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Relating Horizontal Statistics to Vertical Statistics in Clastic Reservoirs

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A numerical reservoir model is required for resource evaluation and input to reservoir flow simulation for production forecasting and recovery calculations. Variations in the facies are the important heterogeneity for reservoir flow simulation performance. Facies are correlated together and exhibit complex auto-dependence with themselves and complex cross-dependence between different facies types. Indicator direct- and cross-variograms have been used to describe the degree of spatial dependence of facies. There is asymmetry between the different categories, that is, the relationship upward is not the same as the relationship downward. Moreover, it is difficult to fit all of the direct and cross variograms simultaneously. Transition probability matrices are used to characterize the spatial variation of facies in the vertical direction aligned with vertical wells. There is a need to predict the transition probabilities in the 3D spatial space for geostatistical modeling. An approach to relate the horizontal transition probabilities to vertical transition probabilities is developed. The basis is the high resolution sequence stratigraphic results which reflect the model's geological setting and the geologist's expert knowledge.

Introduction

The transition probability matrix (TPM) is used to characterize facies spatial variability. Constructing the horizontal transition probability matrix is almost always a challenge, especially in clastic reservoirs where the facies change rapidly. The available well data are not always sufficient to construct a reliable horizontal transition probability matrix especially in the appraisal stage. This is one aim of this research: developing an approach to relate the horizontal transition probabilities to vertical transition probabilities.

The high resolution facies variations are understood in the vertical direction. High resolution chronostratigraphic correlation permits well information to be extended into a predictive 3-D model. The transition probability matrices obtained from the vertical profile along wells have the facies stacking pattern information that can be converted into horizontal directions. In the relating or transforming process, the anisotropy ratio along different sedimentary direction is provided from high resolution chronostratigraphic correlation, which in turn provides a way to integrate the sequence stratigraphy information into the facies modeling.

Transition probability matrix (TPM)

In geology, facies are a volume of rock with specified characteristics. Ideally, a facies is a distinctive rock unit that forms under certain conditions of sedimentation, reflecting a particular process or environment (e.g. river channels, delta systems, submarine fans, reefs). The characteristics of the rock unit come from the depositional environment and composition of the sediments. Sedimentary facies reflect the depositional environment. Each facies is a distinct kind of sediment for that area or environment (Reading, 1996). Consider the facies in the research area as $\{1,2,...,K\}$. The first step in the construction the TPM is to discrete the vertical well profile with equal length segments. For two locations $\mathbf{u}$ and $\mathbf{u}+\mathbf{h}$, the transition probability is defined as: $P(facies(\mathbf{u}+\mathbf{h})=k, facies(\mathbf{u})=k')$ which is a bivariate probability of facies k at location $\mathbf{u}+\mathbf{h}$ and facies k' at location $\mathbf{u}$ at the same time. Later, this bivariate probability will be simplified denoted as $P(\mathbf{h};k,k')$ when the transition probability is independent the specific location in the research domain.

At each lag distance $\mathbf{h}$, the transition probabilities will compose a $K\times K$ matrix which is called transition probability matrix (TPM). The transition probabilities will change as the lag distance increases to larger distances. All the TPM at each step will compose an experimental transition probability curves as the experimental variogram in traditional geostatistics. If the facies at location $\mathbf{u}$ and $\mathbf{u}+\mathbf{h}$ are the same $(k=k')$, it is called a direct-transition probability. And when they are not the same, it is called a cross-transition probability that reflects the cross-correlation between different states. For example, consider 3 facies (1, 2 and 3) in a vertical profile. As the calculation $\mathbf{h}$ distance increases, the transition probability of facies 1 to facies 1 decreases, while the probability of facies 1 to 2 and 1 to 3 increases, as shown in Figure 1.

