Paper 304, CCG Annual Report 11, 2009 (© 2009)

Robust Geostatistical Techniques for Large Mine Planning Problems

Behrang Koushavand

Uncertainty is inevitable with sparse geological data. Conditional simulation algorithms are geostatistical methods used to assess geological uncertainty. The generated realizations are equally probable and represent plausible geological outcomes. Long-term mine planning and the management of future cash flows are vital for surface mining operations. Traditionally the long-term mine plans are generated based on an estimated input geological block model. The most common estimation method used in industry is Kriging; however, Kriging results do not capture uncertainty and may be systematically biased. Mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets.

This short note summarized proposed research focused on transferring geological uncertainty into mine planning to generate a robust schedule. Stochastic goal programming techniques will be used to get an optimum result. A number of realizations generated by geostatistical conditional simulation algorithms will be used as stochastic part of optimization algorithm. The size of optimization is very huge and there are storage, memory and CPU speed limits to use conventional methods. These limitations motivate to propose some robust and fast simulation algorithms that are used on the fly to optimization problem, where no need to save all realization to such a huge file and read again which take too much time. On the other there is no any limitation on number of realization and algorithm will generate realization until to reach to convergence point or program termination criteria will satisfied.

Spectral unconditional simulation algorithm which is using Fast Fourier Transformation (FFT) will be used to generate high resolution unconditional realizations. To conditioning realizations kriging technique is used. At first realization kriging weight are saved at each grid node therefore at next realization, program doesn't need to solve kriging system and just used previously saved kriging weights. This method takes same time as SGS algorithm for first realization but next realizations are much faster.

Introduction

Numerical modeling is the only method to quantify geological complexity. A limited number of samples as a representation for the Domain of Interest are collected and used to build numerical models. Uncertainty is always present with sparse geological data. Geostatistics simulations algorithms are widely used for quantify and assess this uncertainty. The generated realizations are equally probable and represent plausible geological outcomes. (Journel and Huijbregts, 1981; Kesler, 1994; Arik, 2002; Journel and Kyriakidis, 2004).

There are very well established geostatistical methods such as Kriging and sequential simulation algorithms; however these techniques are highly dependent on the quality of the variogram model and a reliable variogram is difficult to infer in presence of sparse data. Fair uncertainty predictions and robust estimates motivate the quantification and use of variogram uncertainty in estimation and simulation.
Meanwhile the scale of samples is always different from the scale of the block required for mine planning. The vertical data spacing is mostly very dense and the vertical variogram can be inferred easily with little uncertainty. The spacing between drill holes is quite large and horizontal variograms are often uncertain. To make Kriging system more robust and decrease its sensitivities to variogram, motivate this research to find more robust new applicable method that is close to behavior of natural phenomenon.

Mine planning is the procedure to find a schedule for a bock model that maximizes the profit at a feasible order of extraction. Accurate prediction of mining reserves is crucial for a successful mining operation, long-term mine planning and the management of future cash flows that are vital for mining operations. Traditionally the long-term mine plans are generated based on an estimated input geological block model. The most common estimation method used in industry is kriging; however, kriging results do not capture uncertainty and may be systematically biased. Mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and

production targets. The mine planning procedure is not a linear process and the mine plan generated based on the Kriging estimate is not the expected result from all of the simulated realizations.
The aim of this research is to get a more realistic and robust estimation and simulation algorithm and transfer geological uncertainty into mine planning.

Production scheduling is a critical process in the surface mine planning. Long-term production scheduling is used to maximize the net present value (NPV) of the project and focuses on the sequencing of materials to be mined in space over time, under technical, financial and environmental constraints. The importance of incorporating uncertainty and risk from the technical, geological and mining sources in mine production schedules, particularly the possible in situ variability of pertinent orebody grade and ore quality characteristics, is well appreciated (Dimitrakopoulos and Ramazan 2004). Discrepancies between planning expectations and actual production may occur at any stage of mining. Vallee (2000) reported that 60% of the mines surveyed had an average rate of production that was less than 70% of the designed capacity in the early years. Others (e.g., Rossi and Parker, 1994) reported shortfalls against predictions of mine production in later stages of production.

