A Short Note on Representative Facies Volume and The Nature of Facies Mixtures

S. H. Derakhshan and Clayton V. Deutsch

The representative elementary volume (REV) is the smallest volume (scale) over which a measurement can be made that will yield a value representative of the whole. The appropriate scale for reservoir simulation is REV. The notion of Representative Elementary Volume (REV) can be used in the context of facies modeling. The equivalent of REV in facies modeling context can be named as Representative Facies Volume (RFV). Beyond RFV, any indicator simulation techniques which benefit the assumption of mutual exclusivity do not represent the heterogeneity of the reservoir well. The reason is the mixing of facies at scale greater than RFV.

Introduction

To simulate the fluid behavior of the reservoir, reservoir engineers consider a scale that can represent the average heterogeneous reservoir properties well. This scale is called The classic definition of representative elementary volume (REV). Reservoir engineers usually upscale the reservoir properties to REV scale. There are different reasons to do that. The first reason is the computational time for reservoir simulation; defining the grid definition at small scales will result in large number of grids that take large amount of time for the simulator to be run. Tracing the fluid movement at field scale is much more important for reservoir engineers than tracing the fluid movement from one pore to the next (pore scale). Although building the geostatiscal models at small scale is a need for upscaling but it is not for flow simulation. Bear (1972) define REV as the volume at which the small scale fluctuations become significant when approaching from large volume. Figure 1 shows the REV schematically. In this figure, the *y*-axis shows the average porosity against the averaging volume, V (*x*-axis). Bear (1972) refers to the large scale fluctuations as heterogeneity.

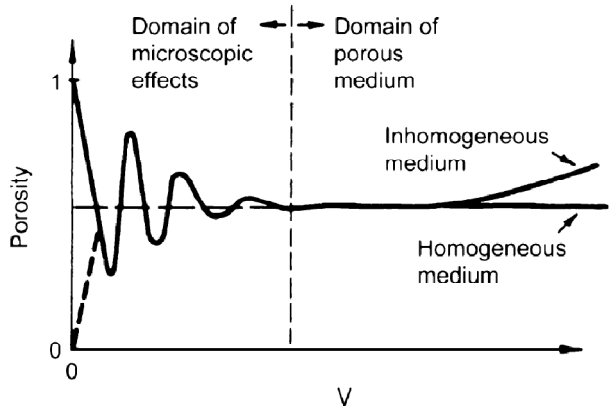

Figure 1 the REV concept applied to porosity; the dotted line shows REV (Bear, 1972)

Furbish (1997) used another way to define REV. Supposed that at point scale the porosity is defined as 0, if the point is within the solid medium and 1, if it is located within pore. At very small scale the probability density function of porosity is bimodal (0 or 1). As scale increases the porosity becomes proportion and a continuous probability density function can be observed. At REV scale (V_* in Figure 2) the probability density function starts to become univariate (global mean of porosity).

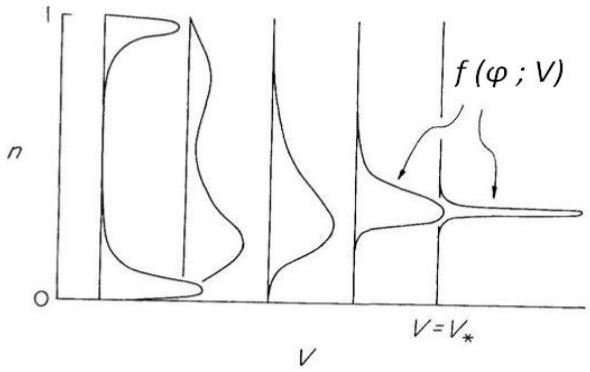

Figure 2 how the probability density function $f(\varphi; V)$ of porosity changes with scale (Furbish, 1997)

Lake and Srinivasan (2004) used mathematical and statistical formulation to define REV. If we plot the variance of the mean versus the scale of averaging on log-log plot then the REV is the defined as the onset of the portion with slope of -1 (Figure 3).