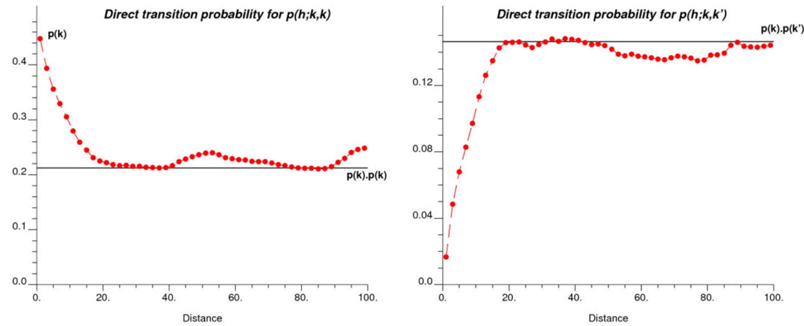

Figure 1 the direct transition probability and the cross transition probability curve

Basic properties of transition probability

1. The transition probability, as a bivariate probability, is nonnegative, so $P(\mathbf{h};k,k')\geq 0$.The transition probability is used as a bivariate probability in this paper which is different than the transition probability that used in the markov transition probability based geostatistics (Carle, 2000; Carle and Fogg, 1996; Dai et al., 2007; Li, 2007; Li and Zhang, 2006). In those researches, the transition probability is a conditional probability and defined as: $P(facies(\mathbf{u}+\mathbf{h}_\alpha)=k \mid facies(\mathbf{u})=k')$.

2. The univariate proportions are already embedded in the transition probability. At any specific lag, values of TPM headed by the same facies sum to the univariate probability of this facies. That is: $\sum_{k=1}^{K} P(\mathbf{h};k,k')=P(k)$.

3. Because of mutually exclusive quality of facies, the nuggets for the direct TPM are always their univariate probability; and the nuggets for cross TPM always are zero. The sill for the direct TPM should be: $P(k)\cdot P(k)$ the sill for the cross TPM should be: $P(k')\cdot P(k)$ as shown in Figure 1.

4. Asymmetric character of the transition probability. Mathematically, bivariate probability is symmetric. That is, for two random variables $k \text{ and } k'$, the bivariate probabilities satisfy $P(k,k')=P(k',k)$. But for the transition probability used in this research, it has the asymmetric properties for its directional dependent. If the head and tail switched, the transition probability will be different. $P(\mathbf{h};k,k')\neq P(\mathbf{h};k',k)$.

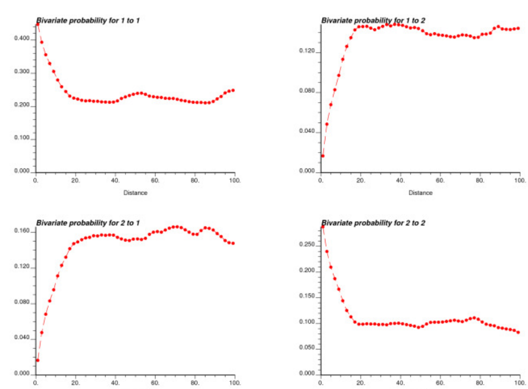

Figure 2 the asymmetric character of the transition probability matrix curve

3D Facies Transition Probability Matrix model

The transition probability can be calculated from the vertical well log profile or well exposed outcrops where it is possible and easy to get high density data sample. In this case, taking advantage of the classic mathematical models that are widely used in modeling direct-variograms (Deutsch, 1998), some theoretical mathematical models as listed in table 1 can be also used in TPM modeling (Weidong Li, 2007; Ritzi, 2000). The spherical, exponential and gaussian TPM models are provided in table 1 which are all satisfy the basic previous TPM requirements. More complex model such as cosine model could be better for modeling more complex geological heterogeneity.

Table 1 some theoretical Transition probability model

Transition Probability model	Theoretical function
Spherical(Direct)	$P(\mathbf{h};k,k) = p(k).\{1-[1-p(k)][1.5(\frac{\mathbf{h}}{a_{kk}})-0.5((\frac{\mathbf{h}}{a_{kk}})^3)]\}$
Spherical(Cross)	$P(\mathbf{h};k,k') = p(k).p(k')[1.5(\frac{\mathbf{h}}{a_{kk}})-0.5((\frac{\mathbf{h}}{a_{kk'}})^3)]$
Exponential(Direct)	$P(\mathbf{h};k,k) = p(k).\{1-(1-p(k))[1-exp(-\frac{3\mathbf{h}}{a_{kk}})]\}$
Exponential(Cross)	$P(\mathbf{h};k,k) = p(k).p(k')[1-exp(-\frac{3\mathbf{h}}{a_{kk'}})]$
Gaussian(Direct)	$P(\mathbf{h};k,k) = p(k).\{1-(1-p(k))[1-exp(-(\frac{3\mathbf{h}}{a_{kk}}))^2]\}$
Gaussian(Cross)	$P(\mathbf{h};k,k) = p(k).p(k')[1-exp(-(\frac{3\mathbf{h}}{a_{kk'}}))^2]$