These shortfalls were mostly attributed to orebody uncertainty. Because traditional production scheduling methods do not consider the risk of not meeting production targets caused by grade variability, they cannot produce optimal results. The detrimental effects of grade uncertainty in optimizing open pit mine design are shown in recent studies. Dimitrakopoulos et al. (2002) showed the substantial conceptual and economic differences of risk-based frameworks compare to the methods ignoring geological risk. Dowd (1997) proposed a framework for risk integration in surface mining projects. Godoy and Dimitrakopoulos (2004) presented a new approach for risk-inclusive cutback designs, which yield substantial NPV increases. Ravenscroft (1992) discussed risk analysis in mine production scheduling, where the use of stochastically simulated orebodies showed the impact of grade uncertainty on production scheduling. Ravenscroft concluded that conventional mathematical programming models cannot accommodate quantified risk, thus there is a need for a new generation of scheduling formulations to overcome infeasible or unrealistic scheduling and account for production risk. Smith and Dimitrakopoulos (1999) showed additional examples using mixed-integer programming to verify the above conclusion in the context of shortterm planning. Kumral and Dowd (2001) used stochastic simulations and optimization in short-term planning. Past efforts to deal with uncertainty attempt to sequentially link stochastic orebody models with conventional optimization formulations, with the exception of Godoy and Dimitrakopoulos (2004). This sequential process is inefficient and, although it assesses risk in a schedule, it does not produce optimal scheduling solutions in the presence of uncertainty. In addition, these efforts do not consider multi-element deposits with complex ore quality constraints, such as nickel laterites, iron ore or magnesium deposits. Furthermore, dealing with orebody uncertainty needs to consider issues of equipment access and mobility in the related “stochastic” optimization formulations.

Common Geostatistical Techniques for Resource Evaluation and Uncertainty Assessment

There are many of geostatistical techniques available for resource evaluation and uncertainty evaluation (Journel, 1989; Cressie, 1993; Deutsch and Journel, 1998; Deutsch, 2002). Among these techniques the most popular are Kriging and Simulation. Both of these techniques are briefly described below.

Kriging is a well established linear estimation technique that provides an estimate of the unsampled value Z(**u**) as a linear combination of neighboring observations $Z(\mathbf{u}_i), i=1,\ldots,n(\mathbf{u})$

The Simple Kriging estimator predicts the value of the variable of interest as the following linear combination

$$Z^*_{SK}(\mathbf{u})=\sum_{i=1}^{n(\mathbf{u})}\lambda_i(\mathbf{u})Z(\mathbf{u}_i) \qquad (1)$$

where $\boldsymbol{\lambda}=(\lambda_1(\mathbf{u}),\ldots,\lambda_{n(\mathbf{u})}(\mathbf{u}))^T$ denotes the vector of the Simple Kriging weights calculated from the normal system of equations for the estimation location **u**

$$\sum_{i=1}^{n(\mathbf{u})} \lambda_i(\mathbf{u})\mathrm{Cov}(Z(\mathbf{u}_i),Z(\mathbf{u}_j))=\mathrm{Cov}(Z(\mathbf{u}),Z(\mathbf{u}_j)), \quad j=1,\ldots,n(\mathbf{u}) \tag{2}$$

where $\mathrm{Cov}(Z(\mathbf{u}_i),Z(\mathbf{u}_j)), \quad i,j=1,\ldots,n(\mathbf{u})$ denotes data-to-data covariance function and $\mathrm{Cov}(Z(\mathbf{u}),Z(\mathbf{u}_j)), \quad j=1,\ldots,n(\mathbf{u})$ is data-to-estimation point covariance function. The covariance function is a required parameter in geostatistics and is calculated under stationarity though the semivariogram model. Simple Kriging is the best linear unbiased estimator, that is, it provides estimates with minimum error variance $\sigma^2_{SK}(\mathbf{u})$ in the least square sense given by

$$\sigma^2_{SK}(\mathbf{u})=\sigma^2-\sum_{i=1}^{n(\mathbf{u})} \lambda_i(\mathbf{u})\mathrm{Cov}(Z(\mathbf{u}),Z(\mathbf{u}_j)) \tag{3}$$

where σ^2 is the population variance. Kriging is Best Linear Unbiased Estimation (BLUE) in the sense of least square error for a given covariance model. Kriging estimates produce have smoother distribution than original data and the covariance between estimated values is not reproduced. Also, kriging variance is independent of the data values and only provides a comparison of alternative geometric data configuration and due to proportional effect (Heteroscedasticity) at data values in original units, kriging variance is usually not measures of local accuracy.

Stochastic Simulation

Stochastic simulation is widely used to quantify uncertainty in mineral resources (Journel, 1974). Stochastic simulation is the process of building alternative, equally probable, high-resolution models of spatial distribution of Z(**u**) (Deutsch and Journel 1997). The realizations reproduce the data values at their locations and given covariance function.

There are many approaches that can be used for geostatistical simulation. The most popular and simplest simulation techniques for continuous variables are based on the assumption of multivariate Gaussianity. The multivariate Gaussian distribution is characterized by the property that all conditional and marginal distributions are also Gaussian. The simple kriging mean and variance are precisely the mean and variance of the local conditional distribution.

It is rare, however, that a geological variable is Gaussian; therefore, the data need to be transformed before analysis. Simulation is conducted in normal score or Gaussian units. When stochastic simulation is completed, the simulated values are back-transformed into original units. There are many algorithms for Gaussian simulation. These include sequential Gaussian simulation, matrix approaches, moving average, turning bands and spectral methods.