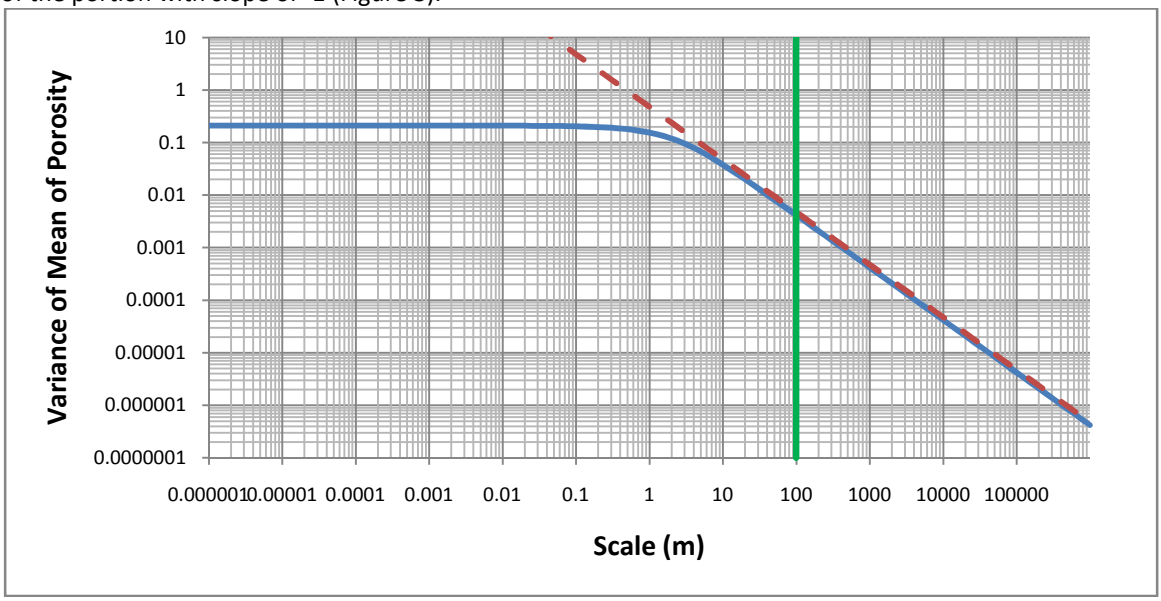

Figure 3 variance of the mean of porosity versus the scale of averaging; the REV is defined as the onset of the portion with slope of -1 (modified after Lake et. al., 2004)

Journel and Huijbregts (1978) derived the variance of the average of Gaussian random variable *Z* over a distance *L* for one dimensional case with standardized covariance function of $\rho(\boldsymbol{h})$ is:

$$Var(\bar{Z}) = \frac{2\sigma^2}{L^2}\int_{\zeta=0}^{\zeta=L}\int_{\eta=0}^{\eta=\zeta}\rho(\eta)d\eta d\zeta$$

The plot in Figure 3 is the log-log plot of $Var(\bar{Z})$ versus *L*.

Scaling Law

Before getting into the scaling law it is needed to recall the definition of the variogram and two important assumptions of first and second order stationarity. The variogram is a two-point statistic that spatially relates two random variables $Z(\boldsymbol{u})$ and $Z(\boldsymbol{u}+\boldsymbol{h})$:

$$2\gamma(\boldsymbol{h}) = E\{[Z(\boldsymbol{u}) - Z(\boldsymbol{u}+\boldsymbol{h})]^2\}$$