There exists a good relationship between variogram model and the transition probability model. For example, the spherical variogram model can be inferred from the direct transition probability as:

$$
\begin{aligned}
\gamma(\mathbf{h};k) &= p(k) - P(\mathbf{h};k,k) \\
&= p(k) - p(k)\cdot\{1-[1-p(k)]\cdot[1.5(\tfrac{\mathbf{h}}{a_{kk}}) - 0.5(\tfrac{\mathbf{h}}{a_{kk}})^3]\} \\
&= p(k)\cdot[1-p(k)]\cdot[1.5(\tfrac{\mathbf{h}}{a_{kk}}) - 0.5(\tfrac{\mathbf{h}}{a_{kk}})^3] \\
&= c\cdot[1.5(\tfrac{\mathbf{h}}{a_{kk}}) - 0.5(\tfrac{\mathbf{h}}{a_{kk}})^3]
\end{aligned}
$$

An example of the TPM models and the related variogram models is shown in Figure 3. The new developed facies modeling algorithm DMPE is based on the TPM (details can be found in the paper 106 of this volume). The estimation results from using the theoretical TPM model and the related variogram model are shown Figure 4. For more information are integrated into the DMPE model from the TPM, the uncertainty in the final model is reduced compared with the traditional indicator approach which is also shown in Figure 4 (uncertainty reduction criteria can be find in the paper 107 of this volume).

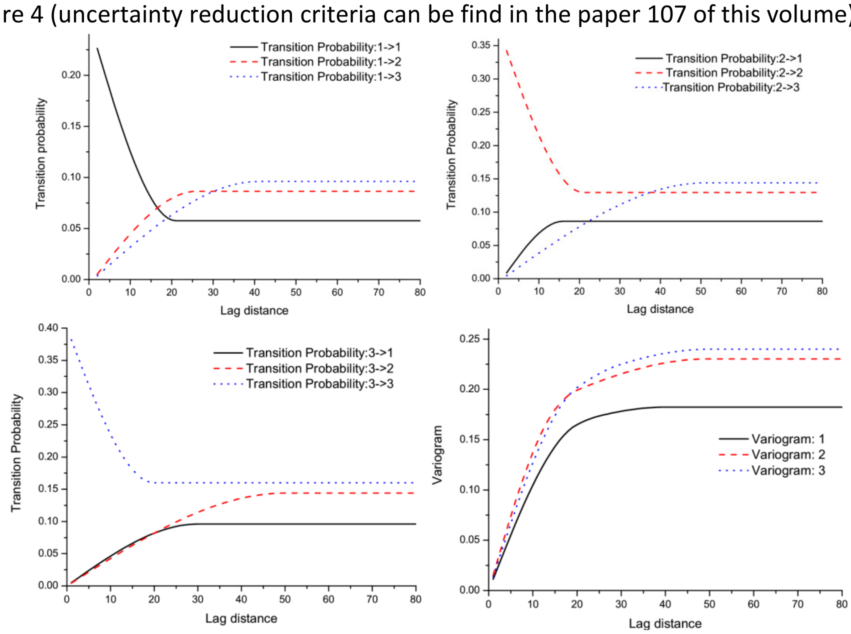

Figure 3 transition probability matrix model and the related variogram model for three categories

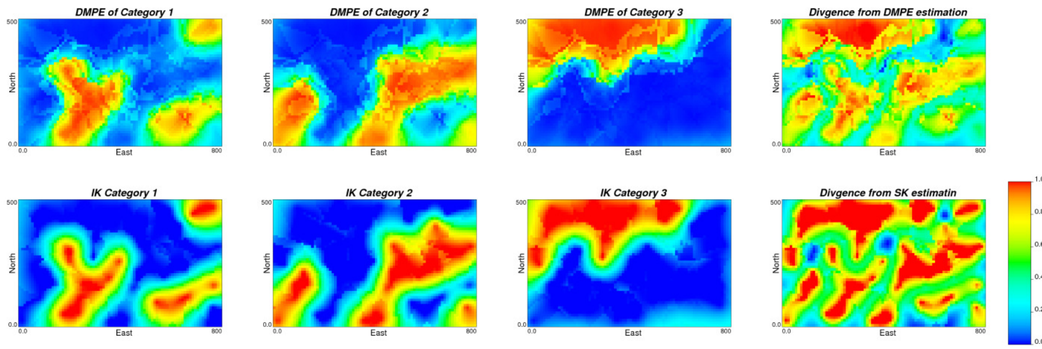

Figure 4 the DMPE estimation and SK estimation results comparison using a theoretical transition probability matrix model and the related variogram model