Proposed Methods and Research Plan

Long-term mine planning and the management of future cash flows are vital for mining operations. Traditionally the long-term mine plans are generated based on an estimated input geological block model. The most common estimation method used in industry is kriging; however, kriging results do not capture uncertainty and may be systematically biased. Mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets. The mine planning procedure is not a linear process and the mine plan generated based on the Kriging estimate is not the expected result from all of the simulated realizations. Generated realizations will be used to get more robust mine planning. Goal programming algorithms will be applied to all realizations at once to reach a target production with minimal variations.

Goal programming is a method of optimization to maximize (or minimize) a function and commonly used when multiple goals are often in conflict with each other. With multiple goals, all goals usually cannot be realized exactly. For example, the twin goals of getting maximum profit with minimum risk are generally incompatible and therefore unachievable. Goal programming does not attempt to maximize or minimize a single objective function as does the linear programming model. Rather, it seeks to minimize the deviations among the desired goals and the actual results according to the priorities assigned. The objective function of a goal programming model is expressed in terms of the deviations from the target goals, in mine planning case target production.

A set of robust methods for geostatistical modeling will be developed and used in stochastic goal programming techniques to get a robust optimum mine planning schedules, where multiple objectives are satisfied such as:

- Maximum profit or net present value (NPV)
- Minimum deviation from target production
- Considering Ergodic fluctuation of variables at a jointly uncertainty manner

Spectral simulation algorithm is used to generate unconditional realization, by using FFT method; a very high resolution grid can be simulated very fast. Kriging is used for conditioning. To increase speed of algorithm, kriging weights are saved at a binary file at first realization. Kriging weights will not change for a particular location at different realizations. Therefore for next realizations only a vector to vector Multiplication needs to make unconditional simulation to be conditioning.

Because stochastic goal programming techniques are applied to each of realization to satisfy multiple objectives, no simulation realization is saved on a file. This method increases the speed of algorithm very efficiently. Reading and writing to the files take too much time. On the other hand, optimization algorithm will stop at some specific stop criteria that user selected. Therefore both of simulation and optimization parts need to run simultaneously until objective functions are satisfy.

A Multi-process program code is a very good idea to generate realization and do optimization on the fly. This is very effective with dual (or more) core CPUS, while the fist one is working on generating realizations the second one working on optimization procedure. Also there isn't any limitation on number of realizations because it is on the fly and no need for saving any high resolution grid nodes.

Spectral simulation method

The Spectral simulation algorithm is wildly used at electrical engineering to generate a random field with a given covariance spectrum (Borgam, Taheri and Hagen, 1984; Gutijahr and others, 1978; Navels, 1993; Gutjhar,Bullerd and Hatch, 1994, 1997). The algorithm is efficiently fast when based on Fast Fourier Transformation (FFT). (McKay,1988; Pardo- Igúzquiza and chica-Olmo, 1993).

According to classical spectral representation theorem, any sequence of N values of z(k) can be expressed as a finite series of Fourier coefficients, a_i, b_i. In 1D, that series is written as, Bracewell (1986):

$$z(k)=\sum_{j=0}^{N-1}\left[a_i\cos\left(2\pi jk/N\right)+b_j\sin\left(2\pi jk/N\right)\right] \quad (11)$$

$$k=0,\ldots,N-1$$

Or, equivalently, using a complex exponential Fourier series:

$$\mathrm{z}(\mathrm{k})=\mathrm{F}^{-1}\left(\mathrm{A}(\mathrm{j})\right)=\sum_{\mathrm{j}=0}^{\mathrm{N}-1}\mathrm{A}(\mathrm{j})\mathrm{e}^{\mathrm{i}2\pi\mathrm{kj/N}} \quad (12)$$

Where $A(j)=a_j-ib_j=\left|A(j)\right|e^{i\varphi(j)}$ is the jth complex Fourier coefficient, $\left|A(j)\right|=\sqrt{a_j^2+b_j^2}$ is the amplitude , and $\varphi(j)=\tan^{-1}\left(-b_j/a_i\right)$ is the phase of the jth Fourier coefficient. The amplitudes are related to the discrete spectral density s(j). FFT is used to transfer covariance function to spectrum (Yao, 1998b).

$$s(j)=F\left(C_Z(h)\right) \quad (13)$$

Spectral simulation does not call for any analytical variogram model as most other simulation algorithms do. The spectrum density map can be obtained directly by FFT of experimental covariance map (Yao, 1998b) or directly from data values by using the Lomb periodogram method.

