation. The idea of scale dependent transforms is used in the technique of Uniform Conditioning. The same ideas are used in the techniques of disjunctive kriging and the discrete Gaussian model. The essential idea is to fit the transform with Hermite polynomials and adjust coefficients according to scale. Hermite polynomials are used in stochastic modeling because they provide flexibility in transferring a variable with skewed distribution into a Gaussian variable.

The Hermite polynomials are based on the Gaussian density function (GDF)

$$g(y) = \frac{1}{\sqrt{2\pi}} e^{(-\frac{x^2}{2})}$$

The Hermite polynomials are defined as the derivatives of the density function:

$$H_p(y) = \frac{p^{th} \, derivative \, of \, GDF}{GDF} = \frac{g^{(p)}(y)}{g(y)} \, , p = 0,1,2, \dots$$

The Hermite polynomials can be standardized by the division of $\sqrt{p!}$, by doing so we arrive at the Rodrigues Formula:

$$H_p(y) = \frac{1}{g(y)\sqrt{p!}} \frac{d^p g(y)}{dy^p}$$

Where $H_p(y)$ is the Hermire polynomial of order p, g(y) is the probability of the y value for standard normal distribution. The first two Hermite polynomials are: $H_0(y)=1$, $H_1(y)=-y$. Higher orders can be found by applying the recursive formula:

$$H_{p+1}(y) = -\frac{1}{\sqrt{p+1}}yH_p(y) - \sqrt{\frac{p}{p+1}}H_{p-1}(y)$$

The Hermite polynomials provide the flexibility to transfer a variable from skewed distribution to a Gaussian distribution. This can be achieved by introducing a non linear function φ that establishes a bijective correspondence between the random variable Z and the Gaussian random variable Y:

$$Z = \varphi\,(Y)$$

In practice the anamorphosis φ will be fitted to the data

$$\varphi_p \approx \sum_{\alpha=2}^{n}(z(u_{\alpha-1}) - z(u_\alpha).\frac{1}{\sqrt{p}}H_{p-1}(y(u_\alpha)).g(y(u_\alpha))$$

Then the transfer of variable with skewed distribution in to Gaussian variable and vice versa can be achieved.

$$z(u) = \sum_{p=0}^{\infty}\varphi_p H_p\,[y(u)]$$

Where φ_p= anamorphosis coefficient for a given order p, H_p= Hermite polynomial for a given order p, and $y(u)$= the normal score value of data point z(u). The variance of each polynomial is 1 and they are independent so that means the variance of $Z(u)$ equal to the summation of the squared anamorphosis coefficients, that is

$$Var\{Z(u)\} = \sum_{p=1}^{\infty}\varphi_p{}^2$$

Then, a change of support coefficient (r) can be introduced to account for the change of support from one scale to another scale.

$$Z(v) = \sum_{p=0}^{\infty}r^p\varphi_p H_p\,[Y(v)]$$

By calculating the value (r) the distribution of scale other than the data scale can be determined.

Vision on how multi-scale modeling could proceed
Assume that we have data from three different scales, seismic, well log and core samples, the probability density function (PDF) can be can be established. **Figure 2** shows a hypothetical PDF sketch for three different types of data. The red PDF represent data collected from large scale (seismic), The yellow PDF represent data collected from a smaller scale (well log) and the blue PDF represent data collected from small scale (core smaples). The aim is to develop a methodology that can provide mapping of point variable Z to the Gaussian variable Y and vice-versa for different scales. **Figure 3** shows a hypothetical sketch of the target chart.

Figure 4 shows a flow chart of how developing a methodology for multi-scale reservoir heterogeneity and uncertainty modeling can be achieved. First a scale dependent transform methodology for data at different scales coming from variety of sources has to be devolved. Then a methodology to simultaneously process all the transformed data has to be developed. Co kriging and/or block kriging under multi-Gaussian model can be used to develop a methodology to simultaneously process all the transformed data. The output of this process is the prediction of a conditional mean and conditional variance at unsampled location. At this stage a conditional distribution at unsampled location can be generated in Gaussian units, it then can be back transformed to obtain a scale dependent conditional distribution in original units which has no artifacts, no biases and handles numerical features of geological data such as nonlinearity and the proportional effect.