The geological pattern captured by facies proportion in Transition probability

In practice, modeling the experimental TPM with the mathematic modeling will be a very tedious process. If there are K facies, totally, K^2 different TPM model should be modeled. Furthermore, although it is relatively easy to obtain the TPM along the direction where enough data are sampled, it is difficult to obtain a reliable experimental TPM along the lateral direction where no sufficient data are sampled. The TPM from spares data sample may not convey reliable information. Interpreting the lateral heterogeneity is always a challenge in geostatistics. Using the soft information is one possible approach, such as seismic data or geological understanding. In this research, mainly focuses on how to incorporate the geological constraints to infer a 3D TPM model.

Sequence stratigraphy is the most recent revolutionary paradigm in the field of sedimentary geology. The sequence stratigraphic approach has led to improve understanding of how stratigraphic units, facies tracts, and depositional elements relate to each other in time and space within sedimentary basins (Catuneanu, 2006). Well correlation and sequence stratigraphic analysis is a routine part of reservoir characterization (Cross et al., 1993; Labourdette et al., 2008; Miall and Miall, 2001; Schwarzacher, 2000). Some efforts that integrate the stratigraphically controlled “nonstationarity” into conditional simulations is through incorporating vertical proportion curve (Langlais et al., 1993), or the locally various facies proportion(Deutsch, 2006) into indicator kriging.

High-resolution sequence stratigraphy can contribute directly to more accurate and predictive TPM model in two ways. The fist will be the facies transition pattern which is characterized by the facies proportions within different sedimentary units. The second will be the anisotropy ratio along lateral, including the sedimentary trend direction. These will form the basis for relating the lateral spatial statistics to vertical statistics proposed in this session.

An important principle in stratigraphy application is the law of superposition—the principle that in any undisturbed deposit the oldest layers are normally located at the lowest level which is also called Walther’s law. Walther’s law is the basement for many sequence stratigraphic theories: sedimentary environments that started out side-by-side will end up above one another over time due to transgressions and regressions. The result is a vertical sequence of facies that is similar to the original lateral distribution of sedimentary environments. As an example, in one parasequence of fluvial dominated delta building sedimentary process, the sedimentary facies will have an up-coarsing pattern along the vertical direction. While along the lateral chronostratigraphical boundary, from the proximal to distal direction, the same pattern can be found which is shown in Figure 5. In Figure 5, the fluvial sand will have a higher proportion in the delta plain sedimentary environment than that in delta front sedimentary environment. This is valid in vertical direction as well as in lateral direction along the lateral chrononstratigraphical boundary direction.

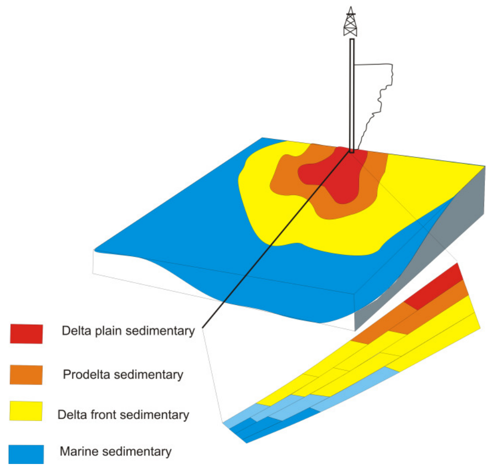

Figure 5 diagram of progradtion parasequence in fluvial-dominated deltaic sedimentary unit (The same facies transition pattern can be found along the vertical direction as along the lateral chronostratigraphical boundary direction)

This vertical and lateral stacking pattern created in the depositional processes can be captured by the vertical and horizontal facies proportion for each genetic sequence. As discussed in previous session, the univariate proportion already embedded in the transition probability. The facies univariate proportion can also be captured by the TPM as shown in Figure 6. In Figure 6, the facies contact pattern is the same while with different facies proportion. From the left to right, proportion of facies 1 increasing trend is clearly shown in those three facies' transition probability changing. From the same facies contact pattern, different transition probability can be obtained by using different proportion to the contact pattern. It is possible to integrate sedimentary conceptual information into the transition probability matrix. In Figure 6, from the left to right, it could be a progradtion process in lateral direction.