For a data obtained at equal space intervals, Fourier based methods are applied to get spectrum density. The Lomb periodogram (Lomb, 1975 and Scargle 1982) has been proposed for processing of irregularly sampled data. Its successful application to detecting periodicities in data and to estimating the spectral density of deterministic signal has been reported in literature (see, e.g. Green et al., 2002, Kaneoke and Vitek, 1996, Fortin and Mackey, 1999, and Laguna et al., 1998). This can be lead to proposed automatic simulation algorithm that does not need to variogram model.

Stochastic Goal programming

Rather than using a typical "best guess" method, the idea behind goal programming is to find solutions that best satisfy an established list of prioritized objectives. One of these objectives might be a profit requirement, while others might arise from social and environmental concerns, productivity targets, use of financial resources, meeting marketing goals, and so on. By utilizing the power of goal programming, it is possible to find a solution that best satisfies the conflicting goals of the organization and comes closest to meeting the stated goals.

Goal programming is a branch of multi-objective optimization, which in turn is a branch of multi-criteria decision analysis (MCDA), also known as multiple-criteria decision making (MCDM). It can be thought of as an extension or generalization of linear programming to handle multiple, normally conflicting objective measures. Each of these measures is given a goal or target value to be achieved. Unwanted deviations from this set of target values are then minimized in an achievement function.

Stochastic programming is the study of procedures for decision making under uncertainty. The aim of stochastic programming is to find optimal decisions in problems which involve uncertain data. The fundamental idea behind stochastic linear programming is the concept of *recourse.* Recourse is the ability to take corrective action after a random event has taken place.

In the simplest model of this type we have two stages:

- In the first stage we make a decision
- In the second stage we see a realisation of the stochastic elements of the problem BUT are allowed to make further decisions to avoid the constraints of the problem becoming infeasible.

References

Cressie, N., 1993, Statistics for Spatial Data, revised edition, Wiley, New York, 900 p.

Deutsch, C.V. and Journel, A.G., 1998, GSLIB: Geostatistical Software Library and User’s Guide, 2nd edition, Oxford University Press, New York, 369 p.

Deutsch, C.V, 2002, Geostatistical Reservoir Modeling, Oxford University Press, New York, 376 p.

Journel, A.G. and Huijbregts, Ch.J., 1981, Mining Geostatistics, Academic Press, London, 600 p.

Journel, A.G. and Kyriakidis, P.C., 2004, Evaluation of Mineral Reserves: A Simulation Approach, Oxford University Press, New York, 216 p.

Journel, A.G., 1989, Fundamentals of Geostatistics in Five Lessons. Volume 8 Short Course in Geology: American Geophysical Union, Washington, D.C., 40 p.

Journel, A.G., 1974, Geostatistics for conditional simulation of ore bodies, Economic Geology, 69(5), pp. 673-687.

Kall, P. and Wallace, S. W., 1994, Stochastic Programming, JOHN WILEY & SONS, 305p

Kesler, S.E., 1994, Mineral Resources, Economics, and the Environment, Maxwell Macmillan International, New York, 391 p.

Kyriakidis, P.C. and Journel, A.G., 2001, Stochastic modeling of atmospheric pollution: a spatial time-series framework. Part II: application to monitoring monthly sulfate deposition over Europe, Atmospheric Environment, 35(13), pp. 2331-2337.

Marti, K., 2008, Stochastic Optimization Methods, Springer, 340p

Marchant, B. P., and Lark, R. M., 2004, Estimation Variogram Uncertainty, Math. Geology, v. 36, no, 8, p. 865-898

Matheron, G., 1971, The theory of regionalized variables and their applications: Ecole des Mines, Fontainbleau, France, 211 p.

Müller, W. G., and Zimmerman, D. L., 1999, Optimal designs for variogram estimation: Environmetrics, v. 10, no. 1, p. 23–37.

Ortiz, C. J., and Deutsch, C. V., 2002, Calculation of uncertainty in the variogram: Math. Geology: v. 34, no. 2, p. 169–183.

Pardo- Igúzquiza, E., and Dowd, P. A., 2001a, Variance–covariance matrix of the experimental variogram: Assessing variogram uncertainty: Math. Geology, v. 33, no. 4, p. 397–419.

Tewarson, Reginald P, 1973, Sparse Matrices (Part of the Mathematics in Science & Engineering series), Academic Press Inc.,

Webster, R., and Oliver, M. A., 1992, Sample adequately to estimate variograms of soil properties: J. Soil Sci., v. 43, no. 1, p. 177–192.

Yao, T., 1998a. Conditional Spectral Simulation with Phase Identification, Math. Geology, 30(3), pp. 285-308.

Yao, T., 1998b. SPECSIM: a Fortran-77 program for conditional spectral simulation in 3D. Computers & Geosciences 24 (10), 911–921.

Yamamoto J.K., 2005, Correcting the Smoothing Effect of Ordinary Kriging Estimates, Math. Geology, 37(1), pp. 69-94.