Getting Started- Understanding correlation coefficient
Understanding the correlation coefficient between different volumes is important. If the marginal distribution of two volumes is said to be Gaussian and the bivariate distribution is Gaussian then the relationship between them is fully captured by the correlation coefficient. The correlation coefficient can

be theoretically calculated, we should be able to verify practice with theory. The correlation coefficient between two variables can be written as follow

$$\rho_{XY} = \frac{\text{Cov}\{X, Y\}}{\sqrt{Var\{X\}\,Var\{Y\}}}$$

$$\text{Cov}\{X, Y\} = E\{X - Y\} - E\{X\}\,E\{Y\} = E\{X - Y\} - 0$$

Lets say X_k are points values and Y are block values

$$Y = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$\text{Cov}\{X, Y\} = E\left\{X_k \frac{1}{n} \sum_{i=1}^{n} X_i\right\} = \frac{1}{n} \sum_{i=1}^{n} E\{X_k X_i\} = \frac{1}{n} \sum_{i=1}^{n} C_{ki} \quad \text{since } E\{X_k X_i\} \text{ is Cov}\{X_k, X_i\}$$

The variance of point values is 1 $\quad Var\{X_k\}=1$. The Variance of the block values $Var\{Y\}= E\{Y^2\}-m^2 = E\{Y^2\}-0$

$$Var\{Y\} = E\left\{\frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} X_i\,X_j\right\} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}$$

$$\rho_{XY} = \frac{C_{ij}}{\sqrt{C_{ij}}}$$

The above equation states that the correlation coefficient between two different volumes equals to the covariance value of the two volumes divided by the square root of the covariance value of the two volumes. Unconditional simulation was performed to create 512 * 512 small scale data set, a 2 * 2 averaging was performed to create a 256 * 256 and a spherical variogram model with a nugget value of 0.1 and a minimum and maximum range of 32 was chosen. Calculating the correlation coefficient from theory will give a result of 0.9351. The correlation coefficient determined from practice will give a result close to the one determined from theory. **Figure 5** shows a summary of the results generated from practice.

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Table 1. Measurements at different scales.

Type	Level	Measurement Scale	Measurements
Micro	Pore	~Millimetre	Pore geometry Grain size Mineralogy
Macro	Core	~ Centimetre	K,kr,Ø,Pc Wetability Saturation
Mega	Grid block	~Metre	Logs Single well tracer
Giga	Interwell	~Kilometre	Well test Surface seismic Interwell tracer test
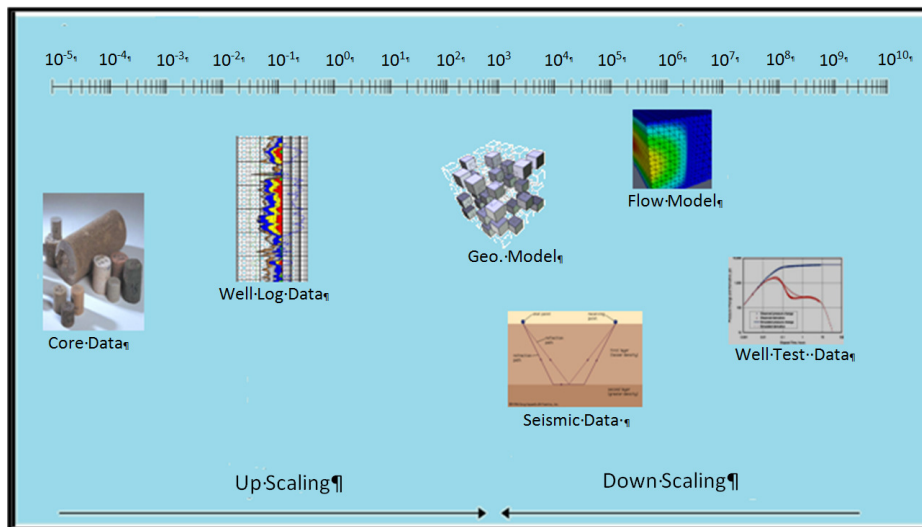
Figure 1. Upscaling , downscaling concept.
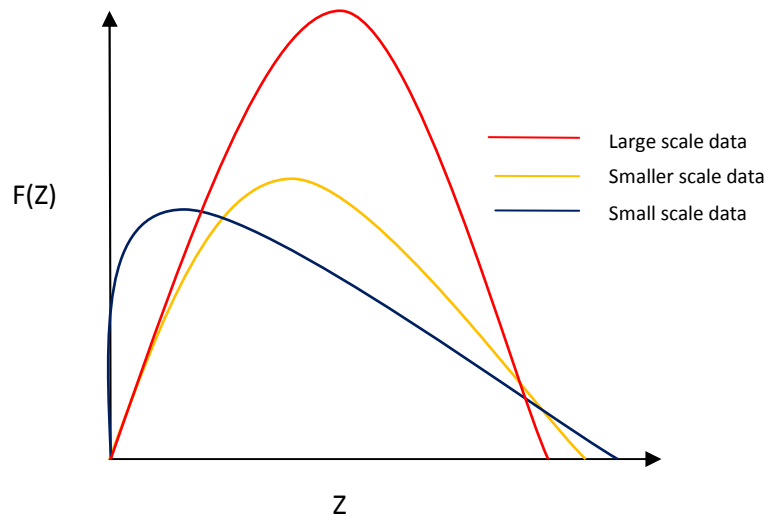
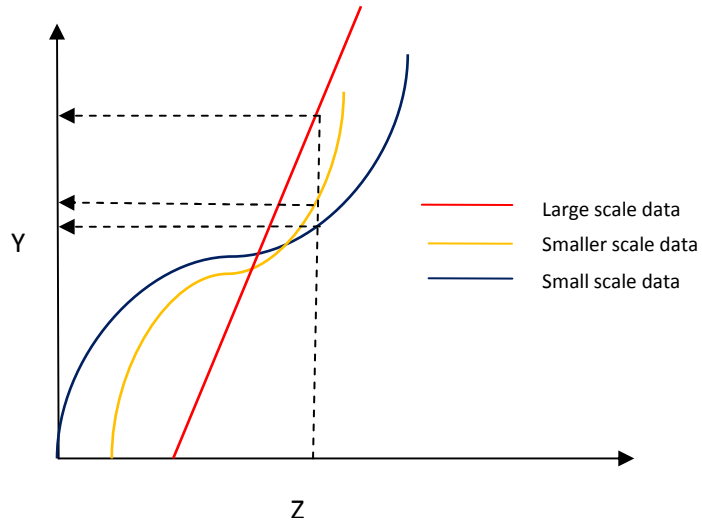Figure 2. Hypothetical distributions for data from different scales.