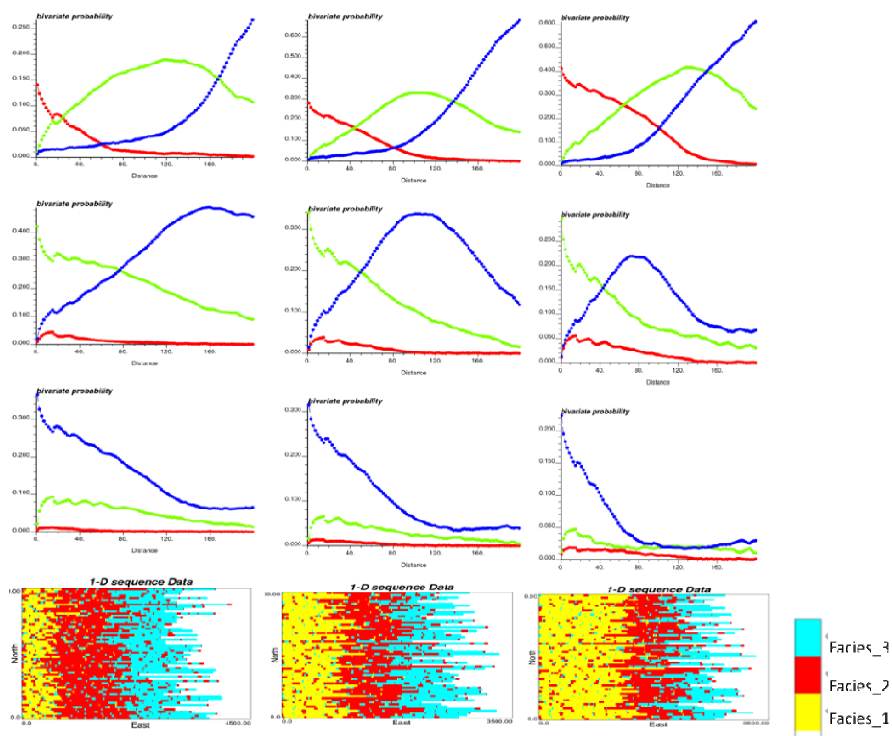

Figure 6 transition probability for same facies contact pattern with different facies proportion (from left to right, the proportion of facies one increase which could be a progradtion process)

The geological pattern captured by anisotropy ratio

Sea-level cycles, or changes of sedimentation sources, lead to significant spatial variation of facies proportions, and body geometry (orientation and size, variable azimuth). Through high-resolution correlation, the facies stacking pattern and the degree of heterogeneity of reservoir along different direction may be approximated and predicted. As an example, in delta sedimentary environment, during the delta-building process, under normal discharge conditions, sediment remains within the channel until it reaches the river mouth. No lateral dispersion of the load occurs on the subaerial delta plain, and because river velocity is so low, waves and currents spread the fine-grained portion of the sediment laterally along the delta front. This facilitates accumulation at the delta front and causes the subaqueous delta to prograde. During floods, however, delta lobe shifting may happen along the coast line as shown in Figure 7. In Figure 7, the facies stacking pattern along the coast line will be different with that perpendicular to the coast line. This kind of sedimentary source changing is very common in clastic sedimentary environment. The heterogeneity or nonstationarity is universal in reality and should be reproduced in our modeling realization. The information from doing sequence analysis can be used in lateral TPM modeling.