Where $\boldsymbol{u}$ and $\boldsymbol{h}$ are location and lag vectors, respectively, in domain of study. The assumption of stationarity requires that the proposed geostatistical model based on our sampled data, can adequately describe the behaviour of the population. The goal is to infer the population based on the sample data. So, we should make an informed decision regarding what information we can use to describe the region

of interest, this is called the decision of stationarity (Kelkar and Perez, 2002). In geostatistical study two kinds of stationarities can be defined, they are first order and second order stationarities. The first order stationarity is as below:

$$E\{Z(\boldsymbol{u})\} = E\{Z(\boldsymbol{u}+\boldsymbol{h})\} = m$$

Where m is the mean of data. It means that the expected value of a random variable at $\boldsymbol{u}$ is the same as the expected value of a random variable $\boldsymbol{h}$ lag distance away. Therefore first order stationarity means that the expected value across the region is the same. If we divide the region into small subregions and calculate the mean within each subregion then the means should be approximately the same in the case of first order stationarity (Kelkar and Perez, 2002). If the mean varies significantly from a subregion to another subregion, then there is a trend in the data. One of the most important parts of geostatistical modelling is to find the correct trend model if the data show a systematic trend. The trend function can be developed by a regression technique, inverse distance weighting and moving window averaging. This trend should be removed before variogram modelling and geostatistical simulation. Second order stationarity uses the variance at each location, and it assumes that the variance is constant across the region. Therefore,

$$Var\{Z(\boldsymbol{u})\} = Var\{Z(\boldsymbol{u}+\boldsymbol{h})\} = \sigma^2$$

By using the first and second order stationarities the relationship between the covariance and variogram can be obtained:

$$\gamma(\boldsymbol{h}) = \sigma^2 - C(\boldsymbol{h})$$

Where $\sigma^2$is the variance of data, $\gamma(\boldsymbol{h})$ is the variogram and $C(\boldsymbol{h})$ is the covariance function.

The scaling law requires the assumption of stationarities. Journel and Huijbregts (1978) developed a series of theoretical concepts and theorems for volume scaling. Given two different volumetric supports v and V, there are three important concepts: dispersion variance $D^2(v,V)$, average variogram $\bar{\gamma}(v,V)$ and mean covariance $\bar{C}(v,V)$ which are defined as (Journel and Huijbregts, 1978):

$$D^2(v,V) = E\{[m_v - m_V]^2\}$$

$$\bar{\gamma}(v,V) = \frac{1}{v.V}\int_v \int_V \gamma(\boldsymbol{y}-\boldsymbol{y}').d\boldsymbol{y}.d\boldsymbol{y}'$$

$$\bar{C}(v,V) = \frac{1}{v.V}\int_v \int_V C(\boldsymbol{y}-\boldsymbol{y}').d\boldsymbol{y}.d\boldsymbol{y}'$$

Where m_v and m_V are average values at the scale v and V respectively. The average values of variogram, $\bar{\gamma}(v,V)$, and covariance, $\bar{C}(v,V)$, are the average values of the point variogram, $\gamma(\boldsymbol{h})$, and covariance, $C(\boldsymbol{h})$, where one extremity of the distance vector $\boldsymbol{h}$ falls in the volume of v and the other extremity independently falls in V. The following is well known (Journel and Huijbregts, 1978):

$$D^2(v,V) = D^2(v,\Omega) - D^2(V,\Omega)$$

$$\sigma^2(v,V) = \bar{C}(v,v) - \bar{C}(V,V) = \bar{\gamma}(V,V) - \bar{\gamma}(v,v)$$

In above relations two assumptions are made, they are: (1) $\sigma^2(\cdot,\cdot) = 0$ and (2) $\bar{\gamma}(\Omega,\Omega) = D^2(\cdot,\Omega)$.

"$\cdot$" denotes the point scale. There are other previous works on scaling law. Isaaks and Srivastava (1989) presented the scaling laws with a practical study. Deutsch and Frykman (1999) discuss variogram modeling at different scales.

Representative Facies Volume (RFV)

Facies are considered to be mutually exclusive at a small scale and become proportions at larger scales. In the case of facies simulation with coarse grids this assumption may not be valid; mixing of facies inevitably occurs at larger scales. The concept of REV can be proposed in context of facies modeling which is representative facies volume (RFV). Beyond RFV, any indicator simulation techniques which benefit the assumption of mutual exclusivity do not work fine and represent the heterogeneity of the reservoir. A schematic diagram is shown in Figure 4 to show the definition of RFV scale. Three regions are defined in RFV plot, (1) mutual exclusive and exhaustive region, where there is no significant mixing of facies (2) mixing region where there exists mixing of facies, the scale of reservoir simulation grid falls in this region (the REV scale falls in region 2, REV is greater than RFV) and (3) large scale region, where all of the facies are mixed and the proportions of the facies reach the global proportion and homogeneity is considered.