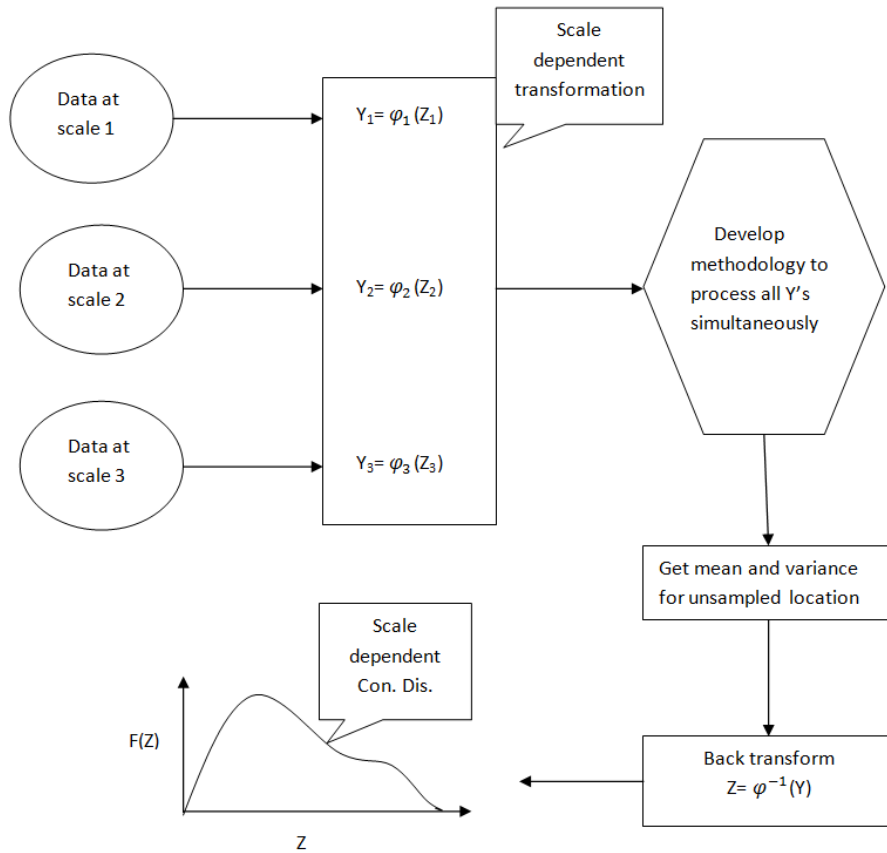Figure 3. Mapping of point variable Z to the Gaussian variable Y

Figure 4. flow chart of methodology for multi-scale reservoir heterogeneity and uncertainty modeling.

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results.out
mean of blocks = -8.030769143436789E-002
mean of points = -7.934977394292925E-002
variance of blocks =  0.869593673537773
variance of points =  0.980587081463705
Covariance of points-blocks =  0.869511723039692
Correlation of points-blocks =  0.9416168
```

Figure 5. 512*512 unconditional simulation and 2*2 averaging results