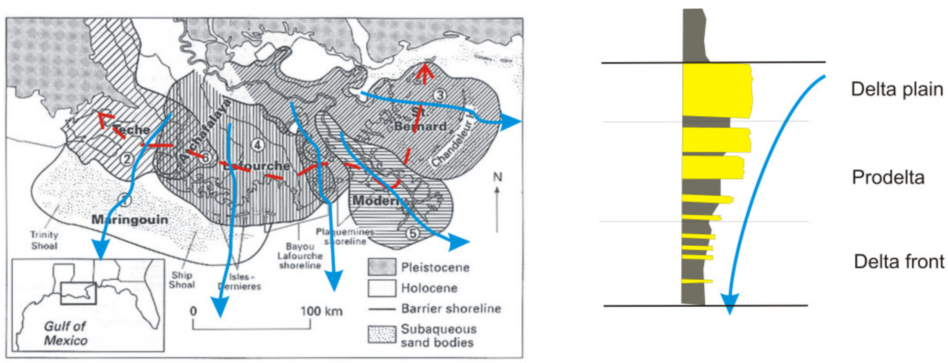

Figure 7 the diagram of lobe shifting in fluvial-dominated deltaic sedimentary environment (Blue: perpendicular to the coast line, the facies transition will be similar with the vertical pattern from up to bottom direction; Red: parallel to the lobe shifting direction, the pattern will be only part of the whole facies transition pattern along the vertical direction)

As discussed by many authors, different heterogeneity scales exist in every sedimentary process (Gibling, 2006; Miall, 1985; Richards and Bowman, 1998; Walker and James, 1984; Ye and Khaleel, 2008). For example in deep-marine clastic depositional systems, the relative larger scale is sedimentary body scale (such as sub-marine fan or lobes complex). The sediments in the bed-set scale (such as a single lobe or channelized sediments) will be the media scale heterogeneity. The small one could be in bed scale. Many factors that control the sedimentary system have contributed to this heterogeneity variety. Getting an anisotropy ratio related with vertical to lateral direction is possible after taking geological analysis efforts.

Relating the lateral spatial statistics to vertical statistics

After high-resolution chronostratigraphic correlation, the facies transition pattern, transition directions and anisotropy ratio along different direction are ready to use in 3D TPM modeling. As shown in Figure 8, assuming the vertical facies TPM are already obtained from location ***a***, along the direction from ***a*** to ***f (***named as ***dip*** direction***)***, it will be more distal, from ***a*** to ***b*** it will be more proximal. Along the direction from ***a*** to ***d (***named as ***strike*** direction***)*** will be just a sedimentary source shifting. Location ***c*** is an arbitrary spatial location departing from location ***a.***

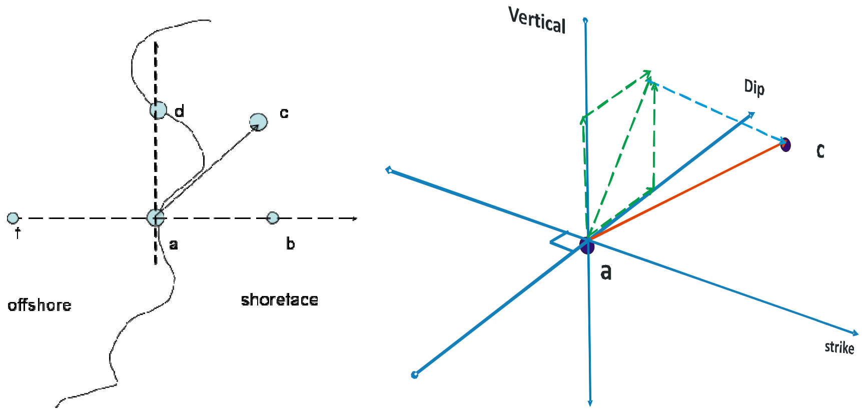

Figure 8 the three main facies transition direction and 3D spatial distance vector decomposition (Left: along the dip from a to f, the sedimentary deposit will be more distal, from *a* to *b* it will be more proximal. From *a* to *d* is the strike direction, the deposit will be similar with each other. Right: Any 3D spatial distance vector decomposition along three main directions)

Based on this information, the later facies transition probability in 3D can be related to vertical direction. Any 3D spatial distance vector will be decomposed into three vectors along three main facies transition directions that is vertical, dip and strike direction. After that, based on the anisotropy ratio, an effective distance is obtained by combining them as:

$$h_{effect} = \sqrt{(\frac{h_{dip}}{a_{dip}})^2 + (\frac{h_{vertical}}{a_{vertical}})^2 + f_{strike} \bullet (\frac{h_{strike}}{a_{strike}})^2} \qquad (1)$$

In Equation (1), a_{dip} and $a_{vertical}$ are the anisotropy ratios along the vertical and dip direction which will transform the 3D spatial distance along the dip and vertical distance. The factor f_{strike} is used to character the sedimentary resource shift process. As the transition pattern will stay as the same, the effective distance calculated from the vertical and dip direction will only modified randomly based on the effective distance along the strike direction. The factor could be modeled as a normal distribution as shown in Figure 9 where a_{strike} is the anisotropy ration along the strike direction. How large the variance should be changed along the strike direction would also depend on the geological environment which is also usually clear before facies modeling.