The concept of an RFV is defined using the probability density function (PDF) of facies at different scale. The change in shape of the PDF from bimodal to unimodal characterizes the RFV. The RFV scale is denoted by V_1 . V_1 and V_2 define the three mentioned regions. The probability contours starts from 0 and 1 and converge to global proportion. They are characteristics of the reservoir and are functions of field size, object size, clustering size of objects.

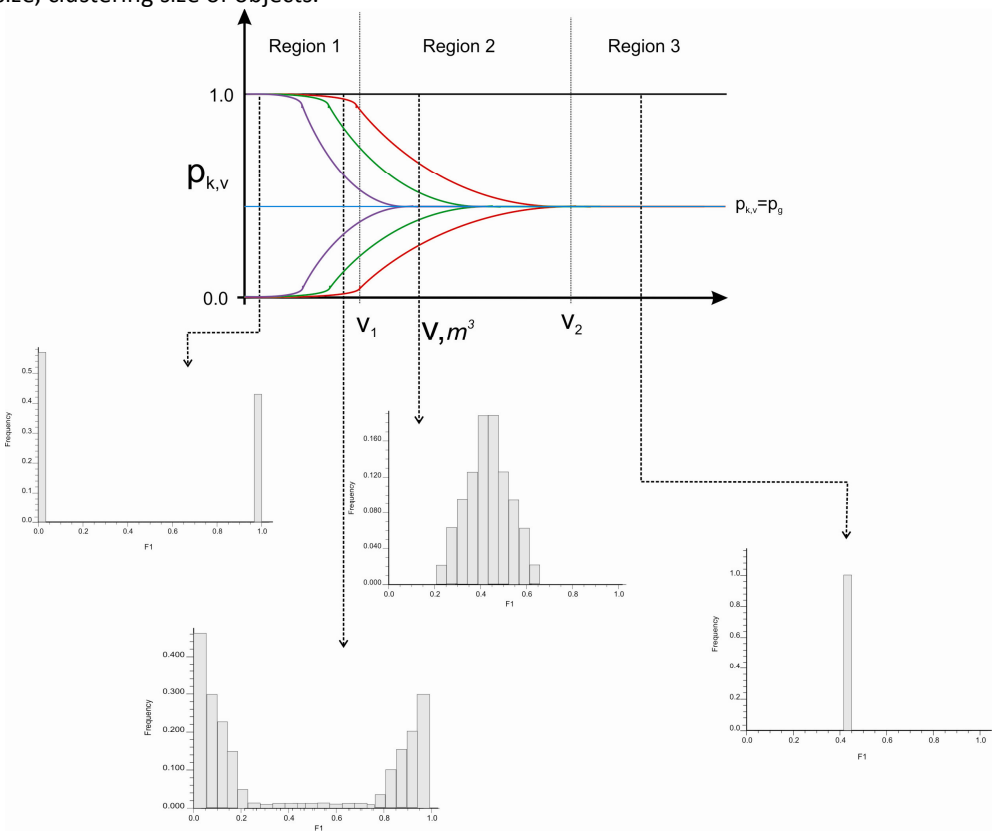

Figure 4 A schematic diagram for illustrating the concept of Representative Facies Volume (RFV)

Conclusion and Future Work

REV and RFV should be defined for the purpose of reservoir flow simulation and geostatistical facies modeling respectively. There are different statistical methods to define REV. This paper introduces the concept of RFV; defining RFV using statistical methods, examining different parameters that affect on RFV (field size, object size, etc) can be considered as future work. Considering RFV concept in geostatistical facies modeling will result in modeling and capturing the heterogeneity better. The current facies modeling techniques do not consider mixing at scales greater than RFV.

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