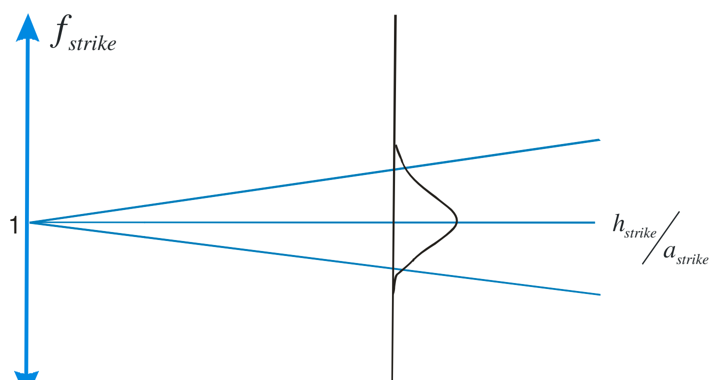

Figure 9 the randomly modification factor distribution along the strike direction

By this approach, the anisotropy ratio along three major directions will compose a 3D anisotropy ratios transform space as shown in Figure 10. All the distance between two locations will be transformed and recombined to an effective distance along the vertical direction by equation 1 and retrieve transition probability from the vertical direction. By this approach, the facies transition patterns which in turn are representative of the depositional processes the 1D wells information was extended into a predictive 3-D model based on the facies model. Modeling accuracy will be increased by introducing these sequence stratigraphic additional relevant constraints and information in the simulation algorithm.

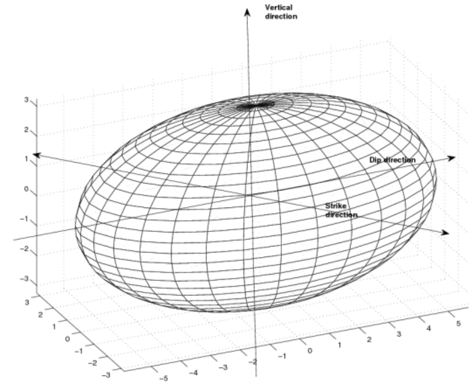

Figure 10 the 3D anisotropy ratio transformation space

Facies model examples

The proposed 3D spatial facies transition probability matrix construction scheme is used in the newly developed direct facies modeling with Direct Multivariate Probability Estimation (DMPE). In DMPE algorithm, the multivariate probability for each unsampled location is estimated directly from the transition probability between each two locations. The transition probabilities, which is the bivariate probability marginal's of estimated multivariate probability, serve as the constraints in the estimation process and satisfied by iteratively modify the estimated multivariate probability.

The DMPE facies modeling algorithm is also a cell-based sequential simulation approach (Deutsch and Journel, 1998) where all simulation grid cells are visited only once along a random path and simulated cell values become conditioning data for cells visited later in the sequence. The needed marginal probability in this DMPE algorithm is obtained from vertical well profile or the outcrop of the aiming modeling area.

From the vertical well profile analysis obtain a pattern and believe this pattern will prevail in certain sedimentary units which may be defined by sequence stratigraphic surface. In this small example, the transition probability model is assumed obtained from some vertical profile and will be used in the later direction data set shown in Figure 11. It could be just one pattern in cyclical sediment. The TPM calculated from top-to-bottom and bottom-to-top can reflect their contact information.

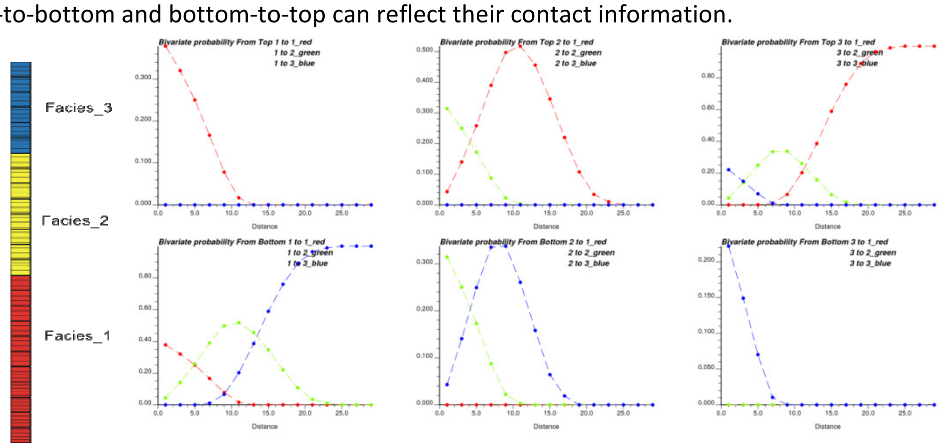

Figure 11 example of 1D pattern (left) and the pattern captured by TPM counting from two opposite directions (right)

Then if it is assumed that in lateral direction from East-to-West exist the same vertical pattern as counting from Top-to-Bottom given the sampled data location in Figure 12, the realizations from DMPE simulations based on the TPM from vertical direction are clear as the same pattern in vertical direction as shown in Figure 12.

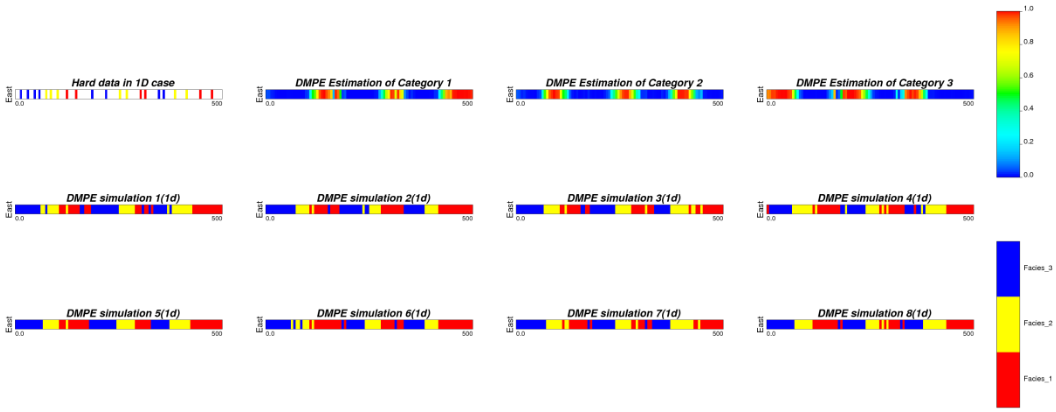

Figure 12 one example of 1D pattern reproduction (Estimation and Simulation results with DMPE)

While for 2D case, for each unsampled location, the spatial relationship is characterized by the distance and relative direction along the dip and strike direction. The bivariate probability will be obtained from the same vertical transition profile in 1D case. Then if define the strike direction is along NE-SW direction, enforce this constraints, the realization from DMPE will clear reflect this geological pattern as shown in Figure 13.

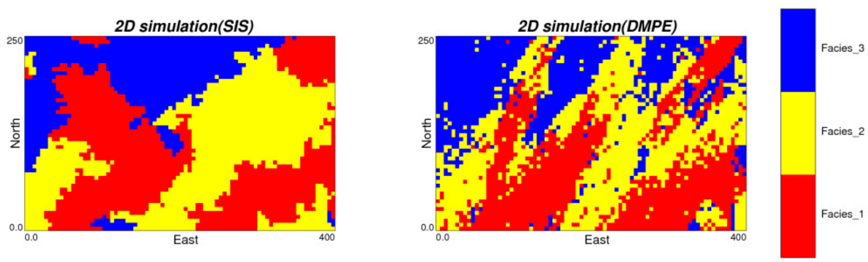

Figure 13 realizations with SIS and DMPE (the strike direction in DMPE is along NE-SW direction)

Conclusions and Future Research

Depositional processes create vertical and lateral facies patterns. These features can be partially captured by the vertical and horizontal transition probability. The transition probability matrices will integrate the sedimentological understanding and act as an additional constraint in geostatistical modeling. The transition probability matrices can be modified by the locally univariate proportions to provide an approach for the non-stationary case. Although, the proposed 3D transition probability transforming scheme and the new developed facies modeling algorithm provide a new promising facies modeling approach, this new method is not very mature.

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